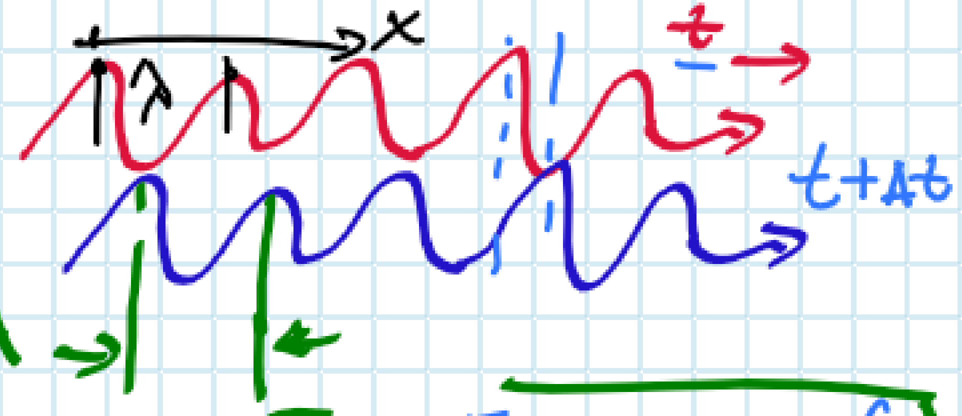
Aula: Óptica de feixes gaussianos



$$\boxed{k = \frac{2\pi}{\lambda}\ ;\ \omega = 2\pi \cdot f}$$

- Eq. de Onda (clássica)

$u = u(x,t)$
$\vec{u} = u(\vec{r},t)$

$$\left(\frac{\partial^2 u}{\partial t^2} = c^2 \cdot \frac{\partial^2 u}{\partial x^2}\right) \rightsquigarrow \boxed{\partial_t^2 u = c^2 \cdot \partial_x^2 u} \quad (1D)$$

c: velocidade

$u = u(x - ct)$ ⇒ ondas ...

→ $\mathrm{sen}(kx - \omega t)$

→ $e^{i(kx - \omega t)}$

$[e^{i\theta} = \cos\theta - i\,\mathrm{sen}\,\theta]$

(3D) $\ddot{u} = c^2 \nabla^2 u$

Laplaciano: $\vec{\nabla}\cdot\vec{\nabla}(\cdot)$

$(\partial_x^2 + \partial_y^2 + \partial_z^2)\,\square\, u$

- Eq. Maxwell

sem cargas livres

(Gauss): $\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0 \rightarrow \rho/\epsilon$ Meios materiais

$\vec{\nabla}\cdot\vec{B} = 0$

(Faraday): $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(Ampere): $\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$

$\mu_0 \to \mu$, $\epsilon_0 \to \epsilon$; $\mu = \mu_0$ materiais não magnéticos

$\vec{\nabla}\times(\vec{\nabla}\times\vec{E})$

$$\nabla^2 E - \epsilon\mu \frac{\partial^2 E}{\partial t^2} - g\mu \frac{\partial E}{\partial t} = 0$$

⇓ $g = 0$ (dielétricos)

(1) $\nabla^2 E - \epsilon\mu \frac{\partial^2 E}{\partial t^2} = 0$

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon\mu}\nabla^2 E$$

$v^2 = \frac{1}{\epsilon\mu} \Rightarrow v = \frac{1}{\sqrt{k_e \cdot k_\mu}}\sqrt{\frac{1}{\epsilon_0\mu_0}}$

índice de refração

$v = \frac{c}{n}$

Vácuo

(2) $$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon_0\mu_0}\nabla^2 E$$

$$\boxed{c^2 = \frac{1}{\epsilon_0\mu_0}}$$

Velocidade no vácuo → $c = \sqrt{\frac{1}{\epsilon_0\mu_0}}$

Procurar Soluções: $\vec{E}(\vec{r},t) = \vec{E}(\vec{r}) \cdot e^{i\omega t}$

$\nabla^2\vec{E} - \epsilon\mu\,\partial_t^2\vec{E} = 0$

Cartesianas

$(\partial_x^2 + \partial_y^2 + \partial_z^2)\vec{E} - \epsilon\mu \cdot \partial_t^2 E = 0$

$e^{i\omega t}\nabla^2 E(r) - E(r)\,\epsilon\mu\,\partial_t^2(e^{i\omega t}) =$

$(i\omega)^2 \cdot e^{i\omega t}$

$e^{i\omega t}\left(\nabla^2 E(r) + \epsilon\mu\omega^2 E(r)\right) = 0$

$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{n}$

qualquer onda satisfaz Eq. de onda.

$\lambda \cdot f = v$

$$\boxed{\nabla^2 E + k^2 E = 0}$$ Eq. de Helmholtz

$\vec{E}(\vec{r})$

$(k = 2\pi/\lambda) \rightarrow k = \omega/v = \frac{\omega \cdot n}{c}$; $\omega = 2\pi \cdot f$

$k^2 = \omega^2\mu\epsilon \longrightarrow k^2(\vec{r},\omega) = \omega^2\mu\,\epsilon(\vec{r},\omega)\left[1 - i\,\sigma(\vec{r},\omega)\right]$

Absorção/ganho

No caso + geral: $\epsilon = \epsilon(\vec{r},\omega)$

$(r,\theta,z) \to (r,z)$

↳ Coordenadas cilíndricas c/ simetria axial

1º Aprox.

↳ $\nabla^2 f \cong \underbrace{\left(\partial_r^2 f + \frac{1}{r}\partial_r f + \partial_z^2 f\right)}$

↳ Soluções propagante na direção do eixo-óptico

$$\boxed{E(r,z) = \underbrace{\psi(r,z)}_{\text{Amplitude}} \cdot e^{ikz}}$$

2º Aprox.

↳ $\nabla_T^2\psi - 2ik\psi' + k^2\psi - \psi'' = 0 \Rightarrow \boxed{\nabla_T^2\psi - 2ik\psi' + k^2\psi = 0}$

$\psi'' \ll k^2\psi,\ k\cdot\psi'$ ← Amplitude Varia lentamente

$\partial_z^2\psi$

3ª Aprox.

↳ Aproximação Paraxial

↳ $\nabla_T^2\psi - 2ik\psi = 0$ ← Angulos pequenos (próximo do eixo óptico) $\theta \ll 1\,\text{rad}$

⇓ soluções do tipo: $\psi(r,z) = \psi_0 \cdot \exp\left[-i\left(P(z) + \frac{Q(z)}{2}\cdot r^2\right)\right]$

$\rightsquigarrow \psi(r,z) = \psi_0 \frac{q_0}{q_0+z} \cdot \exp\left[\frac{-ik}{2(z+q_0)} r^2\right] \Rightarrow q_0 = i z_0$, $z_0 = \frac{w_0^2 \cdot k}{2}$

$$E(r,z) = E_0(r,z)\cdot \exp\left\{-i\left[kz - \eta(z) + \frac{kr^2}{2R(z)}\right]\right\}$$

- $E_0(r,z) = E_0 \cdot \frac{W_0}{W(z)} \cdot \exp\left(\frac{-r^2}{W^2(z)}\right)$ ⇐ Gaussiana !!
- $W(z) = W_0^2\left(1 + \left(\frac{z}{z_0}\right)^2\right)$
- $R(z) = z\left(1 + \left(\frac{z_0}{z}\right)^2\right)$
- $\eta(z) = \tan^{-1}\left(\frac{z}{z_0}\right) = \arctan\left(\frac{\lambda z}{\pi n W_0^2}\right)$

$$z_0 = \frac{w_0^2 \cdot k}{2} = \frac{n\pi W_0^2}{\lambda}$$

comprimento de Rayleigh

Intensidade

$\rightsquigarrow I(r,z) = E^*(r,z)\cdot E(r,z) = |E(r,z)|^2$

↳ $\boxed{I(r,0) = I_0 \cdot e^{-2\left(\frac{r}{w_0}\right)^2}}$ ← gaussiana!