Exercícios

MAP 2110 - Diurno

IME USP

9 de junho

Seria bom ter a imagem do que acontece em dimensão 3 para entender algumas provas:

$$\det(A) = \left(a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \right)$$

onde

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Exercício 1 - pg183

$$\begin{vmatrix} a+px & b+qx & c+rx \\ p+ux & q+vx & r+wx \\ u+ax & v+bx & w+cx \end{vmatrix} =$$

$$\begin{vmatrix} a & b & c \\ p+ux & q+vx & r+wx \\ u+ax & v+bx & w+cx \end{vmatrix} + \begin{vmatrix} px & qx & rx \\ p+ux & q+vx & r+wx \\ u+ax & v+bx & w+cx \end{vmatrix}$$

$$\begin{vmatrix} a & b & c \\ p+ux & q+vx & r+wx \\ u+ax & v+bx & w+cx \end{vmatrix} =$$

$$\begin{vmatrix} a & b & c \\ p & q & r \\ u+ax & v+bx & w+cx \end{vmatrix} + \begin{vmatrix} a & b & c \\ ux & vx & wx \\ u+ax & v+bx & w+cx \end{vmatrix} =$$

$$\begin{vmatrix} a & b & c \\ p & q & r \\ u & v & w \end{vmatrix} + \begin{vmatrix} a & b & c \\ p & q & r \\ ax & bx & cx \end{vmatrix} = \begin{vmatrix} a & b & c \\ p & q & r \\ u & v & w \end{vmatrix}$$

Parte Azul

$$\begin{vmatrix} px & qx & rx \\ p+ux & q+vx & r+wx \\ u+ax & v+bx & w+cx \end{vmatrix} =$$

$$x\begin{vmatrix} p & q & r \\ p & q & r \\ u+ax & v+bx & w+cx \end{vmatrix} + x\begin{vmatrix} p & q & r \\ ux & vx & wx \\ u+ax & v+bx & w+cx \end{vmatrix} =$$

$$x^2\begin{vmatrix} p & q & r \\ u & v & w \\ u & v & w \end{vmatrix} + x^2\begin{vmatrix} p & q & r \\ u & v & w \\ ax & bx & cx \end{vmatrix} = x^3\begin{vmatrix} p & q & r \\ u & v & w \\ a & b & c \end{vmatrix}$$

Juntando as partes

$$\begin{vmatrix} a+px & b+qx & c+rx \\ p+ux & q+vx & r+wx \\ u+ax & v+bx & w+cx \end{vmatrix} = (1+x^3)\begin{vmatrix} a & b & c \\ p & q & r \\ u & v & w \end{vmatrix}$$

Exercício 4 pg 183

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = \begin{vmatrix} 1 & a & a^3 \\ 0 & b-a & b^3-a^3 \\ 0 & c-a & c^3-a^3 \end{vmatrix} = \begin{vmatrix} 1 & a & 0 \\ 0 & b-a & b^3-ba^2 \\ 0 & c-a & c^3-ca^2 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & b-a & b^3-ba^2 \\ 0 & c-a & c^3-ca^2 \end{vmatrix} = (b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & b(b+a) \\ 0 & 1 & c(c+a) \end{vmatrix}$$

Na segunda igualdade subtraimos a^2 vezes a segunda coluna da terceira coluna. e na terceira fizemos a segunda coluna menos a vezes a primeira.

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (b-a)(c-a)(c(c+a) - b(b+a) + cb - cb) =$$

$$(b-a)(c-a)(c-b)(a+b+c)$$

Exercício 5, pg183

$$\begin{vmatrix} 3R_1 + 2R_3 \\ 2R_1 + 5R_2 \end{vmatrix} = \begin{vmatrix} 3R_1 + 2R_3 \\ 2R_1 \end{vmatrix} + \begin{vmatrix} 3R_1 + 2R_3 \\ 5R_2 \end{vmatrix} =$$

$$4\begin{vmatrix} R_3 \\ R_1 \end{vmatrix} + 15\begin{vmatrix} R1 \\ R_2 \end{vmatrix} + 10\begin{vmatrix} R_3 \\ R_2 \end{vmatrix}$$

Se $R_3 = R_2$

$$\begin{vmatrix} 3R_1 + 2R_2 \\ 2R_1 + 5R_2 \end{vmatrix} = \begin{vmatrix} 3R_1^T + 2R_2^T & 2R_1^T + 5R_2^T \end{vmatrix} =$$

$$\det\left(\begin{pmatrix} R_1^T & R_2^T \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 2 & 5 \end{pmatrix}\right) = \begin{vmatrix} R_1 \\ R_2 \end{vmatrix} \begin{vmatrix} 3 & 2 \\ 2 & 5 \end{vmatrix} = 55$$

Exercício 5 pg 148

A é 3×3 e

$$\det(2A^{-1}) = -4 = \det(A^3.{B^{-1}}^T) \implies$$
$$2^3 \det(A^{-1}) = 8/\det(A) = -4 \implies \det(A) = -2$$
$$-4 = \det(A^3)\det(B^{-1}) = \det(A)^3 \det(B)^{-1} \implies$$
$$\det(B) = \frac{1}{-4}\det(A)^3 = 2$$

Exercício 8, pg 148

a)

$$\begin{bmatrix} 2 & 1 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$x = \frac{\begin{vmatrix} 1 & 1 \\ -2 & 7 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 3 & 7 \end{vmatrix}} \text{ e } y = \frac{\begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 3 & 7 \end{vmatrix}} x = \frac{9}{11} \text{ e } y = \frac{-7}{11}$$

$$\begin{bmatrix} 4 & -1 & 3 \\ 6 & 2 & -1 \\ 3 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$x = \frac{\begin{vmatrix} 1 & -1 & 3 \\ 0 & 2 & -1 \\ -1 & 3 & 2 \end{vmatrix}}{\begin{vmatrix} 4 & -1 & 3 \\ 6 & 2 & -1 \\ 3 & 3 & 2 \end{vmatrix}} \text{ e } y = \frac{\begin{vmatrix} 4 & 1 & 3 \\ 6 & 0 & -1 \\ 3 & -1 & 2 \end{vmatrix}}{\begin{vmatrix} 4 & -1 & 3 \\ 6 & 2 & -1 \\ 3 & 3 & 2 \end{vmatrix}} \text{ e } z = \frac{\begin{vmatrix} 4 & -1 & 1 \\ 6 & 2 & 0 \\ 3 & 3 & -1 \end{vmatrix}}{\begin{vmatrix} 4 & -1 & 3 \\ 6 & 2 & -1 \\ 3 & 3 & 2 \end{vmatrix}}$$

$$x = \frac{12}{79} \; y = \frac{-37}{79} \; z = \frac{-2}{79}$$