

A Method for Hesitant Fuzzy Multiple Attribute Decision Making and Its Application to Risk Investment

Xiang Gu, Ying Wang, Bei Yang
Faculty of Construction Management and Real Estate, Chongqing University,
Chongqing 400030 China
cqxgu@163.com
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Abstract

With respect to evaluation model for risk investment with hesitant fuzzy information, some operational laws of hesitant fuzzy numbers are introduced. We utilize the hesitant fuzzy weighted averaging (HFWA) operator to aggregate the hesitant fuzzy information corresponding to each alternative, and then rank the alternatives and select the most desirable one(s) according to the score function. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Keywords: Multiple Attribute Decision Making, Hesitant Fuzzy Number, Hesitant Fuzzy Weighted Averaging (HFWA) Operator, Weight Information

1. Introduction

Atanassov [1,2] introduced the concept of intuitionistic fuzzy set(IFS), which is a generalization of the concept of fuzzy set [3]. The intuitionistic fuzzy set has received more and more attention since its appearance[4-24]. Furthermore, Torra [20] proposed the hesitant fuzzy set which permits the membership having a set of possible values and discussed the relationship between hesitant fuzzy set and intuitionistic fuzzy set, and showed that the envelope of hesitant fuzzy set is an intuitionistic fuzzy set. Xia and Xu[21] gave an intensive study on hesitant fuzzy information aggregation techniques and their application in decision making. They developed some hesitant fuzzy operational rules based on the interconnection between the hesitant fuzzy set and the intuitionsitic fuzzy set. In order to aggregate the hesitant fuzzy information, they proposed a series of operators under various situations and discussed the relationships among them. Moreover, they applied the developed aggregation operators to solve the decision making problems with anonymity.

In the process of MADM problems with hesitant fuzzy information, sometimes, the attribute values take the form of hesitant fuzzy numbers, and the information about attribute weights is completely known. The aim of this paper is to solve the multiple attribute decision making problems to deal with risk investment with completely known information on attribute weights to which the attribute values are given in terms of hesitant fuzzy numbers. The remainder of this paper is set out as follows. In the next section, we introduce some basic concepts related to hesitant fuzzy sets. In Section 3 we introduce the MADM problem to deal with risk investment with hesitant fuzzy information, in which the information about attribute weights is completely known, and the attribute values take the form of hesitant fuzzy numbers. We utilize the hesitant fuzzy weighted averaging (HFWA) operator to aggregate the hesitant fuzzy information corresponding to each partner, and then rank the partners and select the most desirable one(s) according to the score function. In Section 4, an illustrative example with risk investment is pointed out. In Section 5 we conclude the paper and give some remarks.

2. Preliminaries

In the following, we introduce some basic concepts related to intuitionistic fuzzy sets. Atanassov[1-2] extended the fuzzy set to the IFS, shown as follows:

Definition 1. An IFS A in X is given by

$$A = \left\{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \right\} \quad (1)$$

Where $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$, with the condition

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1, \quad \forall x \in X$$

The numbers $\mu_A(x)$ and $\nu_A(x)$ represent, respectively, the membership degree and non-membership degree of the element x to the set A [1-2].

Definition 2. For each IFS A in X , if

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x), \quad \forall x \in X \quad (2)$$

Then $\pi_A(x)$ is called the degree of indeterminacy of x to A [1-2].

However, when giving the membership degree of an element, the difficulty of establishing the membership degree is not because we have a margin of error, or some possibility distribution on the possibility values, but because we have several possible values. For such cases, Torra[20] proposed another generation of FS.

Definition 2[20]. Given a fixed set X , then a hesitant fuzzy set(HFS) on X is in terms of a function that when applied to X returns a subset of $[0,1]$.

To be easily understood, Xu[21] express the HFS by mathematical symbol:

$$E = \left(\langle x, h_E(x) \rangle \mid x \in X \right) \quad (3)$$

where $h_E(x)$ is a set of some values in $[0,1]$, denoting the possible membership degree of the element $x \in X$ to the set E . For convenience, Xu[21] call $h = h_E(x)$ a hesitant fuzzy element(HFE) and H the set of all HFEs.

Definition 4. For a HFE h , $s(h) = \frac{1}{\#h} \sum_{\gamma \in h} \gamma$ is called the score function of h , where $\#h$ is the number of the elements in h . For two HFEs h_1 and h_2 , if $s(h_1) > s(h_2)$, then $h_1 > h_2$; if $s(h_1) = s(h_2)$, then $h_1 = h_2$.

Definition 5[20]. Let $E = \{h_1, h_2, \dots, h_n\}$ be a set of n HFEs, Θ be a function on E , $\Theta : [0,1]^N \rightarrow [0,1]$, then

$$\Theta_E = \bigcup_{\gamma \in \{h_1 \times h_2 \times \dots \times h_n\}} \{\Theta(\gamma)\} \quad (4)$$

Based on the Definition 5 and the defined operations for HFEs, Xia and Xu[21] proposed a series of aggregation operators for HFEs.

Definition 6[21]. Let $h_j (j = 1, 2, \dots, n)$ be a collection of HFEs. The hesitant fuzzy weighted averaging(HFWA) operator is a mapping $H^n \rightarrow H$ such that

$$HFWA(h_1, h_2, \dots, h_n) = \bigoplus_{j=1}^n (\omega_j h_j) = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \dots, \gamma_n \in h_n} \left\{ 1 - \prod_{j=1}^n (1 - \gamma_j)^{\omega_j} \right\} \quad (5)$$

where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of $h_j (j = 1, 2, \dots, n)$, and $\omega_j > 0$, $\sum_{j=1}^n \omega_j = 1$.

3. A method for hesitant fuzzy multiple attribute decision making its application to risk investment

In this section, we apply the hesitant fuzzy weighted averaging (HFWA) operator to multiple attribute decision making problems to deal with risk investment with anonymity. Let $A = \{A_1, A_2, \dots, A_m\}$ be a discrete set of alternatives, and $G = \{G_1, G_2, \dots, G_n\}$ be the set of attributes, $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ is the weighting vector of the attribute $G_j (j = 1, 2, \dots, n)$,

where $\omega_j \in [0, 1]$, $\sum_{j=1}^n \omega_j = 1$. If the decision makers provide several values for the alternative A_i under the attribute G_j with anonymity, these values can be considered as a hesitant fuzzy element h_{ij} . In the case where two decision makers provide the same value, then the value emerges only once in h_{ij} . Suppose that the decision matrix $H = (h_{ij})_{m \times n}$ is the hesitant fuzzy decision matrix, where $h_{ij} (i = 1, 2, \dots, m, j = 1, 2, \dots, n)$ are in the form of HFEs.

In the following, we apply the HFWA operator to multiple attribute decision making to deal with risk investment with hesitant fuzzy information.

Step 1. We utilize the decision information given in matrix R , and the HFWA operator

$$h_i = HFWA(h_{i1}, h_{i2}, \dots, h_{in}) = \bigoplus_{j=1}^n (\omega_j h_{ij}) = \bigcup_{\gamma_1 \in h_{i1}, \gamma_2 \in h_{i2}, \dots, \gamma_n \in h_{in}} \left\{ 1 - \prod_{j=1}^n (1 - \gamma_j)^{\omega_j} \right\}, \quad i = 1, 2, \dots, m. \quad (6)$$

to derive the overall preference values $h_i (i = 1, 2, \dots, m)$ of the alternative A_i .

Step 2. Calculate the scores $S(h_i) (i = 1, 2, \dots, m)$ of the overall hesitant fuzzy preference values $h_i (i = 1, 2, \dots, m)$ to rank all the alternatives $A_i (i = 1, 2, \dots, m)$ and then to select the best one(s).

Step 3. Rank all the alternatives $A_i (i = 1, 2, \dots, m)$ and select the best one(s) in accordance with $S(h_i) (i = 1, 2, \dots, m)$.

4. Illustrative Example

Let us suppose there is a risk investment company, which wants to invest a sum of money in the best option (adapted from [22]). There is a panel with four possible alternatives to invest the money: ① A_1 is a car company; ② A_2 is a food company; ③ A_3 is a computer company; ④ A_4 is a TV company. The investment company must take a decision according to the following four attributes: ① G_1 is the risk analysis; ② G_2 is the growth analysis; ③ G_3 is the social-political impact analysis; ④ G_4 is the environmental impact analysis. In order to avoid influence each other, the decision makers are required to evaluate the four possible alternatives $A_i (i = 1, 2, 3, 4)$ under the above four attributes in anonymity and the decision matrix $H = (h_{ij})_{m \times n}$ is presented in Table 1, where $h_{ij} (i = 1, 2, 3, 4, j = 1, 2, 3, 4)$ are in the form of HFEs.

Table 1. Hesitant fuzzy decision matrix

	G_1	G_2	G_3	G_4
A_1	(0.2,0.3,0.5)	(0.3,0.4)	(0.6,0.7)	(0.5,0.7)
A_2	(0.3,0.5)	(0.2,0.3,0.6)	(0.4,0.6)	(0.3,0.4,0.6)
A_3	(0.2,0.4,0.7)	(0.5,0.6)	(0.2,0.8)	(0.6,0.9)
A_4	(0.1,0.3)	(0.2,0.4,0.7)	(0.6,0.8)	(0.2,0.5)

The information about the attribute weights is known as follows: $\omega = (0.2, 0.1, 0.3, 0.4)$. Procedure for selection of supplier contains the following steps.

Step 1. Utilize the weight vector $\omega = (0.2, 0.1, 0.3, 0.4)$ and by Eq. (6), we obtain the overall values \tilde{r}_i of the alternative $A_i (i = 1, 2, 3, 4)$. Take alternative A_1 for an example, we have

$$\begin{aligned}
 h_1 &= HFWA(h_{11}, h_{12}, h_{13}, h_{14}) = HFWA\{(0.2, 0.3, 0.5), (0.6, 0.7), (0.7, 0.8), (0.5, 0.7)\} \\
 &= \bigoplus_{j=1}^4 (w_j h_j) \\
 &= \bigcup_{\gamma_{11} \in h_{11}, \gamma_{12} \in h_{12}, \gamma_{13} \in h_{13}, \gamma_{14} \in h_{14}} \left\{ 1 - \prod_{j=1}^4 (1 - \gamma_{1j})^{\omega_j} \right\} \\
 &= \bigcup_{\gamma_{11} \in h_{11}, \gamma_{12} \in h_{12}, \gamma_{13} \in h_{13}, \gamma_{14} \in h_{14}} \left\{ 1 - (1 - \gamma_{11})^{0.2} (1 - \gamma_{12})^{0.1} (1 - \gamma_{13})^{0.3} (1 - \gamma_{14})^{0.4} \right\} \\
 &= \{0.4687, 0.4768, 0.4827, 0.4906, 0.5126, 0.5201, 0.5238, 0.5255, 0.5327, 0.5564, 0.5631, 0.5669, \\
 &\quad 0.5735, 0.5783, 0.5848, 0.6027, 0.6058, 0.6088, 0.6118, 0.6132, 0.6191, 0.6383, 0.6439\}
 \end{aligned}$$

Step 2. Calculate the scores $S(h_i)$ of the overall hesitant fuzzy preference values $h_i (i = 1, 2, 3, 4)$

$$S(h_1) = 0.5590, S(h_2) = 0.4509, S(h_3) = 0.6554, S(h_4) = 0.4842$$

Step 3. Rank all the partner $A_i (i = 1, 2, 3, 4)$ in accordance with the scores $S(h_i) (i = 1, 2, 3, 4)$ of the overall hesitant fuzzy preference values $h_i (i = 1, 2, 3, 4)$: $A_3 \succ A_1 \succ A_4 \succ A_2$, and thus the most desirable partner is A_3 .

5. Conclusion

In this paper, we have investigated the multiple attribute decision making problems to deal with risk investment with completely known information on attribute weights to which the attribute values are given in terms of hesitant fuzzy numbers. We utilize the hesitant fuzzy weighted averaging (IFWA) operator to aggregate the hesitant fuzzy information corresponding to each alternative, and then rank the alternatives and select the most desirable one(s) according to the score function. Finally, an illustrative example is given. In the future, we shall continue working in the application of the hesitant fuzzy multiple attribute decision-making to other domains.

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