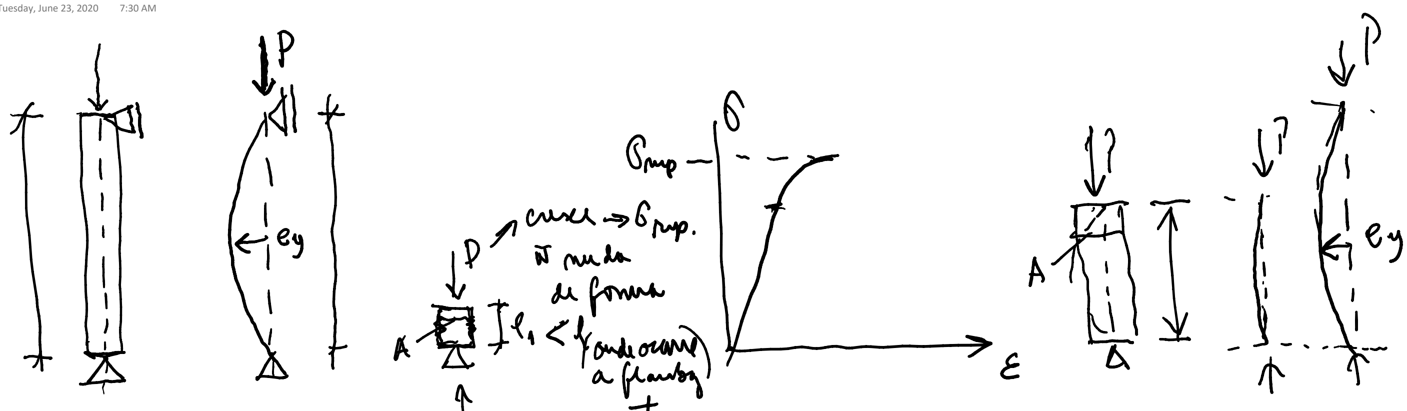
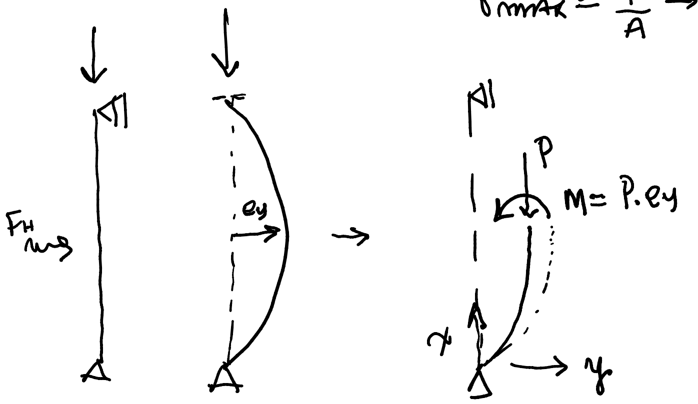
Flambagem 1 - barras prismáticas

Tuesday, June 23, 2020 7:30 AM



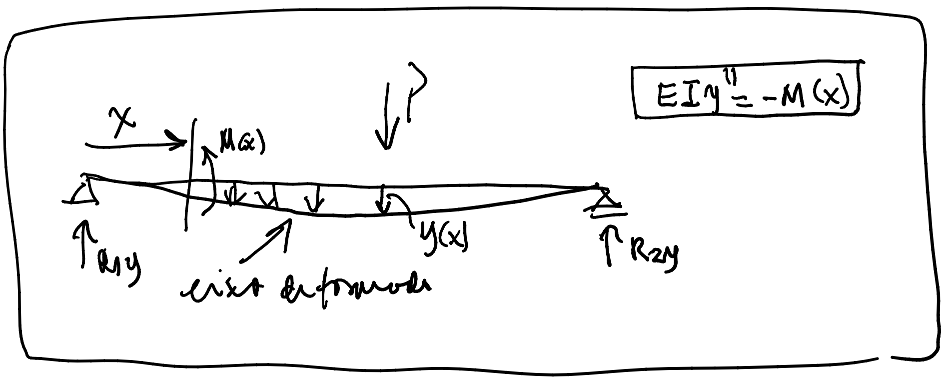
esmagamento

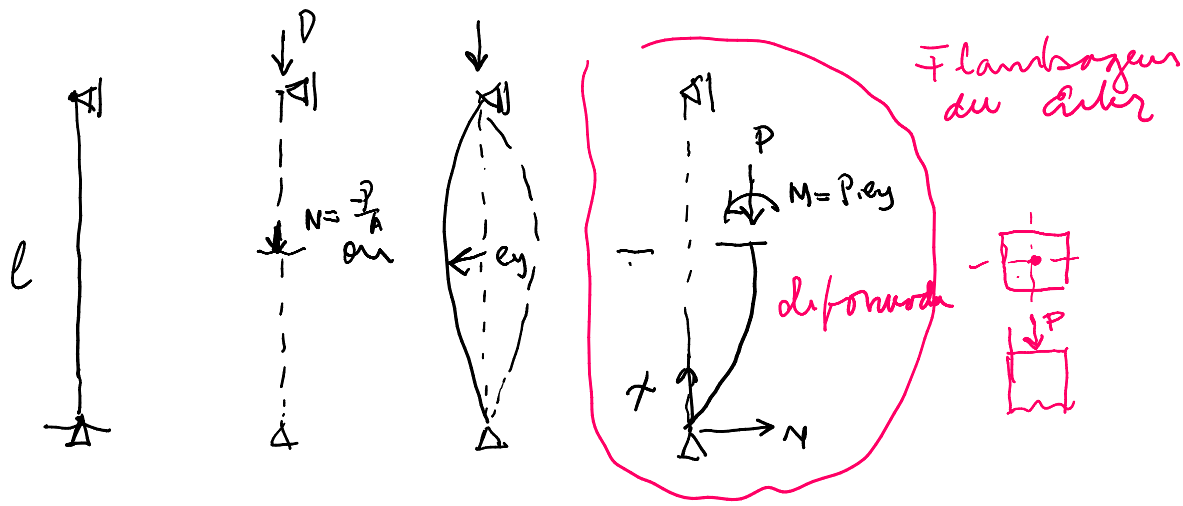
$\sigma_{max} = \frac{P}{A} \rightarrow \sigma_{rup}$.



$\sigma = \sigma_N + \sigma_M$

Instabilidade por bifurcação do ponto de equilíbrio.





Flambagem de Euler

deformada

$M = P \cdot e_y$

Eq. linha elástica $\quad EIy'' = -M(x)$

$EIy'' = -P \cdot e_y$ / y

$$EI\,y'' + P.y = 0$$

$$\frac{EI\,y''}{EI} + \frac{P}{EI}.y = 0 \qquad k^2 = \frac{P}{EI}$$

$$\boxed{y'' + k^2 y = 0} \qquad y'' = -k^2 y$$

$y = C_1 . \sin kx + C_2 \cos kx$ c.c. $y(0) = 0$ e $y(l) = 0$ $C_2 = 0$

$y = C_1 \sin kx$ $C_1 . \sin kx = 0$ $C_1 \neq 0 \longrightarrow$

$\sin k(x = l) = 0$

$\sin kl = 0$ .... $n = 0, 1, 2, 3$

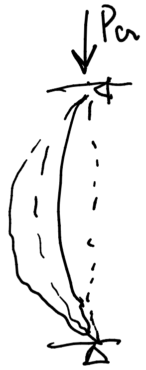
$n = 1$

$k^2 = \frac{P}{EI}$

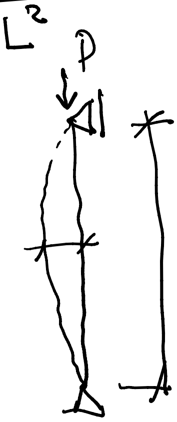
$\sin kl = 0$

$$\boxed{kl = \pi \; (n=1)}$$

$kl = \pi$ e sen $\frac{P}{EI} = \frac{\pi^2}{L^2} \Rightarrow P = \frac{\pi^2 . EI}{L^2}$



$$\longrightarrow P_{cr} = \frac{\pi^2 . EI}{L^2}$$



$\sigma_{cr} = \frac{P_{cr}}{A} \Rightarrow \sigma_{cr} = \frac{\pi^2 E\, I}{L^2 . A}$ $i^2 = \frac{I}{A} \rightarrow i$: raio de giração da seção plana

$i = \sqrt{\frac{I}{A}}$

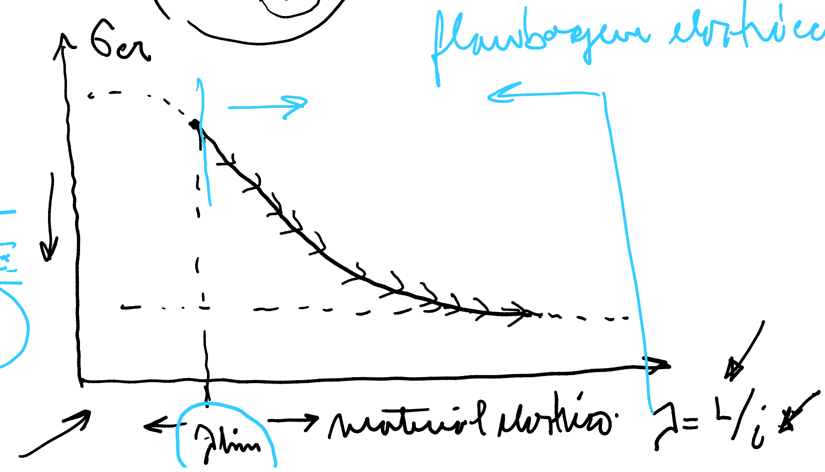
$\sigma_{cr} = \frac{\pi^2 E . i^2}{L^2 .}$ definindo $\lambda = \frac{L}{i}$ parâmetro de esbeltez

$$\boxed{\sigma_{cr} = \frac{\pi^2 E}{\lambda^2}}$$

flambagem elástica

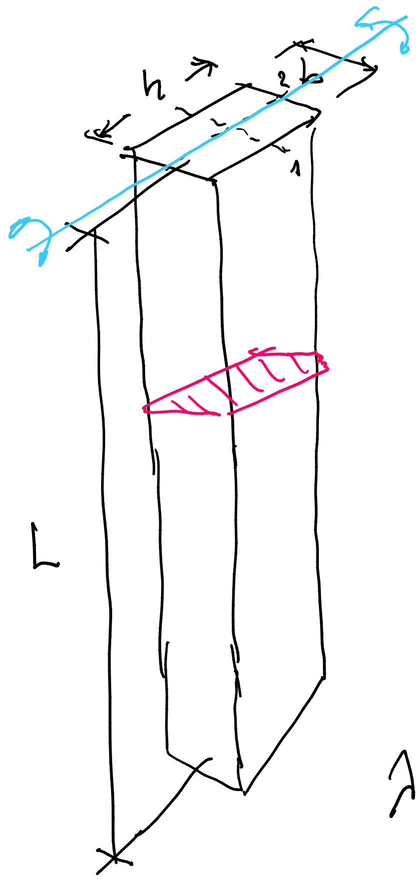
$f_y = \frac{\pi^2 E}{\lambda^2} \Rightarrow \lambda_{lim} = \sqrt{\frac{\pi^2 E}{f_y}}$

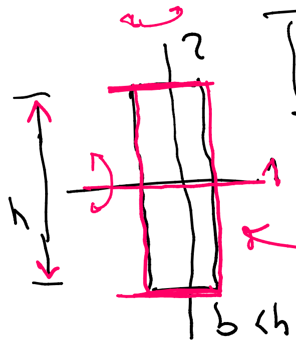
O material escoa



O material emerga — material elástico: $\lambda = L/i$ — λ_{lim}

$$\sigma_{cr} = \frac{P_{cr}}{A} \qquad \text{onde} \quad P_{cr} = \frac{\pi^2 \cdot E \cdot I}{L^2}$$





$$\boxed{I_1 > I_2}$$

$$I = \frac{b\,h^3}{12} \qquad I_1 = \frac{b\,h^3}{12} \qquad I_1 > I_2 \qquad I_2 = \frac{h\,b^3}{12}$$

secção transversal

$b < h$

$$P_{cr} = \frac{\pi^2 E I}{L^2}$$

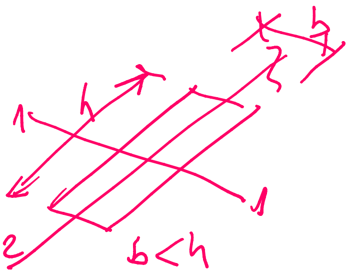
$I_1 > I_2 \Rightarrow P_{cr1} > P_{cr2}$

$I_2 \Rightarrow P_{cr2} < P_{cr1}$

consideremos que a estrutura à flambe primeiro $\boxed{\text{com } P_{cr2}.}$

$\lambda = ?$ $\qquad \lambda = \frac{L}{i}$ e $i_1^2 = \frac{I_1}{A}$, $\quad i_2^2 = \frac{I_2}{A}$

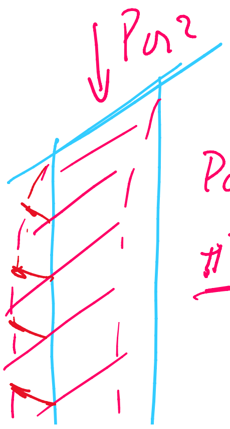
$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2}$$



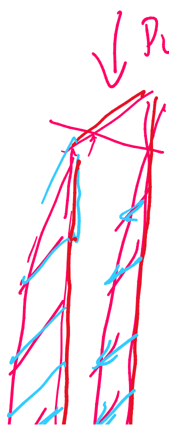
$$\lambda_1 = \frac{L}{i_1}$$

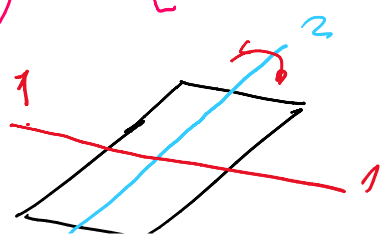
$\lambda_2 = \frac{L}{i_2}$ neste caso $\lambda_2 > \lambda_1$

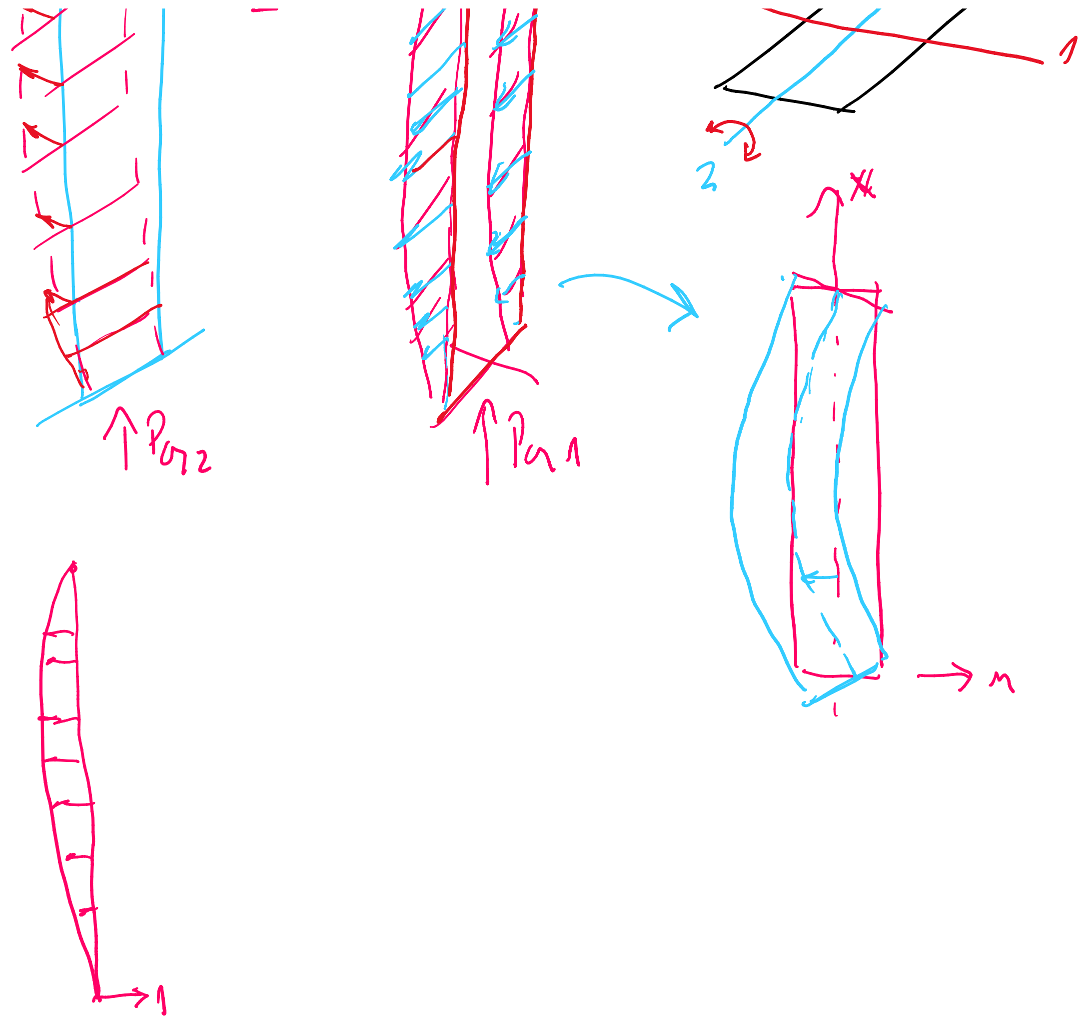
portanto, a barra tem maior esbeltez em torno do eixo 2

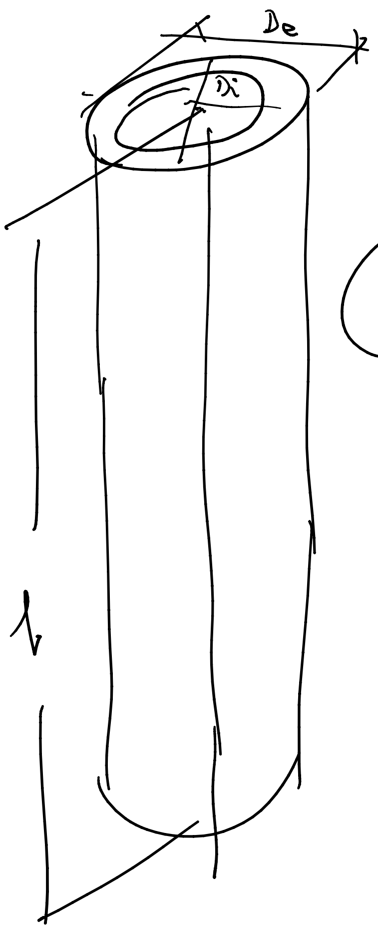


$$P_{cr2} = \frac{\pi^2 E I_2}{L^2} < P_{cr1} = \frac{\pi^2 E I_1}{L^2}$$







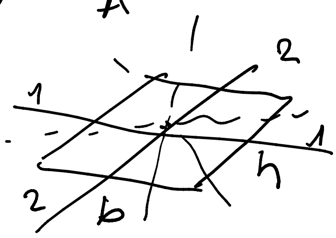


$$I = \frac{\pi D^4}{64}$$

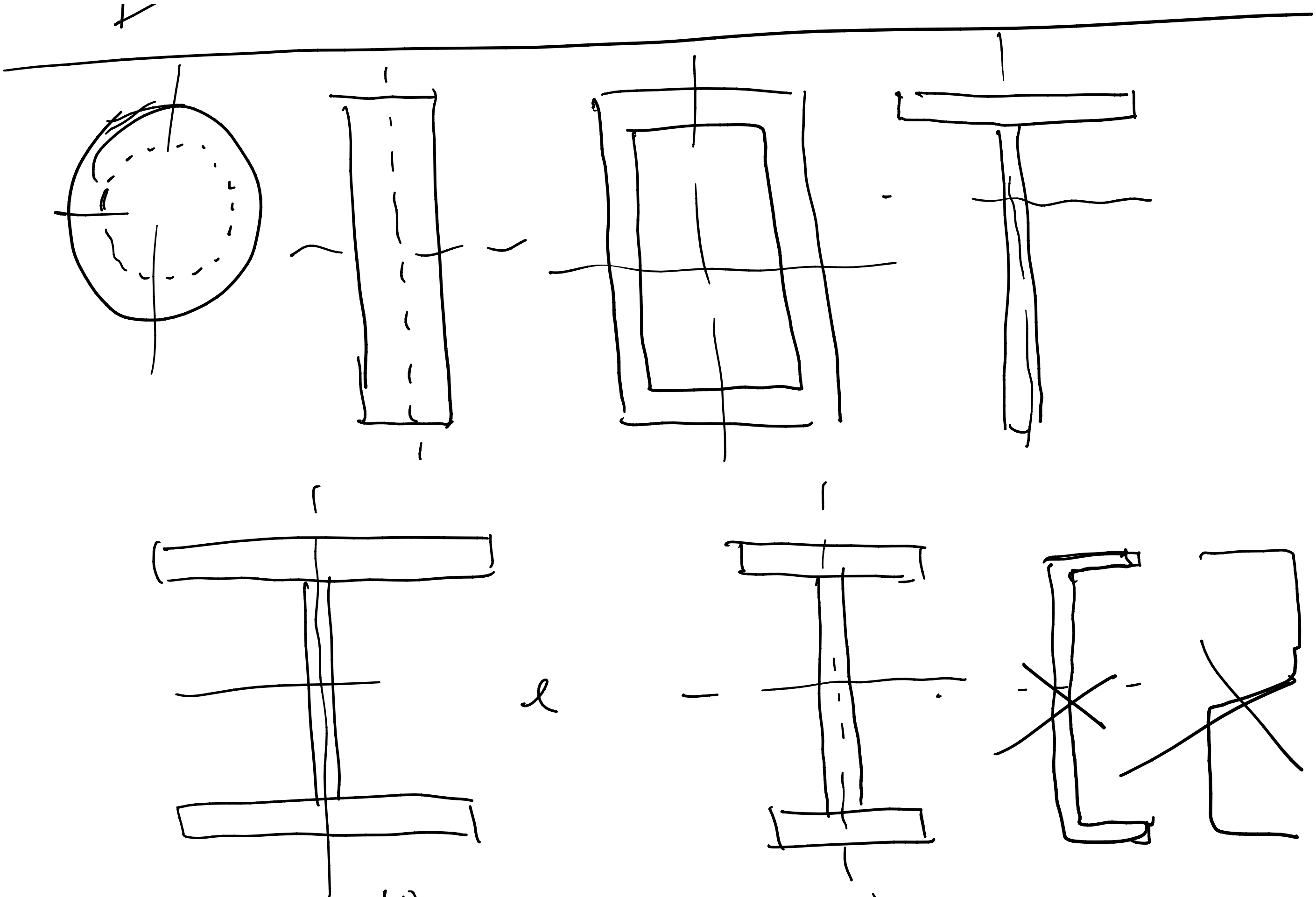
$$A = \frac{\pi}{4}\left(D_e^2 - D_i^2\right)$$

$$I = \frac{\pi}{64}\left(D_e^4 - D_i^4\right)$$

$$\lambda = \frac{I}{A}$$



$b = h$



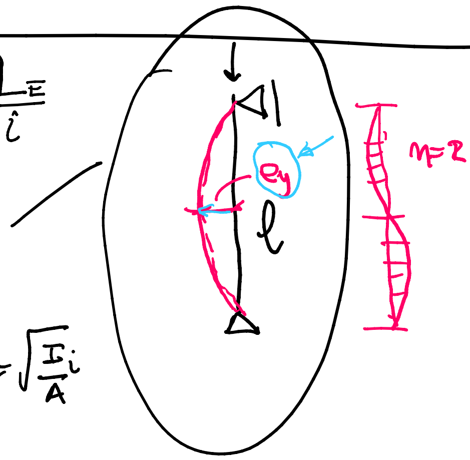
(H) (I)

$\sigma_{cr} = \frac{\pi E}{\lambda^2}$

λ: esbelteza $= \frac{L_E}{i}$

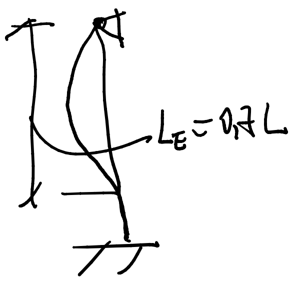
$P_{cr} = \frac{\pi^2 E I}{L^2}$

i: raio de giração $i_i = \sqrt{\frac{I_i}{A}}$



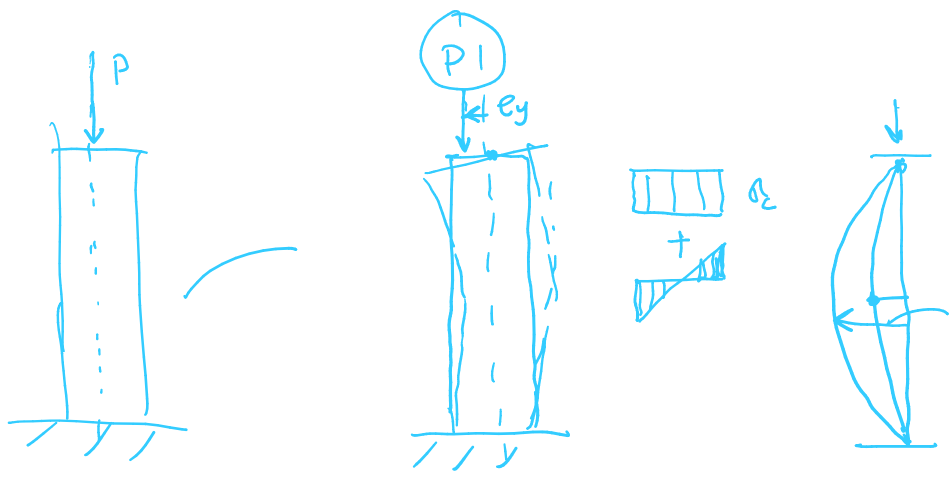
$\boxed{\ell_{\text{comprimento de flambagem}} = 2L}$ — $P_{cr} = \frac{\pi^2 E I}{(L_E = 2L)}$

$\boxed{\lambda_{lim} = \sqrt{\frac{\pi^2 E}{\sigma_y}}}$



1P

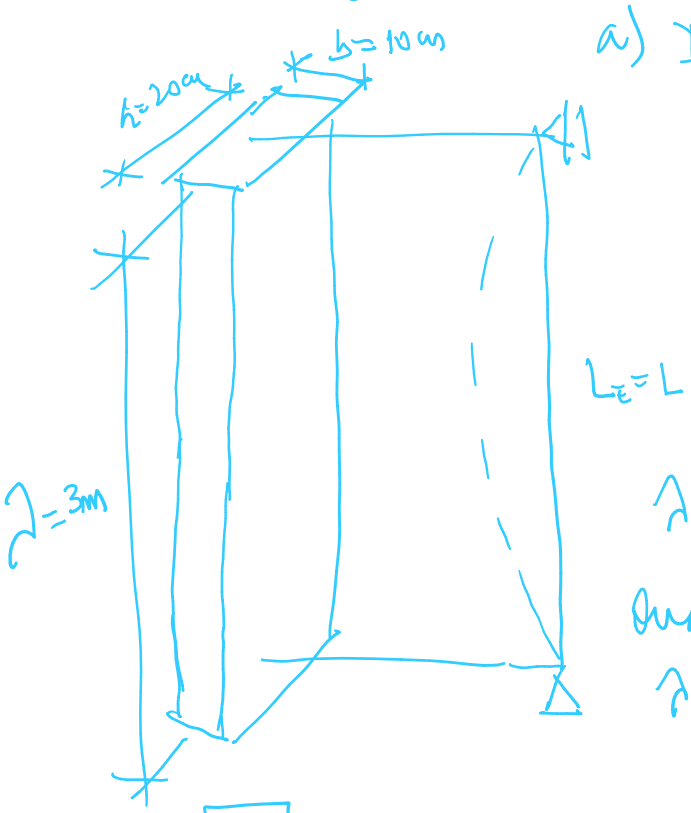
P1

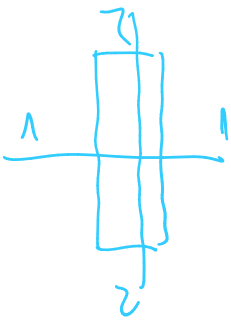


e_y cresce bastante.

Ex. Flambagem - Seção Retangular.

a) Det. a esbeltez da barra.





$I_1 = \frac{b.h^3}{12}$ $\quad b < h$

$I_2 = \frac{h.b^3}{12}$

$i_1^2 = \frac{I_1}{A}$

$i_2^2 = \frac{I_2}{A}$

$\lambda_2 = \frac{L_E}{i_2} > \lambda_1 = \frac{L_E}{i_1}$

Qual é a esbeltez de interesse?

$\lambda_2 = \frac{300cm}{(i_2 = ?)}$

$i_2 = \sqrt{\frac{I_2}{A}}$

$A = b \times h = 10 \times 20 = 200 \, cm^2$

$I_2 = \frac{h.b^3}{12} = \frac{20 \times 10^3}{12} = \frac{20000}{12} = ?$

$i_2 = \sqrt{\frac{\cancel{h} b^{\cancel{3}2}}{12 . \cancel{bh}}} = \frac{b}{\sqrt{12}} = \frac{b}{3{,}46} \Rightarrow \lambda_2 = \frac{300}{b} \times 3{,}46$

$$\boxed{\lambda_2 = \frac{30\cancel{0}}{1\cancel{0}} . 3{,}46 = 103{,}8}$$

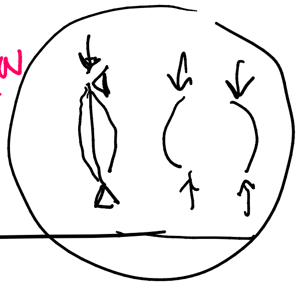
$$\lambda_1 = \frac{300}{h} . 3{,}46 = \frac{3\cancel{0}0}{2\cancel{0}} . 3.46 = 51{,}9$$

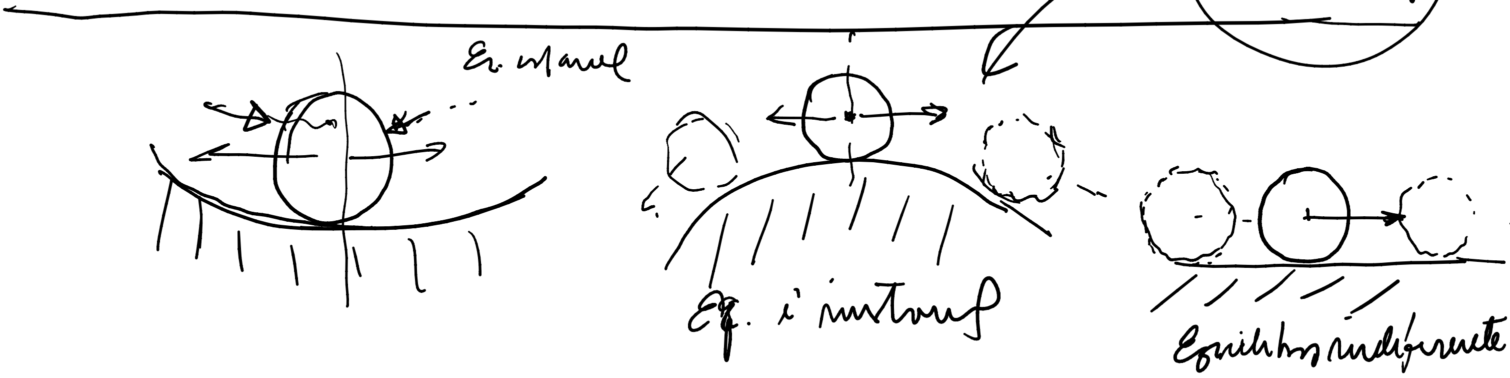
20GPa

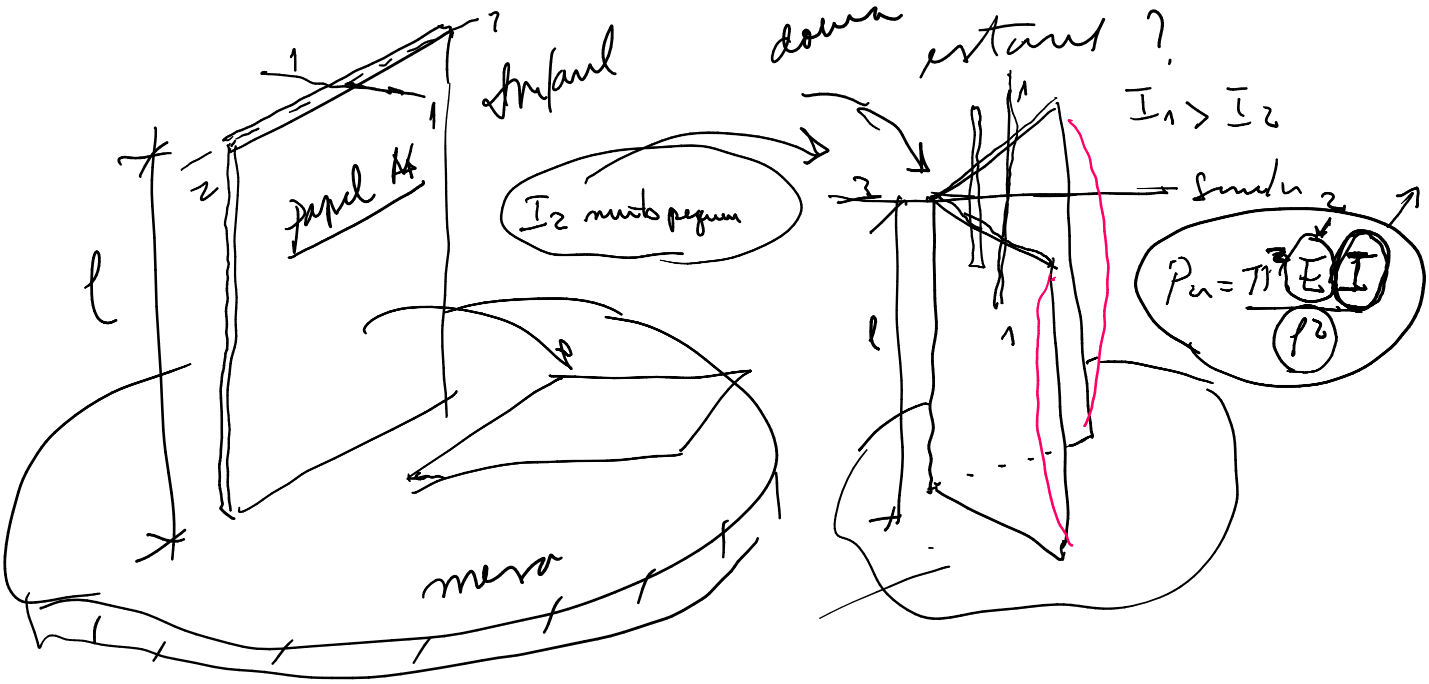
h $\quad$ 20 $\quad$ $//$ $\quad$ 20 GPa

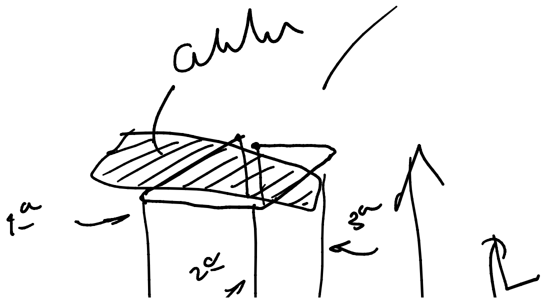
$$P_{cr_2} = \frac{\pi^2 \cdot E I_2}{L^2} = \frac{\pi^2 \cdot E \cdot h \cdot b^3}{12 \times (300)^2} \Rightarrow \text{considerando } E = 2000\ kN/cm^2$$

$$P_{cr2} = \frac{\pi^2 \cdot 2000\ kN/cm^2 \cdot 20\ cm \cdot (10\ cm)^3}{12 \times (300\ cm)^2} = 365{,}5\ kN$$

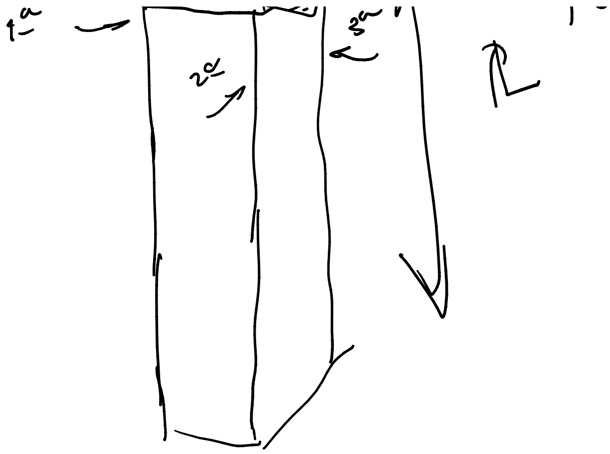








Porquê?



$b := 10cm$ $h := 20cm$ $A := b \cdot h = 0.02\,m^2$ $L := 3m$

$I_2 := \frac{h \cdot b^3}{12} = 1.667 \times 10^3\,cm^4$ $i_2 := \sqrt{\frac{I_2}{A}} = 28.868\,mm$ $\lambda_2 := \frac{L}{i_2} = 103.923$

$E := 20GPa$ modulo de elasticidade da madeira ou do concreto

$P_{cr2} := \frac{\pi^2 \cdot E \cdot I_2}{L^2} = 365.541\,kN$

$P_{cr2} = 37.275$ tor

$I_1 := \frac{b \cdot h^3}{12} = 6.667 \times 10^{-5}\,m^4$ $P_{cr2} = 3.727\times$

$i_1 := \sqrt{\frac{I_1}{A}} = 57.735\,mm$

$\lambda_1 := \frac{L}{i_1} = 51.962$

$P_{cr1} := \frac{\pi^2 \cdot E \cdot I_1}{L^2} = 1.462\,MN$

$P_{cr1} = 149.099$ t

$1MN = 101.972\,tonnef$