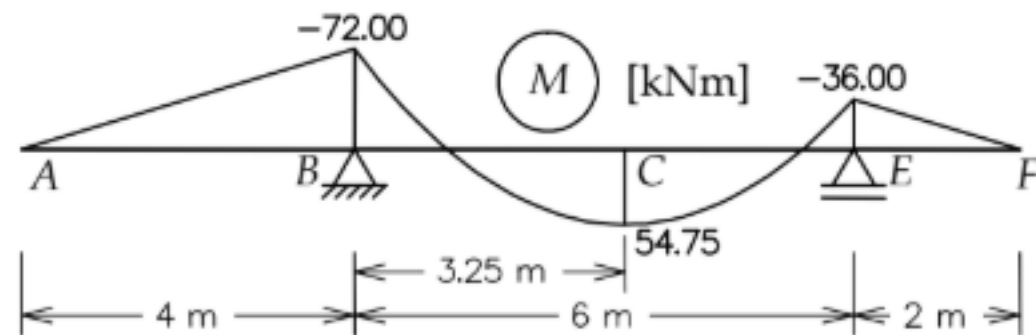


**PEF3208**

**Tensões normais na flexão  
exemplos adicionais**

**Professores Osvaldo Nakao, Guilherme Franzini, Rodrigo Provasi e Martin Schwark**

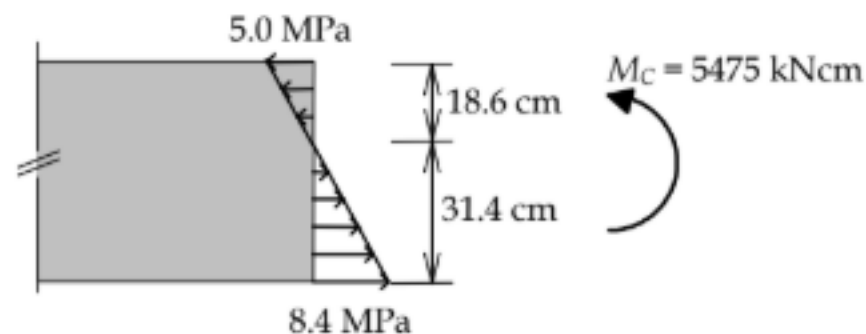
**Maio de 2020**



### Tensões normais na seção C:

$$\sigma_s = -\frac{M_C \cdot y_s}{I} = -\frac{5475 \cdot 18.6}{203697} = -0.50 \text{ kN/cm}^2 = -5.0 \text{ MPa}$$

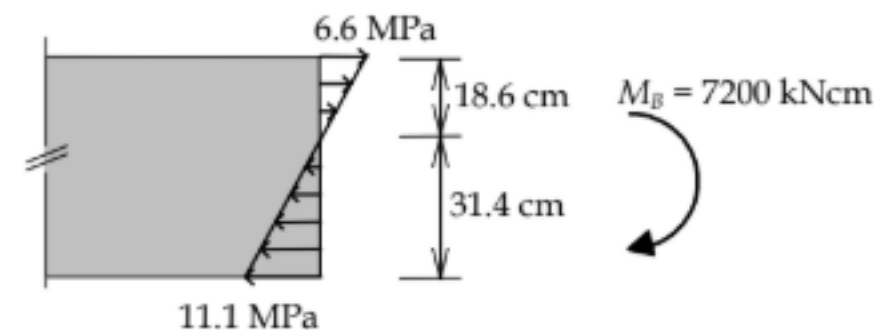
$$\sigma_i = \frac{M_C \cdot y_i}{I} = \frac{5475 \cdot 31.4}{203697} = +0.84 \text{ kN/cm}^2 = +8.4 \text{ MPa}$$



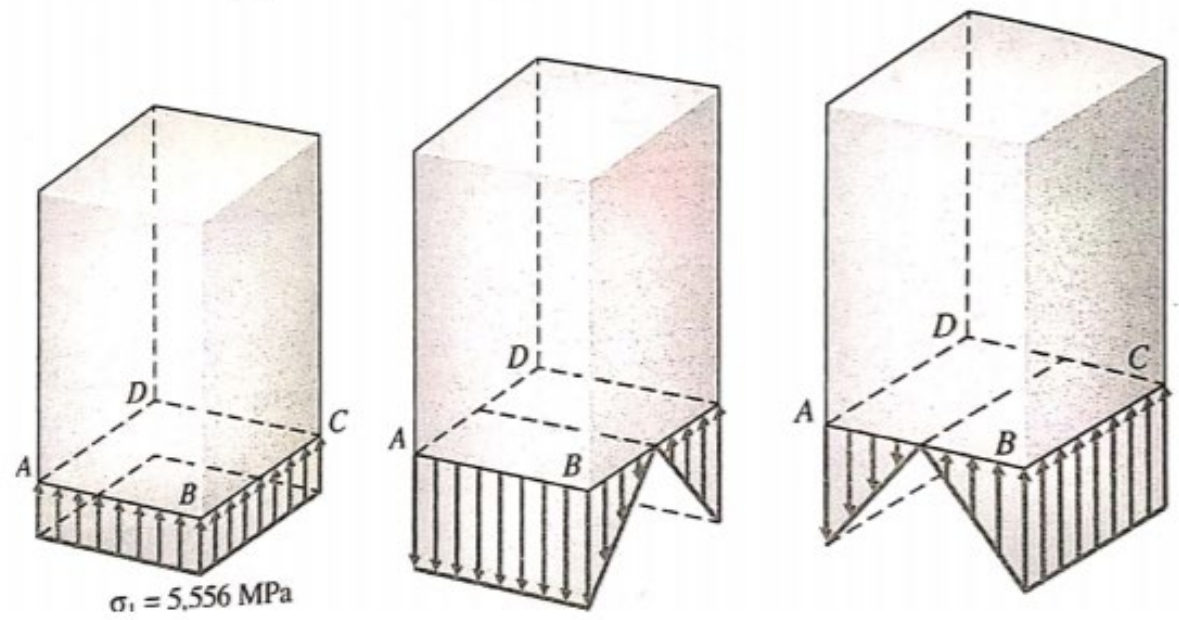
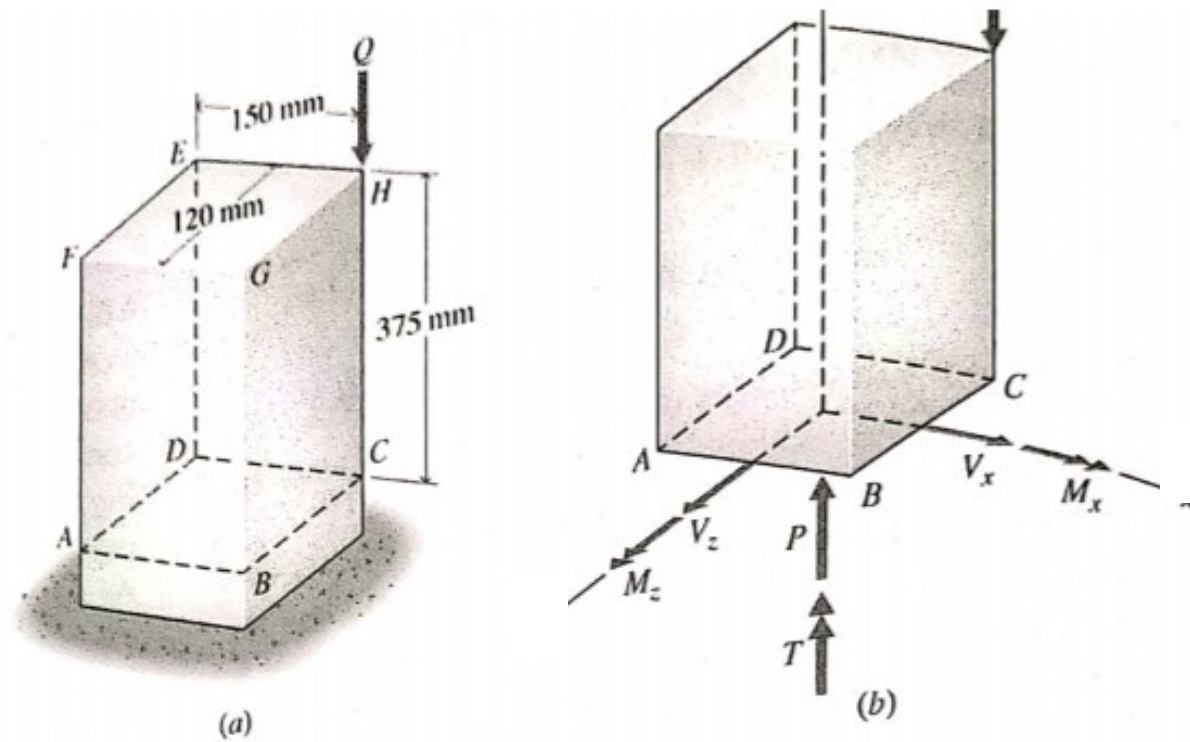
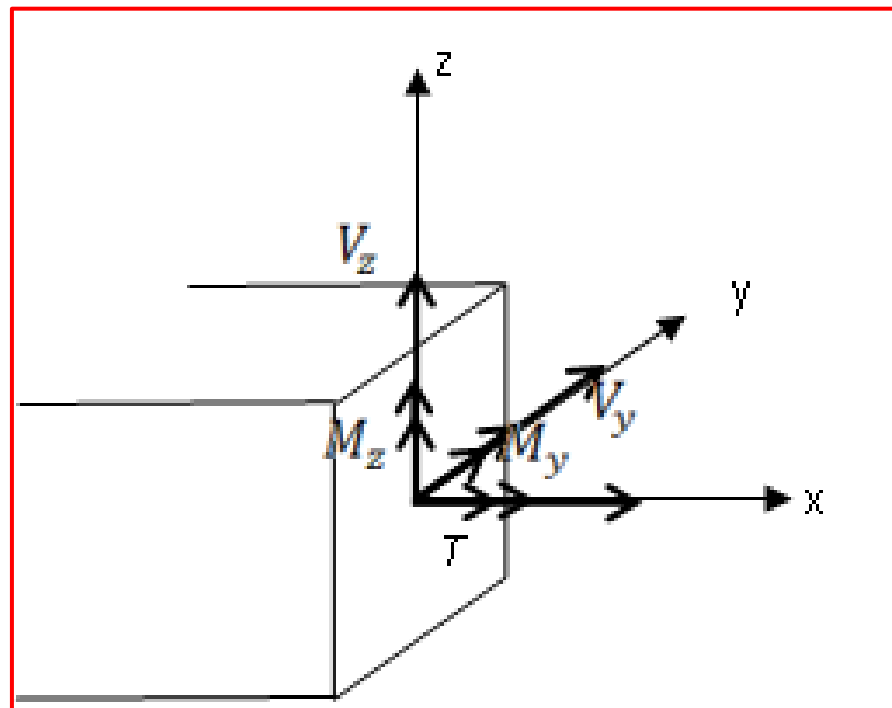
### Tensões normais na seção B:

$$\sigma_s = -\frac{M_B \cdot y_s}{I} = -\frac{-7200 \cdot 18.6}{203697} = +0.66 \text{ kN/cm}^2 = +6.6 \text{ MPa}$$

$$\sigma_i = \frac{M_B \cdot y_i}{I} = \frac{-7200 \cdot 31.4}{203697} = -1.11 \text{ kN/cm}^2 = -11.1 \text{ MPa}$$



$$\sigma = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$

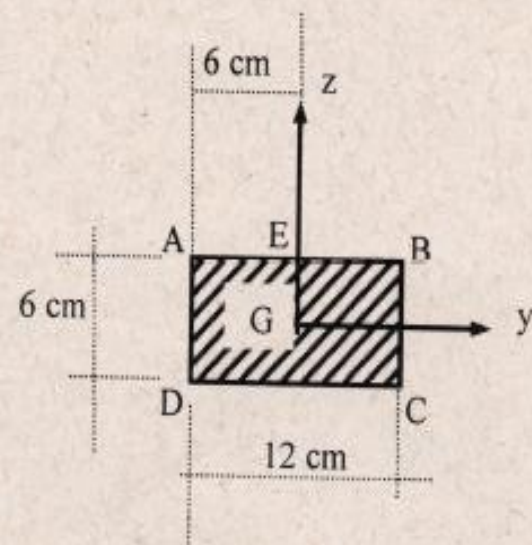
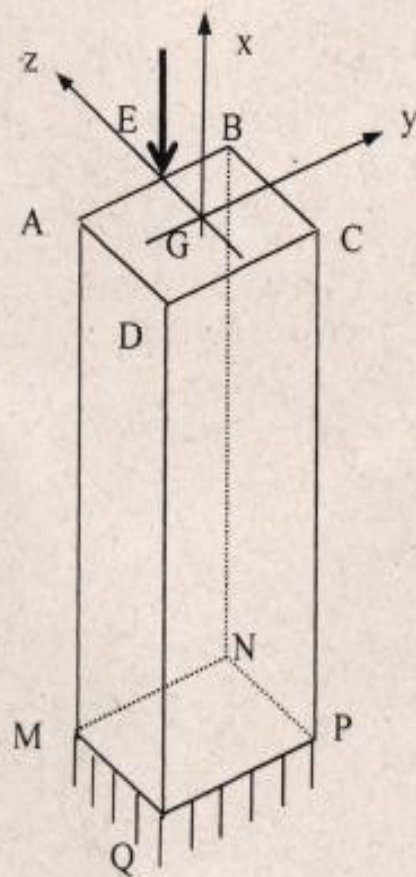


# P2 2019

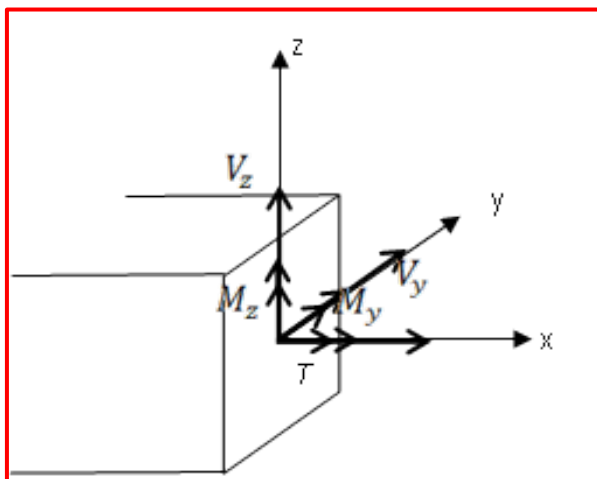
40

3ª Questão – O pilar de seção retangular de área  $S$ , de base  $MNPQ$ , está submetido a uma força vertical de 72 kN aplicada no ponto  $E$  (a 6 cm de  $A$ ) do topo  $ABCD$ , conforme indicado na figura. Sabendo-se que  $S = 72 \text{ cm}^2$ ,  $I_y = 216 \text{ cm}^4$  e  $I_z = 864 \text{ cm}^4$ , determine:

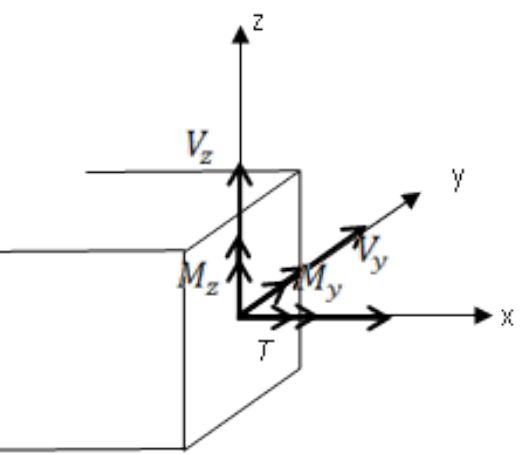
- Os esforços solicitantes na seção de topo, transferindo a força vertical para o centro de gravidade  $G$ ;
- A expressão das tensões normais  $\sigma$  na seção de topo;
- A equação da linha neutra e desenhe a sua posição na seção de topo;
- As tensões normais extremas de tração e de compressão na seção de topo.



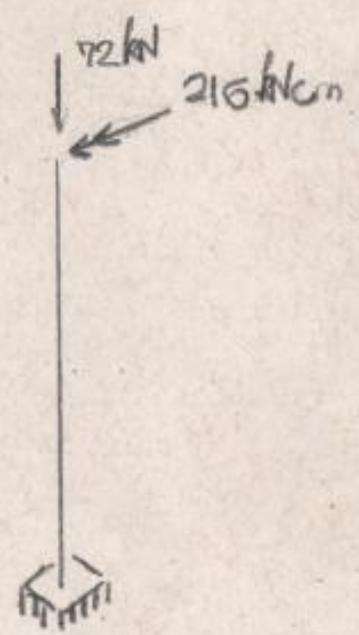
$$\sigma = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$



$$\sigma = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$



a) transferindo:



Assim:

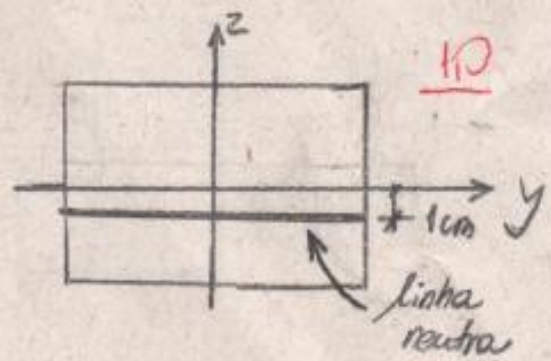
$$\begin{cases} N = -72 \text{ kN} \\ V_y = 0 \\ V_z = 0 \\ T = 0 \\ M_{xy} = -216 \text{ kNcm} \\ M_{yz} = 0 \end{cases} \quad \underline{1,0}$$

b)  $\sigma = -\frac{72}{72} - \frac{216}{216} z = -1 - z$  (em  $\text{kN/cm}^2$ )

ou  $\sigma = -10 - 10z$  (em MPa) ( $z$  em cm) 1,0

c) linha neutra  $\Rightarrow \underline{\underline{\sigma = 0}}$

$$-10 - 10z = 0 \Rightarrow \underline{\underline{z = -1 \text{ cm}}}$$



d) tensões extremas em  $z = -3 \text{ cm}$  (tração) e  $z = 3 \text{ cm}$  (compressão):

$$\sigma_T = -10 - 10(-3) \Rightarrow \underline{\underline{\sigma_T = 20 \text{ MPa}}} \quad \underline{0,5}$$

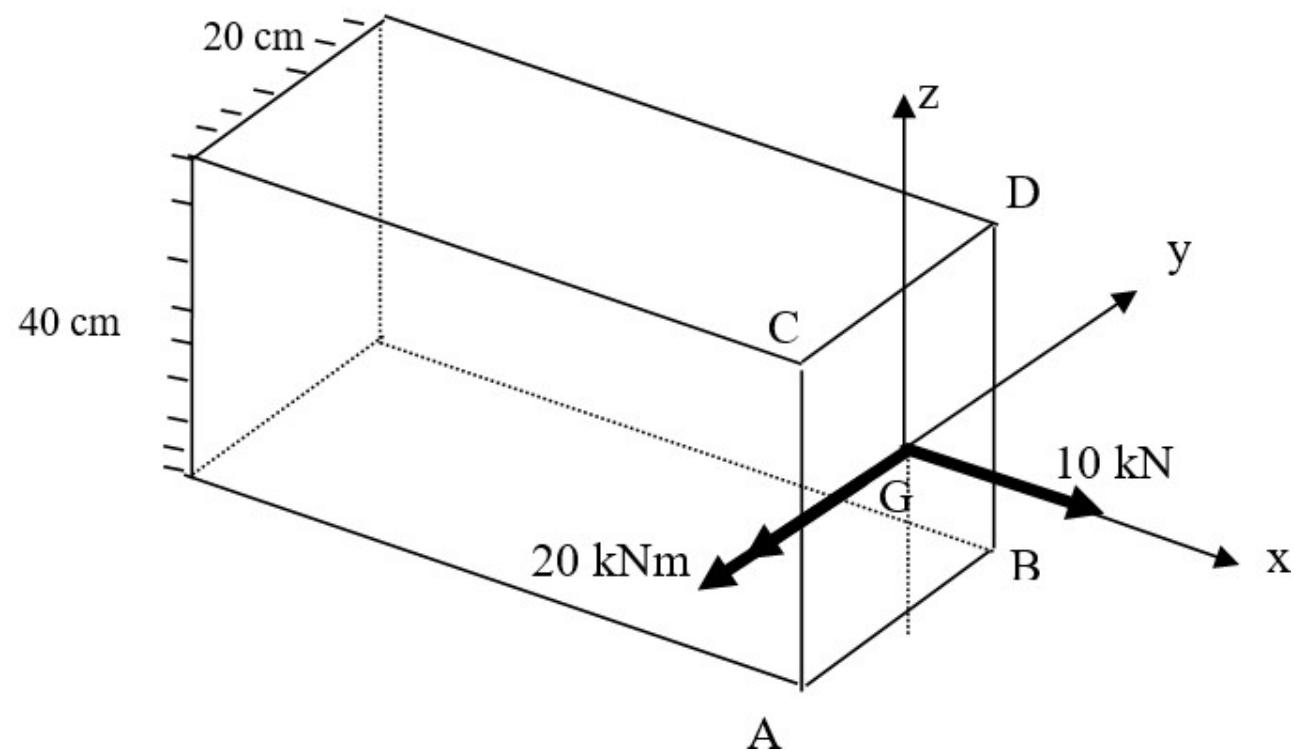
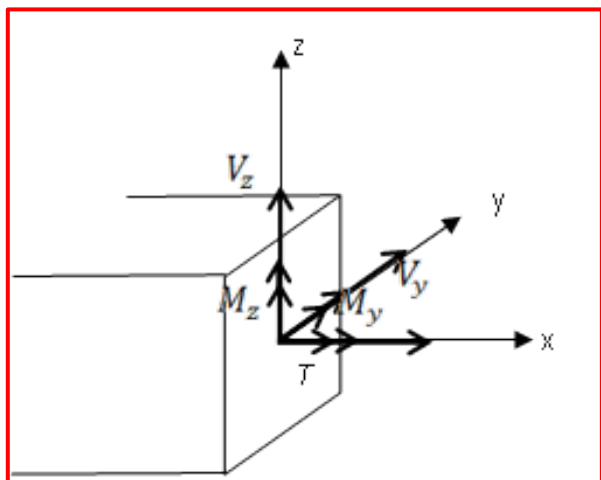
$$\sigma_C = -10 - 10 \cdot 3 \Rightarrow \underline{\underline{\sigma_C = -40 \text{ MPa}}} \quad \underline{0,5}$$

# SUB 2019

Questão D (4 pontos): Para a viga em balanço de seção retangular da figura, determine:

- os esforços solicitantes no engastamento,
- a equação das tensões normais,
- a equação da linha neutra,
- as tensões normais máximas.

$$\sigma = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$



QD

4,0

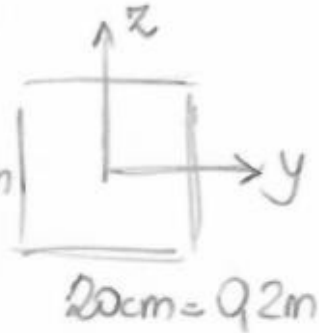
a)  $N = 10 \text{ kN}$      $T = 0$

$V_y = 0$      $M_{fy} = -20 \text{ kNm}$     1,0

$V_z = 0$      $M_{fz} = 0$

b) ST

$0,4 \text{ m} = 40 \text{ cm}$



$20 \text{ cm} = 0,2 \text{ m}$

$A = 0,08 \text{ m}^2$

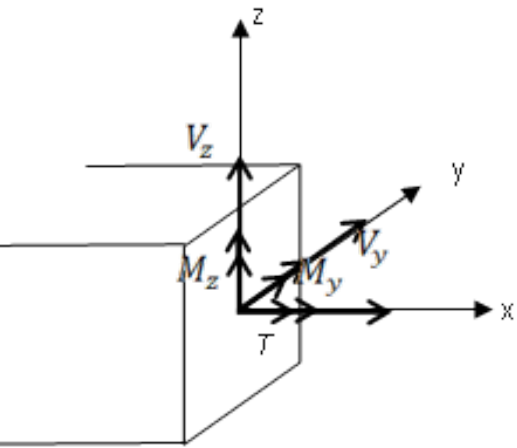
$I_y = \frac{2}{1875} \text{ m}^4$

$I_z = \frac{1}{3750} \text{ m}^4$

$$\sigma = \frac{10 \cdot 10^3}{0,08} + \frac{(-20 \cdot 10^3)}{2/1875} \cdot z \Rightarrow \sigma = 125000 - 1875000z \text{ (em Pa)}$$

ou  $\Rightarrow \sigma = 0,125 - 18,75y \text{ (em MPa, } y \text{ em m)}$     1,0

$$\sigma = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$



c) linha neutra:  $\sigma = 0$ :

$$0,125 - 18,75\bar{y} = 0 \Rightarrow \bar{y} = 1/150 \text{ m} \quad \text{ou} \quad \bar{y} = 0,6\bar{7} \text{ cm} \quad \underline{1,0}$$

d) tensões máximas:

$$\sigma_T = \sigma(z = -0,2 \text{ m}) = 3,875 \text{ MPa} \quad \underline{0,5}$$

$$\sigma_C = \sigma(z = 0,2 \text{ m}) = -3,625 \text{ MPa} \quad \underline{0,5}$$

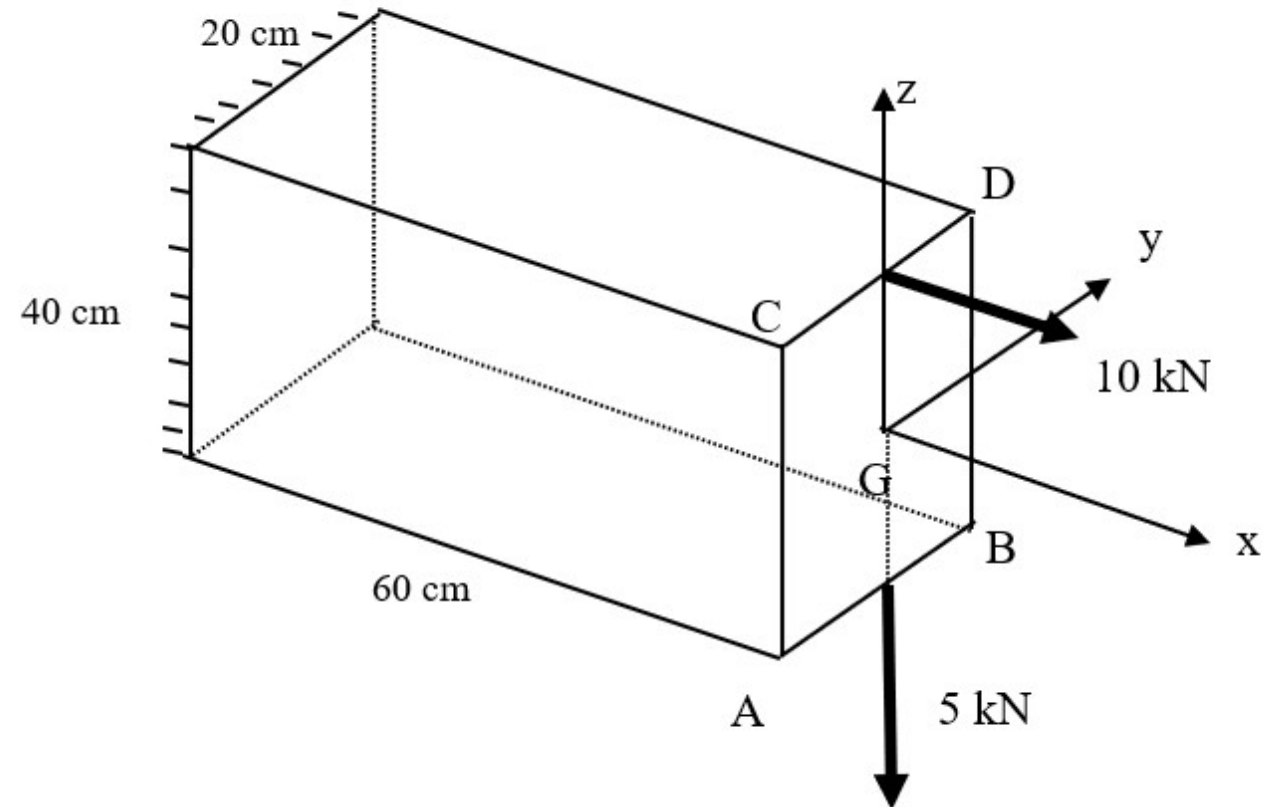
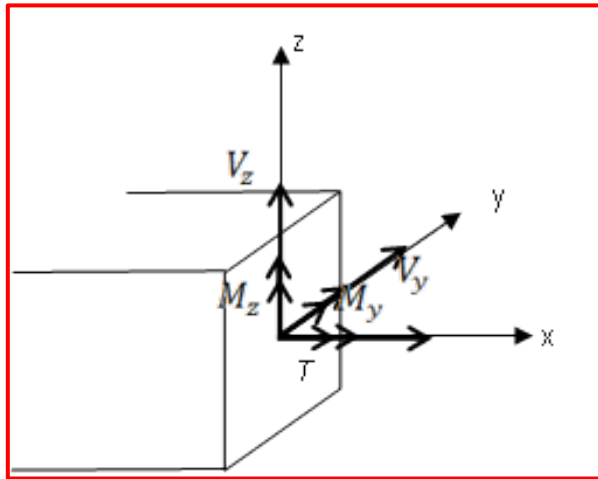


# REC 2019

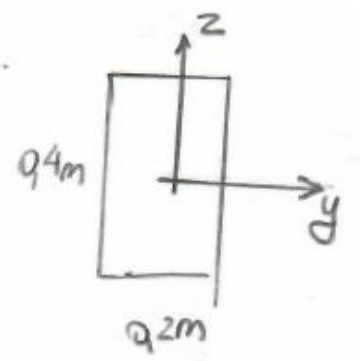
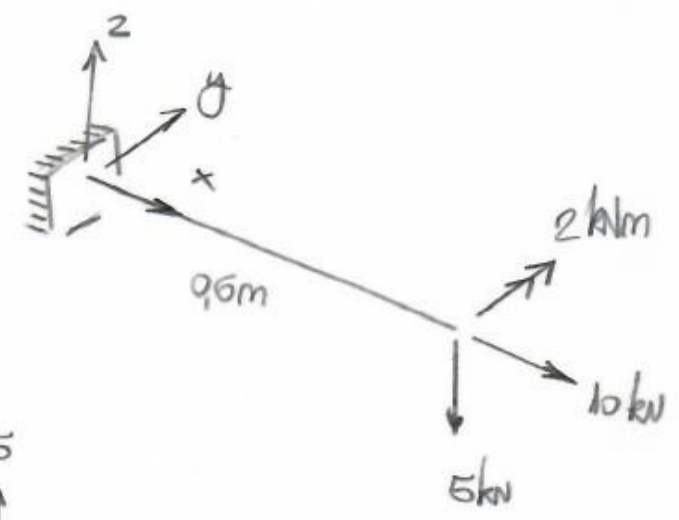
3ª. Questão (4 pontos): A viga em balanço de comprimento 60 cm de seção retangular da figura está submetida a uma força horizontal (direção do eixo x) de 10 kN e a uma força vertical (direção do eixo z) de 5 kN aplicadas na extremidade livre. Determine na seção do engastamento:

- os esforços solicitantes,
- a equação das tensões normais,
- a equação da linha neutra,
- as tensões normais máximas.

$$\sigma = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$



Q3

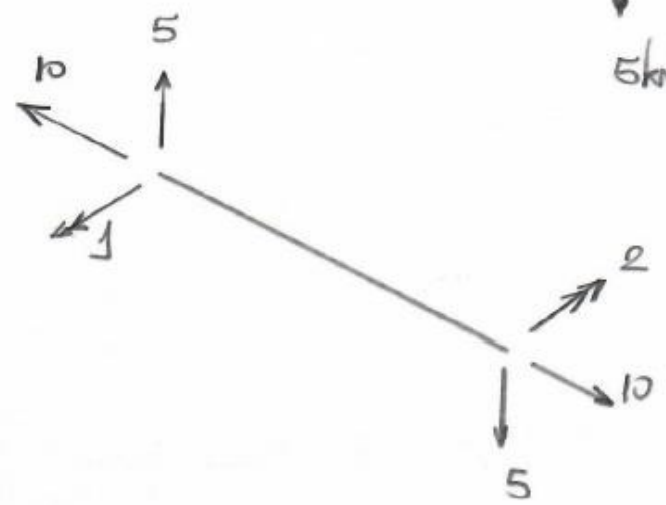
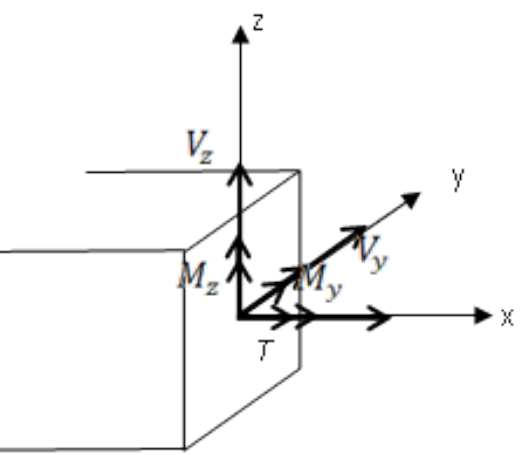


$$A = 908 \text{ m}^2$$

$$I_y = \frac{32}{3} \cdot 10^{-4} \text{ m}^4$$

$$I_z = \frac{8}{3} \cdot 10^{-4} \text{ m}^4$$

$$\sigma = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$



a) Esforços:  $N = 10 \text{ kN}$        $T = 0$   
 $V_y = 0$        $M_y = -1 \text{ kNm}$   
 $V_z = -5 \text{ kN}$        $M_z = 0.$

$$b) \sigma = \frac{10 \cdot 10^3}{908} + \frac{(-1 \cdot 10^3) z}{32/3 \cdot 10^{-4}} = 125000 - 937500 z \quad (\text{em Pa})$$

$$c) \sigma = 0 \Rightarrow 125000 - 937500z = 0 \Rightarrow z = \frac{2}{15} \text{ m } (0,133\overline{3} \text{ m})$$

d) Tensões máximas:

$$\sigma_T = \sigma(z = -0,2 \text{ m}) = 312500 \text{ Pa ou } 0,3125 \text{ MPa}$$

$$\sigma_C = \sigma(z = 0,2 \text{ m}) = -62500 \text{ Pa ou } -0,0625 \text{ MPa.}$$

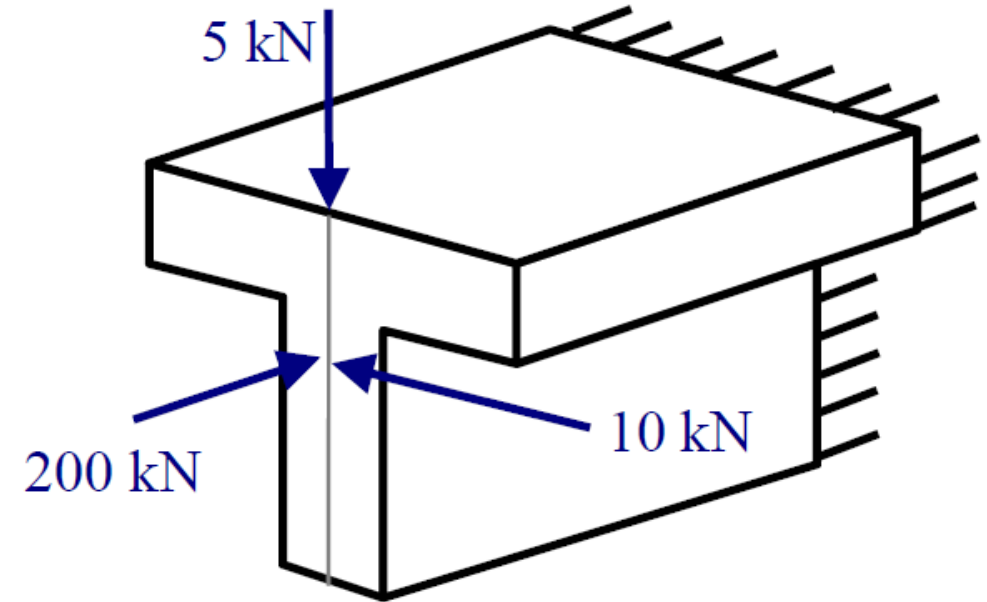
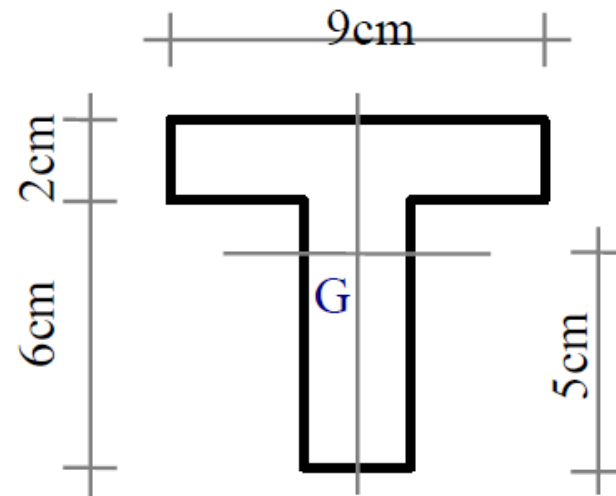
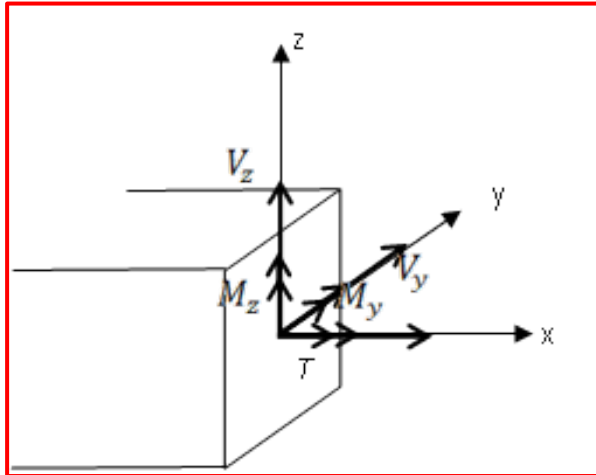
# Lista P2b

PEF 2308 – Fundamentos de Mecânica das Estruturas – Lista de Exercícios P2-B

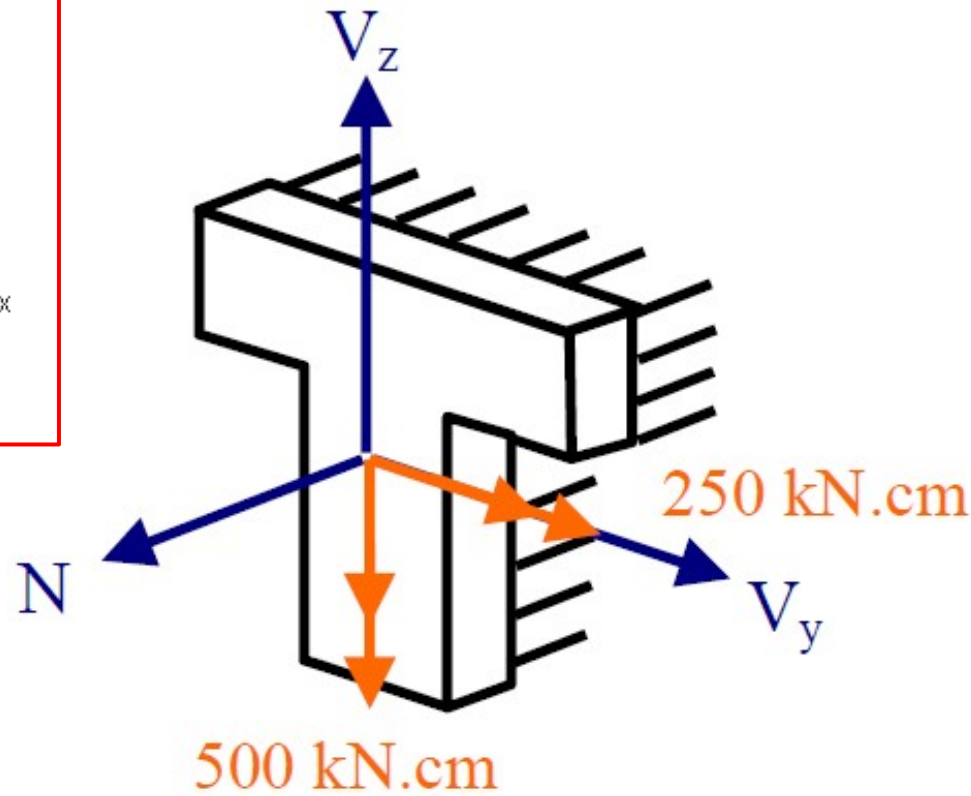
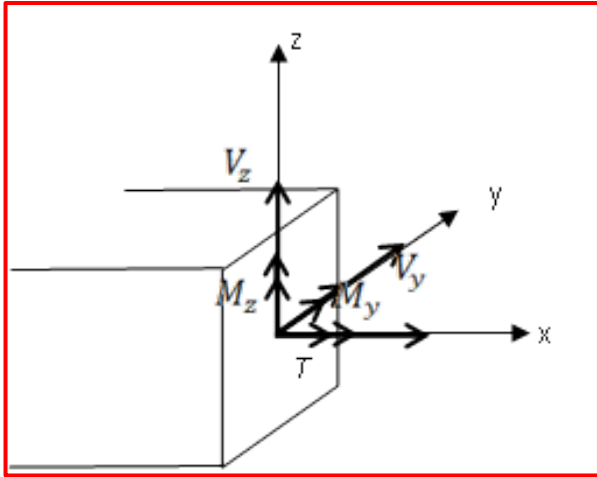
(E4) Para a viga da figura abaixo com  $A = 36 \text{ cm}^2$ ,  $I_y = 204 \text{ cm}^4$ ,  $I_z = 135 \text{ cm}^4$ , Determine:

- Os esforços solicitantes na seção mais solicitada;
- A expressão das tensões normais;
- A posição da linha neutra;
- As tensões normais extremas (máxima e mínima).

$$\sigma = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$



a) No engastamento temos:



$$N = -200 \text{ kN}$$

$$V_y = -10 \text{ kN}$$

$$V_z = -5 \text{ kN}$$

$$M_y = 250 \text{ kN.cm}$$

$$M_z = -500 \text{ kN.cm}$$

$$T = 0$$

b) A tensão normal é dada por:

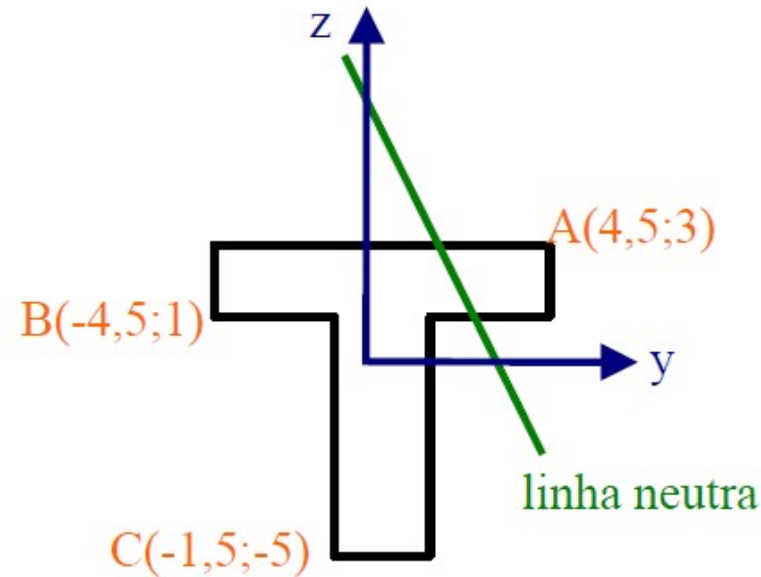
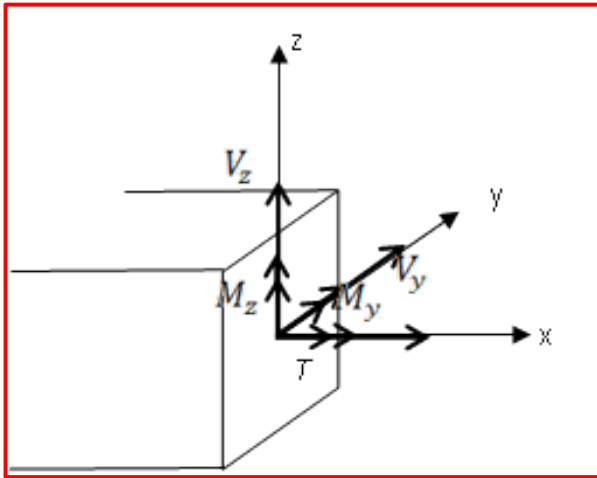
$$\sigma = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$

$$\sigma = -\frac{200}{36} - \frac{500}{135} y + \frac{250}{204} z \quad \rightarrow \quad \sigma = -5,56 + 3,70 y + 1,23 z$$

c) A linha neutra é dada por:

$$\sigma = 0 \quad \rightarrow \quad -5,56 + 3,70 y + 1,23 z = 0 \quad \rightarrow \quad z = -3,01 y + 4,52$$

$$\sigma = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$



d) As tensões normais extremas são nos pontos A e (B ou C):

(TRAÇÃO)

$$\sigma_A^{(y=4,5; z=3)} = -5,56 + 3,70 \cdot 4,5 + 1,23 \cdot 3 = +14,78 \text{ kN.cm}^{-2}$$

(COMPRESSÃO)

$$\sigma_B^{(y=-4,5; z=1)} = -5,56 + 3,70 \cdot (-4,5) + 1,23 \cdot 1 = -20,98 \text{ kN.cm}^{-2}$$

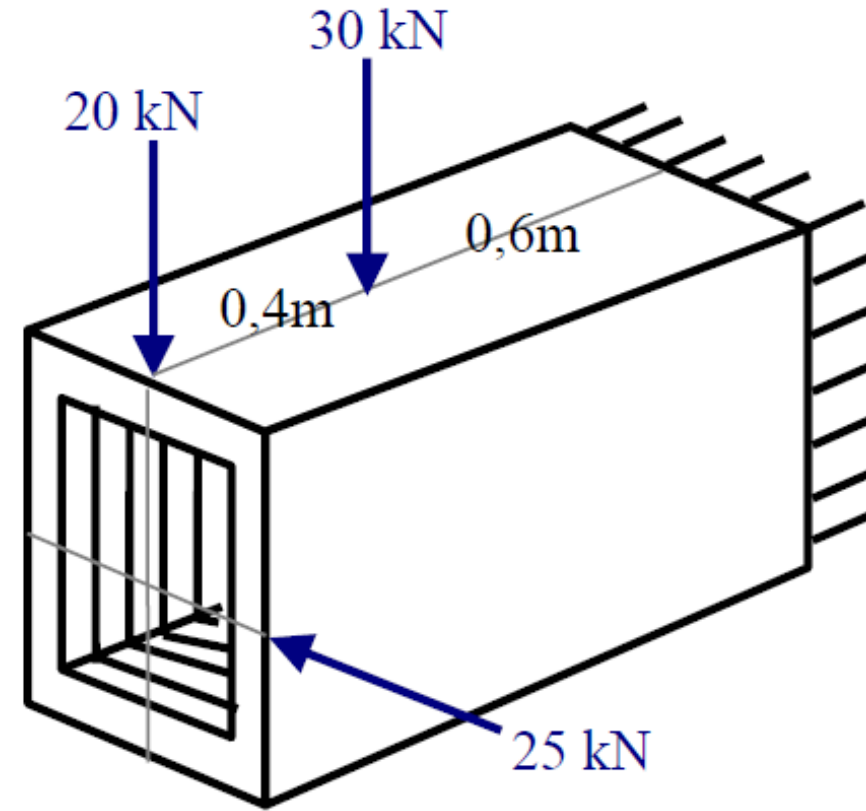
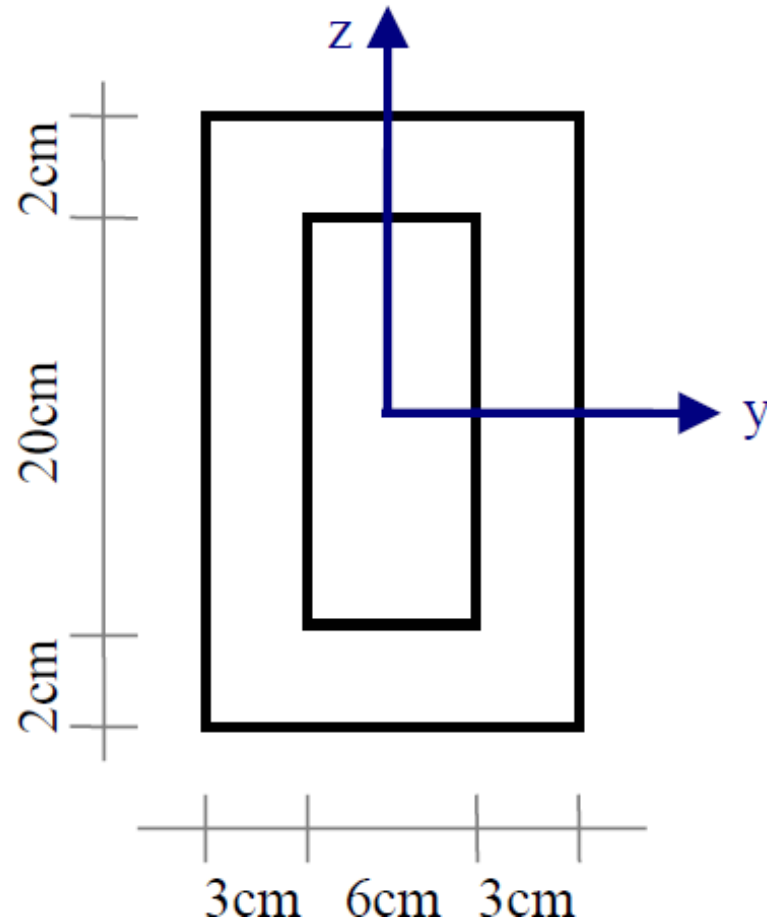
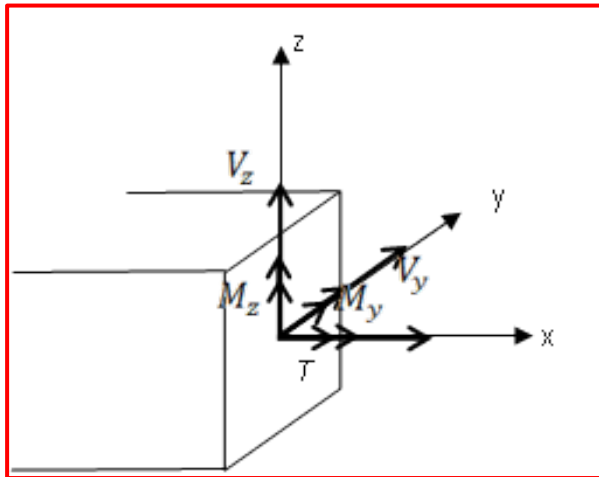
$$\sigma_C^{(y=-1,5; z=-5)} = -5,56 + 3,70 \cdot (-1,5) + 1,23 \cdot (-5) = -17,26 \text{ kN.cm}^{-2}$$

# Lista P2b

(E5) Para a viga da figura abaixo, determine:

- Os esforços solicitantes extremos;
- Na seção mais solicitada, a posição da linha neutra;
- As tensões normais extremas (máxima e mínima).

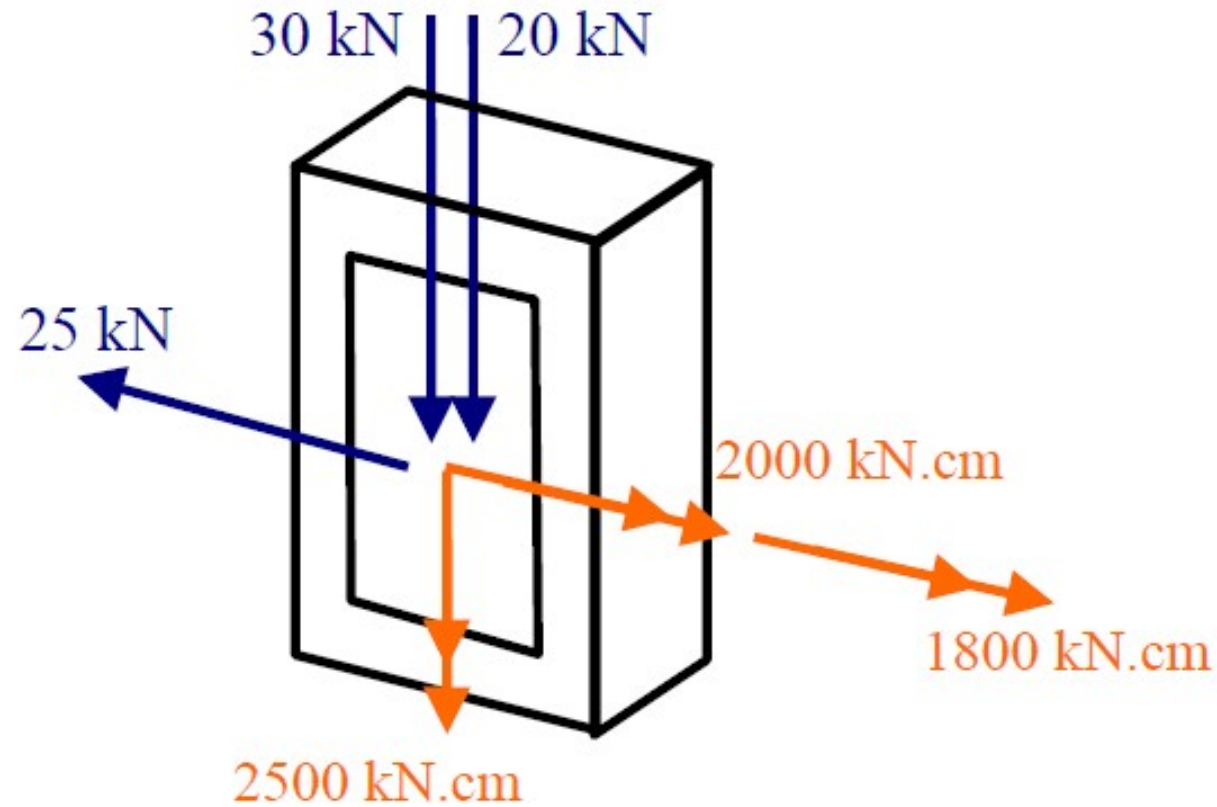
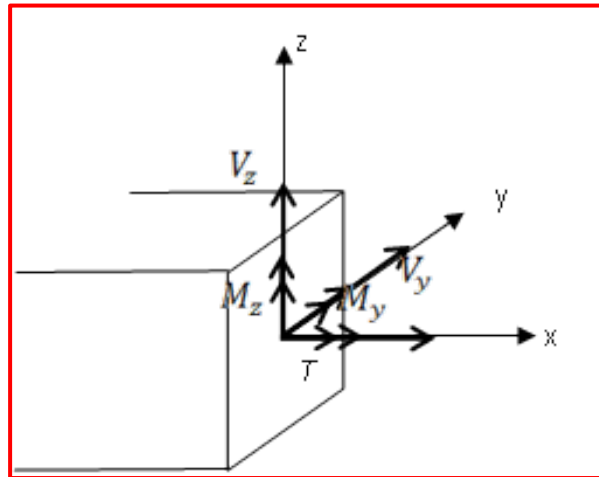
$$\sigma = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$





a) No engastamento temos:

$$\sigma = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$



$$N = 0 \text{ kN}$$

$$|V_y| = 25 \text{ kN}$$

$$|V_z| = 50 \text{ kN}$$

$$M_y = 3800 \text{ kN.cm}$$

$$M_z = -2500 \text{ kN.cm}$$

$$T = 0$$

b) Características da seção:

$$I_y = \left( \frac{b \cdot h^3}{12} \right)_{\text{total-buraco}} = \frac{12 \cdot 24^3}{12} - \frac{6 \cdot 20^3}{12} = 9\,824 \text{ cm}^4$$

$$I_z = \left( \frac{h \cdot b^3}{12} \right)_{\text{total-buraco}} = \frac{24 \cdot 12^3}{12} - \frac{20 \cdot 6^3}{12} = 3\,096 \text{ cm}^4$$

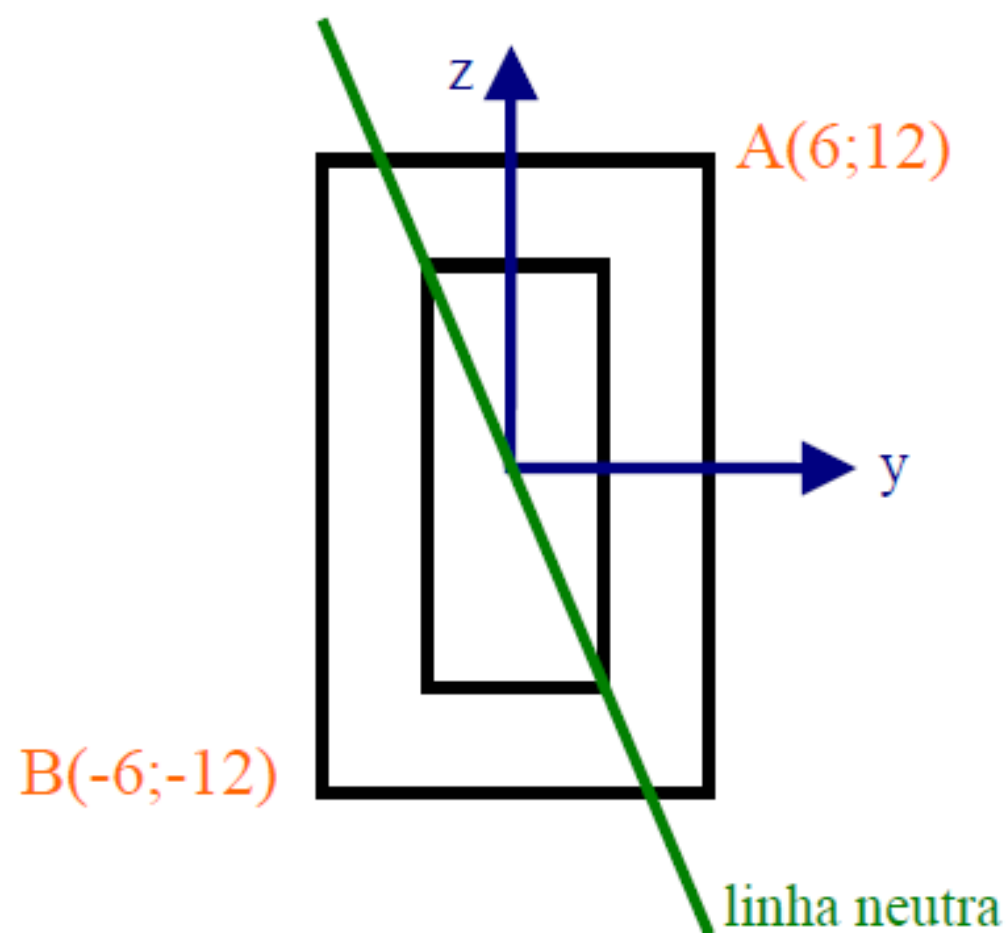
A tensão normal é dada por:

$$\sigma = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$

$$\sigma = 0 - \frac{2500}{3096} y + \frac{3800}{9824} z \quad \rightarrow \quad \sigma = 0,81 y + 0,39 z$$

A linha neutra é dada por:

$$\sigma = 0 \quad \rightarrow \quad 0,81y + 0,39z = 0 \quad \rightarrow \quad z = -2,08y$$



c) As tensões normais extremas são nos pontos A e B:

(TRAÇÃO)

$$\sigma_A^{(y=6; z=12)} = 0,81 \cdot 6 + 0,39 \cdot 12 = +9,54 \text{ kN.cm}^{-2}$$

(COMPRESSÃO)

$$\sigma_B^{(y=-6; z=-12)} = 0,81 \cdot (-6) + 0,39 \cdot (-12) = -9,54 \text{ kN.cm}^{-2}$$