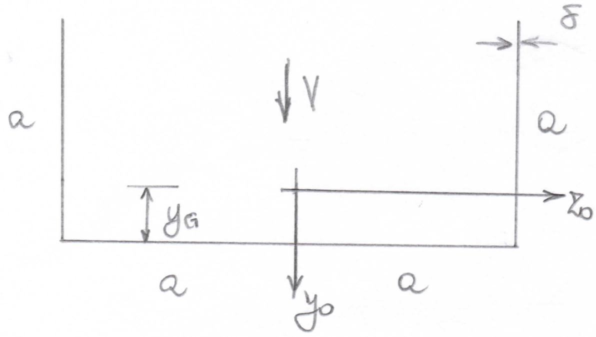
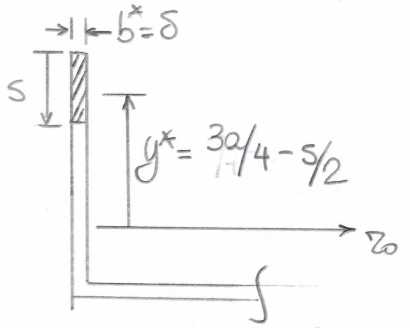
Exemplo: (Apostila cap. 0,2, ex. 11)



, $\delta << a$

$y_G = a/4$, $I_{z0} = \frac{5}{12}\delta a^3$

Calculando as tensões:



$S^* = \delta s$

$M_s^* = \delta s.\left(\frac{3a}{4} - \frac{s}{2}\right) = \frac{\delta s}{4}(3a - 2s)$

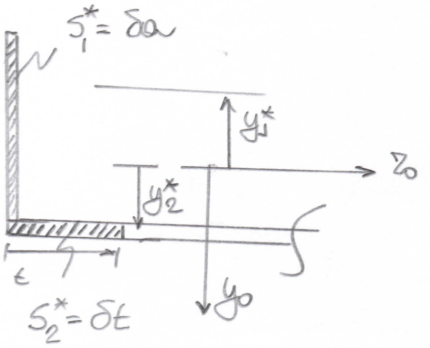
$\tau = \frac{V.M_s^*}{b^* I_{z0}} = \frac{V}{\delta} \cdot \frac{12}{5\delta a^3} \cdot \frac{\delta s}{4}(3a - 2s) = \frac{3}{5}\frac{V}{\delta a^3}(3as - 2s^2)$

$\tau_{máx} \Rightarrow \frac{\partial \tau}{\partial s} = 0 \Rightarrow 3a - 4s = 0 \Rightarrow s = 3/4\, a$ (na linha neutra)

$\tau_{máx} = \frac{3}{5}\frac{V}{\delta a^3}\left(\frac{9a^2}{4} - \frac{9a^2}{8}\right) = \frac{27}{40}\frac{V}{\delta a}$

$\tau(s=0) = 0$

$\tau(s=a) = \frac{3}{5}\frac{V}{\delta a}$ (observe que $y^* = -a/4$)!!



$y_1^* = a/2 - a/4 = a/4$

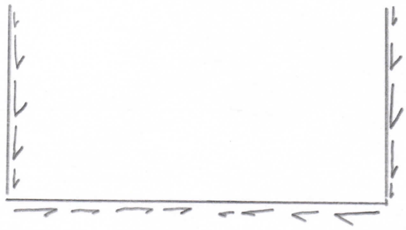
$y_2^* = -a/4$! (coerente com o anterior)!!

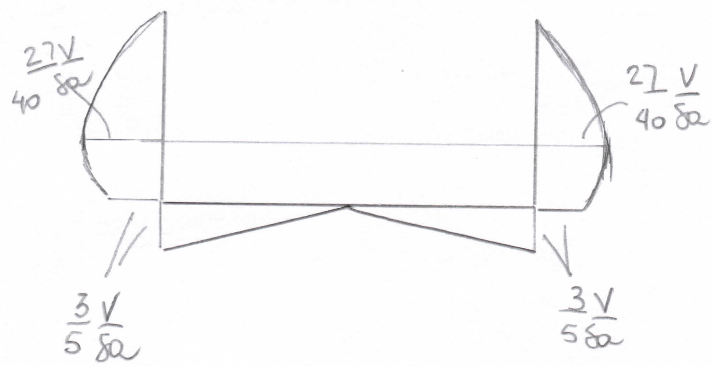
$$M_S^* = S_1 y_1^* + S_2 y_2^* = \frac{\delta a^2}{4} - \frac{\delta a t}{4} = \frac{\delta a}{4}(a-t)$$

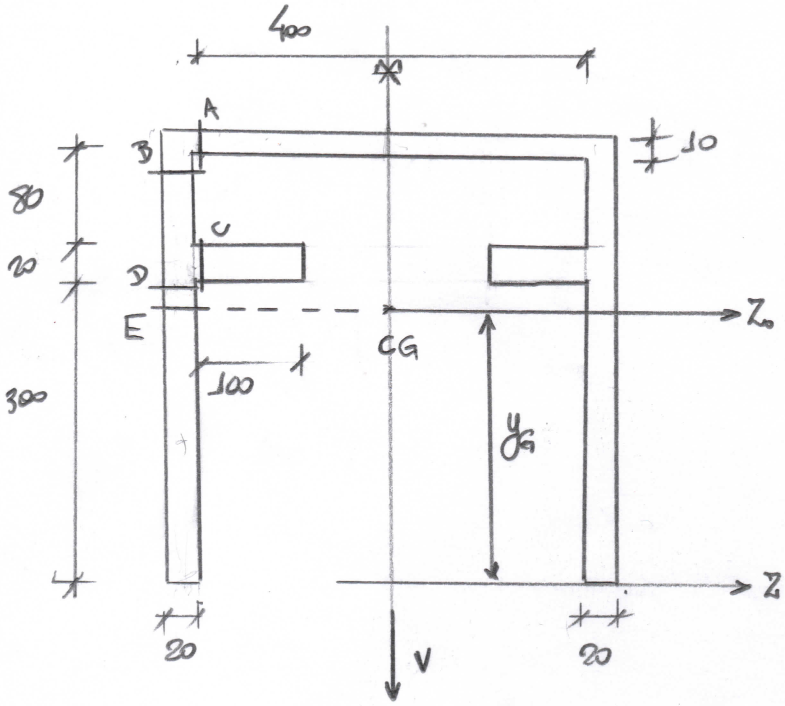
$$\tau = \frac{V \cdot M_S^*}{b^* \cdot I_{z_0}} = \frac{V}{\delta} \cdot \frac{12}{5\delta a^3}\left[\frac{\delta a}{4}(a-t)\right] = \frac{3V}{5\delta a^2}(a-t)$$

$$\tau(t=0) = \frac{3}{5}\frac{V}{\delta a}$$

$$\tau(t=a) = 0$$



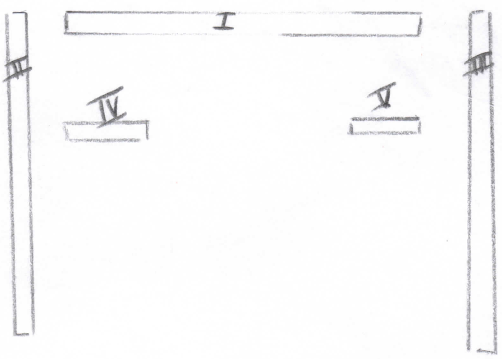




unidades em mm

$V = 225\,kN$

Encontrando y_G e I_{z0}:

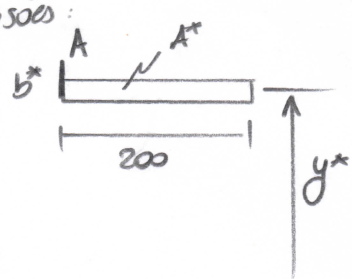


Área	y_G	A	I_{z0}	d	d^2A
I	405	4400	$36,7 \cdot 10^3$	150	$9,9 \cdot 10^7$
II	200	8000	$10,7 \cdot 10^7$	55	$2,42 \cdot 10^7$
III	200	8000	$10,7 \cdot 10^7$	55	$2,42 \cdot 10^7$
IV	310	2000	$6,7 \cdot 10^4$	55	$6,05 \cdot 10^6$
V	310	2000	$6,7 \cdot 10^4$	55	$6,05 \cdot 10^6$
		24400	$214,17 \cdot 10^6$		$159,5 \cdot 10^6$

$y_G = 255\,mm$

$I_{z0} = 37367 \cdot 10^4\,mm^4$

Tensões:



$b^* = 10\,mm$

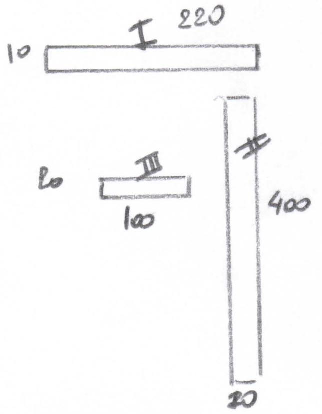
$A^* = 2000\,mm^2$

$y^* = 405 - 255 = 150\,mm$

$M_S^* = 300000\,mm^3$

$$\tau_A = \frac{225 \cdot 10^3 \cdot 3 \cdot 10^5}{10 \cdot 37367 \cdot 10^4} = 18,06 \frac{N}{mm^2} = 18,06\,MPa$$

Podemos usar meia estrutura para obter y_G e I_{zo}:



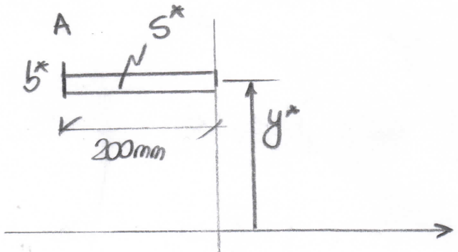
Área	y_G	A	I_{zo}	d	d^2A
I	405	2200	$1,83.10^4$	150	$49,5.10^6$
II	200	8000	$10,7.10^7$	55	$24,2.10^6$
III	310	2000	$6,7.10^4$	55	$6,05.10^6$
sum		12200	$107,08.10^6$		$79,75.10^6$

$y_G = 255 cm$

$I_{zo}^{7} = 186,83.10^6$

$I_{zo} = 2 . I_{zo}^{7} = 373,66.10^6 \ mm^4$

Tensões:



$b^* = 10mm$

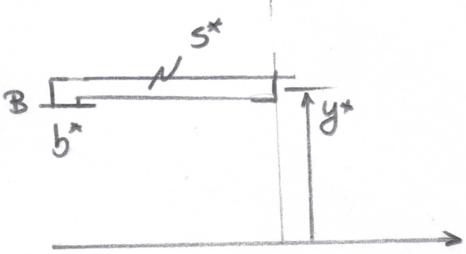
$S^* = 2000 mm^2$

$y^* = 405 - 255 = 150 mm$

$M_S^* = 3.10^5 mm^3$

$\tau_A = \frac{225.10^3 . 3.10^5}{10 . 376,66 . 10^6} = 17,92 \ MPa$

$\boxed{\tau_A \simeq 18 MPa}$



$b^* = 20mm$

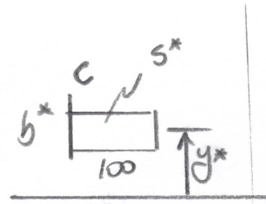
$S^* = 2200 mm^2$

$y^* = 150 mm$

$M_S^* = 33.10^4 mm^3$

$\tau_B = \frac{225.10^3 . 33.10^4}{20 . 376,66 . 10^6} = 9,86 \ MPa$

$\boxed{\tau_B \simeq 10 MPa}$



$b^* = 20\,mm$

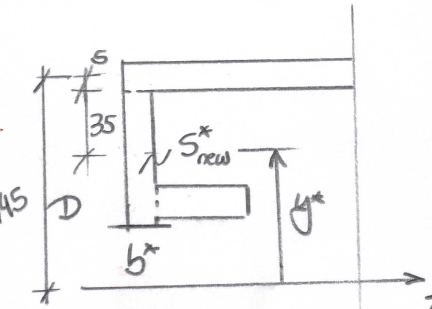
$S^* = 2000\,mm^2$

$y^* = (300-255)+10 = 55\,cm$

$M_S^* = 11.10^4\,mm^3$

$\tau_c = \dfrac{225.10^3.11.10^4}{\frac{20}{2}.376,66.10^6} = 3,28\,MPa$

$\boxed{\tau_c \simeq 3,3\,MPa}$



$M_S^* = M^*_{S,B} + M^*_{S,C} + M^*_{S,D} = 58,7.10^4\,mm^3$

$S^*_{new} = 1400\,mm^2$

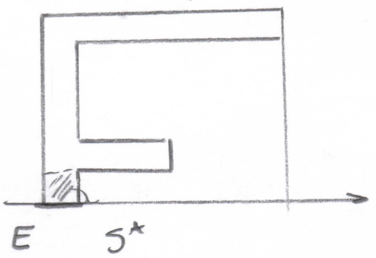
$y^* = 105\,mm$

$M^*_{S,D} = 14,7.10^4\,mm^3$

$b^* = 20\,mm$

$\tau_D = \dfrac{225.10^3.58,7.10^4}{\frac{20}{2}.373,66.10^6} = 17,67\,MPa$

$\boxed{\tau_D \simeq 18\,MPa}$



$b^* = 20\,mm$

$S^* = 45.20 = 900\,mm^2$

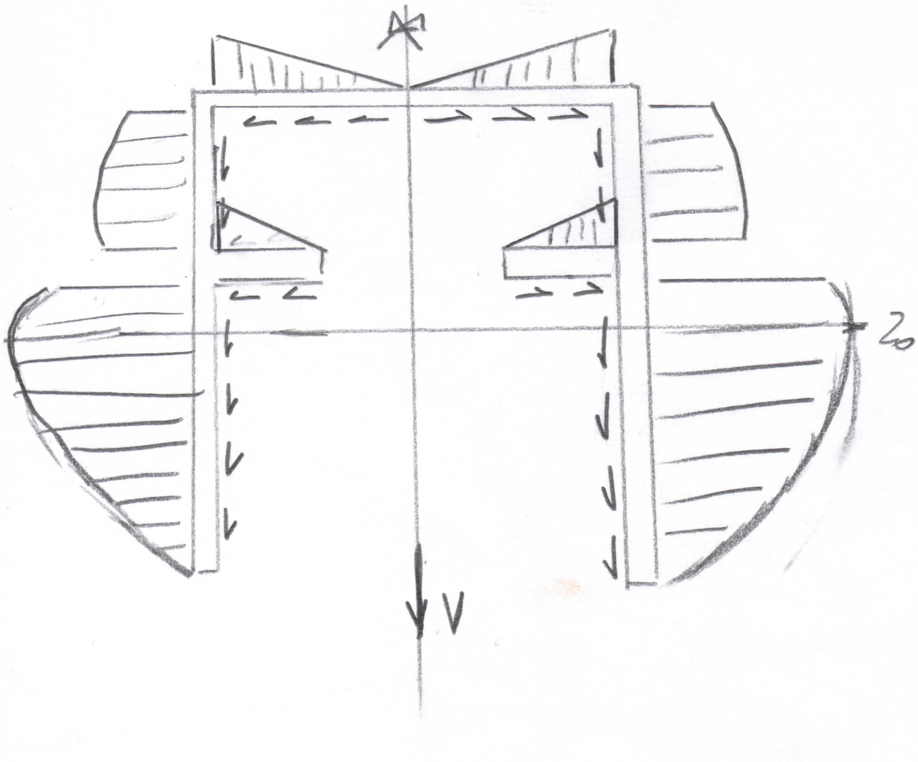
$y^* = 22,5\,mm$

$M_S^* = 2,025.10^4 + 58,7.10^4 = 60,725.10^4$

$\tau_E = \dfrac{225.10^3.60,725.10^4}{\frac{20}{2}.376,66.10^6} = 18,13\,MPa$

$\boxed{\tau_E \simeq 18,1\,MPa}$

Fluxo e diagramas:



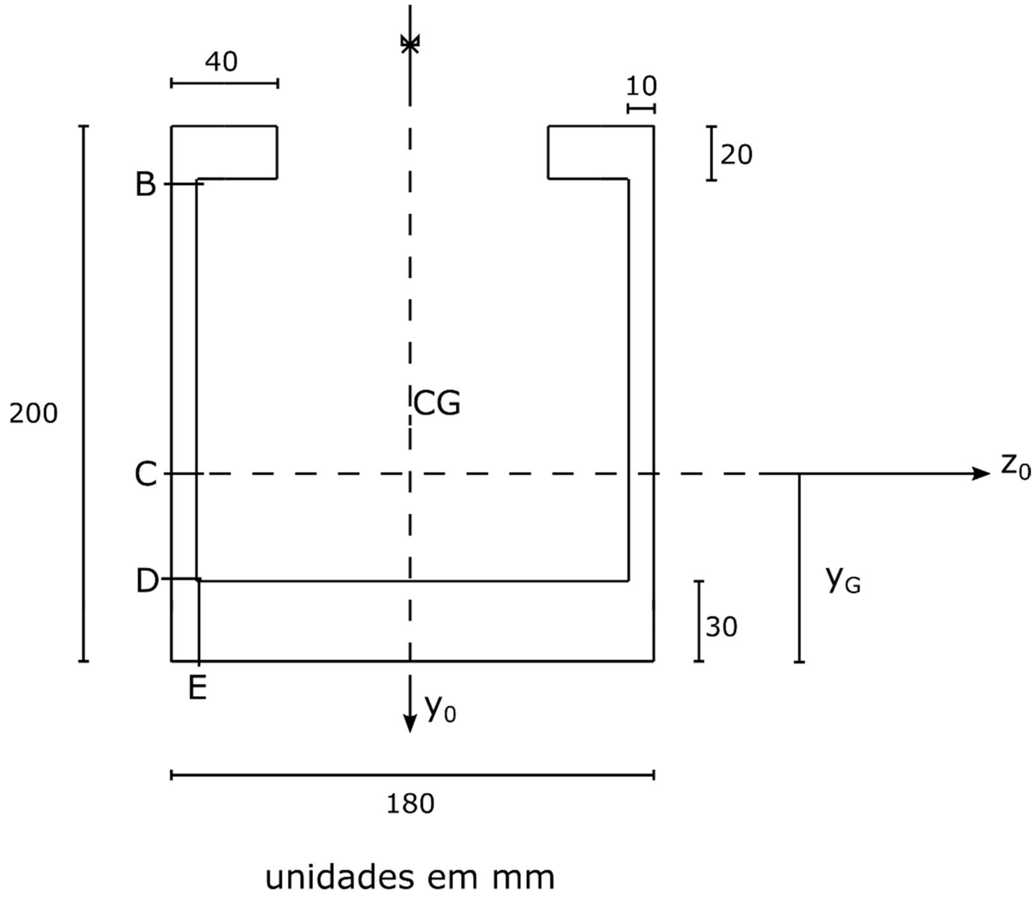
ESCOLA POLITÉCNICA DA UNIVERSIDADE DE SÃO PAULO

Departamento de Engenharia de Estruturas e Geotécnica

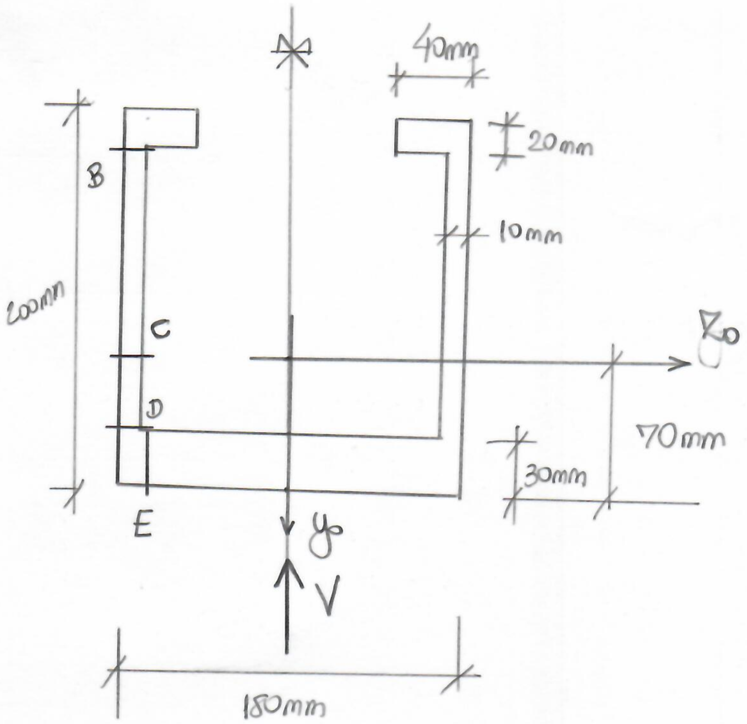
PEF3202 – Introdução à Mecânica dos Sólidos – Terceira Prova (13/06/2019)

Nome: ____ Nº USP _____________

3a. Questão (3,0 pontos) Determine, para a seção abaixo, as tensões de cisalhamento, em MPa, nos pontos B, C, D, E, para uma força cortante de $V = 250.10^3\ N$. Esboce também o diagrama e o fluxo de tensões de cisalhamento ao longo da seção. São dados $I_{z0} = 4913{,}34.10^4\ mm^4$ e $y_G = 70\ mm$.



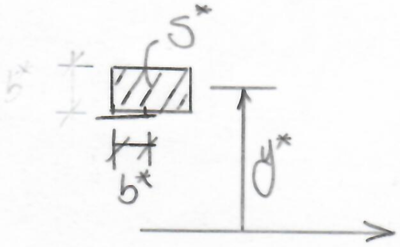
Q3



$I_{z0} = 4913{,}34 \cdot 10^4\ mm^4$

$V = 250 \cdot 10^3\ N$

Tensões em B:



$b^* = 10\ mm$

$S^* = 40 \cdot 20 = 800\ mm^2$

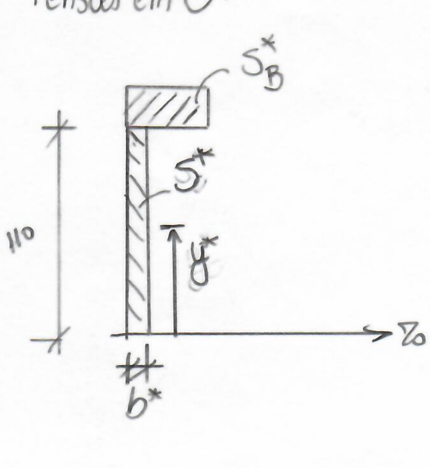
$y^* = 190 - 70 = 120\ mm$

$M_S^* = 96000\ mm^3$

$$\tau_B = \frac{250 \cdot 10^3 \cdot 96000}{4913{,}34 \cdot 10^4 \cdot 10}$$

$$\boxed{\tau_B = 48{,}85\ MPa}$$

Tensões em C:



$b^* = 10\ mm$

$S^* = 1100\ mm^2$

$y^* = 55\ mm$

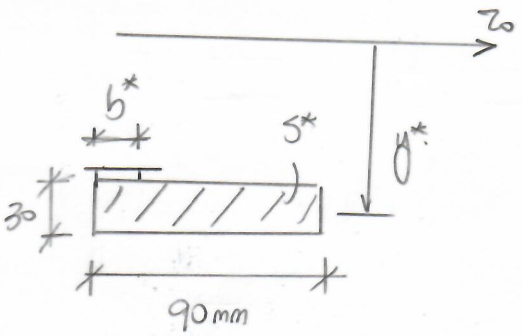
$M_S^* = 60500 + 96000$

$M_S^* = 156500\ mm^3$

$$\tau_C = \frac{250 \cdot 10^3 \cdot 156500}{4913{,}34 \cdot 10^4 \cdot 10}$$

$$\boxed{\tau_C = 79{,}63\ MPa}$$

Tensões em D:



$b^* = 10mm$

$S^* = 2700mm^2$

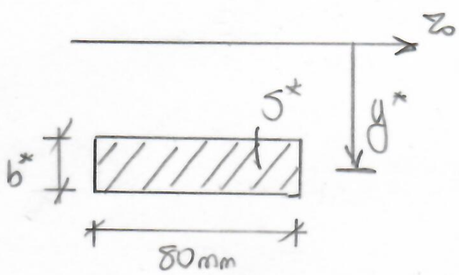
$y^* = 70 - 15 = 55mm$

$M_S^* = 148500\,mm^3$

$$\tau_D = \frac{250.10^3 . 148500}{4913,34 . 10^4 . 10}$$

$\boxed{\tau_D = 75,56MPa}$

Tensões em E:



$b^* = 30mm$

$S^* = 2400mm$

$y^* = 55mm$

$M_S^* = 132000mm^3$

$$\tau_E = \frac{250.10^3 . 132000}{4913,34 . 10^4 . 30}$$

$\boxed{\tau_E = 22,39MPa}$

Fluxo/Diagrama: