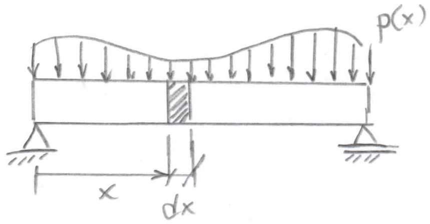
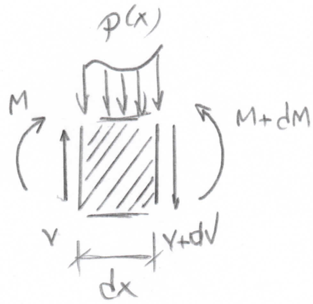
Cisalhamento na Flexão





Lembrando:

A equação de equilíbrio na vertical fornece: $\frac{dV}{dx} = -p$

E a equação de equilíbrio de momentos: $\frac{dM}{dx} = V$

Integrando as tensões na área para obter os esforços solicitantes:

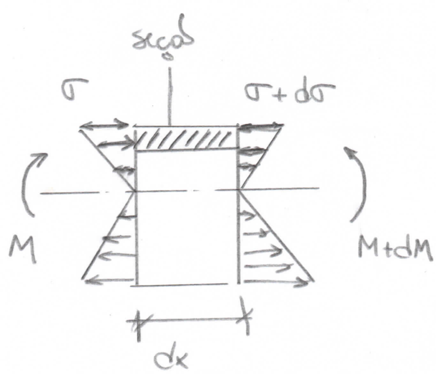
$$N = \int_A \sigma dA = 0$$

$$M = \int_A \sigma y dA$$

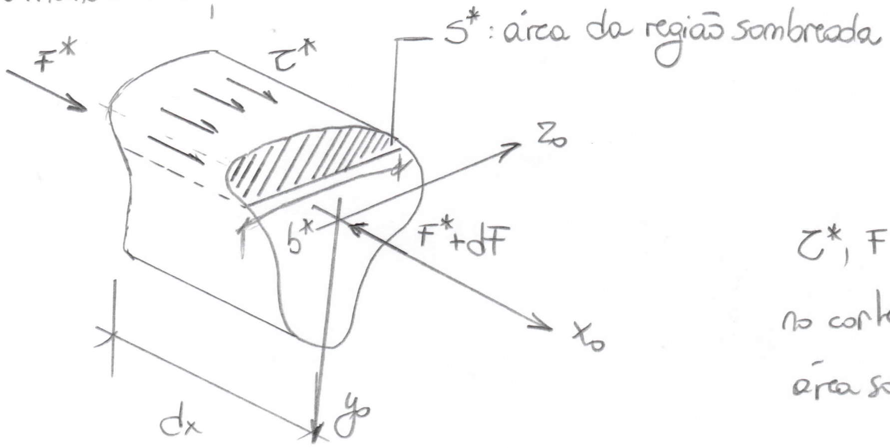
$$V = \int_A \tau dA$$

Digamos que:

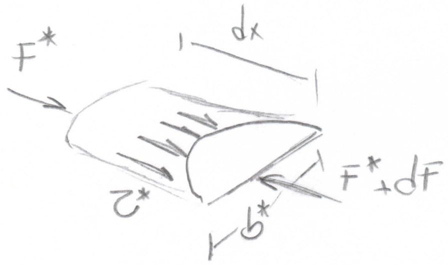
$$\frac{dM}{dx} = V > 0$$



Olhando a seção em 3D:



τ^*, F^*, F^*+dF atuando no corte (delimitado pela área sombreada).



Fazendo o equilíbrio:

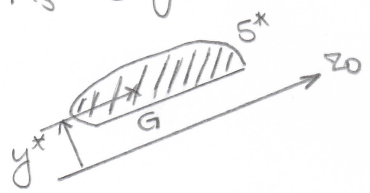
$$F^* - (F^* + dF) + \tau^* . (b^* dx) = 0$$

$$dF = \tau^* b^* dx \Rightarrow \tau^* = \frac{1}{b^*}\frac{dF^*}{dx}$$

Mas: $F^* = \int_{S^*} \sigma dA = \int_{S^*} \frac{M}{I_{z0}} y dA = \frac{M}{I_{z0}} \int_{S^*} y dA = \frac{M}{I_{z0}} . M_S^*$

M_S^*: momento estático da área sombreada em relação ao eixo z0.

$M_S^* = S^* . y^*$ (S^*: área da região sombreada; y^*: distância de S^* até z_0)



Logo:

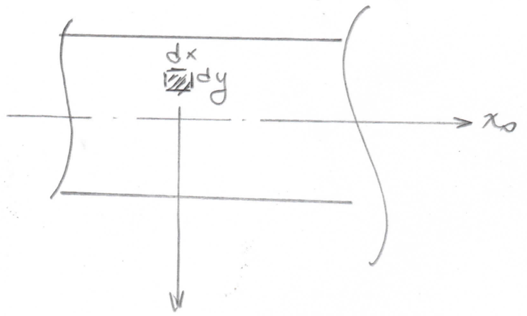
$$F^* = \frac{M}{I_{z0}} . M_S^*$$

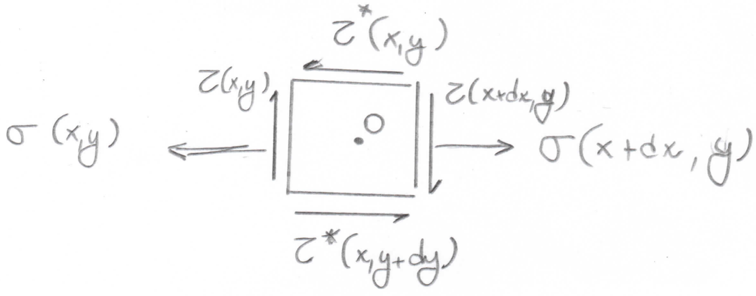
Derivando F^* em relação a x:

$$\frac{dF^*}{dx} = \frac{d}{dx}\left(\frac{M}{I_{z0}} . M_S^*\right) = \frac{dM}{dx} . \frac{M_S^*}{I_{z0}} = \frac{V M_S^*}{I_{z0}}$$

e assim: $\tau^* = \frac{1}{b^*}\frac{dF^*}{dx} \Rightarrow \boxed{\tau^* = \frac{V . M_S^*}{b^* I_{z0}}}$

Voltando a barra:





Fazendo o equilíbrio de momentos em O:

$$-\tau(x,y).\frac{dx}{2}+\tau^*(x,y).\frac{dy}{2}-\tau^*(x,y+dy).\frac{dy}{2}+\tau(x+dx,y)\frac{dx}{2}=0$$

É possível mostrar que:

$$\boxed{\tau=\tau^*}$$

Logo: $\tau = \dfrac{V M_s^*}{b^* I_{zo}}$

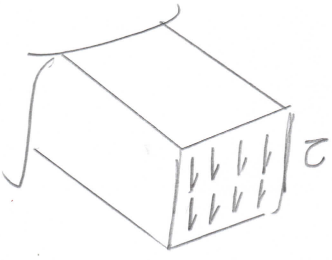
Lembrando que:

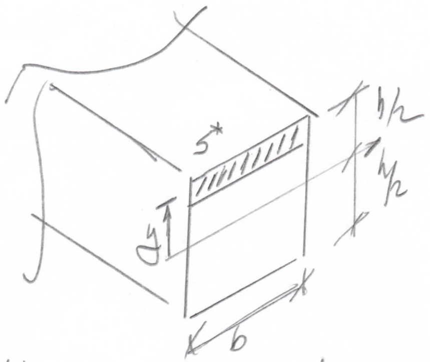
$$V=\int \tau \, dA = \int \frac{V}{b^*}\frac{M}{I_{zo}} dA = \frac{V}{I_{zo}}\int \frac{M_s^*}{b^*} dA$$

Pode-se demonstrar que:

$$I_{zo} = \int \frac{M_s^*}{b^*} dA$$

Seção Retangular





$$M_s^* = \int_y^{h/2} y\,dA = \int_y^{h/2} y\,(b\,dy) = b\left[\frac{y^2}{2}\right]_y^{h/2} = \frac{b}{2}\left[\frac{h^2}{4} - y^2\right]$$

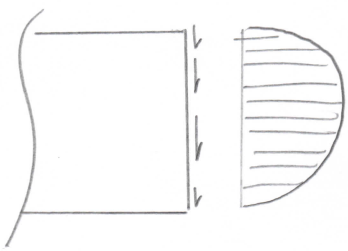
$b^* = b$

Logo:

$$\tau = \frac{V M_s^*}{b^* I_{zo}} = \frac{V}{\cancel{b}} \cdot \frac{1}{bh^3/12} \cdot \frac{\cancel{b}}{2}\left[\frac{h^2}{4} - y^2\right] = \frac{12V}{2bh^3} \cdot \frac{h^2}{4}\left[1 - \frac{4y^2}{h^2}\right]$$

$$\tau = \frac{3}{2}\frac{V}{bh}\left[1 - \frac{4y^2}{h^2}\right]$$

Vista lateral

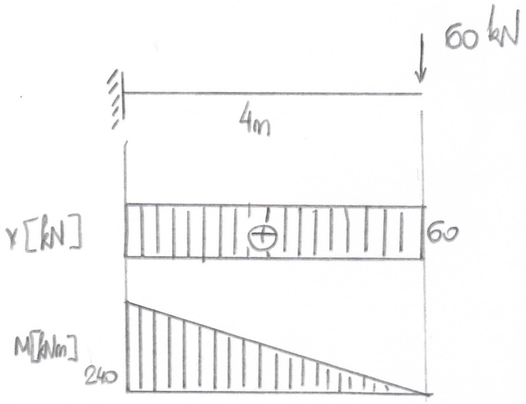


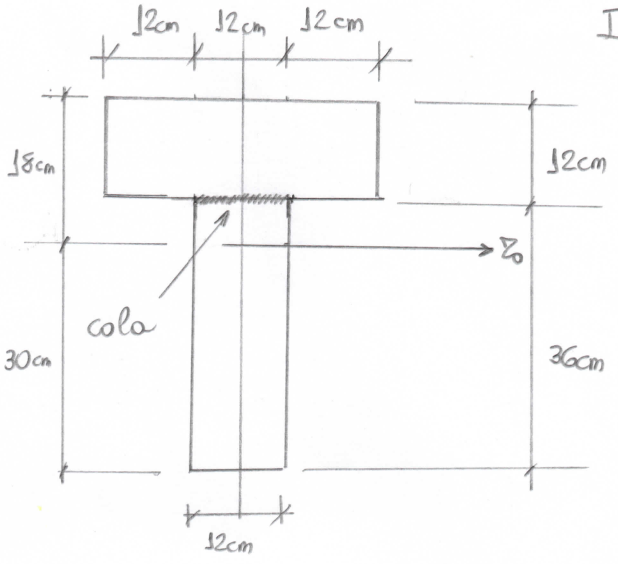
$$\tau_{máx} = \frac{3}{2}\frac{V}{bh} = \frac{3}{2}\tau_{médio}$$

$$\tau_{máx} = f \cdot \tau_{médio}$$

↑ fator de correção para τ.

Exemplo: Determinar a máxima tensão de ruptura de cisalhamento na cola para que o fator de segurança seja 2.





$I_{z0} = 176256\,cm^4$

$$\tau^* = \frac{V.\,M_s^*}{b^* I_{z0}} \qquad b^* = 12cm \text{ (cola)}$$

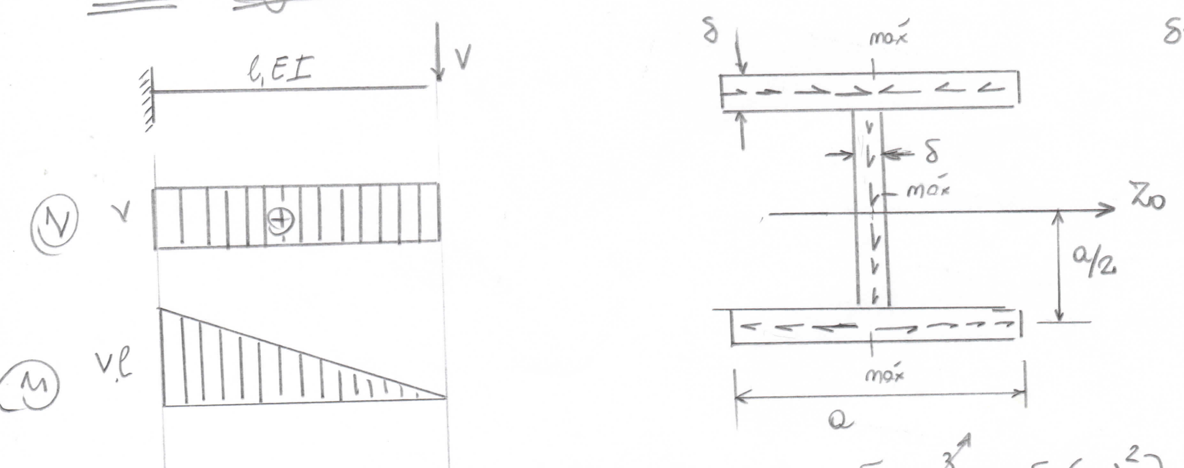
$$M^* = S^* y^* = (36.12).12 = 5184\,cm^3$$

$$\tau^* = \frac{60.\,5184}{12.\,176256} = 0{,}147\,\frac{kN}{cm^2} = 0{,}147.\,\frac{10^3 N}{10^{-4} m^2} = 0{,}147.\,10MPa \Rightarrow \tau^* \simeq 1{,}5MPa$$

Para $s = 2$:

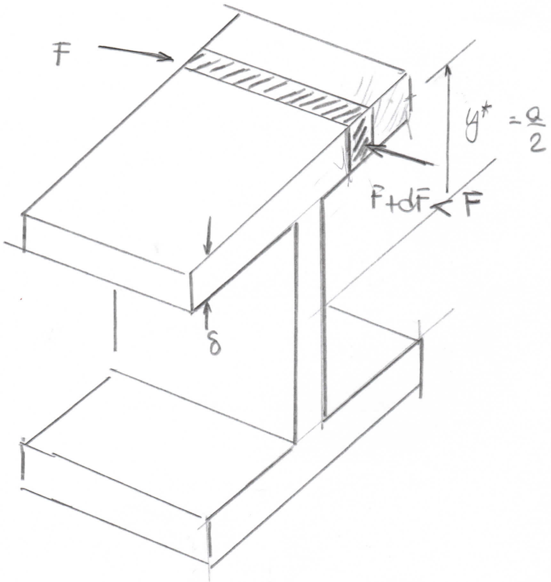
$$\frac{\tau_R}{s} = \bar{\tau} \Rightarrow \tau_R = s.\bar{\tau} = 2.1{,}5 \Rightarrow \boxed{\tau_R \simeq 3MPa}$$

Seções Delgadas Simétricas

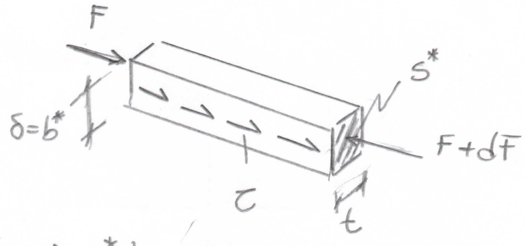


$$I = I_{z0} \simeq \frac{\delta a^3}{12} + 2\left[\cancel{\frac{a\delta^3}{12}} + a.\delta.\left(\frac{a}{2}\right)^2\right] = \frac{7}{12}\delta a^3$$

desprezível



Tensões nas flanges



$$s^* = b^*.t = \delta t$$

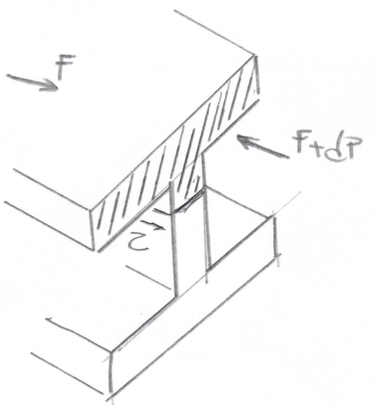
$$y^* = \frac{a}{2},\ \forall t$$

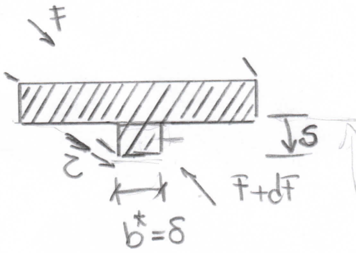
$$M^* = s^* y^* = \frac{t\delta a}{2}$$

$$\tau = \frac{V.M_s^*}{b^*.I} = \frac{V.\ t\delta a/2}{\delta.(7/12\,\delta a^3)} = \frac{12}{14}\frac{Vt\delta a}{\delta^2 a^3} \Rightarrow \tau = \frac{6}{7}\frac{Vt}{\delta a^2}$$

Logo: $\begin{cases} \tau(t=0) = 0 \\ \tau(t=a/2) = 3/7\,\dfrac{V}{\delta a} \end{cases}$

Tensões na alma:





$$y^* = \frac{a-s}{2}$$

$$M_s^* = M_1^* + M_2^* \Rightarrow \begin{cases} M_1^* = a.\delta\frac{a}{2} = \frac{\delta a^2}{2} \\ M_2^* = (s.\delta).y^* = \frac{s\delta(a-s)}{2} \end{cases}$$

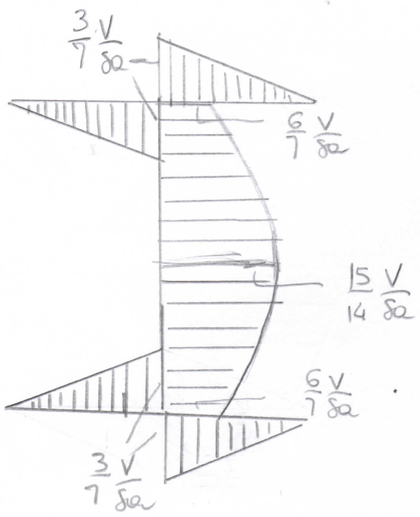
$$M_s^* = \frac{\delta a^2}{2} + \frac{s\delta a}{2} - \frac{\delta s^2}{2} = \frac{\delta}{2}(a^2+sa-s^2)$$

$$\tau^* = \frac{VM_s^*}{b^*.I} = \frac{V}{\delta}.\frac{\delta/2\,(a^2+sa-s^2)}{7/12\,\delta a^3} = \frac{6}{7}V\left(\frac{a^2+sa-s^2}{\delta a^3}\right)$$

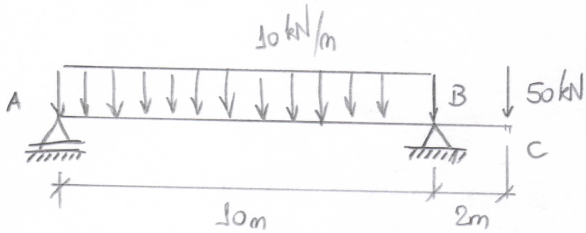
$$\tau(s=0) = 6/7\ V/\delta a$$

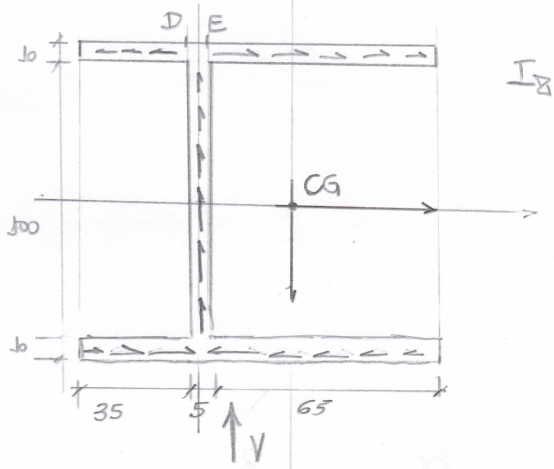
$$\tau(s=a/2) = \frac{15}{14}\frac{V}{\delta a} = \tau_{máximo}.$$

Montando uma figura com o fluxo de cisalhamento:



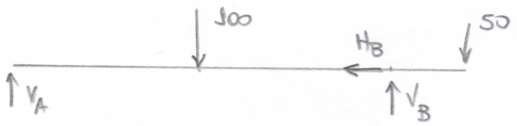
Exercício. Determine o fluxo de cisalhamento para a seção de maior esforço cortante e a tensão em D e E





$I_z = 6483.10^3 m^4$

Calculando as reações de apoio



$\Sigma F_H = 0 \cdot H_B = 0$

$\Sigma F_V = 0 : V_A + V_B = 150$

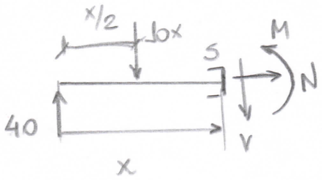
$\Sigma M_A = 0 : -10\cancel{0}.5 + V_B.1\cancel{0} - 5\cancel{0}.12 = 0$

$V_B = 50 + 60 \Rightarrow \underline{V_B = 110 \, kN}$

$V_A = 150 - V_B \Rightarrow \underline{V_A = 40 \, kN}$

Diagramas:

Trecho AB:

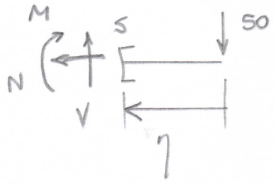


$\Sigma F_H = 0 \Rightarrow \boxed{N = 0}$

$\Sigma F_V = 0 \Rightarrow 40 - 10x - V = 0 \Rightarrow \underline{V = 40 - 10x}$

$\Sigma M_S = 0 \Rightarrow M - 40x + 10x.x/2 = 0 \Rightarrow \underline{M = 40x - 5x^2}$

Trecho BC

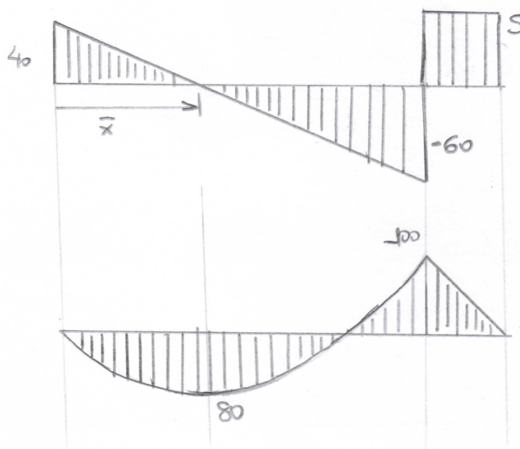


$\Sigma F_H = 0 \Rightarrow N = 0$

$\Sigma F_V = 0 \Rightarrow V - 50 = 0 \Rightarrow V = 50\ kN$

$\Sigma M_S = 0 \Rightarrow -M - 50\eta = 0 \Rightarrow M = -50\eta$

Diagramas:



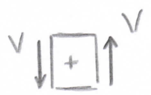
$\bar{x} = 4m$

$M(\bar{x}) = 160 - 5.16 = 80$

$M(0) = 0 \; ; \; M(10) = -100$

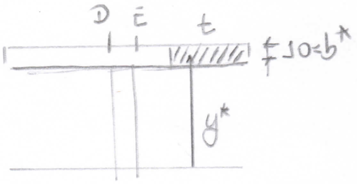
maior força cortante ⇒ imediatamente antes de B $(V = -60\ kN)$.

Em B



Lembrando que $\tau = \frac{V M_s^*}{I_{zo} b^*} = \left(\frac{V}{I_{zo}}\right)\left(\frac{M_s^*}{b^*}\right)$ com $\frac{V}{I_{zo}} = \frac{60.10^3}{6483.10^3} = 9{,}255.10^{-3}\ \frac{N}{mm^4}$

Considerando a flange superior/inferior:



$s^* = 10t$

$y^* = 55$

$b^* = 10$

$M_s^* = 550t$

$p/ t = 0 : M_s^* = 0$

$p/ t = 30 : M_s^* = 16500\ mm^3$

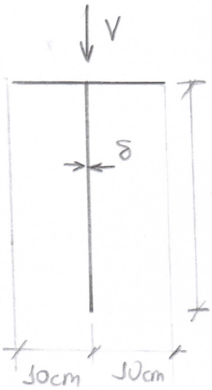
$p/ t = 65 : M_s^* = 35750\ mm^3$

$$\tau_D = \frac{V}{I_{zo}} \cdot \left(\frac{M_s^*(t=35)}{b^*}\right) = 9{,}255.10^{-3} \cdot \frac{16500}{10} \Rightarrow \tau_D = 15{,}27\ \frac{N}{mm^2} = 15{,}27\ MPa$$

$$\tau_E = \frac{V}{I_{zo}} \left(\frac{M^*(t=65)}{b^*}\right) = 9{,}255.10^{-3} \cdot \frac{35750}{10} = 33{,}09\ MPa$$

Apostila (ex. 12, cap. 5.2)

Sendo $V = 79.200\,kgf$, achar a distribuição de cisalhamento. $\delta = 2cm$.



encontrando o CG:

$$y_G = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$A_1 = \delta . 20 = 40cm$ $\quad y_1 = 30cm$

$A_2 = \delta . 30 = 60cm$ $\quad y_2 = 15cm$

$$y_G = \frac{40.30 + 60.15}{40+60} = \frac{1200+900}{100} = 21\,cm$$

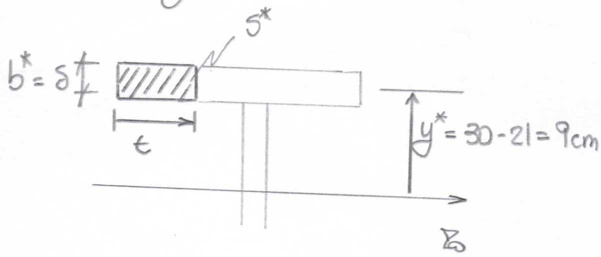
encontrando o momento de inércia:

$$I_{z_0} = I_z^1 + I_z^2 = \left[\frac{20.2^3}{12} + (40).(30-21)^2\right] + \left[\frac{2.30^3}{12} + 60(21-15)^2\right] =$$

$$= \left[\frac{40}{3} + 3240\right] + \left[4500 + 2160\right] \Rightarrow I_{z_0} = 9900\,cm^4$$

Encontrando a distribuição de cisalhamento:

- flange:



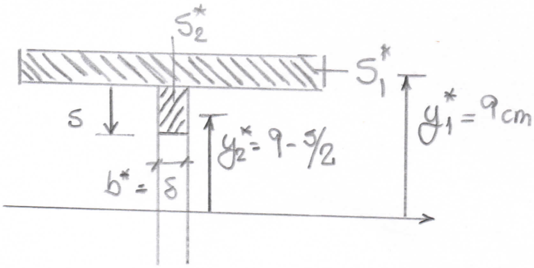
$S^* = \delta t = 2t$

$M_S^* = S^* y^* = 18t$

$$\tau = \frac{V . M_S^*}{b^* I_{z_0}} = \frac{79200 . 18t}{2 . 9900} = 72t$$

$$\begin{cases} \tau(t=0) = 0 \\ \tau(t=10) = 720\,kgf/cm^2 \end{cases}$$

- alma



$S_1^* = \delta . 20 = 40cm$

$S_2^* = \delta s = 2s$

$$M_S^* = S_1^* y_1^* + S_2^* y_2^* = 40.9 + 2s(9 - s/2)$$

$$= 360 + 18s - s^2$$