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An approach to multiple attribute decision making problems based on hesitant fuzzy set

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Abstract. In this paper, we investigate the multiple attribute decision making with hesitant fuzzy information. Motivated by the ideal of dependent aggregation, in this paper, we develop some dependent hesitant fuzzy aggregation operators: the dependent hesitant fuzzy ordered weighted averaging (DHFOWA) operator and the dependent hesitant fuzzy ordered weighted geometric (DHFOWG) operator, in which the associated weights only depend on the aggregated hesitant fuzzy arguments and can relieve the influence of unfair hesitant fuzzy arguments on the aggregated results by assigning low weights to those "false" and "biased" ones and then apply them to develop some approaches for multiple attribute group decision making with hesitant fuzzy information. Finally, an illustrative example for supplier selection is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Keywords: Multiple attribute decision making, hesitant fuzzy information, dependent hesitant fuzzy ordered weighted averaging (DHFOWA) operator, dependent hesitant fuzzy ordered weighted geometric (DHFOWG) operator, supplier selection

1. Introduction

Atanassov [1, 2] proposed the definition of intuitionistic fuzzy set (IFS) based on the concept of fuzzy set [3]. The intuitionistic fuzzy set has received more and more attention since its appearance [4–18]. Furthermore, Torra [19] proposed the hesitant fuzzy set which permits the membership having a set of possible values and discussed the relationship between hesitant fuzzy set and intuitionistic fuzzy set, and showed that the envelope of hesitant fuzzy set is an intuitionistic fuzzy set. Xia and Xu [20] proposed some hesitant fuzzy information aggregation operators and their application to multiple attribute decision making. Xu and Xia [21, 22] developed the distance and correlation measures with hesitant fuzzy information. Wang et al. [23] proposed the generalized hesitant fuzzy hybrid weighted distance (GHFHWD) measure based on the generalized hesitant fuzzy weighted distance (GHFWD) measure and the generalized hesitant fuzzy ordered weighted distance (GHFOWD) measure [21, 22]. Xu et al. [23] developed some aggregation operators for hesitant fuzzy information. For more studies with hesitant fuzzy information, please refer to references [24–30].

In this paper, we investigate the multiple attribute decision making with hesitant fuzzy information. Motivated by the ideal of dependent aggregation [31, 32], in this paper, we develop some dependent hesitant fuzzy aggregation operators: the dependent hesitant fuzzy ordered weighted averaging (DHFOWA) operator and the dependent hesitant fuzzy ordered weighted geometric (DHFOWG) operator, in which the associated weights only depend on the aggregated hesitant fuzzy arguments and can relieve the influence of unfair hesitant fuzzy arguments on the aggregated results by assigning low weights to those "false" and "biased"

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ones and then apply them to develop some approaches for multiple attribute group decision making with hesitant fuzzy information. The remainder of this paper is set out as follows. In the next section, we introduce some basic concepts related to hesitant fuzzy sets. In Section 3 we propose some dependent aggregation operators with hesitant fuzzy information. In Section 4 we introduce the MADM problem with hesitant fuzzy information based on the dependent hesitant fuzzy ordered weighted averaging (DHFOWA) operator and the dependent hesitant fuzzy ordered weighted geometric (DHFOWG) operator. In Section 5, an illustrative example for supplier selection is pointed out. In Section 6 we conclude the paper and give some remarks.

2. Preliminaries

In the following, we introduce some basic concepts related to hesitant fuzzy sets.

Definition 1. [19] Given a fixed set X, then a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a subset of [0, 1]. To be easily understood, Xia and Xu [21] express the HFS by mathematical symbol:

$$E = \left(\langle x,\ h_E(x)\rangle \,|\, x \in X\right), \tag{1}$$

where $h_E(x)$ is a set of some values in [0, 1], denoting the possible membership degree of the element $x \in X$ to the set E. For convenience, Xia and Xu [20] call $h = h_E(x)$ a hesitant fuzzy element (HFE) and H the set of all HFEs.

Definition 2. [20] For a HFE h, $s(h) = \frac{1}{\#h}\sum_{\gamma\in h}\gamma$ is called the score function of h, where $\#h$ is the number of the elements in h. For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then $h_1 > h_2$; if $s(h_1) = s(h_2)$, then $h_1 = h_2$.

Based on the relationship between the HFEs and IFVs, Xia and Xu [20] define some new operations on the HFEs h, h_1 and h_2:

(1) $h^\lambda = \cup_{\gamma\in h}\left\{\gamma^\lambda\right\}$;

(2) $\lambda h = \cup_{\gamma\in h}\left\{1-(1-\gamma)^\lambda\right\}$;

(3) $h_1 \oplus h_2 = \cup_{\gamma_1\in h_1,\gamma_2\in h_2}\{\gamma_1+\gamma_2-\gamma_1\gamma_2\}$;

(4) $h_1 \otimes h_2 = \cup_{\gamma_1\in h_1,\gamma_2\in h_2}\{\gamma_1\gamma_2\}$.

Based on the Definition 1 and the defined operations for HFEs, Xia and Xu [21] proposed a series of aggregation operators for HFEs as listed below:

Let $h_j\,(j = 1, 2, \cdots, n)$ be a collection of HFEs, then

(1) The hesitant fuzzy weighted averaging (HFWA) operator

$$\begin{aligned}\text{HFWA}(h_1, h_2, \cdots, h_n) &= \mathop{\oplus}_{j=1}^{n}\left(\omega_j h_j\right)\\ &= \cup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_n\in h_n}\left\{1-\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\omega_j}\right\}\end{aligned} \tag{2}$$

where $\omega = (\omega_1,\ \omega_2, \cdots,\ \omega_n)^T$ be the weight vector of $h_j\,(j = 1,\ 2, \cdots,\ n)$, and $\omega_j > 0$, $\sum_{j=1}^{n}\omega_j = 1$.

(2) The hesitant fuzzy weighted geometric (HFWG) operator

$$\begin{aligned}\text{HFWG}(h_1, h_2, \cdots, h_n) &= \mathop{\otimes}_{j=1}^{n}\left(h_j\right)^{\omega_j}\\ &= \cup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_n\in h_n}\left\{\prod_{j=1}^{n}\left(\gamma_j\right)^{\omega_j}\right\}.\end{aligned} \tag{3}$$

where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ be the weight vector of $h_j\,(j = 1, 2, \cdots, n)$, and $\omega_j > 0$, $\sum_{j=1}^{n}\omega_j = 1$.

(3) The hesitant fuzzy ordered weighted averaging (HFOWA) operator

$$\begin{aligned}\text{HFOWA}(h_1, h_2, \cdots, h_n) &= \mathop{\oplus}_{j=1}^{n}\left(w_j h_{\sigma(j)}\right)\\ &= \cup_{\gamma_{\sigma(1)}\in h_{\sigma(1)},\gamma_{\sigma(2)}\in h_{\sigma(2)},\cdots,\gamma_{\sigma(n)}\in h_{\sigma(n)}}\\ &\quad\left\{1-\prod_{j=1}^{n}\left(1-\gamma_{\sigma(j)}\right)^{w_j}\right\}\end{aligned} \tag{4}$$

where $(\sigma(1),\ \sigma(2), \cdots,\ \sigma(n))$ is a permutation of $(1,\ 2, \cdots,\ n)$, such that $h_{\sigma(j-1)} \geq h_{\sigma(j)}$ for all $j = 2, \cdots, n$, and $w = (w_1,\ w_2, \cdots,\ w_n)^T$ is the aggregation-associated weight vector such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

(4) The hesitant fuzzy ordered weighted geometric (HFOWG) operator

$$\begin{aligned}\text{HFOWG}(h_1, h_2, \cdots, h_n) &= \mathop{\otimes}_{j=1}^{n}\left(h_{\sigma(j)}\right)^{w_j}\\ &= \cup_{\gamma_{\sigma(1)}\in h_{\sigma(1)},\gamma_{\sigma(2)}\in h_{\sigma(2)},\cdots,\gamma_{\sigma(n)}\in h_{\sigma(n)}}\left\{\prod_{j=1}^{n}\left(\gamma_{\sigma(j)}\right)^{w_j}\right\}\end{aligned} \tag{5}$$

where $(\sigma(1),\ \sigma(2), \cdots,\ \sigma(n))$ is a permutation of $(1, 2, \cdots, n)$, such that $h_{\sigma(j-1)} \geq h_{\sigma(j)}$ for all $j = 2, \cdots, n$, and $w = (w_1, w_2, \cdots, w_n)^T$ is the aggregation-associated weight vector such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

3. Some dependent aggregation operators with hesitant fuzzy information

Definition 3. Let $h_j\,(j = 1, 2, \cdots, n)$ be a collection of HFEs, the average value of the score function of $h_j\,(j = 1, 2, \cdots, n)$ is computed as

$$s(\bar{h}) = \frac{\sum_{j=1}^{n} s\left(h_j\right)}{n} \tag{6}$$

Definition 4. Let h_1 and h_2 be two HFSs on $X = \{x_1,\ x_2, \cdots,\ x_n\}$, then the distance measure between h_1 and h_2 is defined as

$$d\left(h_1,\ h_2\right) = \frac{1}{l_{l_{h_2}} l_{h_1}} \sum_{\lambda=1}^{l_{h_1}} \sum_{\mu=1}^{l_{h_2}} \left| h_1^{\sigma(\lambda)} - h_2^{\sigma(\mu)} \right| \tag{7}$$

where $h_1^{\sigma(k)}$ and $h_2^{\sigma(k)}$ are kth largest values in h_1 and h_2, respectively, which will be used thereafter.

Definition 5. Let $h_j\,(j = 1, 2, \cdots, n)$ be a collection of HFEs, and $s(\bar{h})$ the arithmetic mean of these hesitant fuzzy values, then we define the standard deviation of these hesitant fuzzy values as

$$\sigma = \sqrt{\frac{1}{n} \sum_{j=1}^{n} d\left(s(h_j), s\left(\bar{h}\right)\right)^2} \tag{8}$$

Definition 6. Let $h_j\,(j = 1, 2, \cdots, n)$ be a collection of HFEs, then we call

$$sim\left(s\left(h_{\sigma(j)}\right), s\left(\bar{h}\right)\right) = 1 - \frac{d\left(s\left(h_{\sigma(j)}\right),\ s\left(\bar{h}\right)\right)}{\sum_{j=1}^{n} d\left(s\left(h_{\sigma(j)}\right),\ s\left(\bar{h}\right)\right)}, \ j = 1,\ 2, \cdots,\ n. \tag{9}$$

the degree of similarity between the *jth* largest hesitant fuzzy values $s\left(h_{\sigma(j)}\right)$ and the mean $s\left(\bar{h}\right)$, where $(\sigma(1),\ \sigma(2), \cdots,\ \sigma(n))$ is a permutation of $(1,\ 2, \cdots,\ n)$, such that $s\left(h_{\sigma(j-1)}\right) \geq s\left(h_{\sigma(j)}\right)$ for all $j = 2, \cdots, n$.

In real-life situations, the hesitant fuzzy values $h_j\,(j = 1,\ 2, \cdots,\ n)$ usually take the form of a collection of n preference values provided by n different individuals. Some individuals may assign unduly high or unduly low preference values to their preferred or repugnant objects. In such a case, we shall assign very low weights to these "false" or "biased" opinions, that is to say, the closer a preference value (argument) is to the mid one(s), the more the weight. As a result, based on (9), we define the HFOWA weights as

$$w_j = \frac{sim\left(s\left(h_{\sigma(j)}\right),\ s\left(\bar{h}\right)\right)}{\sum_{j=1}^{n} sim\left(s\left(h_{\sigma(j)}\right),\ s\left(\bar{h}\right)\right)}, \quad j = 1,\ 2, \cdots,\ n \tag{10}$$

Obviously, $w_j \geq 0$, $j = 1,\ 2, \cdots,\ n$ and $\sum_{j=1}^{n} w_j = 1$.

Especially, if $s\left(h_k\right) = s\left(h_l\right)$, for all $i, j = 1,\ 2, \cdots,\ n$, then by (11), we have $w_j = \frac{1}{n}$, for all $j = 1,\ 2, \cdots,\ n$.

By (4), we have

$$\text{HFOWA}\left(h_1, h_2, \cdots, h_n\right) = \mathop{\oplus}_{j=1}^{n} \left(w_j h_{\sigma(j)}\right) = \cup_{\gamma_{\sigma(1)} \in h_{\sigma(1)}, \gamma_{\sigma(2)} \in h_{\sigma(2)}, \cdots, \gamma_{\sigma(n)} \in h_{\sigma(n)}} \left\{ 1 - \prod_{j=1}^{n} \left(1 - \gamma_{\sigma(j)}\right)^{\frac{sim\left(s\left(h_{\sigma(j)}\right), s\left(\bar{h}\right)\right)}{\sum_{j=1}^{n} sim\left(s\left(h_{\sigma(j)}\right), s\left(\bar{h}\right)\right)}} \right\} \tag{11}$$

Since

$$\sum_{j=1}^{n} sim\left(s\left(h_{\sigma(j)}\right),\ s\left(\bar{h}\right)\right) h_{\sigma(j)} = \sum_{j=1}^{n} sim\left(s\left(h_j\right),\ s\left(\bar{h}\right)\right) h_j$$

and

$$\sum_{j=1}^{n} sim\left(s\left(h_{\sigma(j)}\right), s\left(\bar{h}\right)\right) = \sum_{j=1}^{n} sim\left(s\left(h_j\right),\ s\left(\bar{h}\right)\right)$$

then we replace (11) by

$$\text{HFOWA}\,(h_1,\ h_2,\cdots,\ h_n)=\mathop{\oplus}_{j=1}^{n}\left(w_j h_{\sigma(j)}\right)$$
$$=\cup_{\gamma_{\sigma(1)}\in h_{\sigma(1)},\gamma_{\sigma(2)}\in h_{\sigma(2)},\cdots,\gamma_{\sigma(n)}\in h_{\sigma(n)}}\left\{1-\prod_{j=1}^{n}\left(1-\gamma_{\sigma(j)}\right)^{\frac{sim\left(s\left(h_{\sigma(j)}\right),s(\bar h)\right)}{\sum_{j=1}^{n} sim\left(s\left(h_{\sigma(j)}\right),s(\bar h)\right)}}\right\}$$
$$=\cup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_n\in h_n}\left\{1-\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\frac{sim\left(s\left(h_j\right),s(\bar h)\right)}{\sum_{j=1}^{n} sim\left(s\left(h_j\right),s(\bar h)\right)}}\right\} \tag{12}$$

We call (12) a dependent hesitant fuzzy ordered weighted averaging (DHFOWA) operator, which is a generalization of the dependent ordered weighted averaging (DOWA) operator Xu [31]. Consider that the aggregated value of the DHFOWA operator is independent of the ordering, thus it is also a neat operator.

Based on the HFOWG and Equation (10), we developed the dependent hesitant fuzzy ordered weighted geometric (DHFOWG) operator as follows:

$$\text{HFWG}\,(h_1,h_2,\cdots,h_n)=\mathop{\otimes}_{j=1}^{n}\left(h_j\right)^{w_j}$$
$$=\cup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_n\in h_n}\left\{\prod_{j=1}^{n}\left(\gamma_j\right)^{\frac{sim\left(s\left(h_j\right),s(\bar h)\right)}{\sum_{j=1}^{n} sim\left(s\left(h_j\right),s(\bar h)\right)}}\right\} \tag{13}$$

Similar to Xu [31, 32], we have the following result:

Theorem 1. *Let $h_j\,(j=1,\ 2,\cdots,\ n)$ be a collection of HFEs, and let $s\left(\bar h\right)$ the average value of the score function of $h_j\,(j=1,\ 2,\cdots,\ n)$, $(\sigma(1),\ \sigma(2),\cdots,\ \sigma(n))$ is a permutation of $(1,\ 2,\cdots,\ n)$, such that $s\left(h_{\sigma(j-1)}\right)\geq s\left(h_{\sigma(j)}\right)$ for all $j=2,\cdots,n$. If $sim\left(s\left(h_i\right),\ s\left(\bar h\right)\right)\geq sim\left(s\left(h_j\right),\ s\left(\bar h\right)\right)$, then $w_i\geq w_j$.*

The normal distribution is one of the most commonly observed and is the starting point for modeling many natural process, it is usually found in events that are the aggregation of many smaller, but independent random events. Xu [32] introduced a normal distribution based method to determine some dependent uncertain ordered weighted aggregation operators, in which the associated weights only depend on the aggregated arguments. Motivated by the idea, in the following, we give another method for deriving the DHFOWA weights:

$$w_j=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{\left(d\left(s\left(h_{\pi(j)}\right),s(\bar h)\right)\right)^2}{2\sigma^2}},\quad j=2,\cdots,n. \tag{14}$$

where $s\left(\bar h\right)$ and σ are the arithmetic mean and the standard deviation of these hesitant fuzzy arguments variables $h_j\,(j=1,\ 2,\cdots,\ n)$, respectively, $(\pi(1),\ \pi(2),\cdots,\ \pi(n))$ is a permutation of $(1,\ 2,\cdots,\ n)$, such that $s\left(h_{\sigma(j-1)}\right)\geq s\left(h_{\sigma(j)}\right)$ for all $j=2,\cdots,n$.

Consider that $w_j\geq 0,\quad j=1,2,\cdots,n$ and $\sum_{j=1}^{n}w_j=1$, then by (14), we have

$$w_j=\frac{\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{\left(\left(d\left(s\left(h_{\pi(j)}\right),s(\bar h)\right)\right)\right)^2}{2\sigma^2}}}{\sum_{j=1}^{n}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{\left(\left(d\left(s\left(h_{\pi(j)}\right),s(\bar h)\right)\right)\right)^2}{2\sigma^2}}},$$
$$=\frac{e^{-\frac{\left(\left(d\left(s\left(h_{\pi(j)}\right),s(\bar h)\right)\right)\right)^2}{2\sigma^2}}}{\sum_{j=1}^{n}e^{-\frac{\left(\left(d\left(s\left(h_{\pi(j)}\right),s(\bar h)\right)\right)\right)^2}{2\sigma^2}}}\quad j=1,\ 2,\cdots,\ n. \tag{15}$$

then by (5), we have

$$\text{HFOWA}\,(h_1,\ h_2,\cdots,\ h_n)$$
$$=\mathop{\oplus}_{j=1}^{n}\left(w_j h_{\sigma(j)}\right)$$
$$=\mathop{\oplus}_{j=1}^{n}\left(\frac{e^{-\frac{\left(\left(d\left(s\left(h_{\pi(j)}\right),s(\bar h)\right)\right)\right)^2}{2\sigma^2}}}{\sum_{j=1}^{n}e^{-\frac{\left(\left(d\left(s\left(h_{\pi(j)}\right),s(\bar h)\right)\right)\right)^2}{2\sigma^2}}}h_{\sigma(j)}\right)$$
$$=\cup_{\gamma_{\sigma(1)}\in h_{\sigma(1)},\gamma_{\sigma(2)}\in h_{\sigma(2)},\cdots,\gamma_{\sigma(n)}\in h_{\sigma(n)}}\left\{1-\prod_{j=1}^{n}\left(1-\gamma_{\sigma(j)}\right)^{e^{-\frac{\left(\left(d\left(s\left(h_{\pi(j)}\right),s(\bar h)\right)\right)\right)^2}{2\sigma^2}}\Big/\sum_{j=1}^{n}e^{-\frac{\left(\left(d\left(s\left(h_{\pi(j)}\right),s(\bar h)\right)\right)\right)^2}{2\sigma^2}}}\right\} \tag{16}$$

Since

$$\sum_{j=1}^{n} e^{-\frac{\left(\left(d\left(s\left(h_{\pi(j)}\right),s(\bar{h})\right)\right)\right)^2}{2\sigma^2}} h_{\pi(j)} = \sum_{j=1}^{n} e^{-\frac{\left(\left(d\left(s\left(h_j\right),s(\bar{h})\right)\right)\right)^2}{2\sigma^2}} h_j$$

and

$$\sum_{j=1}^{n} e^{-\frac{\left(\left(d\left(s\left(h_{\pi(j)}\right),s(\bar{h})\right)\right)\right)^2}{2\sigma^2}} = \sum_{j=1}^{n} e^{-\frac{\left(\left(d\left(s\left(h_{\pi(j)}\right),s(\bar{h})\right)\right)\right)^2}{2\sigma^2}}$$

then, (16) can be rewritten as

$$\begin{aligned}
&\text{HFOWA}\left(h_1,\ h_2, \cdots,\ h_n\right) \\
&= \overset{n}{\underset{j=1}{\oplus}} \left(w_j h_{\sigma(j)}\right) \\
&= \overset{n}{\underset{j=1}{\oplus}} \left(w_j h_j\right) \\
&= \overset{n}{\underset{j=1}{\oplus}} \left(\frac{e^{-\frac{\left(\left(d\left(s\left(h_j\right),s(\bar{h})\right)\right)\right)^2}{2\sigma^2}}}{\sum_{j=1}^{n} e^{-\frac{\left(\left(d\left(s\left(h_j\right),s(\bar{h})\right)\right)\right)^2}{2\sigma^2}}} h_j \right) \\
&= \cup_{\gamma_j \in h_j, \gamma_2 \in h_2, \cdots, \gamma_n \in h_n} \\
&\left\{ 1 - \prod_{j=1}^{n} \left(1-\gamma_j\right)^{e^{-\frac{\left(\left(d\left(s\left(h_j\right),s(\bar{h})\right)\right)\right)^2}{2\sigma^2}} \Big/ \sum_{j=1}^{n} e^{-\frac{\left(\left(d\left(s\left(h_j\right),s(\bar{h})\right)\right)\right)^2}{2\sigma^2}}} \right\}
\end{aligned} \tag{17}$$

Obviously, (17) is also a neat and dependent hesitant fuzzy ordered weighted averaging (DHFOWA) operator.

Based on the HFOWG and Equation (15), we developed the dependent dependent hesitant fuzzy ordered weighted geometric (DHFOWG) operator as follows:

$$\begin{aligned}
&\text{DHFOWG}\left(h_1,\ h_2, \cdots,\ h_n\right) = \overset{n}{\underset{j=1}{\otimes}} \left(h_j\right)^{w_j} \\
&= \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \cdots, \gamma_n \in h_n} \\
&\left\{ \prod_{j=1}^{n} \left(\gamma_j\right)^{e^{-\frac{\left(\left(d\left(s\left(h_j\right),s(\bar{h})\right)\right)\right)^2}{2\sigma^2}} \Big/ \sum_{j=1}^{n} e^{-\frac{\left(\left(d\left(s\left(h_j\right),s(\bar{h})\right)\right)\right)^2}{2\sigma^2}}} \right\}
\end{aligned} \tag{18}$$

From (10) and (15), we know that all the associated weights of the DHFOWA operator and DHFOWG operator only depend on the aggregated hesitant fuzzy variables, and can relieve the influence of unfair arguments on the aggregated results by assigning low weights to those "false" and "biased" ones, and thus make the aggregated results more reasonable in the practical applications.

4. Approaches to hesitant fuzzy multiple attribute decision making with incomplete weight information

The following assumptions or notations are used to represent the hesitant fuzzy MADM problems. Let $A = \{A_1,\ A_2, \cdots,\ A_m\}$ be a discrete set of alternatives and $G = \{G_1,\ G_2, \cdots,\ G_n\}$ be a set of attributes. If the decision makers provide several values for the alternative A_i under the state of nature G_j with anonymity, these values can be considered as a hesitant fuzzy element h_{ij}. In the case where two decision makers provide the same value, then the value emerges only once in h_{ij}. Suppose that the decision matrix $H = \left(h_{ij}\right)_{m \times n}$ is the hesitant fuzzy decision matrix, where $h_{ij}\ (i = 1,\ 2, \cdots,\ m,\ j = 1,\ 2, \cdots,\ n)$ are in the form of HFEs.

Based on the above models, we develop a practical method for solving the MADM problems with hesitant fuzzy information. The method involves the following steps:

Step 1. Utilize the DHFOWA operator:

$$\begin{aligned}
h_i &= \text{DHFOWA}\left(h_{i1},\ h_{i2}, \cdots,\ h_{in}\right) \\
&= \overset{n}{\underset{j=1}{\oplus}} \left(w_j h_{ij}\right) \\
&= \cup_{\gamma_{i1} \in h_{i1}, \gamma_{i2} \in h_{i2}, \cdots, \gamma_{in} \in h_{in}} \\
&\left\{ 1 - \prod_{j=1}^{n} \left(1 - \gamma_{ij}\right)^{\frac{sim\left(s\left(h_{ij}\right),s\left(\bar{h}_i\right)\right)}{\sum_{j=1}^{n} sim\left(s\left(h_{ij}\right),s\left(\bar{h}_i\right)\right)}} \right\}
\end{aligned} \tag{19}$$

or the DHFOWG operator:

$$\begin{aligned}
h_i &= \text{DHFOWG}\left(h_{i1},\ h_{i2}, \cdots,\ h_{in}\right) \\
&= \overset{n}{\underset{j=1}{\otimes}} \left(h_{ij}\right)^{w_j} \\
&= \cup_{\gamma_{i1} \in h_{i1}, \gamma_{i2} \in h_{i2}, \cdots, \gamma_{in} \in h_{in}} \\
&\left\{ \prod_{j=1}^{n} \left(\gamma_j\right)^{sim\left(s\left(h_{ij}\right),s\left(\bar{h}_i\right)\right) \Big/ \sum_{j=1}^{n} sim\left(s\left(h_{ij}\right),s\left(\bar{h}_i\right)\right)} \right\}
\end{aligned} \tag{20}$$

to derive the overall preference values $h_i\,(i = 1,\ 2, \cdots,\ m)$ of the alternative A_i, where $s\left(\bar{h}_i\right) = \frac{1}{n}\sum_{j=1}^{n} s\left(h_{ij}\right)$.

Step 2. calculate the scores $S\left(\tilde{h}_i\right)$ of the overall hesitant fuzzy preference value $\tilde{h}_i$ $(i = 1, 2, \cdots, m)$ to rank all the alternatives $A_i\,(i = 1,\ 2, \cdots,\ m)$ and then to select the best one(s).

Step 3. Rank all the alternatives $A_i\,(i = 1, 2, \cdots, m)$ and select the best one(s) in accordance with $S\left(\tilde{h}_i\right)$ $(i = 1,\ 2, \cdots,\ m)$.

Step 4. End.

5. Numerical example

Supplier selection is the process by which firms identify, evaluate, and contract with suppliers. The supplier selection process deploys a tremendous amount of a firm's financial resources. In return, firms expect significant benefits from contracting with suppliers offering high value. Thus, in this section we shall present a numerical example for supplier selection with hesitant fuzzy information in order to illustrate the method proposed in this paper. There is a panel with five possible suppliers $A_i\,(i = 1,\ 2,\ 3,\ 4,\ 5)$ to select. The experts selects four attribute to evaluate the five possible suppliers: ①G_1 is the product quality; ②G_2 is the service; ③G_3 is the delivery; ④G_4 is the price. In order to avoid influence each other, the decision makers are required to evaluate the five possible suppliers $A_i\,(i = 1,\ 2,\ 3,\ 4,\ 5)$ under the above four attributes in anonymity and the decision matrix $H = \left(h_{ij}\right)_{4\times 4}$ is presented in Table 1, where $h_{ij}\,(i = 1,\ 2,\ 3,\ 4, j = 1,\ 2,\ 3,\ 4)$ are in the form of HFEs.

In the following, we utilize the approach developed for supplier selection with hesitant fuzzy information.

Utilize the DHFOWA operator and DHFOWG operator to derive the overall preference values $h_i\,(i = 1, 2, 3, 4, 5)$ of the suppliers A_i. The results are shown in Table 2.

Table 1
Hesitant fuzzy decision matrix

	G_1	G_2	G_3	G_4
A_1	(0.3, 0.5)	(0.6, 0.7, 0.8)	(0.7, 0.8)	(0.8, 0.9)
A_2	(0.3, 0.4, 0.5)	(0.6, 0.9)	(0.6, 0.7)	(0.4, 0.5)
A_3	(0.4, 0.6)	(0.7, 0.8)	(0.3, 0.5, 0.7)	(0.6, 0.7)
A_4	(0.7, 0.9)	(0.3, 0.4)	(0.5, 0.7)	(0.3, 0.5)
A_5	(0.2, 0.3)	(0.6, 0.7)	(0.5, 0.6)	(0.7, 0.8, 0.9)

Table 2
The score values of the suppliers

	A_1	A_2	A_3	A_4	A_5
DHFOWA	0.32	0.15	0.26	0.47	0.16
DHFOWG	0.29	0.14	0.21	0.42	0.13

Table 3
Ordering of the suppliers

	Ordering
DHFOWA	$A_4 > A_1 > A_3 > A_5 > A_2$
DHFOWG	$A_4 > A_1 > A_3 > A_2 > A_3$

According to the aggregating results shown in Table 2, the ordering of the suppliers are shown in Table 3. Note that > means "preferred to". As we can see, depending on the aggregation operators used, the ordering of the suppliers is slightly different. Therefore, depending on the aggregation operators used, the results may lead to different decisions. However, the best suppliers is A_4.

6. Conclusion

In this paper, we investigate the multiple attribute decision making with hesitant fuzzy information. Motivated by the ideal of dependent aggregation, in this paper, we develop some dependent hesitant fuzzy aggregation operators: the dependent hesitant fuzzy ordered weighted averaging (DHFOWA) operator and the dependent hesitant fuzzy ordered weighted geometric (DHFOWG) operator, in which the associated weights only depend on the aggregated hesitant fuzzy arguments and can relieve the influence of unfair hesitant fuzzy arguments on the aggregated results by assigning low weights to those "false" and "biased" ones and then apply them to develop some approaches for multiple attribute group decision making with hesitant fuzzy information. Finally, an illustrative example for supplier selection is given to verify the developed approach and to demonstrate its practicality and effectiveness.

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