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Lista 1 - exercício 13

Calcule, caso existam, os seguintes limites

a) $\lim\limits_{x \to 2} \dfrac{x^2+x-1}{x-1}$ $\qquad D_f = \{x \in \mathbb{R} / x \neq 1\}$

$\boxed{f(x) = \dfrac{x^2+x-1}{x-1}, x \neq 1 \text{ é uma racional}}$ e $\lim\limits_{x \to 2}(x-1) = 2-1 = 1 \neq 0$

$$\lim_{x \to 2} \frac{x^2+x-1}{x-1} = \frac{\lim\limits_{x \to 2}(x^2+x-1)}{\lim\limits_{x \to}(x-1)} = \frac{2^2+2-1}{2-1} = \frac{5}{1} = 5$$

Lembrar: Se $p(x)$ uma função polinomial, $x_0 \in \mathbb{R}$

$$\boxed{\lim_{x \to x_0} p(x) = p(x_0)}$$

p é uma função contínua em x_0

Como p é contínua para todo $x_0 \in \mathbb{R}$, simplesmente dizemos que p é uma função contínua.

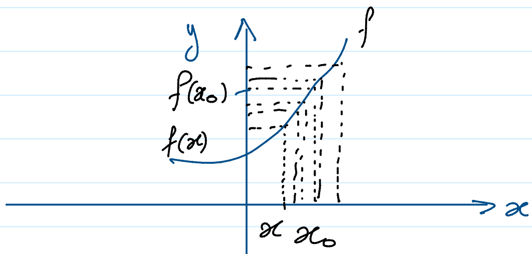
Se f uma função racial, isto é, existem p e q funções polinomiais tais que $f(x) = \dfrac{p(x)}{q(x)}$,

$D_f = \{x \in \mathbb{R} / q(x) \neq 0\}$

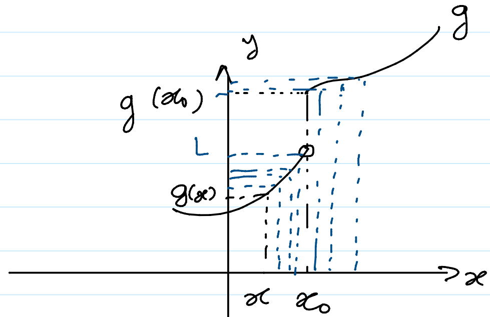
Se $x_0 \in D_f$ $(q(x_0) \neq 0)$

$$\lim_{x \to x_0} f(x) = \lim_{x \to x_0} \frac{p(x)}{q(x)} \overset{q(x_0) \neq 0}{=} \frac{\lim\limits_{x \to x_0} p(x)}{\lim\limits_{x \to x_0} q(x)} = \frac{p(x_0)}{q(x_0)} = f(x_0)$$

$$\Longrightarrow \boxed{\lim_{x \to x_0} f(x) = f(x_0)} \Rightarrow f \text{ é contínua em } x_0$$



f é contínua em x_0



g não é contínua em x_0

f é contínua em x_0

$\lim_{x \to x_0} f(x)$ existe e $\lim_{x \to x_0} f(x) = f(x_0)$

em x_0

$\lim_{x \to x_0} g(x)$ não existe

$\lim_{x \to x_0^-} g(x) = L$

$\lim_{x \to x_0^+} g(x) = g(x_0)$

$L \neq g(x_0)$

Limites laterais

$\lim_{\substack{x \to x_0^+ \\ x > x_0}} f(x)$, limite lateral à direita de x_0

$\lim_{\substack{x \to x_0^- \\ x < x_0}} f(x)$, limite lateral à esquerda de x_0

$\boxed{\text{Teorema:}}$ $\lim_{x \to x_0} f(x) = L \iff \lim_{x \to x_0^+} f(x) = L$ e $\lim_{x \to x_0^-} f(x) = L$

$\boxed{\text{Corolário:}}$ Se $\lim_{x \to x_0^+} f(x) = L_1$, $\lim_{x \to x_0^-} f(x) = L_2$, com $\boxed{L_1 \neq L_2}$,

então $\lim_{x \to x_0} f(x)$ não existe

Exemplo: $\lim_{\substack{x \to 0 \\ x \neq 0}} \frac{|x|}{x}$

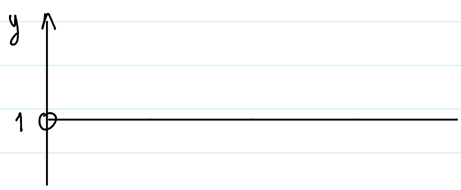
Lembrando: $|x| = x$, se $x \geq 0$ e $|x| = -x$, se $x < 0$

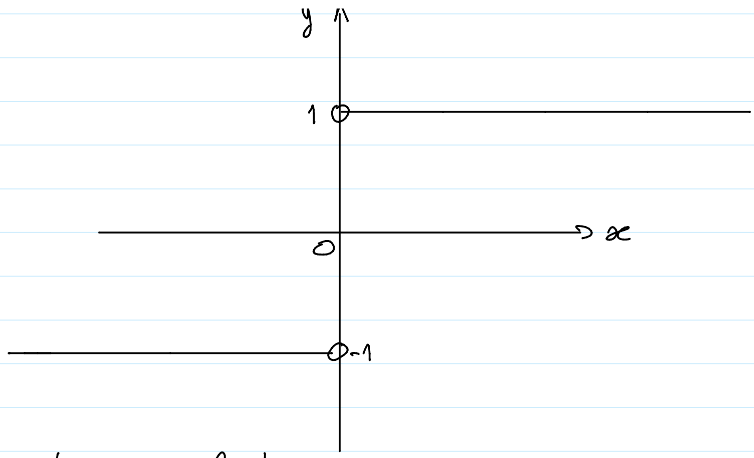
$\boxed{x > 0} \Rightarrow |x| = x \Rightarrow \boxed{\frac{|x|}{x} = \frac{x}{x} = 1}$

$x < 0 \Rightarrow |x| = -x \Rightarrow \frac{|x|}{x} = \frac{-x}{x} = -1$

$\lim_{\substack{x \to 0^+ \\ x > 0}} \frac{|x|}{x} = 1$, $\lim_{\substack{x \to 0^- \\ x < 0}} \frac{|x|}{x} = -1$

$\lim_{x \to 0^+} \frac{|x|}{x} \neq \lim_{x \to 0^-} \frac{|x|}{x} \Rightarrow \lim_{x \to 0} \frac{|x|}{x}$ não existe





voltando para a lista 1

b) $$\lim_{x\to+\infty}\frac{x^3-x+2}{x^3-3x}=$$

$$=\lim_{\boxed{x\to+\infty}}\frac{\cancel{x^3}\left(1-\frac{1}{x^2}+\frac{2}{x^3}\right)}{\cancel{x^3}\left(1-\frac{3}{x^2}\right)}$$

$$=\lim_{x\to+\infty}\frac{1-\underbrace{\frac{1}{x^2}}_{\to 0}+\underbrace{\frac{2}{x^3}}_{\to 0}}{1-\underbrace{\frac{3}{x^2}}_{\to 0}}=\frac{1}{1}=1$$

$$\lim_{x\to-\infty}\frac{x^3-x+2}{x^3-3x}=\lim_{x\to-\infty}\frac{1-\underbrace{\frac{1}{x^2}}_{\to 0}+\underbrace{\frac{2}{x^3}}_{\to 0}}{1-\underbrace{\frac{3}{x}}_{\to 0}}=1$$

$$\lim_{x\to+\infty}\frac{1}{x^k}=0=\lim_{x\to-\infty}\frac{1}{x^k},\ k>0$$

$$\lim_{x\to0^+}\underbrace{\frac{1}{x}}_{>0}=+\infty,\quad \lim_{x\to0^-}\underbrace{\frac{1}{x}}_{<0}=-\infty$$

d) $$\lim_{x\to+\infty}\frac{x^3-x+4}{x^4+x^3-x-5}=\lim_{x\to+\infty}\frac{\cancel{x^3}\left(1-\frac{1}{x^2}+\frac{4}{x^3}\right)}{x^4\left(1+\frac{1}{x}-\frac{1}{x^3}-\frac{5}{x^4}\right)}$$

$$=\lim_{x\to+\infty}\underbrace{\frac{1}{x}}_{\to 0}\cdot\frac{1-\underbrace{\frac{1}{x^2}}_{\to 0}+\underbrace{\frac{4}{x^3}}_{\to 0}}{1+\underbrace{\frac{1}{x}}_{\to 0}-\underbrace{\frac{1}{x^3}}_{\to 0}-\underbrace{\frac{5}{x^4}}_{\to 0}}=0\cdot\frac{1}{1}=0$$

$$\lim_{x\to+\infty}\frac{x^4+x^3-x-5}{x^3-x+4}=\lim_{x\to+\infty}\frac{\cancel{x^4}\left(1+\frac{1}{x}-\frac{1}{x^3}+\frac{4}{x^4}\right)}{\cancel{x^3}\left(1-\frac{1}{x^2}+\frac{4}{x^3}\right)}$$

$$=\lim_{x\to+\infty}\underbrace{x}_{\to+\infty}\cdot\frac{1+\underbrace{\frac{1}{x}}_{\to 0}-\underbrace{\frac{1}{x^3}}_{\to 0}+\underbrace{\frac{4}{x^4}}_{\to 0}}{1-\underbrace{\frac{1}{x^2}}_{\to 0}+\underbrace{\frac{4}{x^3}}_{\to 0}}=+\infty\quad(+\infty\cdot1)$$

Indeterminações: $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0.\infty$, $\infty-\infty$

voltando para a lista 1

j) $\lim\limits_{x\to 2^-} \frac{x^2-2x}{x^2-4x+4}$ $\qquad x \to 2$

$x<2$

$$\lim_{x\to 2^-} (x^2-4x+4) = 4-8+4=0$$

$$\lim_{x\to 2^-} (x^2-2x) = 4-4=0$$, estou no caso $\frac{0}{0}$

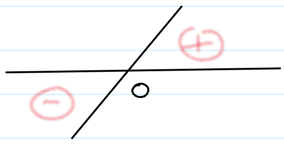
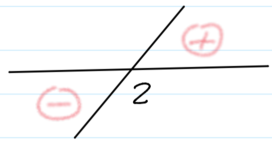
$$\frac{x^2-2x}{x^2-4x+4} = \frac{x\cancel{(x-2)}}{(x-2)^{\cancel{2}}} = \frac{x}{x-2}$$

$$\lim_{x\to 2^-} x = 2$$

$$\lim_{x\to 2^-} (x-2) = 0$$

$$\lim_{x\to 2^-} \frac{x^2-2x}{x^2-4x+4} = \lim_{x\to 2^-} \frac{x}{x-2} = ?\infty$$

Estudemos o sinal de $y = \frac{x}{x-2}$

$y = x$  , $y = x-2$ 

		0		2	
x	$-$		$+$		$+$
$x-2$	$-$		$-$		$+$
$\frac{x}{x-2}$	$+$		$-$		$+$

$$\lim_{x\to 2^+} \frac{x^2-2x}{x^2-4x+4} = +\infty$$

$$\boxed{\sqrt{x^2} = |x|}$$

$$\lim_{x\to 2^-} \frac{x^2-2x}{x^2-4x+4} = -\infty$$

o) $\lim\limits_{x\to -\infty} \frac{\sqrt{7x^6+5x^4+7}}{x^4+2} = \lim\limits_{x\to -\infty} \frac{\sqrt{x^6\left(7+\frac{5}{x^2}+\frac{7}{x^6}\right)}}{x^4\left(1+\frac{2}{x^4}\right)}$

$$\sqrt{(x^3)^2} = |x^3|$$

$$= \lim_{x\to -\infty} \frac{\sqrt{x^6}\sqrt{7+\frac{5}{x^2}+\frac{7}{x^6}}}{x^4\left(1+\frac{2}{x^4}\right)} = \lim_{\substack{x\to -\infty \\ x<0}} \frac{|x^3|\sqrt{7+\frac{5}{x^2}+\frac{7}{x^6}}}{x^4\left(1+\frac{2}{\ldots}\right)}$$

$$\lim_{x\to-\infty} \frac{\ldots}{x^4\left(1+\frac{2}{x^4}\right)} = \lim_{\substack{x\to-\infty \\ x<0}} \frac{|x| \sqrt{\ldots \frac{\ldots}{x^2} \ldots \frac{\ldots}{x^6}}}{x^4\left(1+\frac{2}{x^4}\right)}$$

$$\boxed{\begin{array}{l} |x| = x, \text{ se } x \geq 0 \\ |x| = -x, \text{ se } x < 0 \end{array}}$$

$$= \lim_{x\to-\infty} \frac{-\cancel{x^3}\sqrt{7+\frac{5}{x^2}+\frac{7}{x^6}}}{\cancel{x^4}\left(1+\frac{2}{x^4}\right)}$$

$$= -\lim_{x\to-\infty} \underbrace{\frac{1}{x}}_{\to 0} \frac{\sqrt{7+\overbrace{\frac{5}{x^2}}^{\to 0}+\overbrace{\frac{7}{x^6}}^{\to 0}}}{1+\underbrace{\frac{2}{x^4}}_{\to 0}} = -0\cdot\frac{\sqrt{7}}{1} = 0$$