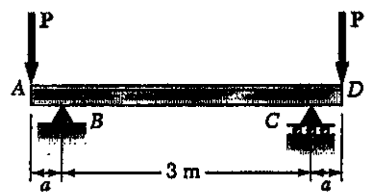
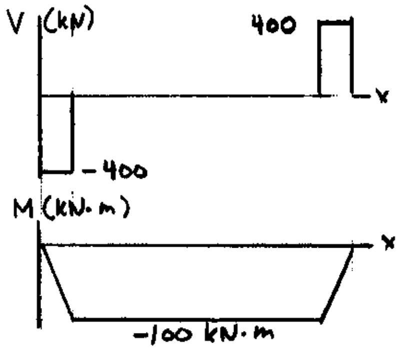
CHAPTER 8

PROBLEM 8.1



8.1 An overhanging W250 × 58 rolled-steel beam supports two loads as shown. Knowing that $P = 400$ kN, $a = 0.25$ m, and $\sigma_{all} = 250$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of aflange and the web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned..



$|V|_{max} = 400 \text{ kN} = 400\times10^3 \text{ N}$

$|M|_{max} = (400\times10^3)(0.25) = 100\times10^3 \text{ N·m}$

For W 250 × 58 rolled steel section

$d = 252$ mm. $\quad b_f = 203$ mm $\quad t_f = 13.5$ mm

$t_w = 8.0$ mm $\quad I_x = 87.3\times10^6 \text{ mm}^4 \quad S_x = 693\times10^3 \text{ mm}^3$

$c = \frac{1}{2}d = 126$ mm $\quad y_b = c - t_f = 112.5$ mm.

(a) $\sigma_m = \dfrac{|M|_{max}}{S_x} = \dfrac{100\times10^3}{693\times10^{-6}} = 144.3\times10^6 \text{ Pa} = 144.3 \text{ MPa}$

$\sigma_b = \dfrac{y_b}{c}\sigma_m = \dfrac{112.5}{126}(144.3) = 128.84$ MPa

$A_f = b_f t_f = (203)(13.5) = 2740.5 \text{ mm}^2$

$\bar{y}_f = \frac{1}{2}(c + y_b) = 119.25 \text{ mm}^3$

$Q_b = A_f \bar{y}_f = 326.80\times10^3 \text{ mm}^3 = 326.80\times10^{-6} \text{ m}^3$

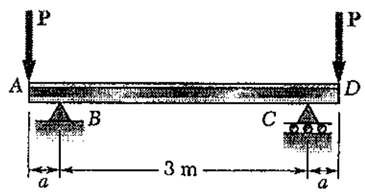
$\tau_{xy} = \dfrac{|V|_{max} Q_b}{I_x t_w} = \dfrac{(400\times10^3)(326.80\times10^{-6})}{(87.3\times10^{-6})(8\times10^{-3})} = 187.2\times10^6 \text{ Pa} = 187.2 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 197.97$ MPa

(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 262$ MPa

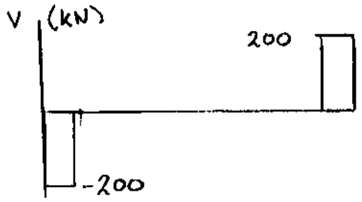
(c) Since $\sigma_{max} > 250$ MPa, W250 × 58 is <u>not acceptable</u>.

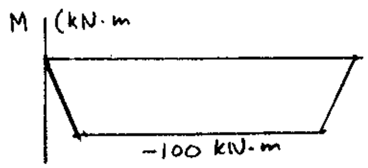
PROBLEM 8.2



8.1 An overhanging W250 × 58 rolled-steel beam supports two loads as shown. Knowing that $P = 400$ kN, $a = 0.25$ m, and $\sigma_{all} = 250$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of aflange and the web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned..

8.2 Solve Prob. 8.1, assuming that $P = 200$ kN and $a = 0.5$ m.





$|V|_{max} = 200 \text{ kN} = 200 \times 10^3 \text{ N}$

$|M|_{max} = (200 \times 10^3)(0.5) = 100 \times 10^3 \text{ N}\cdot\text{m}$

For W250 × 58 rolled steel section

$d = 252$ mm $\quad b_f = 203$ mm $\quad t_f = 13.5$ mm.

$t_w = 8.0$ mm $\quad I_x = 87.3 \times 10^6 \text{ mm}^3$ $\quad S_x = 693 \times 10^3 \text{ mm}^3$

$c = \frac{1}{2} d = 126$ mm $\quad y_b = c - t_f = 112.5$ mm

(a) $\sigma_m = \dfrac{|M|_{max}}{S_x} = \dfrac{100 \times 10^3}{693 \times 10^{-6}} = 144.3 \times 10^6 \text{ Pa}$
$= 144.3$ MPa

$\sigma_b = \dfrac{y_b}{c} \sigma_m = \dfrac{112.5}{126}(144.3) = 128.84$ MPa

$A_f = b_f t_f = (203)(13.5) = 2740.5 \text{ mm}^2$

$\bar{y}_f = \frac{1}{2}(c + y_b) = 119.75$ mm

$Q_b = A_f \bar{y}_f = 326.80 \times 10^3 \text{ mm}^3 = 326.80 \times 10^{-6} \text{ m}^3$

$\tau_{xy} = \dfrac{|V|_{max} Q_b}{I_x t_w} = \dfrac{(200 \times 10^3)(326.80 \times 10^{-6})}{(87.3 \times 10^{-6})(8 \times 10^{-3})} = 93.6 \times 10^6 \text{ Pa} = 93.6 \text{ MPa}$

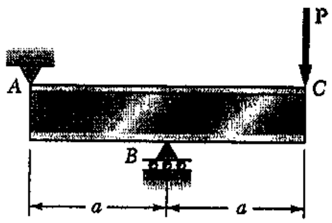
$R = \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 113.63$ MPa

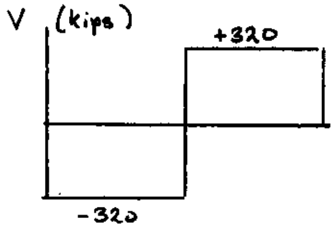
(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 178.0$ MPa

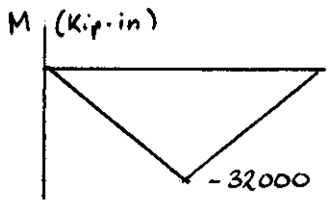
(c) Since $\sigma_{max} < 250$ MPa, W 250×58 is <u>acceptable</u>.

PROBLEM 8.3

8.3 An overhanging W36 × 300 rolled-steel beam supports a load **P** as shown. Knowing that P = 320 kips, a = 100 in., and σ_{all} = 29 ksi, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.







$|V|_{max} = 320$ kips

$|M|_{max} = (320)(100) = 32000$ kip·in.

For W 36 × 300 rolled steel beam

$d = 36.74$ in $\quad b_f = 16.655$ in. $\quad t_f = 1.680$ in.

$t_w = 0.945$ in. $\quad I_x = 20300$ in^4 $\quad S_x = 1110$ in^3

$c = \frac{1}{2}d = 18.37$ in. $\quad y_b = c - t_f = 16.69$ in.

(a) $\sigma_m = \dfrac{|M|_{max}}{S_x} = \dfrac{32000}{1110} = 28.8$ ksi ◄

$\sigma_b = \dfrac{y_b}{c}\sigma_m = \dfrac{(16.69)}{(18.37)}(28.8) = 26.2$ ksi

$A_f = b_f t_f = 27.98$ in^2

$\bar{y}_f = \frac{1}{2}(c + y_b) = 17.53$ in.

$Q_b = A_f \bar{y}_f = 490.49$ in^3

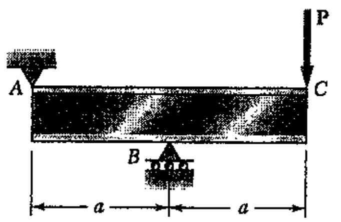
$\tau_{xy} = \dfrac{|V|_{max} Q_b}{I_x t_w} = \dfrac{(320)(490.49)}{(20300)(0.945)} = 8.18$ ksi

$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{26.2}{2}\right)^2 + (8.18)^2} = 15.44$ ksi

(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 28.5$ ksi ◄

(c) Since 28.5 ksi $< \sigma_{all}$, W 36 × 300 is <u>acceptable</u>. ◄

PROBLEM 8.4

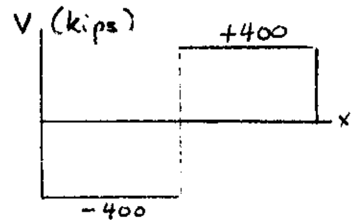


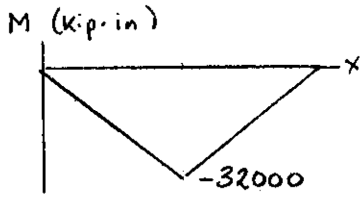
8.3 An overhanging W36 × 300 rolled-steel beam supports a load **P** as shown. Knowing that $P = 320$ kips, $a = 100$ in., and $\sigma_{all} = 29$ ksi, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

8.4 Solve Prob. 8.3, assuming that $P = 400$ kips and $a = 80$ in.

$|V|_{max} = 400$ kips

$|M|_{max} = (400)(80) = 32000$ kip·in.





For W 36 × 300 rolled steel section

$d = 36.74$ in $\quad b_f = 16.655$ in $\quad t_f = 1.680$ in

$t_w = 0.945$ in. $\quad I_x = 20300$ in^4 $\quad S_x = 1110$ in^3

$c = \frac{1}{2}d = 18.37$ in $\quad y_b = c - t_f = 16.69$ in

(a) $\sigma_m = \dfrac{|M|_{max}}{S_x} = \dfrac{32000}{1110} = 28.8$ ksi ◄

$\sigma_b = \dfrac{y_b}{c}\sigma_m = \dfrac{(16.69)}{(18.37)}(28.8) = 26.2$ ksi

$A_f = b_f t_f = 27.98$ in^2

$\bar{y}_f = \frac{1}{2}(c + y_b) = 17.53$ in.

$Q_b = A_f \bar{y}_f = 490.49$ in^3

$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(400)(490.49)}{(20300)(0.945)} = 10.23 \text{ ksi}$$

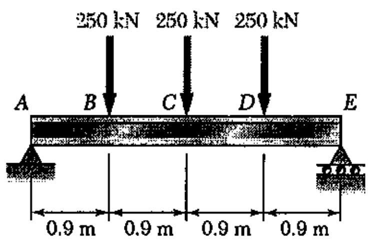
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(13.1)^2 + (10.23)^2} = 16.62 \text{ ksi}$$

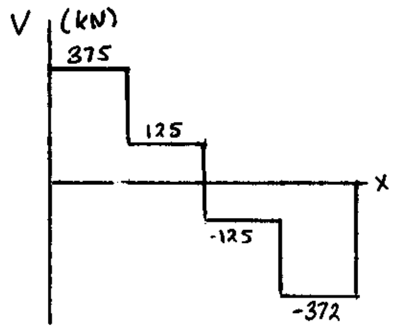
(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 29.7$ ksi ◄

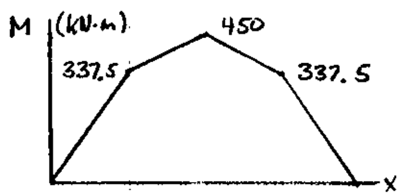
(c) Since 29.7 ksi $> \sigma_{all}$. W 36 × 300 is <u>not acceptable</u> ◄

PROBLEM 8.5

8.5 and 8.6 (*a*) Knowing that $\sigma_{all} = 160$ MPa and $\tau_{all} = 100$ MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.







Reactions: $R_A = 375$ kN ↑ , $R_E = 375$ kN ↑

$|V|_{max} = 375$ kN

$|M|_{max} = 450$ kN·m

$|V|$ at point C 125 kN

$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{450 \times 10^3}{160 \times 10^6} = 2.8125 \times 10^{-3}\ m^3$$
$$= 2812.5 \times 10^3\ mm^3$$

Shape	S_x ($10^3\ mm^3$)
W 840 × 176	5890
W 760 × 147	4410
→ W 690 × 125	3510
W 610 × 155	4220
W 530 × 150	3720
W 460 × 158	3340
W 360 × 216	3800

(a) Use W 690 × 125 ◄

$d = 678$ mm

$t_f = 16.30$ mm

$t_w = 11.7$ mm

$$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{450 \times 10^3}{3510 \times 10^{-6}} = 128.2 \times 10^6\ Pa = 128.2\ MPa$$ ◄

$$\tau_m = \frac{|V|_{max}}{A_w} = \frac{|V|_{max}}{d\,t_w} = \frac{375 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} = 47.3 \times 10^6\ Pa = 47.3\ MPa$$ ◄

At point C $$\tau_w = \frac{V}{A_w} = \frac{125 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} = 15.76 \times 10^6\ Pa$$
$$= 15.76\ MPa$$

$c = \frac{1}{2}d = \frac{678}{2} = 339$ mm $y_b = c - t_f = 339 - 16.30 = 322.7$ mm

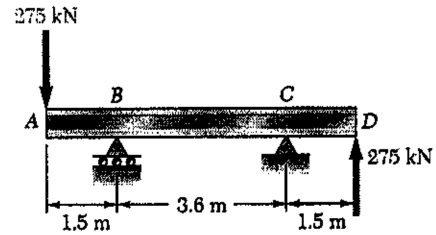
$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{322.7}{339}\right)(128.2) = 122.0\ MPa$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_w^2} = \sqrt{(61.0)^2 + (15.76)^2} = 63.0\ MPa$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 61.0 + 63.0 = 124.0\ MPa$$ ◄

PROBLEM 8.6

8.5 and 8.6 (*a*) Knowing that $\sigma_{all} = 160$ MPa and $\tau_{all} = 100$ MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

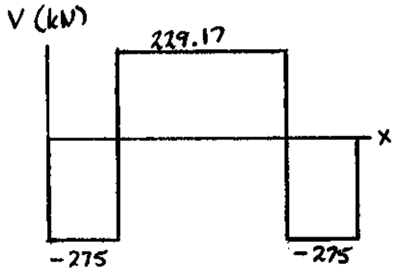


$R_B = 504.17$ kN ↑ $R_C = 504.17$ ↓

$|V|_{max} = 275$ kN

$|M|_{max} = 412.5$ kN·m

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{412.5\times10^3}{160\times10^6} = 2578\times10^{-6}\ \text{m}^3 = 2578\times10^3\ \text{mm}^3$$



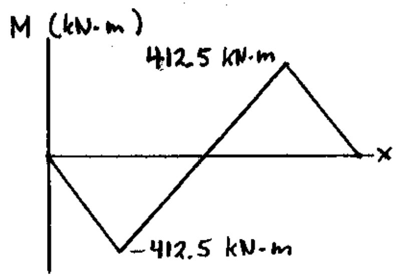
Shape	S_x (10^3 mm^3)
W 760 × 147	4410
→ W 690 × 125	3510
W 530 × 150	3720
W 460 × 158	3340
W 360 × 216	3800

(a) Use W 690 × 125 ◄

$d = 678$ mm

$t_f = 16.30$ mm

$t_w = 11.7$ mm



$$\sigma_m = \frac{|M|_{max}}{S} = \frac{412.5\times10^3}{3150\times10^{-6}} = 117.5\times10^6\ \text{Pa} = 117.5\ \text{MPa} \quad ◄$$

$$\tau_m = \frac{|V|_{max}}{A_w} = \frac{|V|_{max}}{d\,t_w} = \frac{275\times10^3}{(678\times10^{-3})(11.7\times10^{-3})} = 34.7\times10^6\ \text{Pa} = 34.7\ \text{MPa} \quad ◄$$

$c = \frac{1}{2}d = \frac{678}{2} = 339$ mm $t_f = 16.30$ mm $y_b = c - t_f = 339 - 16.30 = 322.7$ mm

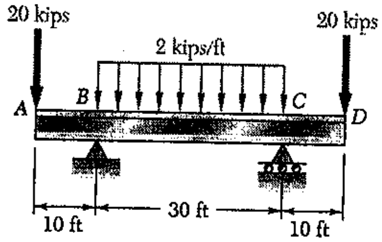
$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{322.7}{339}\right)(117.5) = 111.85\ \text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{(55.925)^2 + (34.7)^2} = 65.815\ \text{MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 55.925 + 65.815 = 121.7\ \text{MPa} \quad ◄$$

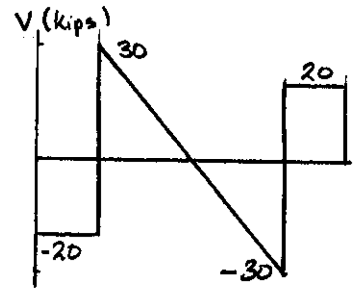
PROBLEM 8.7

8.7 **and** *8.8* (*a*) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.



$R_A = 50$ kips ↑ $\qquad R_D = 50$ kips ↑

$|V|_{max} = 30$ kips



$|M|_{max} = 200$ kip·ft $= 2400$ kip·in.

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{2400}{24} = 100 \text{ in}^3$$

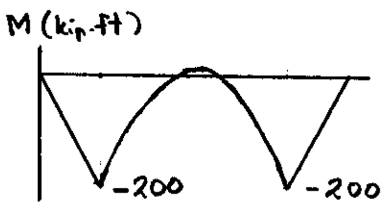
Shape	S (in³)
W 24×68	154
→ W 21×62	127
W 18×76	146
W 16×77	134
W 12×96	103
W 10×112	131

(a) Use W 21×62 ◄

$d = 20.99$ in.

$t_f = 0.615$ in.

$t_w = 0.400$ in.



$$\sigma_m = \frac{|M|_{max}}{S} = \frac{2400}{127} = 18.90 \text{ ksi} \quad ◄$$

$$\tau_m = \frac{|V|_{max}}{d\,t_w} = \frac{30}{(20.99)(0.400)} = 3.57 \text{ ksi} \quad ◄$$

$$c = \tfrac{1}{2}d = \frac{20.99}{2} = 10.495 \text{ in.} \qquad y_b = c - t_f = 10.495 - 0.615 = 9.88 \text{ in.}$$

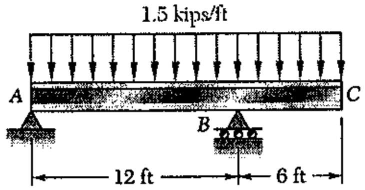
$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{9.88}{10.495}\right)(18.90) = 17.79 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{(8.896)^2 + (3.57)^2} = 9.586 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.896 + 9.586 = 18.48 \text{ ksi} \quad ◄$$

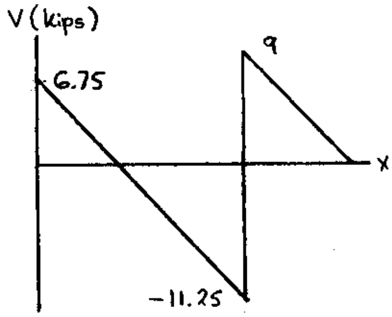
PROBLEM 8.8

8.7 and 8.8 (*a*) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.



$$+\circlearrowleft \Sigma M_B = 0 \qquad -12R_A + (1.5)(18)(3) = 0 \qquad R_A = 6.75 \text{ kips} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0 \qquad 12R_B + (1.5)(18)(9) = 0 \qquad R_B = 20.25 \text{ kips} \uparrow$$



$$|V|_{max} = 11.25 \text{ kips}$$

$$|M|_{max} = 27 \text{ kip}\cdot\text{ft} = 324 \text{ kip}\cdot\text{in}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{324}{24} = 13.5 \text{ in}^3$$

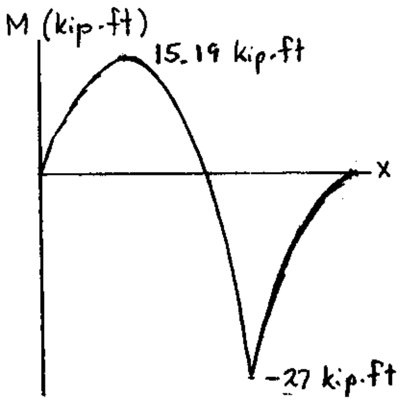
Shape	S (in³)
W 12 × 16	17.1
→ W 10 × 15	13.8
W 8 × 18	15.2
W 6 × 20	13.4

(a) Use W 10 × 15 ◀

$d = 9.99$ in.

$t_f = 0.270$ in.

$t_w = 0.230$ in.



$$\text{(b)} \quad \sigma_m = \frac{|M|_{max}}{S} = \frac{324}{13.8} = 23.5 \text{ ksi} \quad ◀$$

$$\tau_m = \frac{|V|_{max}}{d\,t_w} = \frac{11.25}{(9.99)(0.230)} = 4.90 \text{ ksi} \quad ◀$$

$$c = \tfrac{1}{2}d = \frac{9.99}{2} = 4.995 \text{ in.}$$

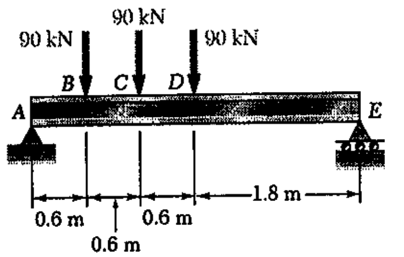
$$y_b = c - t_f = 4.995 - 0.270 = 4.725 \text{ in.}$$

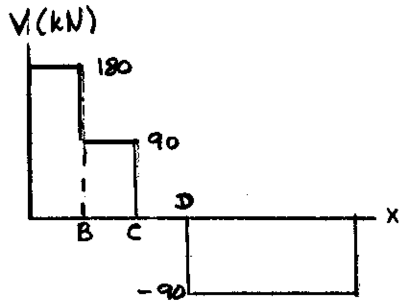
$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{4.725}{4.995}\right)(23.5) = 22.2 \text{ ksi}$$

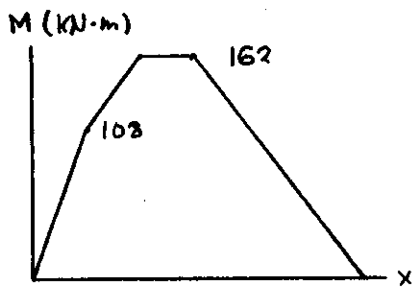
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\frac{22.2}{2}\right)^2 + (4.90)^2} = 12.1 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = \frac{22.2}{2} + 12.1 = 23.2 \text{ ksi} \quad ◀$$

PROBLEM 8.9







8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.9 Loading of Prob. 5.81 and selected W410 × 60 shape.

From Problem 5.81 $\quad \sigma_{all} = 160$ MPa

$|M|_{max} = 162$ kN-m at C and D

$|V| = 90$ kN at C and D

For W 410 × 60 rolled steel section

$d = 407$ mm, $\quad b_f = 178$ mm, $\quad t_f = 12.80$ mm

$t_w = 7.7$ mm, $\quad I_z = 216 \times 10^6$ mm^4, $\quad S_z = 1060 \times 10^3$ mm^3

$c = \frac{1}{2} d = 203.5$ mm

$$\sigma_m = \frac{|M|_{max}}{S} = \frac{162 \times 10^3}{1060 \times 10^{-6}} = 152.8 \text{ MPa}$$

$y_b = c - t_f = 190.7$ mm

$\sigma_b = \frac{y_b}{c} \sigma_m = 143.2$ MPa

$A_f = b_f t_f = 2278$ mm^2

$\bar{y} = \frac{1}{2}(c + y_b) = 197.1$ mm

$Q = A_f \bar{y} = (2278)(197.1) = 449 \times 10^3$ mm^3

$$\tau_b = \frac{VQ}{I t_w} = \frac{(90 \times 10^3)(449 \times 10^{-6})}{(216 \times 10^{-6})(7.7 \times 10^{-3})} = 24.3 \text{ MPa}$$

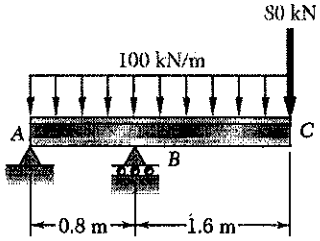
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{71.6^2 + 24.3^2} = 75.6 \text{ MPa}$$

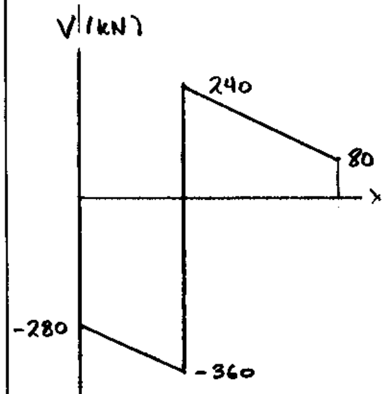
$$\sigma_{max} = \frac{\sigma_b}{2} + R = \sqrt{71.6 + 75.6} = 147.2 \text{ MPa}$$

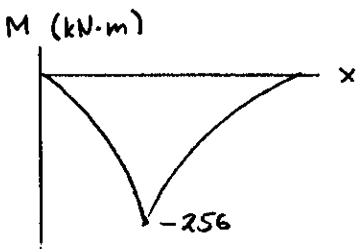
PROBLEM 8.10

8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.10 Loading of Prob. 5.86 and selected S 510 × 98.3 shape.







From Problem 5.86 $\sigma_{all} = 160$ MPa

$|M|_{max} = 256$ kN·m at point B

$|V| = 360$ kN at B

For S 510 × 98.3 rolled steel section

$d = 508$ mm, $b_f = 159$ mm, $t_f = 20.2$ mm

$t_w = 12.8$ mm, $I_x = 495 \times 10^6$ mm^4, $S_x = 1950 \times 10^3$ mm^3

$c = \frac{1}{2} d = 254$ mm

$$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{256 \times 10^3}{1950 \times 10^{-6}} = 131.3 \text{ MPa}$$

$y_b = c - t_f = 233.8$

$\sigma_b = \frac{y_b}{c} \sigma_m = 120.9$ MPa $\frac{\sigma_b}{2} = 60.45$ MPa

$A_f = b_f t_f = 3212$ mm^2

$\bar{y} = \frac{1}{2}(c + y_b) = 243.9$ mm

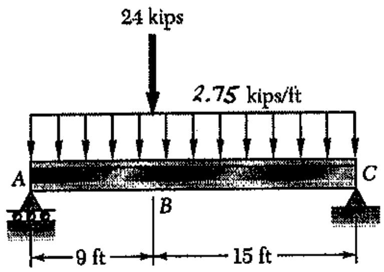
$Q = A_f \bar{y} = 783.4 \times 10^3$ mm^3

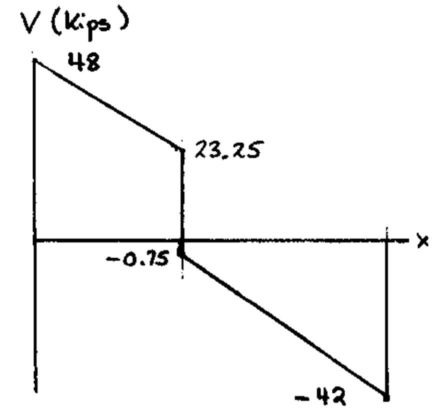
$$\tau_b = \frac{VQ}{I t_w} = \frac{(360 \times 10^3)(783.4 \times 10^{-6})}{(495 \times 10^{-6})(12.8 \times 10^{-3})} = 44.5 \text{ MPa}$$

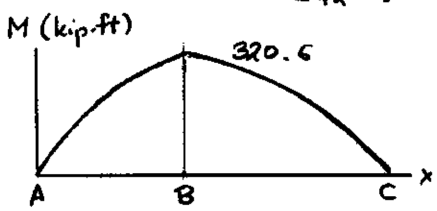
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{60.45^2 + 44.5^2} = 75.06 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 60.45 + 75.06 = 135.5 \text{ MPa}$$

PROBLEM 8.11







8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.11 Loading of Prob. 5.83 and selected W27 × 84 shape.

From Problem 5.83 $\quad \sigma_{all} = 24$ ksi

$$|M|_{max} = 320.6 \text{ kip·ft} = 3847 \text{ kip·in}$$

At B- $\quad |V| = 23.25$ kips

For W 27 × 84 rolled steel section

$d = 26.71$ in, $\quad b_f = 9.960$ in, $\quad t_f = 0.640$ in

$t_w = 0.460$ in, $\quad I_z = 2850 \text{ in}^4$, $\quad S_z = 213 \text{ in}^3$

$$c = \tfrac{1}{2}d = 13.355 \text{ in.}$$

$$\sigma_m = \frac{|M|_{max}}{S} = \frac{3847}{213} = 18.06 \text{ ksi}$$

$$y_b = c - t_f = 12.715 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = 17.20 \text{ ksi} \qquad \frac{\sigma_b}{2} = 8.60 \text{ ksi}$$

$$A_f = b_f t_f = (9.960)(0.640) = 6.3744 \text{ in}^2$$

$$\bar{y} = \tfrac{1}{2}(c + y_b) = 13.305 \text{ in}^2$$

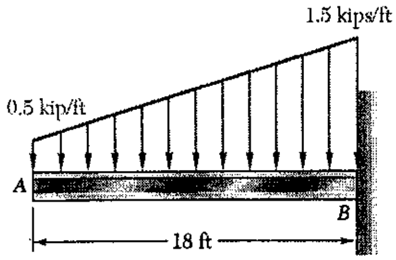
$$Q = A_f \bar{y} = 83.09 \text{ in}^3$$

$$\tau_b = \frac{VQ}{I_z t_w} = \frac{(23.25)(83.09)}{(2850)(0.460)} = 1.47 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{(8.60)^2 + (1.47)^2} = 8.72 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.60 + 8.72 \text{ ksi} = 17.32 \text{ ksi}$$

PROBLEM 8.12



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.12 Loading of Prob. 5.84 and selected W18 × 50 shape.

From Problem 5.84 $\quad \sigma_{all} = 24$ ksi

$|M|_{max} = 135$ kip·ft $= 1620$ kip·in at B

$|V|_{max} = 18$ kips at B

For W18×50 shape $\quad d = 17.99$ in, $b_f = 7.495$ in, $t_f = 0.570$ in.

$t_w = 0.355$ in, $I_z = 800$ in^4, $S_z = 88.9$ in^3, $c = \frac{1}{2}d = 8.995$ in

$$\sigma_m = \frac{|M|_{max}}{S_z} = 18.22 \text{ ksi}$$

$$y_b = c - t_f = 8.425 \text{ in}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = 17.07 \text{ ksi} \qquad \frac{\sigma_b}{2} = 8.535 \text{ ksi}$$

$$A_f = b_f t_f = 4.272 \text{ in}^2$$

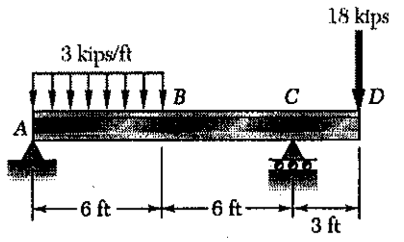
$$\bar{y} = \tfrac{1}{2}(c + y_b) = 8.71 \text{ in} \qquad Q = A_f \bar{y} = 37.21 \text{ in}^3$$

$$\tau_b = \frac{VQ}{I_z t_w} = \frac{(18)(37.21)}{(800)(0.355)} = 2.36 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.535^2 + 2.36^2} = 8.855 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.535 + 8.855 = 17.39 \text{ ksi}$$

PROBLEM 8.13



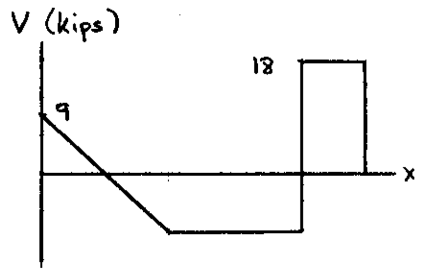
8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.13 Loading of Prob. 5.87 and selected S12 × 31.8 shape.

From Problem 5.87 $\sigma_{all} = 24$ ksi

$|M|_{max} = 54$ kip·ft $= 648$ kip·in at C

At C $|V| = 18$ kips

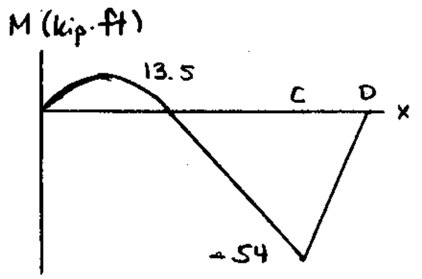


For S12 × 31.8

$d = 12.00$ in, $b_f = 5.00$ in, $t_f = 0.544$ in

$t_w = 0.350$ in $I_z = 218$ in^4, $S_z = 36.4$ in^3

$c = \frac{1}{2}d = 6.00$ in



$$\sigma_m = \frac{|M|}{S_z} = \frac{648}{36.4} = 17.80 \text{ ksi}$$

$y_b = c - t_f = 5.456$ in

$\sigma_b = \frac{y_b}{c}\sigma_m = 16.186$ ksi $\qquad \frac{\sigma_b}{2} = 8.093$ ksi

$A_f = b_f t_f = 2.72$ in^2

$\bar{y} = \frac{1}{2}(c + y_b) = 5.728$ in

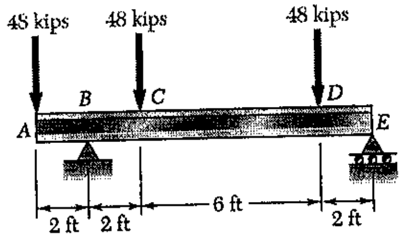
$Q = A_f \bar{y} = 15.58$ in^3

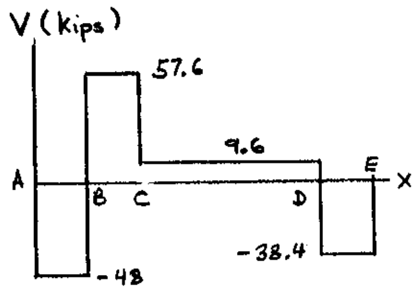
$$\tau_b = \frac{VQ}{I_z t_w} = \frac{(18)(15.58)}{(218)(0.350)} = 3.675 \text{ ksi}$$

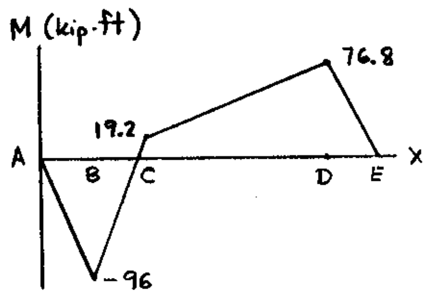
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.093^2 + 3.675^2} = 8.889 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.093 + 8.889 = 16.98 \text{ ksi}$$

PROBLEM 8.14







8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.14 Loading of Prob. 5.88 and selected S15 × 42.9 shape.

From Problem 5.88 $\quad \sigma_{all} = 24$ ksi

$|M|_{max} = 96$ kip·ft $= 1152$ kip·in at D

At D $\quad |V| = 38.4$ kips

For S 15 × 42.9 shape

$d = 15.00$ in, $\quad b_f = 5.501$ in. $\quad t_f = 0.622$ in

$t_w = 0.411$ in, $\quad I_z = 447$ in^4, $\quad S_z = 59.6$ in^3

$c = \frac{1}{2} d = 7.5$ in

$$\sigma_m = \frac{|M|}{S} = \frac{1152}{59.6} = 19.33 \text{ ksi}$$

$y_b = c - t_f = 6.878$ in.

$\sigma_b = \frac{y_b}{c} \sigma_m = 17.73$ ksi $\qquad \frac{\sigma_b}{2} = 8.86$ ksi

$A_f = b_f t_f = 3.4216$ in^2

$\bar{y} = \frac{1}{2}(c + y_b) = 7.189$ in

$Q = A_f \bar{y} = 24.60$ in^3

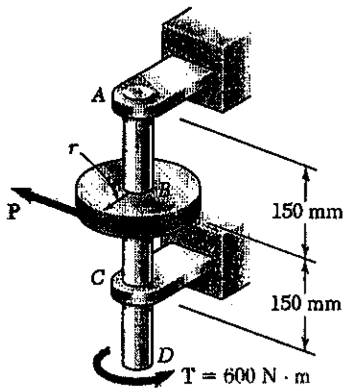
$$\tau_b = \frac{VQ}{I_z t_w} = \frac{(57.6)(24.60)}{(447)(0.411)} = 7.71 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.86^2 + 7.71^2} = 11.74 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.86 + 11.74 = 20.6 \text{ ksi}$$

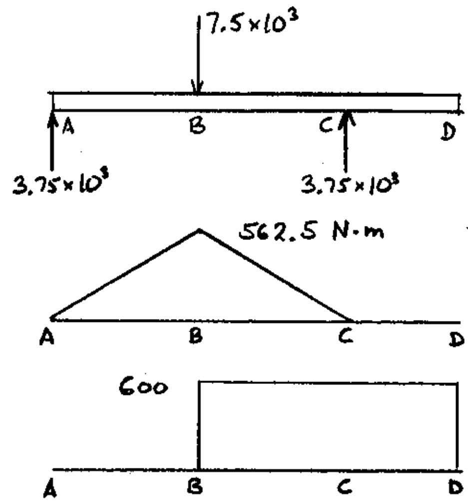
PROBLEM 8.15

8.15 Determine the smallest allowable diameter of the solid shaft *ABCD*, knowing that $\tau_{all} = 60$ MPa and that the radius of disk *B* is $r = 80$ mm.



SOLUTION

$\Sigma M_{axis} = 0 \qquad T - Pr = 0 \qquad P = \frac{T}{r} = \frac{600}{80 \times 10^{-3}} = 7.5 \times 10^{3}$ N



$R_A = R_C = \frac{1}{2}P$

$= 3.75 \times 10^{3}$ N

$M_B = (3.75 \times 10^{3})(150 \times 10^{-3})$

$= 562.5$ N·m

Bending moment

Torque

Critical section lies at point B $\qquad M = 562.5$ N·m, $T = 600$ N·m

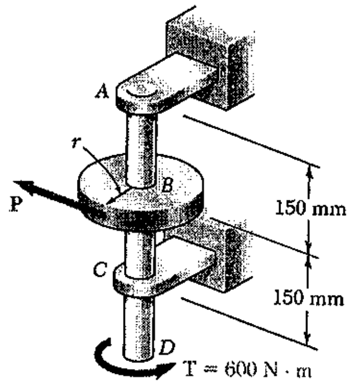
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{(\sqrt{M^2+T^2})_{max}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi}\frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{2}{\pi}\frac{\sqrt{(562.5)^2+(600)^2}}{60 \times 10^{6}} = 8.726 \times 10^{-6} \text{ m}^3$$

$c = 20.58 \times 10^{-3}$ m $\qquad d = 2c = 41.2 \times 10^{-3}$ m $= 41.2$ mm ◄

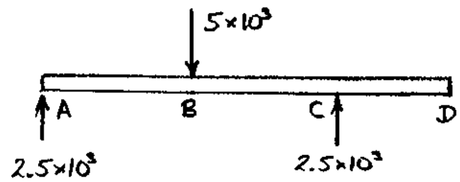
PROBLEM 8.16

8.16 Determine the smallest allowable diameter of the solid shaft *ABCD*, knowing that $\tau_{all} = 60$ MPa and that the radius of disk *B* is $r = 120$ mm



SOLUTION

$\Sigma M_{AD} = 0 \qquad T - Pr = 0 \qquad P = \frac{T}{r} = \frac{600}{120\times10^{-3}} = 5\times10^3$ N

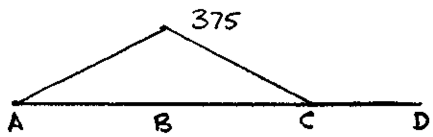


$R_A = R_C = \frac{1}{2}P$

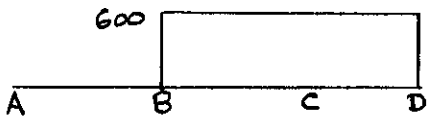
$= 2.5\times10^3$ N

$M_B = (2.5\times10^3)(0.150\times10^{-3})$

$= 375$ N·m



Bending moment



Torque

Critical section lies at point B $\qquad M = 375$ N·m , $T = 600$ N·m

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{(\sqrt{M^2+T^2})_{max}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi}\frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{2}{\pi}\frac{\sqrt{375^2+600^2}}{60\times10^6} = 7.507\times10^{-6}\ \text{m}^3$$

$c = 19.58\times10^{-3}$ m $\qquad d = 2c = 39.2\times10^{-3}$ m $= 39.2$ mm

PROBLEM 8.17

8.17 Using the notation of Sec. 8.3 and neglecting the effect of shearing stresses caused by transverse loads, show that the maximum normal stress in a cylindrical shaft can be expressed as

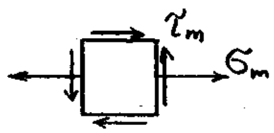
$$\sigma_{\max} = \frac{c}{J}\left[\left(M_y^2 + M_z^2\right)^{\frac{1}{2}} + \left(M_y^2 + M_z^2 + T^2\right)^{\frac{1}{2}}\right]_{\max}$$

SOLUTION

Maximum bending stress $\sigma_m = \frac{|M|c}{I} = \frac{\sqrt{M_y^2 + M_z^2}\, c}{I}$

Maximum torsional stress $\tau_m = \frac{Tc}{J}$

$$\frac{\sigma_m}{2} = \frac{\sqrt{M_y^2 + M_z^2}\, c}{2I} = \frac{c}{J}\sqrt{M_y^2 + M_z^2}$$



Using Mohr s circle

$$R = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \tau_m^2} = \sqrt{\frac{c^2}{J^2}(M_y^2 + M_z^2) + \frac{T^2 c^2}{J^2}}$$

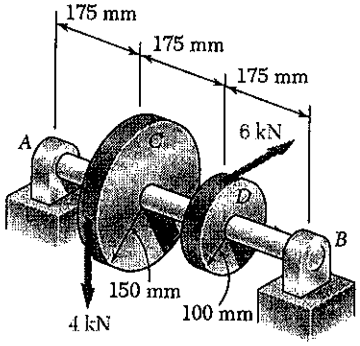
$$= \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$\sigma_{max} = \frac{\sigma_m}{2} + R = \frac{c}{J}\sqrt{M_y^2 + M_z^2} + \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

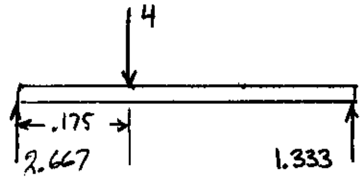
$$= \frac{c}{J}\left[(M_y^2 + M_z^2)^{\frac{1}{2}} + (M_y^2 + M_z^2 + T^2)^{\frac{1}{2}}\right] \blacktriangleleft$$

PROBLEM 8.18

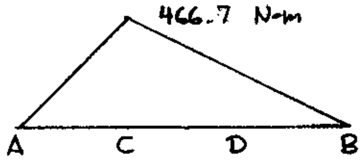
8.18 Use the expression given in Prob. 8.17 to determine the maximum normal stress in the solid shaft AB, knowing that its diameter is 36 mm.



SOLUTION

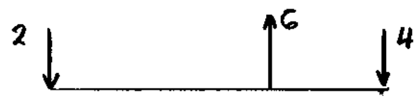


Vertical forces, kN

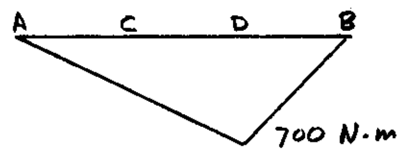


Bending moment M_z

$$M_{zC} = (0.175)(2.667 \times 10^3) = 466.7 \text{ N}\cdot\text{m}$$

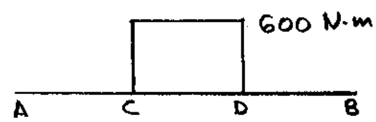


Horizontal forces, kN



Bending moment M_y

$$M_{yD} = (0.175)(4 \times 10^3) = 700 \text{ N}\cdot\text{m}$$



Torque

$$T = (6 \times 10^3)(100 \times 10^{-3}) = 600 \text{ N}\cdot\text{m}$$

At point C $\quad \sqrt{M_y^2 + M_z^2} = \sqrt{350^2 + 466.7^2} = 583.3 \text{ N}\cdot\text{m}$

At point D $\quad \sqrt{M_y^2 + M_z^2} = \sqrt{700^2 + 233.3^2} = 737.9 \text{ N}\cdot\text{m}$

Point D is critical

$$c = \tfrac{1}{2} d = 18 \text{ mm} = 18 \times 10^{-3} \text{ m}$$

$$J = \frac{\pi}{2} c^4 = 164.90 \times 10^3 \text{ mm}^4 = 164.90 \times 10^{-9} \text{ m}^4$$

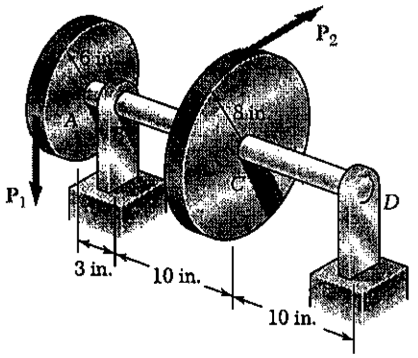
$$\sigma_{max} = \frac{c}{J}\left[\sqrt{M_y^2 + M_z^2} + \sqrt{M_y^2 + M_z^2 + T^2}\right]$$

$$= \frac{18 \times 10^{-3}}{164.90 \times 10^{-9}}\left[737.9 + \sqrt{737.9^2 + 600^2}\right] = 184.4 \times 10^6 \text{ Pa}$$

$$= 184.4 \text{ MPa}$$

PROBLEM 8.19

8.19 The vertical force $\mathbf{P}_1$ and the horizontal force $\mathbf{P}_2$ are applied as shown to disks welded to the solid shaft *AD*. Knowing that the diameter of the shaft is 1.75 in. and that $\tau_{all} = 8$ ksi, determine the largest permissible magnitude of the force $\mathbf{P}_2$.



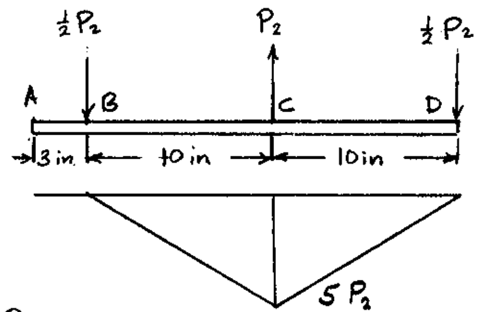
SOLUTION

Let P_2 be in kips.

$\Sigma M_{shaft} = 0 \qquad 6P_1 - 8P_2 = 0 \qquad P_1 = \frac{4}{3}P_2$

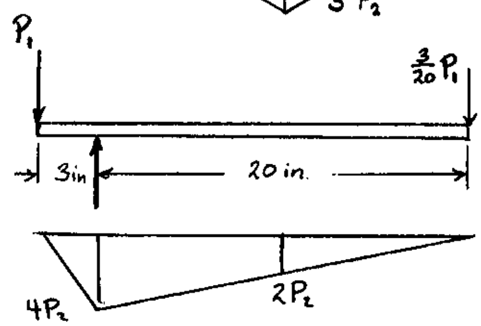
Torque over portion ABC $\qquad T = 8P_2$

Bending in horizontal plane



$M_{Cy} = 10 \cdot \frac{1}{2}P_2$
$= 5P_2$

Bending in vertical plane



$M_{Bz} = 3P_1$
$= 3 \cdot \frac{4}{3}P_2$
$= 4P_2$

Critical point is just to the left of point C.

$T = 8P_2 \qquad M_y = 5P_2 \qquad M_z = 2P_2$

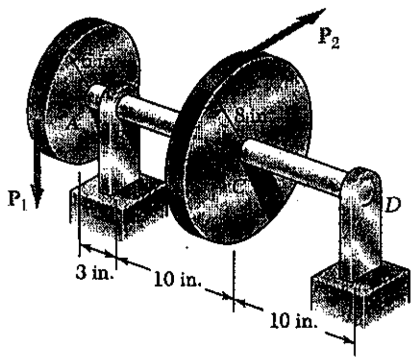
$d = 1.75$ in $\qquad c = \frac{1}{2}d = 0.875$ in $\qquad J = \frac{\pi}{2}(0.875)^4 = 0.92077$ in^4

$$\tau_{all} = \frac{c}{J}\sqrt{T^2 + M_y^2 + M_z^2}$$

$$8 = \frac{0.875}{0.92077}\sqrt{(8P_2)^2 + (5P_2)^2 + (2P_2)^2} = 9.164\ P_2$$

$P_2 = 0.873$ kips $= 873$ lb. ◄

PROBLEM 8.20



8.19 The vertical force $\mathbf{P}_1$ and the horizontal force $\mathbf{P}_2$ are applied as shown to disks welded to the solid shaft *AD*. Knowing that the diameter of the shaft is 1.75 in. and that $\tau_{all} = 8$ ksi, determine the largest permissible magnitude of the force $\mathbf{P}_2$.

8.20 Solve Prob. 8.19, assuming that the solid shaft *AD* has been replaced by a hollow shaft of the same material and of inner diameter 1.50 in. and outer diameter 1.75 in.

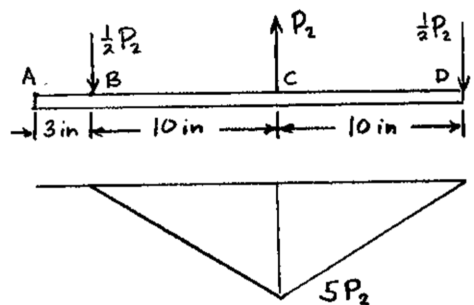
SOLUTION

Let P_2 be in kips

$\Sigma M_{shaft} = 0 \qquad 6P_1 - 8P_2 = 0 \qquad P_1 = \frac{4}{3} P_2$

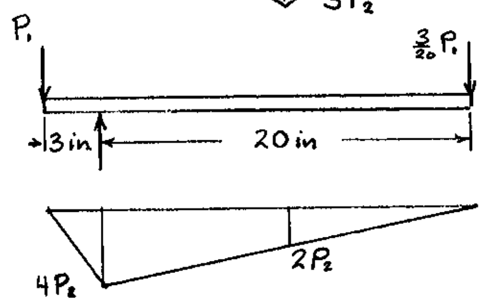
Torque over portion ABC $\qquad T = 8 P_2$

Bending in horizontal plane.



$M_{cy} = 10 \cdot \frac{1}{2} P_2$
$= 5 P_2$

Bending in vertical plane.



$M_{Bz} = 3 P_1$
$= 3 \cdot \frac{4}{3} P_2$
$= 4 P_2$

Critical point is just to the left of point C

$T = 8 P_2 \qquad M_y = 5 P_2 \qquad M_z = 2 P_2$

$c_o = \frac{1}{2} d_o = 0.875 \text{ in.} \qquad c_i = \frac{1}{2} d_i = 0.750 \text{ in}$

$J = \frac{\pi}{2}(c_o^4 - c_i^4) = 0.42376 \text{ in}^4$

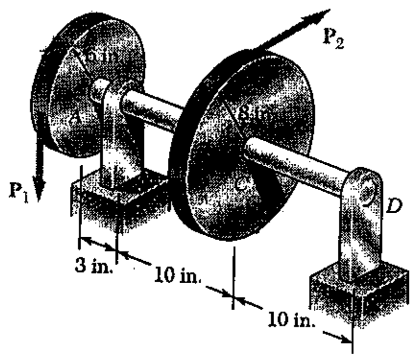
$\tau_{all} = \frac{c_o}{J}\sqrt{T^2 + M_y^2 + M_z^2}$

$8 = \frac{0.875}{0.42376}\sqrt{(8P_2)^2 + (5P_2)^2 + (2P_2)^2} = 19.913 P_2$

$P_2 = 0.402$ kips $= 402$ lb.

PROBLEM 8.22

8.22 Assuming that the magnitudes of the forces applied to disks A and C of Prob. 8.19 are, respectively, $P_1 = 1080$ lb and $P_2 = 810$ lb, and using the expressions given in Prob. 8.21, determine the values of τ_H and τ_K in a section (a) just to the left of B, (b) just to the left of C.



SOLUTION

From Prob. 8.19, shaft diameter = 1.75 in.

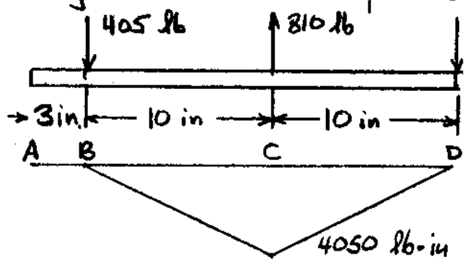
$c = \frac{1}{2}d = 0.875$ in

$J = \frac{\pi}{2}c^4 = 0.92077$

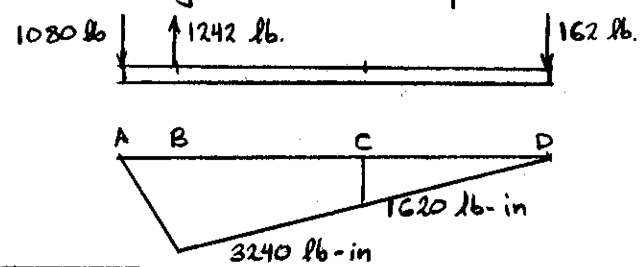
Torque over portion ABC

$T = (6)(1080) = (8)(810) = 6480$ lb-in

Bending in horizontal plane

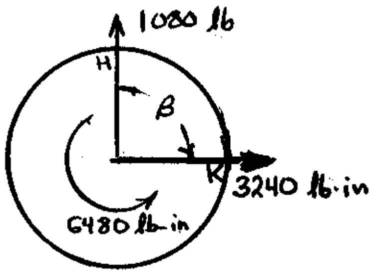


Bending in vertical plane



(a) Just to the left of point B $V = 1080$ lb. $M = 3240$ lb-in

$\beta = 90°$ $T = 6480$ lb-in



$$\tau_H = \frac{c}{J}\sqrt{M^2 + T^2} = \frac{0.875}{0.92077}\sqrt{(3240)^2 + (6480)^2} = 6880 \text{ psi}$$

$$\tau_K = \frac{c}{J}\sqrt{(M\cos\beta)^2 + (\tfrac{2}{3}Vc + T)^2} = \frac{c}{J}\left[\tfrac{2}{3}Vc + T\right]$$

$$= \frac{0.875}{0.92077}\left[(\tfrac{2}{3})(1080)(0.875) + 6480\right] = 6760 \text{ psi}$$

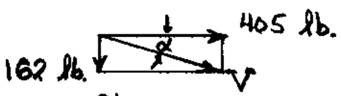
(b) Just to the left of point C

$V = \sqrt{(162)^2 + (405)^2} = 436.2$ lb.

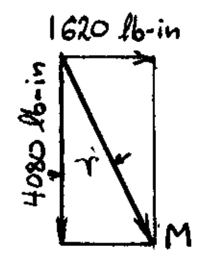
$\alpha = \tan^{-1}\frac{162}{405} = 21.80°$

$M = \sqrt{(1620)^2 + (4050)^2} = 4362$ lb-in

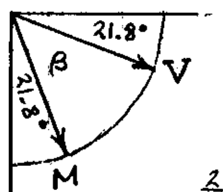
$\gamma = \tan^{-1}\frac{1620}{4050} = 21.80°$



Shear



Bending moment



$\beta = 90° - 21.8° - 21.8° = 46.4°$

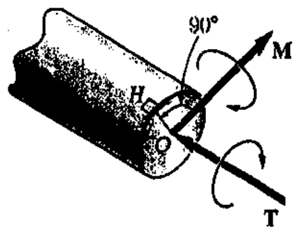
$$\tau_H = \frac{0.875}{0.92077}\sqrt{(6480)^2 + (4362)^2} = 7420 \text{ psi}$$

$\frac{2}{3}Vc + T = (\frac{2}{3})(436.2)(0.875) + 6480 = 6734$ lb-in

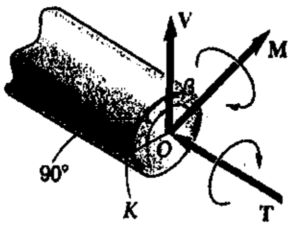
$M\cos\beta = 4362\cos 46.4° = 3008$ lb-in

$$\tau_K = \frac{0.875}{0.92077}\sqrt{(3008)^2 + (6734)^2} = 7010 \text{ psi}$$

PROBLEM 8.21



(*a*)



(*b*)

8.21 It was stated in Sec. 8.3 that the shearing stresses produced in a shaft by the transverse loads are usually much smaller than those produced by the torques. In the preceding problems their effect was ignored and it was assumed that the maximum shearing stress in a given section occurred at point *H* (Fig. P8.21*a*) and was equal to the expression obtained in Eq. (8.5), namely,

$$\tau_H = \frac{c}{J}\sqrt{M^2 + T^2}$$

Show that the maximum shearing stress at point *K* (Fig. P8.21*b*), where the effect of the shear *V* is greatest, can be expressed as

$$\tau_K = \frac{c}{J}\sqrt{\left(M\cos\beta\right)^2 + \left(\tfrac{2}{3}cV + T\right)^2}$$

where β is the angle between the vectors **V** and **M**. It is clear that the effect of the shear *V* cannot be ignored when $\tau_K \geq \tau_H$. (*Hint.* Only the component of **M** along **V** contributes to the shearing stress at *K*.)

SOLUTION

Shearing stress at point k

Due to V: For a semicircle $Q = \frac{2}{3}c^3$

For a circle cut across its diameter $t = d = 2c$

For a circular section $I = \frac{1}{2}J$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(V)(\frac{2}{3}c^3)}{(\frac{1}{2}J)(2c)} = \frac{2}{3}\frac{Vc^2}{J}$$

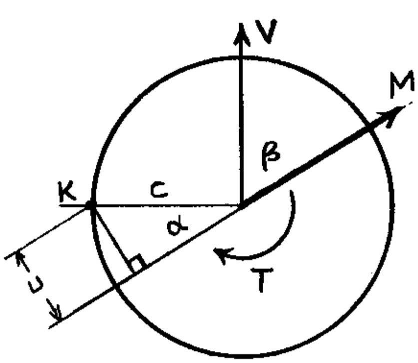
Due to T $\tau_{xy} = \frac{Tc}{J}$

Since these shearing stresses have the same orientation

$$\tau_{xy} = \frac{c}{J}\left(\tfrac{2}{3}Vc + T\right)$$

Bending stress at point K. $\sigma_x = \frac{Mu}{I} = \frac{2Mu}{J}$

where u is distance between point K and the neutral axis,



cross-section

$$u = c\sin\alpha = c\sin\left(\tfrac{\pi}{2} - \beta\right) = c\cos\beta$$

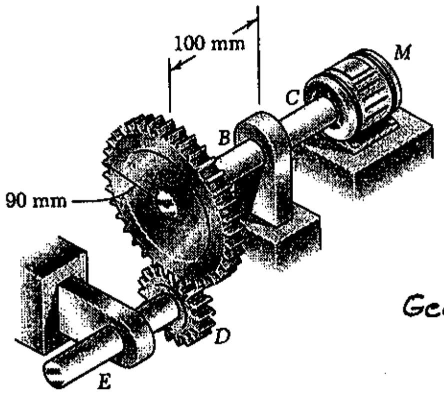
$$\sigma_x = \frac{2Mc\cos\beta}{J}$$

Using Mohr's circle

$$\tau_K = R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{c}{J}\sqrt{(M\cos\beta)^2 + \left(\tfrac{2}{3}Vc + T\right)^2}$$

PROBLEM 8.23



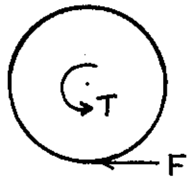
8.23 The solid shaft *ABC* and the gears shown are used to transmit 10 kW from the motor *M* to a machine tool connected to gear *D*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of shaft *ABC*.

SOLUTION

$$f = \frac{240 \text{ rpm}}{60 \text{ sec/min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10\times10^3}{(2\pi)(4)} = 397.89 \text{ N}\cdot\text{m}$$

Gear A



$$F r_A - T = 0$$

$$F = \frac{T}{r_A} = \frac{397.89}{90\times10^{-3}} = 4421 \text{ N}$$

Bending moment at B $\qquad M_B = L_{AB}F = (100\times10^{-3})(4421) = 442.1 \text{ N}\cdot\text{m}$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2+T^2}$$

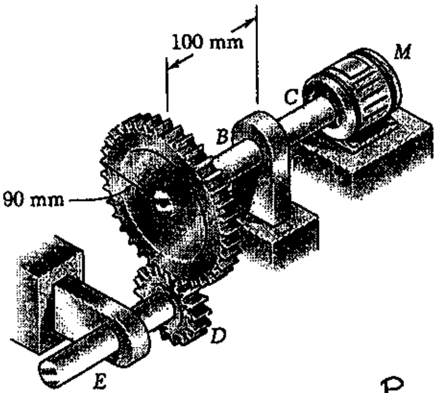
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi}\frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{(2)\sqrt{442.1^2+397.89^2}}{\pi(60\times10^6)} = 6.3108\times10^{-6} \text{ m}^3$$

$c = 18.479\times10^{-3}$ m $\qquad d = 2c = 37.0\times10^{-3}$ m $= 37.0$ mm ◄

PROBLEM 8.24

8.24 Assuming that shaft *ABC* of Prob. 8.23 is hollow and has an outer diameter of 50 mm, determine the largest permissible inner diameter of the shaft .



SOLUTION

From Prob. 8.23

Power tramitted $P = 10\ \text{kW}$

Motor speed $= 240\ \text{rpm} = 4\ \text{Hz}$

$\tau_{all} = 60\ \text{MPa}$

$$T = \frac{P}{2\pi f} = \frac{10\times10^3}{(2\pi)(4)} = 397.89\ \text{N}\cdot\text{m}$$

Gear A

$$Fr_A - T = 0$$

$$F = \frac{T}{r_A} = \frac{397.89}{90\times10^{-3}} = 4421\ \text{N}$$

Bending moment at B $\quad M_B = L_{AB}F = (100\times10^{-3})(4421) = 442.1\ \text{N}\cdot\text{m}$

$$\tau_{all} = \frac{c_o}{J}\sqrt{M^2+T^2} \qquad c_o = \tfrac{1}{2}d_o = 25\times10^{-3}\ \text{m}$$

$$\frac{J}{c_o} = \frac{\pi}{2}\frac{(c_o^4 - c_i^4)}{c_o} = \frac{\sqrt{M^2+T^2}}{\tau_{all}}$$

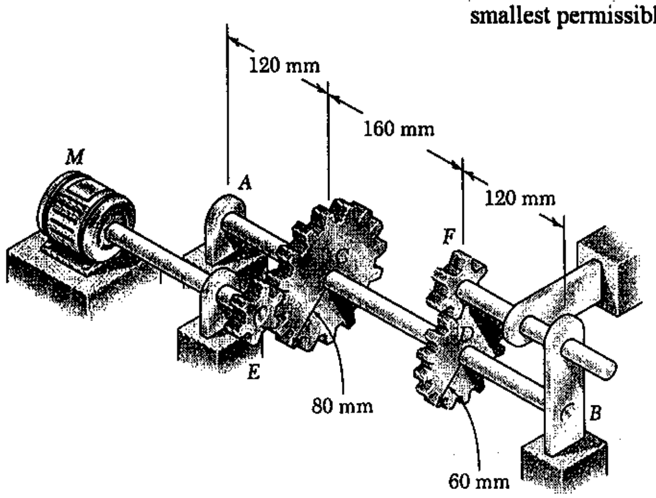
$$c_i^4 = c_o^4 - \frac{2c_o\sqrt{M^2+T^2}}{\pi\,\tau_{all}} = (25\times10^{-3})^4 - \frac{(2)(25\times10^{-3})\sqrt{442.1^2+397.89^2}}{\pi\,(60\times10^6)}$$

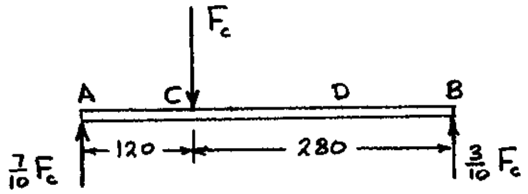
$$= 390.625\times10^{-9} - 157.772\times10^{-9} = 232.85\times10^{-9}$$

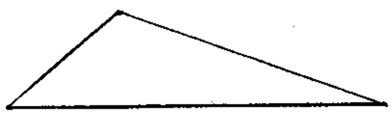
$$c_i = 21.967\times10^{-3}\ \text{m} \qquad d_i = 2c_i = 43.93\times10^{-3}\ \text{m} = 43.9\ \text{mm}$$

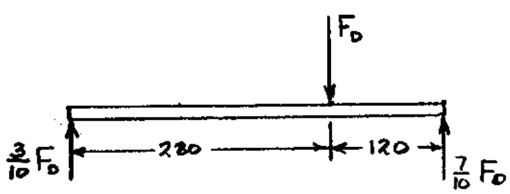
PROBLEM 8.25

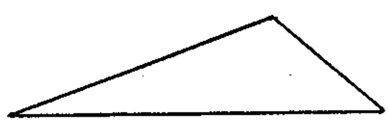
8.25 The solid shaft *AB* rotates at 600 rpm and transmits 80 kW from the motor *M* to a machine tool connected to gear *F*. Knowing that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of shaft *AB*.











SOLUTION

$$f = \frac{600\ \text{rpm}}{60\ \text{sec/min}} = 10\ \text{Hz}$$

$$T = \frac{P}{2\pi f} = \frac{80\times 10^3}{(2\pi)(10)} = 1273.2\ \text{N}\cdot\text{m}$$

Gear C $\quad F_c = \frac{T}{r_c}$

$$F_c = \frac{1273.2}{80\times 10^{-3}} = 15.913\times 10^3\ \text{N}$$

Gear D $\quad F_D = \frac{T}{r_D}$

$$F_D = \frac{1273.2}{60\times 10^{-3}} = 21.221\times 10^3\ \text{N}$$

Forces in vertical plane

$$M_{Cz} = (120\times 10^{-3})(\tfrac{7}{10}F_c) = 1336.7\ \text{N}\cdot\text{m}$$

$$M_{Dz} = \frac{120}{280}M_{Cz} = 572.9\ \text{N}\cdot\text{m}$$

Forces in horizontal plane

$$M_{Dy} = (120\times 10^{-3})(\tfrac{7}{10}F_D) = 1782.6\ \text{N}\cdot\text{m}$$

$$M_{Cy} = \frac{120}{280}M_{Dy} = 764.0\ \text{N}\cdot\text{m}$$

At C: $\sqrt{M_y^2 + M_z^2 + T^2} = 1997.9\ \text{N}\cdot\text{m}$

At D: $\sqrt{M_y^2 + M_z^2 + T^2} = 2264.3\ \text{N}\cdot\text{m}$

$$\tau_{all} = \frac{c}{J}\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max}$$

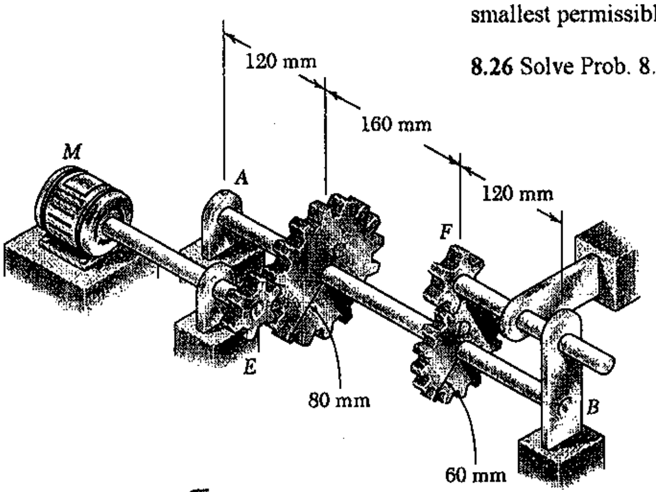
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max}}{\tau_{all}} = \frac{2264.3}{60\times 10^6} = 37.738\times 10^{-6}\ \text{m}^3$$

$c = 28.85\times 10^{-3}$ m $\qquad d = 2c = 57.7\times 10^{-3}$ m $= 57.7$ mm ◄

PROBLEM 8.26

8.25 The solid shaft AB rotates at 600 rpm and transmits 80 kW from the motor M to a machine tool connected to gear F. Knowing that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of shaft AB.

8.26 Solve Prob. 8.25, assuming that shaft AB rotates at 720 rpm



SOLUTION

$$f = \frac{720 \text{ rpm}}{60 \text{ sec/min}} = 12 \text{ Hz}$$

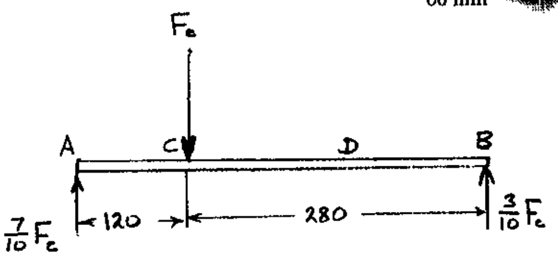
$$T = \frac{P}{2\pi f} = \frac{80\times10^3}{(2\pi)(12)} = 1061.0 \text{ N}\cdot\text{m}$$

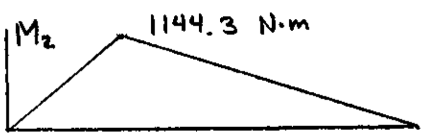
Gear C $\quad F_C = \frac{T}{r_C}$

$$F_C = \frac{1061.0}{80\times10^{-3}} = 13.262\times10^3 \text{ N}$$

Gear D $\quad F_D = \frac{T}{r_D}$

$$F_D = \frac{1061.0}{60\times10^{-3}} = 17.684\times10^3 \text{ N}$$

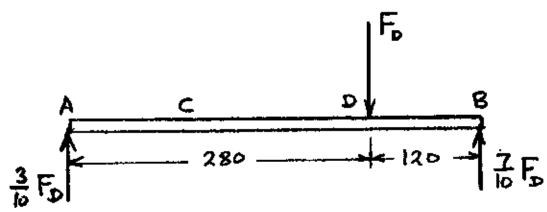


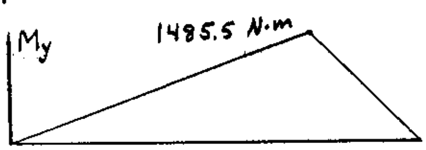


Forces in vertical plane

$$M_{Cz} = (120\times10^{-3})(\tfrac{7}{10}F_C) = 1114.0 \text{ N}\cdot\text{m}$$

$$M_{Dz} = \frac{120}{280}M_{Cz} = 477.4 \text{ N}\cdot\text{m}$$





Forces in horizontal plane

$$M_{Dy} = (120\times10^{-3})(\tfrac{7}{10}F_D) = 1485.5 \text{ N}\cdot\text{m}$$

$$M_{Cy} = \frac{120}{280}M_{Dy} = 636.6 \text{ N}\cdot\text{m}$$

At C: $\sqrt{M_y^2 + M_z^2 + T^2} = 1664.9$ N·m

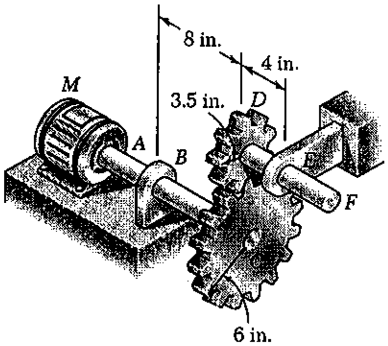
At D: $\sqrt{M_y^2 + M_z^2 + T^2} = 1886.9$ N·m

$$\tau_{all} = \frac{c}{J}\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max}}{\tau_{all}} = \frac{1886.9}{60\times10^6} = 31.448\times10^{-6} \text{ m}^3$$

$c = 27.15\times10^{-3}$ m $\qquad d = 2c = 54.3\times10^{-3}$ m $= 54.3$ mm ◄

PROBLEM 8.27



8.27 The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 20 hp from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 7.5$ ksi, determine the smallest permissible diameter of (*a*) shaft *ABC*, (*b*) shaft *DEF*.

SOLUTION

$$20 \text{ hp} = (20)(6600) = 132 \times 10^3 \text{ in}\cdot\text{lb/s}$$

$$240 \text{ rpm} = \frac{240}{60} = 4 \text{ Hz}$$

(a) Shaft ABC $\quad T = \dfrac{P}{2\pi f} = \dfrac{132 \times 10^3}{(2\pi)(4)} = 5252 \text{ in}\cdot\text{lb}$

Gear C $\quad F_{CD} = \dfrac{T}{r_c} = \dfrac{5252}{6} = 875.4 \text{ lb.}$

Bending moment at B $\quad M_B = (8)(875.4) = 7003 \text{ in}\cdot\text{lb.}$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{\sqrt{(5252)^2 + (7003)^2}}{7500} = 1.1671 \text{ in}^3$$

$c = 0.9057$ in $\qquad d = 2c = 1.811$ in. ◄

(b) Shaft DEF $\quad T = r_D F_{CD} = (3.5)(875.4) = 3064 \text{ in}\cdot\text{lb.}$

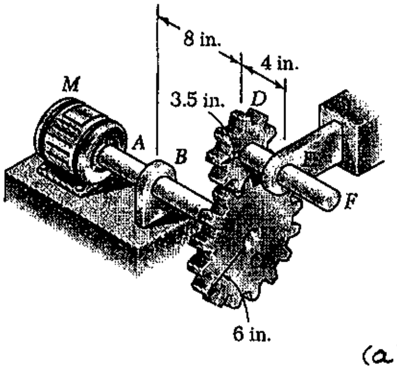
Bending moment at E $\quad M_E = (4)(875.4) = 3502 \text{ in}\cdot\text{lb.}$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{\sqrt{(3502)^2 + (3064)^2}}{7500} = 0.6204 \text{ in}^3$$

$c = 0.7337$ in $\qquad d = 2c = 1.467$ in. ◄

PROBLEM 8.28



8.27 The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 20 hp from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 7.5$ ksi, determine the smallest permissible diameter of (*a*) shaft *ABC*, (*b*) shaft *DEF*.

8.28 Solve Prob. 8.27, assuming that the motor rotates at 360 rpm.

SOLUTION

$$20 \text{ hp} = (20)(6600) = 132 \times 10^3 \text{ in}\cdot\text{lb/s}$$

$$360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

(a) Shaft ABC $\quad T = \frac{P}{2\pi f} = \frac{132 \times 10^3}{(2\pi)(6)} = 3501 \text{ in}\cdot\text{lb}$

Gear C $\quad F_{CD} = \frac{T}{r_C} = \frac{3501}{6} = 583.6 \text{ lb.}$

Bending moment at B $\quad M_B = (8)(583.6) = 4669 \text{ in}\cdot\text{lb.}$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{\sqrt{4669^2 + 3501^2}}{7500} = 0.77806 \text{ in}^3$$

$c = 0.791$ in. $\qquad d = 2c = 1.582$ in. ◄

(b) Shaft DEF $\quad T = r_D F_{CD} = (3.5)(583.6) = 2043 \text{ in}\cdot\text{lb.}$

Bending moment at E $\quad M_E = (4)(583.6) = 2334 \text{ in}\cdot\text{lb.}$

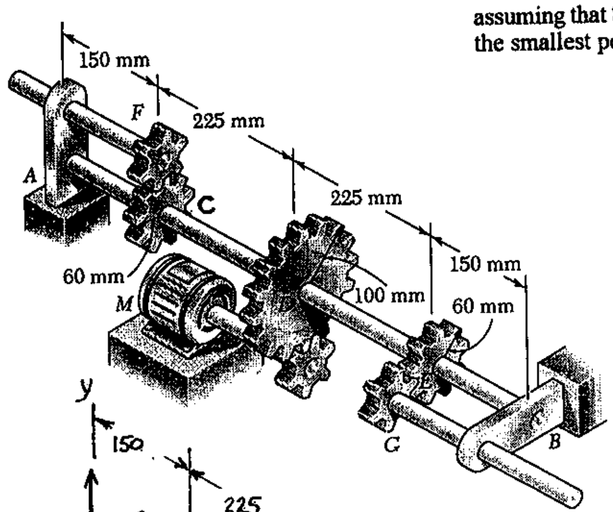
$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

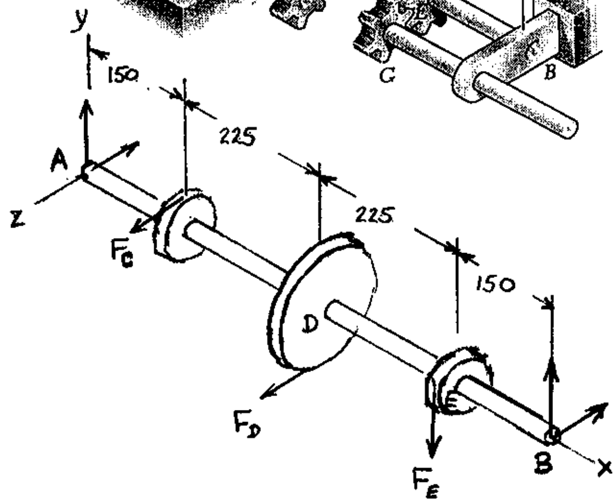
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{\sqrt{2334^2 + 2043^2}}{7500} = 0.41362 \text{ in}^3$$

$c = 0.6410$ in $\qquad d = 2c = 1.282$ in. ◄

PROBLEM 8.29

8.29 The solid shaft AB rotates at 450 rpm and transmits 20 kW from the motor M to machine tools connected to gears F and G. Knowing that $\tau_{all} = 55$ MPa and assuming that 8 kW is taken off at gear F and 12 kW is taken off at gear G, determine the smallest permissible diameter of shaft AB.





SOLUTION

$$f = \frac{450}{60} = 7.5 \text{ Hz}$$

Torque applied at D

$$T_D = \frac{P}{2\pi f} = \frac{20\times10^3}{(2\pi)(7.5)} = 424.41 \text{ N·m}$$

Torques on gears C and E

$$T_C = \frac{8}{20} T_D = 169.76 \text{ N·m}$$

$$T_E = \frac{12}{20} T_D = 254.65 \text{ N·m}$$

Forces on gears

$$F_D = \frac{T_D}{r_D} = \frac{424.41}{100\times10^{-3}} = 4244 \text{ N}$$

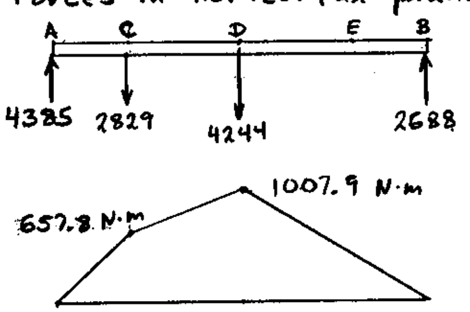
$$F_C = \frac{T_C}{r_C} = \frac{169.76}{60\times10^{-3}} = 2829 \text{ N}$$

$$F_E = \frac{T_E}{r_E} = \frac{254.65}{60\times10^{-3}} = 4244 \text{ N}$$

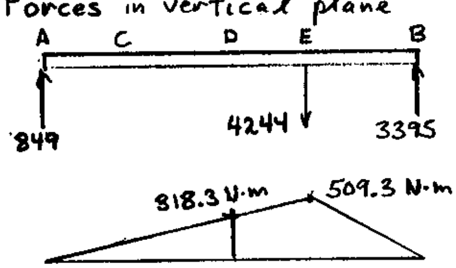
Torques in various parts

AC : $T = 0$
CD : $T = 169.76$ N·m
DE : $T = 254.65$ N·m
EB : $T = 0$

Forces in horizontal plane:



Forces in vertical plane



Critical point lies just the right of D

$T = 254.65$ N·m

$M_y = 1007.9$ N·m

$M_z = 318.3$ N·m

$$\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max} = 1087.2 \text{ N·m}$$

$$\tau_{all} = \frac{c}{J}\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max}}{\tau_{all}} = \frac{1087.2}{55\times10^6}$$

$$= 19.767\times10^{-3} \text{ m}$$

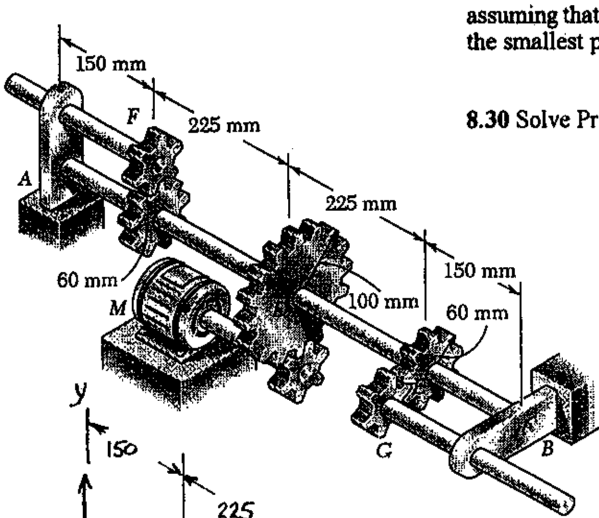
$$c = 23.26\times10^{-3} \text{ m}$$

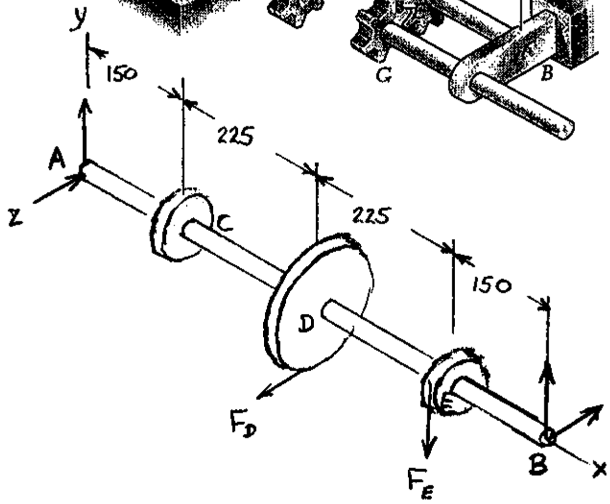
$$d = 2c = 46.5\times10^{-3} \text{ m} = 46.5 \text{ mm}$$

PROBLEM 8.30

8.29 The solid shaft AB rotates at 450 rpm and transmits 20 kW from the motor M to machine tools connected to gears F and G. Knowing that $\tau_{all} = 55$ MPa and assuming that 8 kW is taken off at gear F and 12 kW is taken off at gear G, determine the smallest permissible diameter of shaft AB.

8.30 Solve Prob. 8.29, assuming that the entire 20 kW is taken off at gear G.





SOLUTION

$$f = \frac{450}{60} = 7.5 \text{ Hz}$$

Torque applied at D

$$T_D = \frac{P}{2\pi f} = \frac{20\times10^3}{(2\pi)(7.5)} = 424.41 \text{ N}\cdot\text{m}$$

Torque on gear E

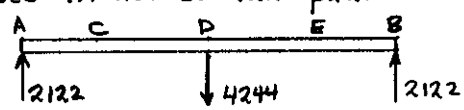
$$T_E = T_D = 424.41 \text{ N}\cdot\text{m}$$

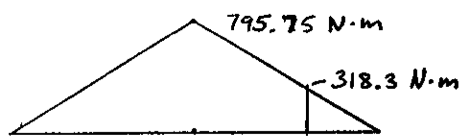
Forces on gears D and E

$$F_D = \frac{T_D}{r_D} = \frac{424.41}{100\times10^{-3}} = 4244 \text{ N}$$

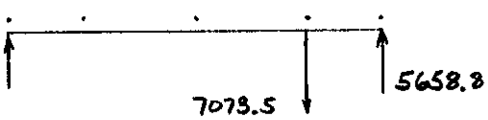
$$F_E = \frac{T_E}{r_D} = \frac{424.41}{60\times10^{-3}} = 7073.5 \text{ N}$$

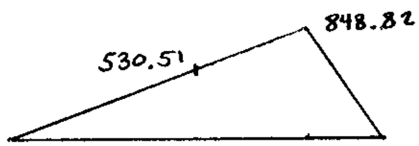
Forces in horizontal plane





Forces in vertical plane





Bending moments

$$M_D = \sqrt{530.51^2 + 795.75^2} = 956.4 \text{ N}\cdot\text{m}$$

$$M_E = \sqrt{848.82^2 + 318.3^2} = 906.5 \text{ N}\cdot\text{m}$$

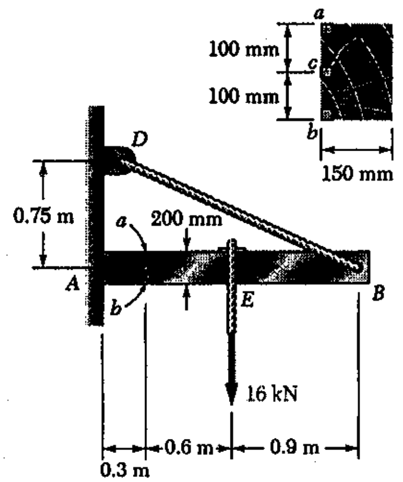
$$\left(\sqrt{M^2+T^2}\right)_{max} = \sqrt{956.4^2 + 424.41^2} = 1046.3 \text{ N}\cdot\text{m}$$

$$\tau_{all} = \frac{c}{J}\left(\sqrt{M^2+T^2}\right)_{max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{1046.3}{55\times10^6} = 19.024\times10^{-6} \text{ m}^3$$

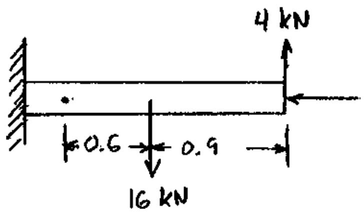
$$c = 22.96\times10^{-3} \text{ m} \qquad d = 2c = 45.9\times10^{-3} \text{ m} = 45.9 \text{ mm}$$

PROBLEM 8.31



8.31 The cantilever beam *AB* has a rectangular cross section of 150 × 200 mm. Knowing that the tension in cable *BD* is 10.4 kN and neglecting the weight of the beam, determine the normal and shearing stresses at the three points indicated.

SOLUTION



$\overline{DB} = \sqrt{.75^2 + 1.8^2}$
$= 1.95 \text{ m}$

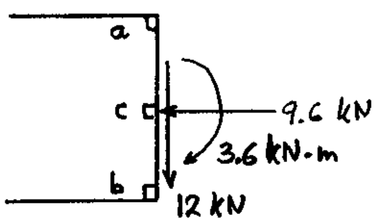
Vertical component of T_{DB}
$\left(\frac{0.75}{1.95}\right)(10.4) = 4 \text{ kN}$

Horizontal component of T_{DB} $\quad \left(\frac{1.8}{1.95}\right)(10.4) = 9.6 \text{ kN}$

At section containing points a, b, and c

$P = -9.6 \text{ kN} \qquad\qquad 16 - 4 = 12 \text{ kN}$

$M = (1.5)(4) - (0.6)(16) = -3.6 \text{ kN}\cdot\text{m}$



Section properties

$A = (0.150)(0.200) = 0.030 \text{ m}^2$

$I = \frac{1}{12}(0.150)(0.200)^3 = 100 \times 10^{-6} \text{ m}^4$

$c = 0.100 \text{ m}$

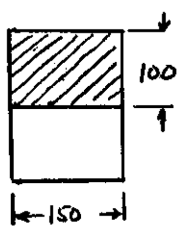
At point a $\quad \sigma_x = -\frac{P}{A} + \frac{Mc}{I} = -\frac{9.6 \times 10^3}{0.030} + \frac{(3.6 \times 10^3)(0.100)}{100 \times 10^{-6}} = 3.28 \text{ MPa}$

$\tau_{xy} = 0$

At point b $\quad \sigma_x = -\frac{P}{A} + \frac{Mc}{I} = -\frac{9.6 \times 10^3}{0.030} - \frac{(3.6 \times 10^3)(0.100)}{100 \times 10^{-6}} = -3.92 \text{ MPa}$

$\tau_{xy} = 0$

At point c $\quad \sigma_x = -\frac{P}{A} = -\frac{9.6 \times 10^3}{0.030} = -0.320 \text{ MPa}$

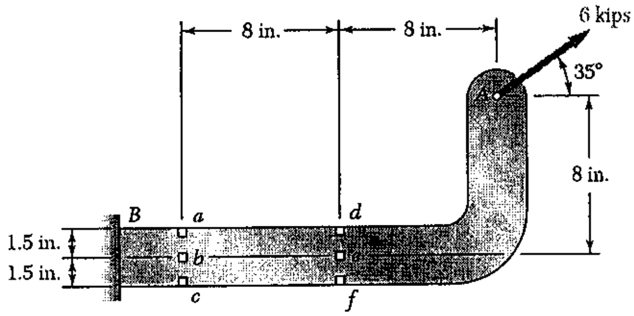


$Q = (150)(100)(50) = 750 \times 10^3 \text{ mm}^3 = 750 \times 10^{-6} \text{ m}^3$

$\tau_{xy} = -\frac{VQ}{It} = -\frac{(12 \times 10^3)(750 \times 10^{-6})}{(100 \times 10^{-6})(0.150)} = -0.600 \text{ MPa}$

PROBLEM 8.32

8.32 A 6-kip force is applied to the machine element *AB* as shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.



SOLUTION

thickness = 0.8 in.

At the section containing points a, b, and c

$P = 6\cos 35° = 4.9149$ kips $\qquad V = 6\sin 35° = 3.4415$ kips

$M = (6\sin 35°)(16) - (6\cos 35°)(8) = 15.744$ kip·in.

$A = (0.8)(3.0) = 2.4\ \text{in}^2$

$I = \frac{1}{12}(0.8)(3.0)^3 = 1.80\ \text{in}^4$

(a) At point a $\qquad \sigma_x = \frac{P}{A} - \frac{Mc}{I} = \frac{4.9149}{2.4} - \frac{(15.744)(1.5)}{1.80} = -11.07$ ksi

$\tau_{xy} = 0$

(b) At point b $\qquad \sigma_x = \frac{P}{A} = \frac{4.9149}{2.4} = 2.05$ ksi

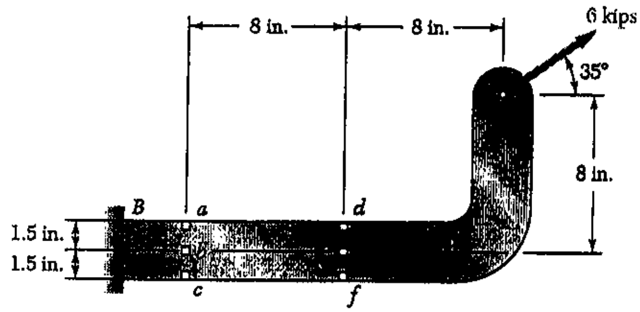
$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\cdot\frac{3.4415}{2.4} = 2.15$ ksi

(c) At point c $\qquad \sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{4.9149}{2.4} + \frac{(15.744)(1.5)}{1.8} = 15.17$ ksi

$\tau_{xy} = 0$

PROBLEM 8.33

8.33 A 6-kip force is applied to the machine element *AB* as shown. Determine the normal and shearing stresses at (*a*) point *d*, (*b*) point *e*, (*c*) point *f*



SOLUTION

thickness = 0.8 in

At the section containing points d, e, and f

$P = 6\cos 35° = 4.9149$ kips $\qquad V = 6\sin 35° = 3.4415$ kips

$M = (6\sin 35°)(8) - (6\cos 35°)(8) = -11.788$ kip·in.

$A = (0.8)(3.0) = 2.4$ in^2

$I = \frac{1}{12}(0.8)(3.0)^3 = 1.80$ in^4

(a) At point d $\qquad \sigma_x = \frac{P}{A} - \frac{Mc}{I} = \frac{4.9149}{2.4} + \frac{(11.788)(1.5)}{1.8} = 11.87$ ksi ◀

$\tau_{xy} = 0$ ◀

(b) At point e $\qquad \sigma_x = \frac{P}{A} = \frac{4.9149}{2.4} = 2.05$ ksi ◀

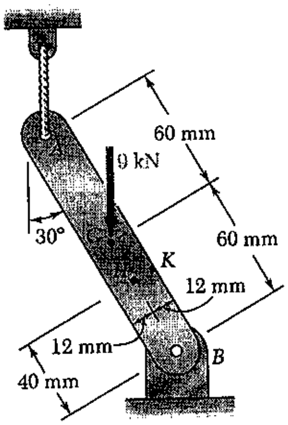
$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\cdot\frac{3.4415}{2.4} = 2.15$ ksi ◀

(c) At point f $\qquad \sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{4.9149}{2.4} - \frac{(11.788)(1.5)}{1.8} = -7.78$ ksi ◀

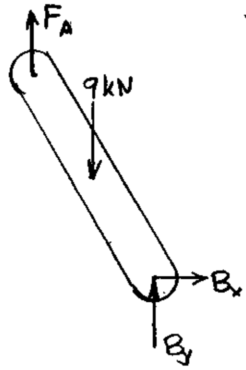
$\tau_{xy} = 0$ ◀

PROBLEM 8.34

8.34 through 8.36 Member *AB* has a uniform rectangular cross section of 10 × 24 mm. For the loading shown, determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.



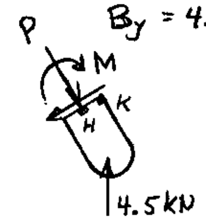
SOLUTION



$\Sigma F_x = 0 \qquad B_x = 0$

$\Sigma M_A = 0 \qquad B_y(120 \sin 30° - 9(60 \sin 30°) = 0$

$B_y = 4.5 \text{ kN}$



At the section containing points H and K

$P = 4.5 \cos 30° = 3.897 \text{ kN} \qquad V = 4.5 \sin 30° = 2.25 \text{ kN}$

$M = (4.5 \times 10^3)(40 \times 10^{-3} \sin 30°) = 90 \text{ N·m}.$

$A = 10 \times 24 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$

$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{ mm}^4 = 11.52 \times 10^{-9} \text{ m}^4$

(a) At point H $\qquad \sigma_x = -\frac{P}{A} = -\frac{3.897 \times 10^3}{240 \times 10^{-6}} = -16.24 \text{ MPa}$ ◀

$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25 \times 10^3}{240 \times 10^{-6}} = 14.06 \text{ MPa}$ ◀

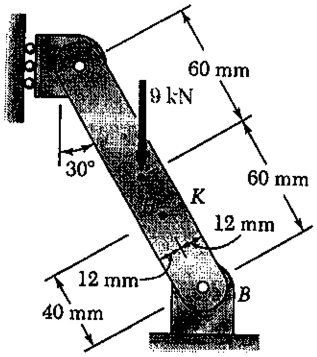
(b) At point K $\qquad \sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{3.897 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$

$- 110.0 \text{ MPa}$ ◀

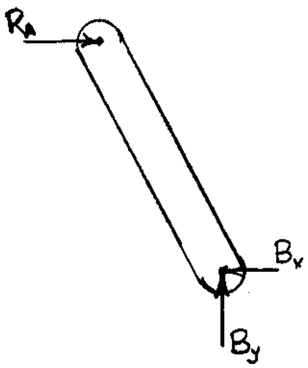
$\tau_{xy} = 0$ ◀

PROBLEM 8.35

8.34 through 8.36 Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stresses at (*a*) point H, (*b*) point K.



SOLUTION



$$\circlearrowleft \Sigma M_B = 0$$

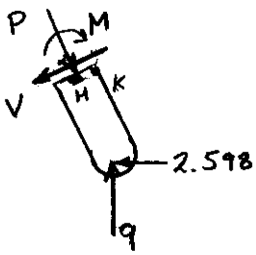
$$(120 \cos 30°) R_A - (60 \sin 30°)(9) = 0$$

$$R_A = 2.598 \text{ kN}$$

$+\uparrow \Sigma F_y = 0 \qquad B_y - 9 = 0 \qquad B_y = 9 \text{ kN} \uparrow$

$+\rightarrow \Sigma F_x = 0 \qquad 2.598 - B_x = 0$

$$B_x = 2.598 \text{ kN} \leftarrow$$



At the section containing points H and K

$$P = 9 \cos 30° + 2.598 \sin 30° = 9.093 \text{ kN}$$

$$V = 9 \sin 30° - 2.598 \cos 30° = 2.25 \text{ kN}$$

$$M = (9 \times 10^3)(40 \times 10^{-3} \sin 30°) - (2.598 \times 10^3)(40 \times 10^{-3} \cos 30°) = 90 \text{ N}\cdot\text{m}$$

$$A = 10 \times 240 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{ mm}^4 = 11.52 \times 10^{-9} \text{ m}^4$$

(a) At point H

$$\sigma_x = -\frac{P}{A} = -\frac{9.093 \times 10^3}{240 \times 10^{-6}} = -37.9 \text{ MPa} \blacktriangleleft$$

$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25 \times 10^3}{240 \times 10^{-6}} = 14.06 \text{ MPa} \blacktriangleleft$$

(b) At point K

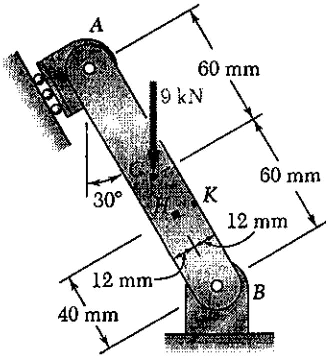
$$\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{9.093 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$$

$$= -131.6 \text{ MPa} \blacktriangleleft$$

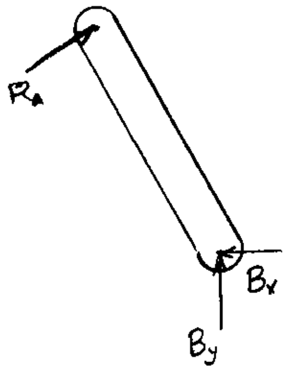
$$\tau_{xy} = 0 \blacktriangleleft$$

PROBLEM 8.36

8.34 through 8.36 Member *AB* has a uniform rectangular cross section of 10 × 24 mm. For the loading shown, determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.



SOLUTION



$+\circlearrowleft \Sigma M_B = 0$

$(9)(60 \sin 30^\circ) - 120 R_A = 0$

$R_A = 2.25 \text{ kN}$

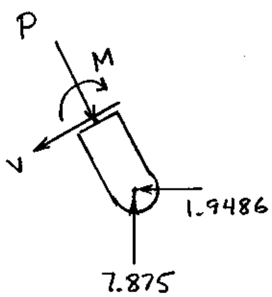
$\xrightarrow{+} \Sigma F_x = 0 \qquad 2.25 \cos 30^\circ - B_x = 0$

$B_x = 1.9486 \text{ kN} \leftarrow$

$+\uparrow \Sigma F_y = 0$

$2.25 \sin 30^\circ - 9 + B_y = 0$

$B_y = 7.875 \text{ kN} \uparrow$



At the section containing points H and K

$P = 7.875 \cos 30^\circ + 1.9486 \sin 30^\circ = 7.794 \text{ kN}$

$V = 7.875 \sin 30^\circ - 1.9486 \cos 30^\circ = 2.25 \text{ kN}$

$M = (7.875\times10^3)(40\times10^{-3} \sin 30^\circ) - (1.9486\times10^3)(40\times10^{-3}\cos 30^\circ) = 90 \text{ N·m}$

$A = 10 \times 24 = 240 \text{ mm}^2 = 240\times10^{-6} \text{ m}^2$

$I = \frac{1}{12}(10)(24)^3 = 11.52\times10^3 \text{ mm}^4 = 11.52\times10^{-9} \text{ m}^4$

(a) At point H $\quad \sigma_x = -\frac{P}{A} = -\frac{7.794\times10^3}{240\times10^{-6}} = -32.5 \text{ MPa}$ ◄

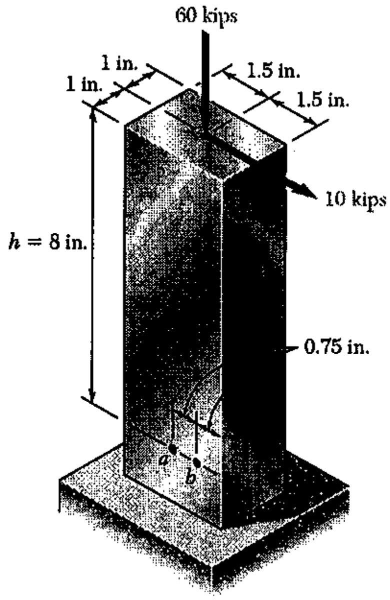
$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\cdot\frac{2.25\times10^3}{240\times10^{-6}} = 14.06 \text{ MPa}$ ◄

(b) At point K $\quad \sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{7.794\times10^3}{240\times10^{-6}} - \frac{(90)(12\times10^{-3})}{11.52\times10^{-9}}$

$= -126.2 \text{ MPa}$ ◄

$\tau_{xy} = 0$ ◄

PROBLEM 8.37



8.37 Two forces are applied to the bar shown. At point *a*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.

SOLUTION

At the section containing point a and b.

$V = 10$ kips $\qquad P = 60$ kips (compression)

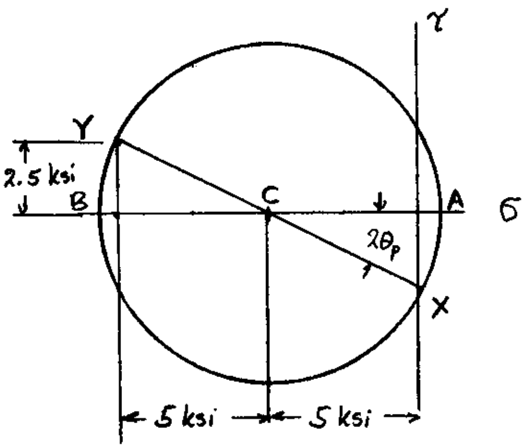
$M = (8)(10) = 80$ kip-in

$A = (2)(3) = 6 \text{ in}^2$

$I = \frac{1}{12}(2)(3)^3 = 4.5$ kip·in

At point a $\quad \sigma_y = -\frac{P}{A} = -\frac{60}{6} = -10$ ksi

$\tau = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{(10)}{6} = 2.5$ ksi, $\sigma_x = 0$



Use Mohr's circle

$\sigma_c = -5$ ksi

$R = \sqrt{5^2 + 2.5^2} = 5.590$ ksi

$\sigma_a = \sigma_c + R = 0.590$ ksi ◄

$\sigma_b = \sigma - R = -10.59$ ksi ◄

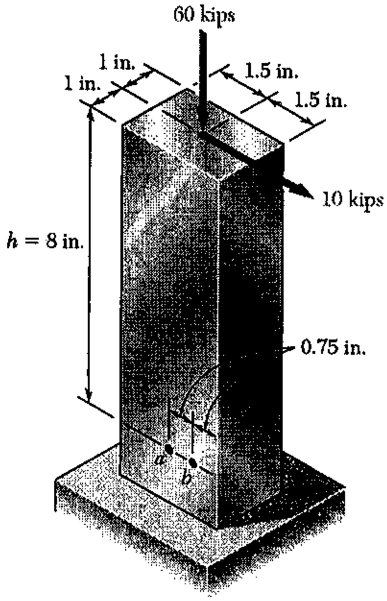
$\tan 2\theta_p = \frac{2.5}{5} = 0.5$

$\theta_p = 13.3°, \; 103.3°$ ◄

$\tau_{max} = R = 5.59$ ksi ◄

PROBLEM 8.38

8.38 Two forces are applied to the bar shown. At point *b*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.



SOLUTION

At the section containing points a and b

$V = 10$ kips , $P = 60$ kips (compression)

$M = (8)(10) = 80$ kip·in.

$A = (2)(3) = 6\ in^2$

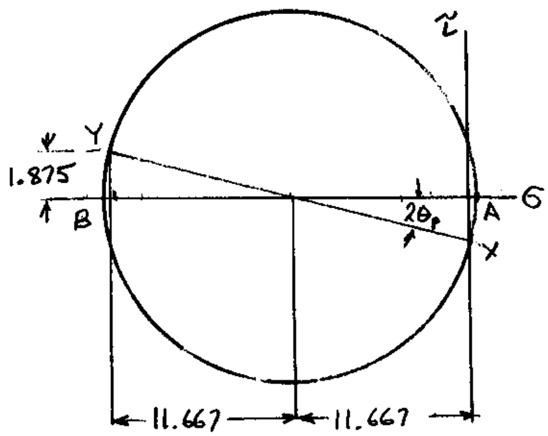
$I = \frac{1}{12}(2)(3)^3 = 4.5\ in^3$

At point b $\sigma_x = 0$

$$\sigma_y = -\frac{P}{A} - \frac{Mx}{I} = -\frac{60}{6} - \frac{(80)(0.75)}{4.5} = -23.33 \text{ ksi}$$

$$\tau = \frac{VQ}{It} = \frac{(10)(2)(0.75)(1.125)}{(4.5)(2)} = 1.875 \text{ ksi}$$

Use Mohr's circle



$\sigma_c = -11.667$ ksi

$R = \sqrt{11.667^2 + 1.875^2} = 11.8164$ ksi

$\sigma_a = \sigma_c + R = 0.150$ ksi ◀

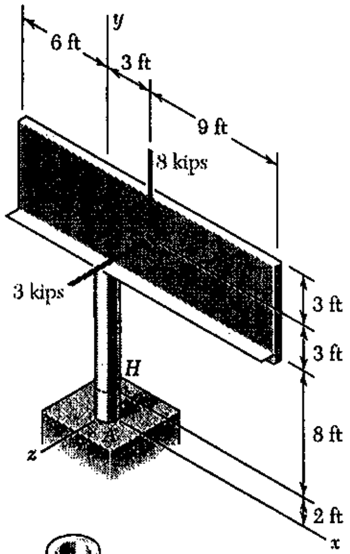
$\sigma_b = \sigma_c - R = -23.5$ ksi ◀

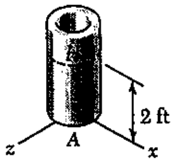
$\tan 2\theta_p = \frac{1.875}{11.667} = 0.16071$

$\theta_p = 4.6°,\ 94.6°$ ◀

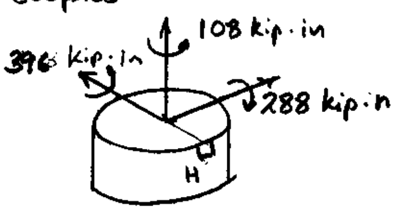
$\tau_{max} = R = 11.82$ ksi ◀

PROBLEM 8.39





Couples



8.39 The billboard shown weighs 8,000 lb and is supported by a structural tube that has a 15-in. outer diameter and a 0.5-in. wall thickness. At a time when the resultant of the wind pressure is 3 kips located at the center *C* of the billboard, determine the normal and shearing stresses at point *H*.

SOLUTION

At section containing point H

$P = 8$ kips (compression)

$T = (3)(3) = 9 \text{ kip·ft} = 108 \text{ kip·in}$

$M_x = -(11)(3) = -33 \text{ kip·ft} = -396 \text{ kip·in.}$

$M_z = -(3)(8) = -24 \text{ kip·ft} = -288 \text{ kip·in}$

$V = 3 \text{ kip}$

Section properties.

$d_o = 15 \text{ in.} \quad c_o = \frac{1}{2} d_o = 7.5 \text{ in} \quad c_i = c_o - t = 7.0 \text{ in}$

$A = \pi (c_o^2 - c_i^2) = 22.777 \text{ in}^2$

$I = \frac{\pi}{4} (c_o^4 - c_i^4) = 599.31 \text{ in}^4$

$J = 2I = 1198.62 \text{ in}^4$

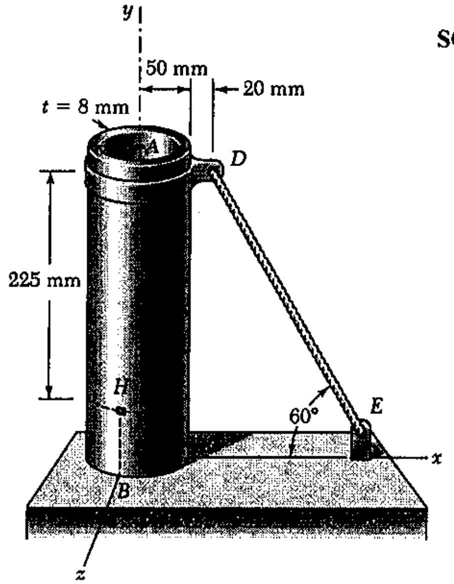
$Q = \frac{2}{3}(c_o^3 - c_i^3) = 52.583 \text{ in}^3$

$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{8}{22.777} - \frac{(288)(7.5)}{599.31} = -0.351 - 3.604 = -3.96 \text{ ksi}$$

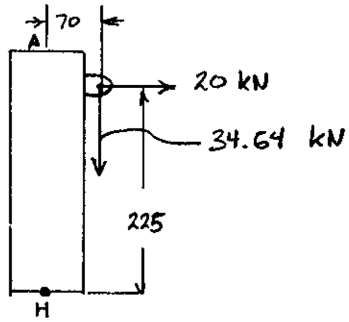
$$\tau = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(108)(7.5)}{1198.62} + \frac{(3)(52.583)}{(599.31)(1.0)} = 0.675 + 0.268 = 0.938 \text{ ksi}$$

PROBLEM 8.40

8.40 The steel pipe *AB* has a 100-mm outer diameter and an 8-mm wall thickness. Knowing that the tension in the cable is 40 kN, determine the normal and shearing stresses at point *H*.



SOLUTION



Vertical force
$40 \cos 30° = 34.64$ kN

Horizontal force
$40 \sin 30° = 20$ kN

Point H lies on neutral axis of bending

Section properties

$d_o = 100$ mm $\quad c_o = \frac{1}{2} d_o = 50$ mm $\quad c_i = c_o - t = 42$ mm

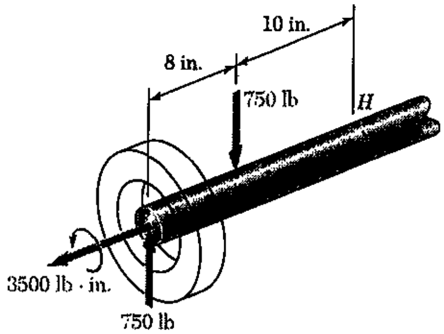
$$A = \pi(c_o^2 - c_i^2) = 2.312 \times 10^3 \text{ mm}^2 = 2.312 \times 10^{-3} \text{ m}^2$$

$$\sigma = -\frac{P}{A} = -\frac{34.64 \times 10^3}{2.312 \times 10^{-6}} = -14.98 \text{ MPa}$$

For thin pipe $\quad \tau = 2\frac{V}{A} = \frac{(2)(20 \times 10^3)}{2.314 \times 10^{-3}} = 17.29 \text{ MPa}$

PROBLEM 8.41

8.41 The axle of a small truck is acted upon by the forces and couple shown. Knowing that the diameter or the axle is 1.42 in., determine the normal and shearing stresses at point H located on the top of the axle.



SOLUTION

The bending moment causing normal stress at point H is

$$M = (8)(750) = 6000 \text{ lb}\cdot\text{in.}$$

$$c = \tfrac{1}{2}d = 0.71 \text{ in.}$$

$$I = \tfrac{\pi}{4}c^4 = 0.19958 \text{ in}^4, \qquad J = 2I = 0.39916 \text{ in}^4$$

Normal stress at H $\quad \sigma_H = -\dfrac{Mc}{I} = -\dfrac{(6000)(0.71)}{0.19958} = -21.3 \times 10^3 \text{ psi}$

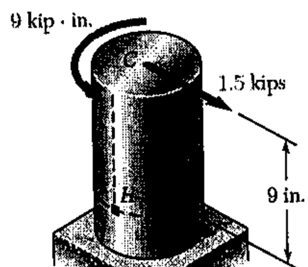
$$= -21.3 \text{ ksi}$$

At the section containing point H $\quad V = 0, \quad T = 3500 \text{ lb}\cdot\text{in}$

$$\tau_H = \frac{Tc}{J} = \frac{(3500)(0.71)}{0.39916} = 6.23 \text{ ksi}$$

PROBLEM 8.42

8.42 A 1.5-kip force and a 9-kip·in. couple areappliedat the top of the cast-iron post shown. Determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.



SOLUTION $\qquad$ diameter = 2.5 in.

At the section containing points H and K.

$$P = 0 \qquad V = 1.5 \text{ kips}$$

$$T = 9 \text{ kip}\cdot\text{in} \qquad M = (1.5)(9) = 13.5 \text{ kip}\cdot\text{in}$$

$$d = 2.5 \text{ in} \qquad c = \tfrac{1}{2}d = 1.25 \text{ in}$$

$$A = \pi c^2 = 4.909 \text{ in}^2 \qquad I = \tfrac{\pi}{4}c^4 = 1.9175 \text{ in}^4 \qquad J = 2I = 3.835 \text{ in}^4$$

For a semicircle $\quad Q = \tfrac{2}{3}c^3 = 1.3021 \text{ in}^3$

(a) At point H $\qquad \sigma_H = 0$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(9)(1.25)}{3.835} + \frac{(1.5)(1.3021)}{(1.9175)(2.5)} = 2.934 + 0.407$$

$$= 3.34 \text{ ksi}$$

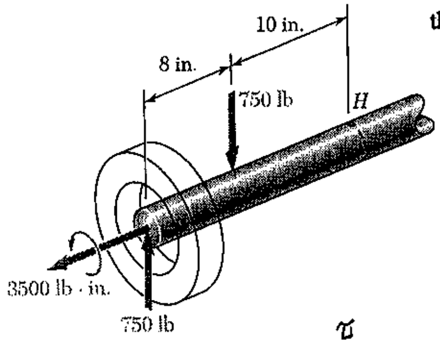
(b) At point K $\qquad \sigma_K = -\dfrac{Mc}{I} = -\dfrac{(13.5)(1.25)}{1.9175} = -8.80 \text{ ksi}$

$$\tau_K = \frac{Tc}{J} = \frac{(9)(1.25)}{3.835} = 2.93 \text{ ksi}$$

PROBLEM 8.43

8.41 The axle of a small truck is acted upon by the forces and couple shown. Knowing that the diameter or the axle is 1.42 in., determine the normal and shearing stresses at point *H* located on the top of the axle.

8.43 For the truck axle and loading of Prob. 8.41, determine the principal stresses and the maximum shearing stress at point *H.*

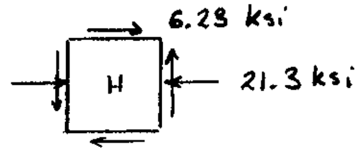


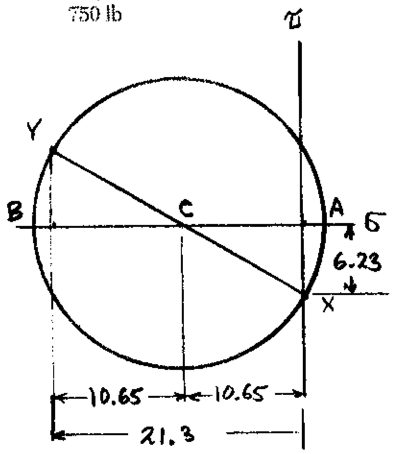
SOLUTION

From the solution of Prob. 8.41

$\sigma_H = -21.3$ ksi

$\tau_H = 6.23$ ksi





$\sigma_c = -\frac{21.3}{2} = -10.65$ ksi

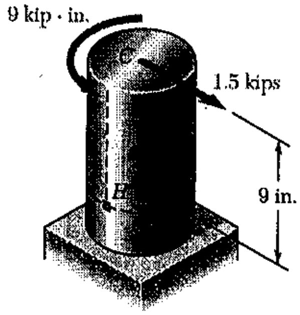
$R = \sqrt{\left(\frac{21.3}{2}\right)^2 + (6.23)^2} = 12.34$ ksi

$\sigma_a = \sigma_c + R = 1.69$ ksi ◄

$\sigma_b = \sigma_c - R = -23.0$ ksi ◄

$\tau_{max} = R = 12.34$ ksi ◄

PROBLEM 8.44



8.42 A 1.5-kip force and a 9-kip·in. couple are allied at the top of the cast-iron post shown. Determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.

8.44 For the post and loading of Prob. 8.42, determine the principal stresses and the maximum shearing stress at (*a*) point *H*, (*b*) point *K*.

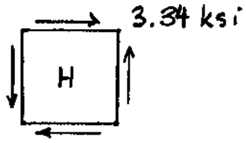
SOLUTION

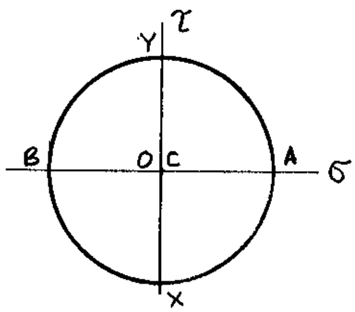
From the solution of Prob. 8.42

(a) $\sigma_H = 0$, $\tau_H = 3.34$ ksi

(b) $\sigma_K = -8.80$ ksi, $\tau_K = 2.93$ ksi

(a) Point H





$\sigma_c = 0$

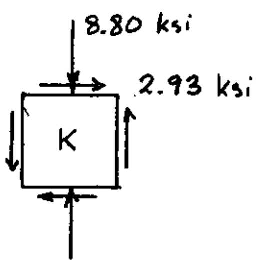
$R = 3.34$ ksi

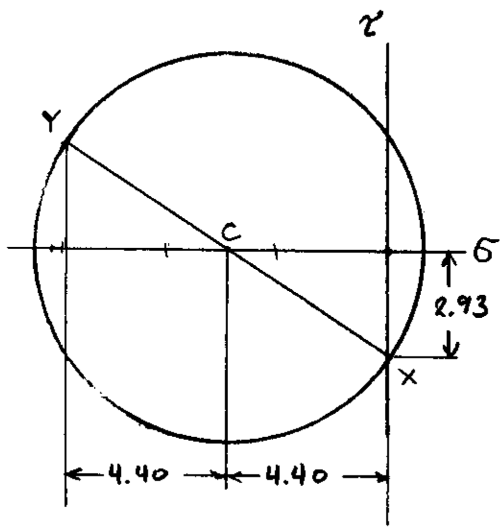
$\sigma_a = \sigma_c + R = 3.34$ ksi

$\sigma_b = \sigma_c - R = -3.34$ ksi

$\tau_{max} = R = 3.34$ ksi

(b) Point K





$$\sigma_c = -\frac{8.80}{2} = -4.40 \text{ ksi}$$

$$R = \sqrt{\left(\frac{8.80}{2}\right)^2 + 2.93^2} = 5.29 \text{ ksi}$$

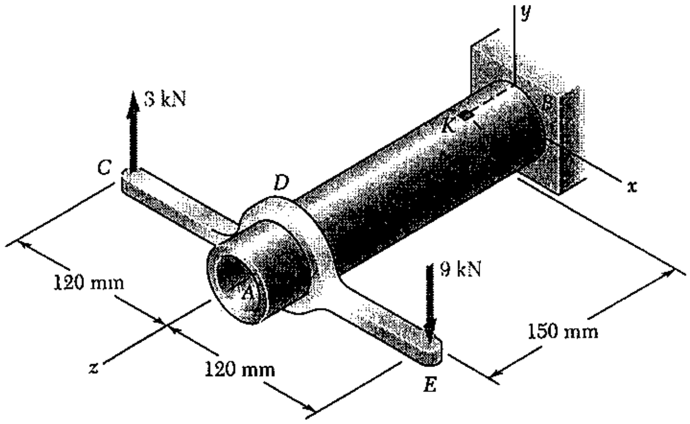
$\sigma_a = \sigma_c + R = 0.89$ ksi

$\sigma_b = \sigma_c - R = -9.69$ ksi

$\tau_{max} = R = 5.29$ ksi

PROBLEM 8.45

8.45 The steel pipe *AB* has a 72-mm outer diameter and a 5-mm wall thickness. Knowing that arm *CDE* is rigidly attached to the pipe, determine the principal stresses, principal planes, and maximum shearing stress at point *H*.

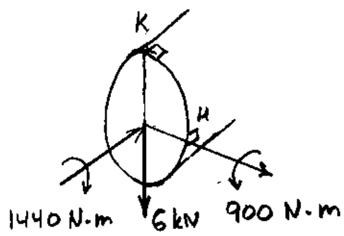


SOLUTION

Replace the forces at C and E by an equivalent force-couple system at D.

$$F_D = 9 - 3 = 6\ kN \downarrow$$

$$T_D = (9\times10^3)(120\times10^{-3}) + (3\times10^3)(120\times10^{-3}) = 1440\ N\cdot m$$



At the section containing points H and K

$P = 0$, $V = 6\ kN$, $T = 1440\ N\cdot m$

$M = (6\times10^3)(150\times10^{-3}) = 900\ N\cdot m$

Section properties: $d_o = 72\ mm$ $\quad c_o = \frac{1}{2}d_o = 36\ mm$ $\quad c_i = c_o - t = 31\ mm$

$A = \pi(c_o^2 - c_i^2) = 1.0524\times10^3\ mm^2 = 1.0524\times10^{-3}\ m^2$

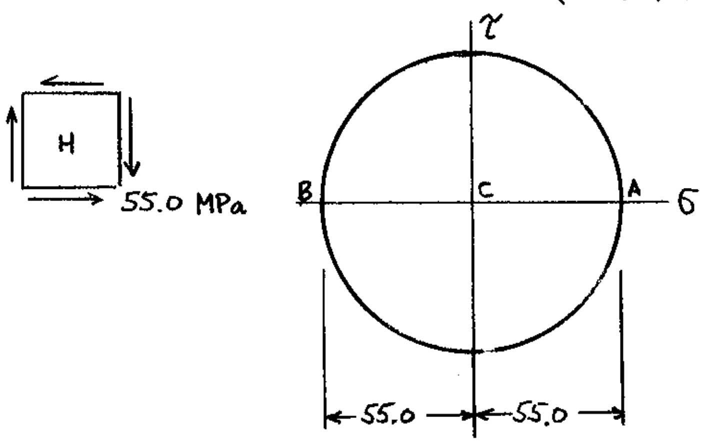
$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 593.84\times10^{-3}\ mm^4 = 593.84\times10^{-9}\ m^4$

$J = 2I = 1.1877\times10^{-6}\ m^4$

For half-pipe $\quad Q = \frac{2}{3}(c_o^3 - c_i^3) = 11.243\times10^3\ mm^3 = 11.243\times10^{-6}\ m^3$

At point H $\quad$ Point H lies on the neutral axis of bending. $\sigma_H = 0$.

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(1440)(36\times10^{-3})}{1.1877\times10^{-6}} + \frac{(6\times10^3)(11.243\times10^{-6})}{(593.84\times10^{-9})(10\times10^{-3})} = 55.0\ MPa$$



$\sigma_c = 0$

$R = 55.0\ MPa$

$\sigma_a = \sigma_c + R = 55.0\ MPa$

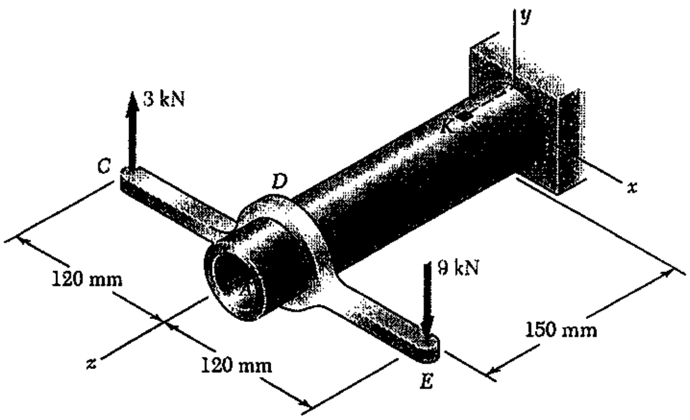
$\sigma_b = \sigma_c - R = -55.0\ MPa$

$\theta_a = -45^\circ$, $\theta_b = +45^\circ$

$\tau_{max} = R = 55.0\ MPa$

PROBLEM 8.46

8.46 The steel pipe *AB* has a 72-mm outer diameter and a 5-mm wall thickness. Knowing that arm *CDE* is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point *K*.



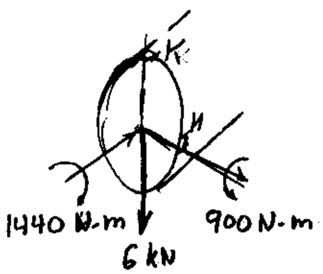
SOLUTION

Replace the forces at C and E by an equivalent force - couple system at D.

$F_D = 9 - 3 = 6 \text{ kN} \downarrow$

$T_D = (9\times10^3)(120\times10^{-3}) + (3\times10^3)(120\times10^{-3}) = 1440 \text{ N}\cdot\text{m}$

At the section containing points H and K



$P = 0,\quad V = 6 \text{ kN} \qquad T = 1440 \text{ N}\cdot\text{m}$

$M = (6\times10^3)(150\times10^{-3}) = 900 \text{ N}\cdot\text{m}$

Section properties: $d_o = 72$ mm $\quad c_o = \frac{1}{2}d_o = 36$ mm $\quad c_i = c_o - t = 31$ mm

$A = \pi(c_o^2 - c_i^2) = 1.0524\times10^3 \text{ mm}^2 = 1.0524\times10^{-3} \text{ m}^3$

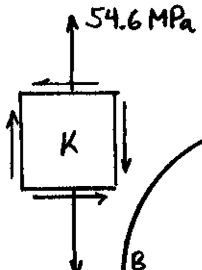
$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 593.84\times10^{-3} \text{ mm}^4 = 593.84\times10^{-9} \text{ m}^4$

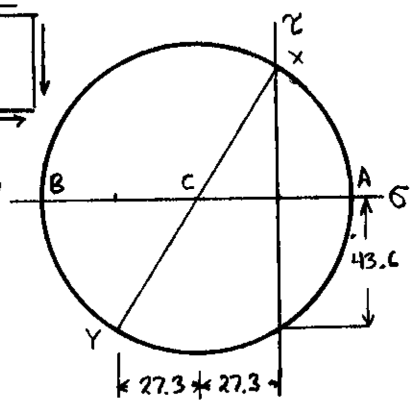
$J = 2I = 1.1877\times10^{-6} \text{ m}^4$

For half-pipe $\quad Q = \frac{2}{3}(c_o^3 - c_i^3) = 11.243\times10^3 \text{ mm}^3 = 11.243\times10^{-6} \text{ m}^3$

At point K $\qquad \sigma_K = \dfrac{Mc}{I} = \dfrac{(900)(36\times10^{-3})}{(593.84\times10^{-9})} = 54.6 \text{ MPa}$

$\tau_K = \dfrac{Tc}{J} = \dfrac{(1440)(36\times10^{-3})}{1.1877\times10^{-6}} = 43.6 \text{ MPa}$





$\sigma_c = -\dfrac{54.6}{2} = -27.3 \text{ MPa}$

$R = \sqrt{\left(\frac{54.6}{2}\right)^2 + 43.6^2} = 51.4 \text{ MPa}$

$\sigma_a = \sigma_c + R = 24.1 \text{ MPa}$ ◄

$\sigma_b = \sigma_c - R = -78.7 \text{ MPa}$ ◄

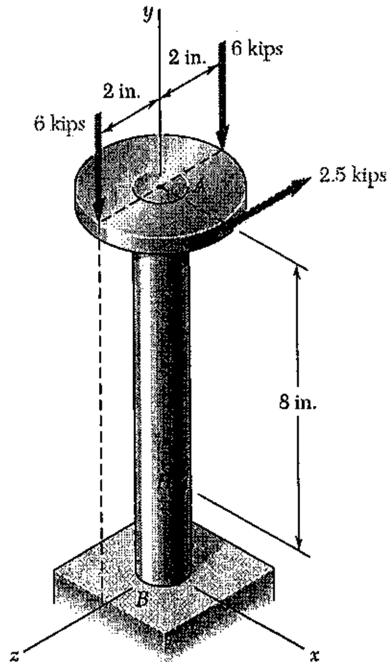
$\tan 2\theta_p = \dfrac{43.6}{27.3} = 1.597$

$\theta_a = 57.9°,\ \theta_b = -32.1°$

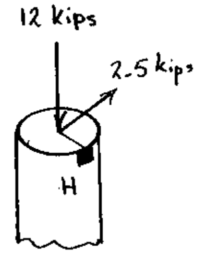
$\tau_{max} = R = 51.4 \text{ MPa}$ ◄

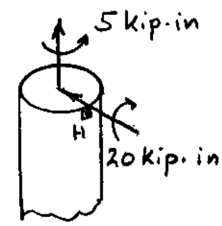
PROBLEM 8.47

8.47 Three forces are applied to 4-in.-diameter plate that is attached to the solid 1.8-in. diameter shaft AB. At point H, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.



SOLUTION





At the section containing point H

$P = 12$ kips (compression)

$V = 2.5$ kips

$T = (2)(2.5) = 5$ kip·in

$M = (8)(2.5) = 20$ kip·in.

$d = 1.8$ in $\quad c = \frac{1}{2}d = 0.9$ in.

$A = \pi c^2 = 2.545 \text{ in}^2$

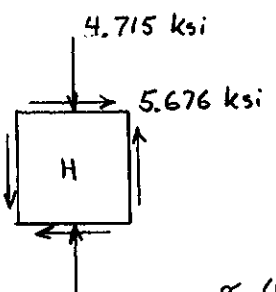
$I = \frac{\pi}{4}c^4 = 0.5153 \text{ in}^4$

$J = 2I = 1.0306 \text{ in}^4$

For a semicircle $\quad Q = \frac{2}{3}c^3 = 0.486 \text{ in}^3$

Point H lies on neutral axis of bending $\quad \sigma_H = \frac{P}{A} = -\frac{12}{2.545} = -4.715$ ksi

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(5)(0.9)}{1.0306} + \frac{(2.5)(0.486)}{(0.5153)(1.8)} = 5.676 \text{ ksi}$$



$\sigma_c = \frac{1}{2}(-4.715) = -2.3575$ ksi

$R = \sqrt{\left(\frac{4.715}{2}\right)^2 + 5.676^2} = 6.1461$

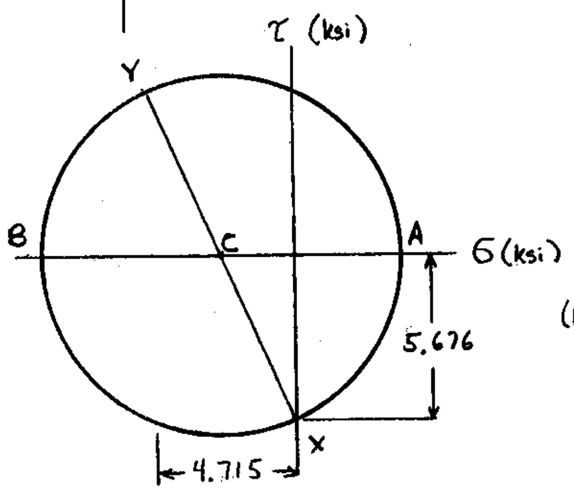
(a) $\sigma_a = \sigma_c + R = 3.79$ ksi ◄

$\sigma_b = \sigma_c - R = -8.50$ ksi ◄

$\tan 2\theta_p = \frac{(2)(5.676)}{4.715} = 2.408$

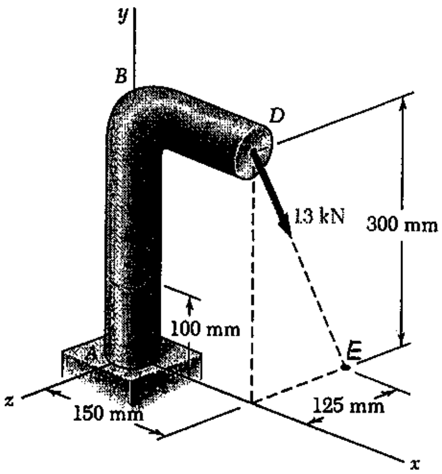
$\theta_a = 33.7°$ ◄ $\quad \theta_b = 123.7°$ ◄

(b) $\tau_{max} = R = 6.15$ ksi ◄



PROBLEM 8.48

8.48 A 13-kN force is applied as shown to the 60-mm-diameter cast-iron post *ABD*. At point *H*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.



SOLUTION

$DE = \sqrt{125^2 + 300^2} = 325 \text{ mm}$

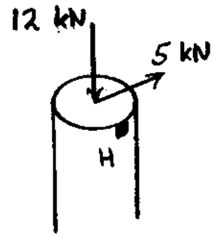
At point D $\qquad F_x = 0$

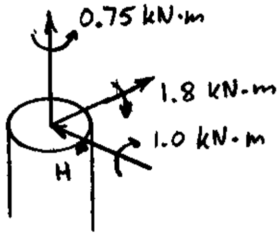
$F_y = -\left(\frac{300}{325}\right)(13) = -12 \text{ kN}$

$F_z = -\left(\frac{125}{300}\right)(13) = -5 \text{ kN}$

Moment of equivalent force-couple system at C, the centroid of the section containing point H

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & k \\ 0.150 & 0.200 & 0 \\ 0 & -12 & -5 \end{vmatrix} = -1.00\,\vec{i} + 0.75\,\vec{j} - 1.8\,k \quad \text{kN}\cdot\text{m}$$





Section properties

$d = 60 \text{ mm} \qquad c = \frac{1}{2}d = 30 \text{ mm}$

$A = \pi c^2 = 2.8274 \times 10^3 \text{ mm}^2$

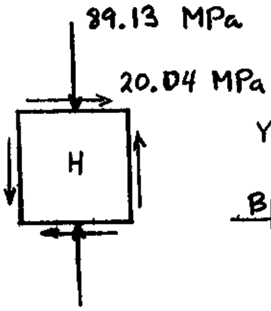
$I = \frac{\pi}{4}c^4 = 636.17 \times 10^3 \text{ mm}^4$

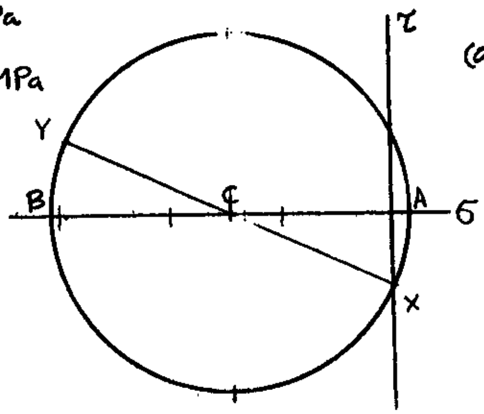
$J = 2I = 1.2723 \times 10^6 \text{ mm}^4$

For a semicircle $\qquad Q = \frac{2}{3}c^3 = 18.00 \times 10^3 \text{ mm}^3$

At point H $\qquad \sigma_H = -\frac{P}{A} - \frac{Mc}{I} = -\frac{12 \times 10^3}{2.8274 \times 10^{-3}} - \frac{(1.8 \times 10^3)(30 \times 10^{-3})}{636.17 \times 10^{-9}} = -89.13 \text{ MPa}$

$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(0.75 \times 10^3)(30 \times 10^{-3})}{1.2723 \times 10^{-6}} + \frac{(5 \times 10^3)(18.00 \times 10^{-6})}{(636.17 \times 10^{-9})(60 \times 10^{-3})} = 20.04 \text{ MPa}$





(a) $\sigma_c = \frac{\sigma_H}{2} = -44.565 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 48.863 \text{ MPa}$

$\sigma_a = \sigma_c + R = 4.3 \text{ MPa}$ ◀

$\sigma_b = \sigma_c - R = -93.4 \text{ MPa}$ ◀

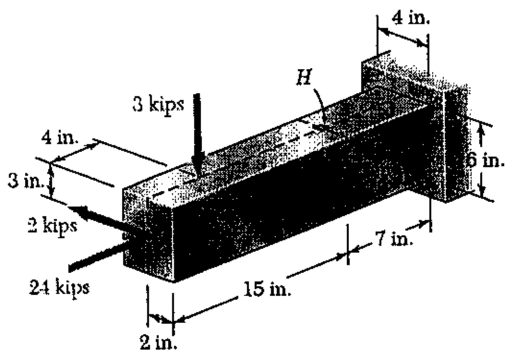
$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = 0.4497$

$\theta_a = 12.1°, \quad \theta_b = 102.1°$ ◀

(b) $\tau_{max} = R = 48.9 \text{ MPa}$ ◀

PROBLEM 8.49

8.49 Three forces are applied to the cantilever beam shown. Determine the normal and shearing stresses at point *H*.



SOLUTION

At the section containing points H and K. the axial and shearing forces are:

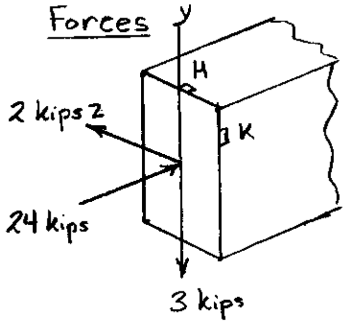
$$P = 24 \text{ kips}, \quad V = \begin{matrix} 3 \text{ kips vertical} \\ 2 \text{ kips horizontal} \end{matrix}$$

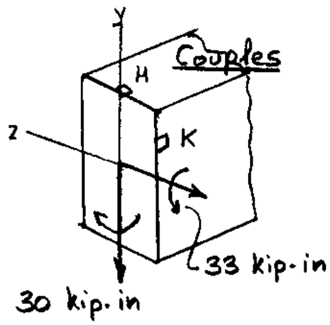
The bending moment components are:

about horizontal axis: $M = (15-4)(3) = 33$ kip.in

about vertical axis $M = (15)(2) = 30$ kip. in.

Forces





Section properties

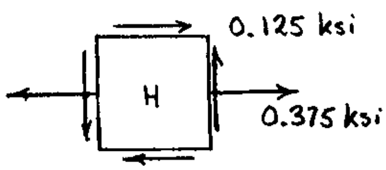
$$A = (4)(6) = 24 \text{ in}^2$$

$$I_z = \tfrac{1}{12}(4)(6)^3 = 72 \text{ in}^4$$

$$I_y = \tfrac{1}{12}(6)(4)^3 = 32 \text{ in}^4$$

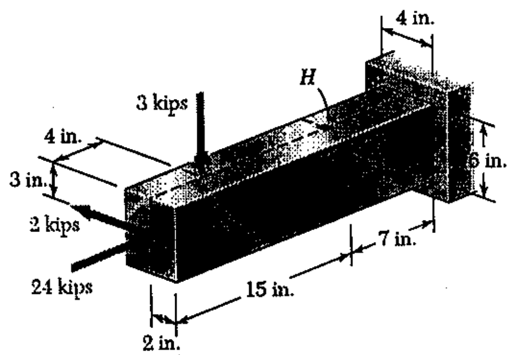
At point H $\quad \sigma_H = -\frac{P}{A} + \frac{Mc}{I} = -\frac{24}{24} + \frac{(33)(3)}{72} = 0.375$ ksi ◄

$$\tau_H = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2}{24} = 0.125 \text{ ksi}$$ ◄



PROBLEM 8.50

8.50 **Three forces are applied to the cantilever beam shown. Determine the normal and shearing stresses at point *K*.**



SOLUTION

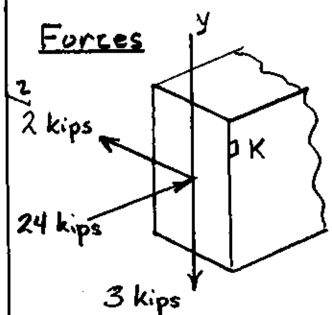
At the section containing points H and K the axial and shearing forces are

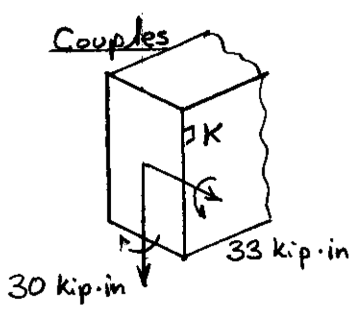
$$P = 24 \text{ kips}, \qquad V = \begin{matrix} 3 \text{ kips vertical} \\ 2 \text{ kips horizontal} \end{matrix}$$

The bending moment components are

about horizontal axis: $M = (15-4)(3) = 33$ kip·in

about vertical axis: $M = (15)(2) = 30$ kip·in





Section properties

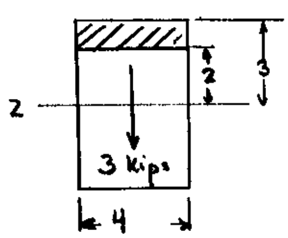
$A = (4)(6) = 24 \text{ in}^2$

$I_z = \frac{1}{12}(4)(6)^3 = 72 \text{ in}^4$

$I_y = \frac{1}{12}(6)(4)^3 = 32 \text{ in}^4$

At point K

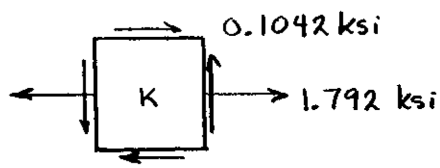
$$\sigma_K = -\frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = -\frac{24}{24} - \frac{(-33)(2)}{72} + \frac{(-30)(-2)}{32} = 1.792 \text{ ksi}$$



$A^* = (1)(4) = 4 \text{ in}^2 \qquad \bar{y} = 2.5 \text{ in.}$

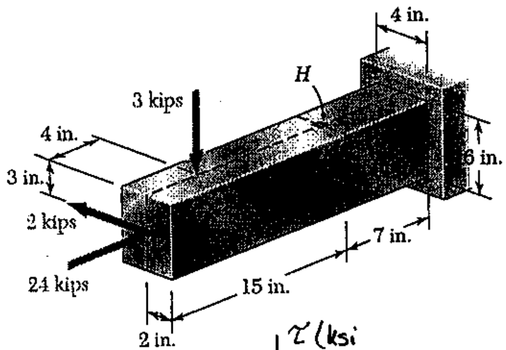
$Q = A^*\bar{y} = (4)(2.5) = 10 \text{ in}^3$

$$\tau_K = \frac{VQ}{It} = \frac{(3)(10)}{(72)(4)} = 0.1042 \text{ ksi}$$



PROBLEM 8.51

8.51 For the beam and loading of Prob. 8.49, determine the principal stresses and the maximum shearing stress at point *H*.

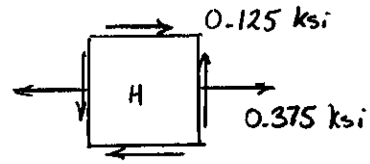


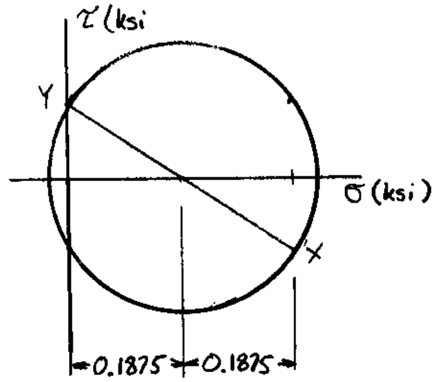
SOLUTION

From the solution of Prob. 8.49

$\sigma_H = 0.375$ ksi

$\tau_H = 0.125$ ksi





$\sigma_c = \dfrac{0.375}{2} = 0.1875$ ksi

$R = \sqrt{\left(\frac{0.375}{2}\right)^2 + (0.125)^2} = 0.2253$ ksi

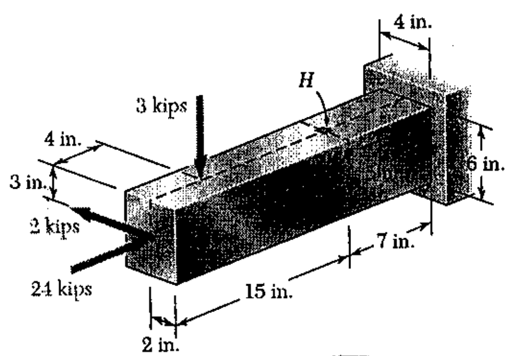
$\sigma_a = \sigma_c + R = 0.413$ ksi ◄

$\sigma_b = \sigma_c - R = -0.0378$ ksi ◄

$\tau_{max} = R = 0.225$ ksi ◄

PROBLEM 8.52

8.52 For the beam and loading of Prob. *8.50*, determine the principal stresses and the maximum shearing stress at point *K*.

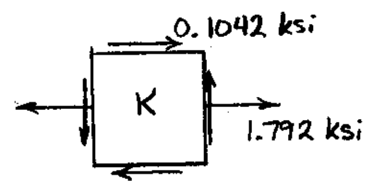


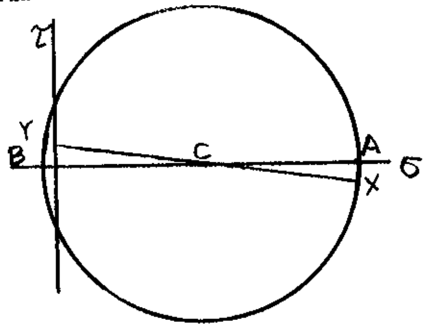
SOLUTION

From the solution of Prob. 8.50

$\sigma_H = 1.792$ ksi

$\tau_H = 0.1042$ ksi





$\sigma_c = \dfrac{1.792}{2} = 0.896$ ksi

$R = \sqrt{\left(\frac{1.792}{2}\right)^2 + (0.1042)^2} = 0.902$ ksi

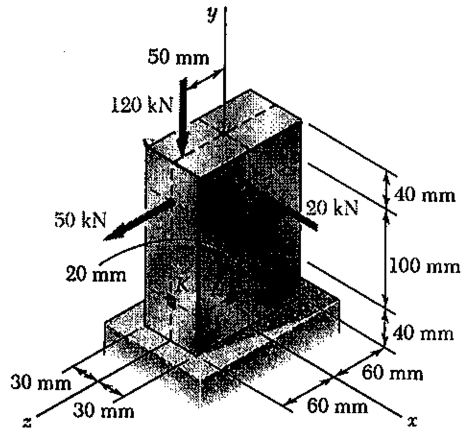
$\sigma_a = \sigma_c + R = 1.798$ ksi ◄

$\sigma_b = \sigma_c - R = -0.006$ ksi ◄

$\tau_{max} = R = 0.902$ ksi ◄

PROBLEM 8.53

8.53 Three forces are applied to a steel post as shown. Determine the normal and shearing stresses at point *H*.



SOLUTION

$$A = (120)(60) = 7.2\times10^{3}\ \text{mm}^2 = 7.2\times10^{-3}\ \text{m}^2$$

$$I_x = \tfrac{1}{12}(60)(120)^3 = 8.64\times10^{6}\ \text{mm}^4 = 8.64\times10^{-6}\ \text{m}^4$$

$$I_z = \tfrac{1}{12}(120)(60)^3 = 2.16\times10^{6}\ \text{mm}^4 = 2.16\times10^{-6}\ \text{m}^4$$

At the section containing points H and K.

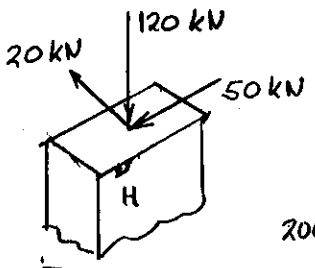
$$P = 120\ \text{kN}\ \ (\text{compression})$$

$$V_x = -20\ \text{kN}$$

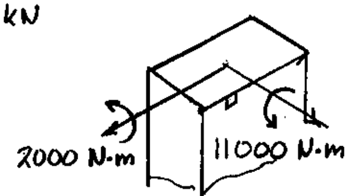
$$V_z = 50\ \text{kN}$$

$$M_z = (20\times10^{3})(100\times10^{-3}) = 2000\ \text{N}\cdot\text{m}$$

$$M_x = (120\times10^{3})(50\times10^{-3}) + (50\times10^{3})(100\times10^{-3}) = 11000\ \text{N}\cdot\text{m}$$



Forces

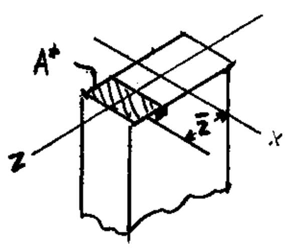


Couples

Stresses at point H

$$\sigma_H = -\frac{P}{A} - \frac{M_x z}{I_x} + \frac{M_z x}{I_z} = -\frac{120\times10^{3}}{7.2\times10^{-3}} - \frac{(11000)(20\times10^{-3})}{8.64\times10^{-6}} + \frac{(2000)(30\times10^{-3})}{2.16\times10^{-6}}$$

$$= -16.67\ \text{MPa} - 25.46\ \text{MPa} + 27.78\ \text{MPa} = -14.35\ \text{MPa}$$

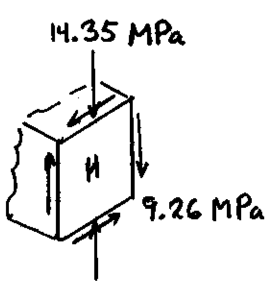


$$A^* = (60)(60-20) = 2.4\times10^{3}\ \text{mm}^2$$

$$\bar{z} = \left(20 + \tfrac{40}{2}\right) = 40\ \text{mm}$$

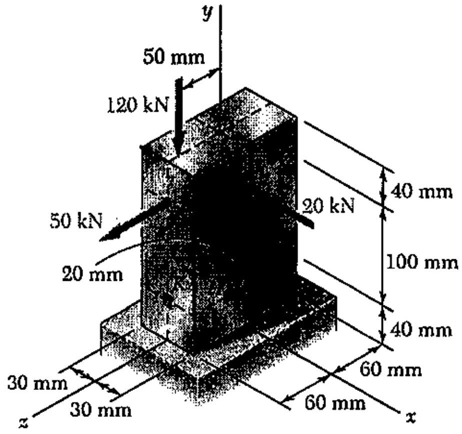
$$\bar{Q}_x = A^*\bar{z} = 96\times10^{3}\ \text{mm}^3 = 96\times10^{-6}\ \text{m}^3$$

$$\tau_H = \frac{V_z Q_x}{I_x t} = \frac{(50\times10^{3})(96\times10^{-6})}{(8.64\times10^{-6})(60\times10^{-3})} = 9.26\ \text{MPa}$$



PROBLEM 8.54

8.54 Three forces are applied to a steel post as shown. Determine the normal and shearing stresses at point *K*.



SOLUTION

$A = (120)(60) = 7.2\times10^{3}\ \text{mm}^2 = 7.2\times10^{-3}\ \text{m}^2$

$I_x = \frac{1}{12}(60)(120)^3 = 8.64\times10^{6}\ \text{mm}^4 = 8.64\times10^{-6}\ \text{m}^4$

$I_z = \frac{1}{12}(120)(60)^3 = 2.16\times10^{6}\ \text{mm}^4 = 2.16\times10^{-6}\ \text{m}^4$

At the section containing points H and K.

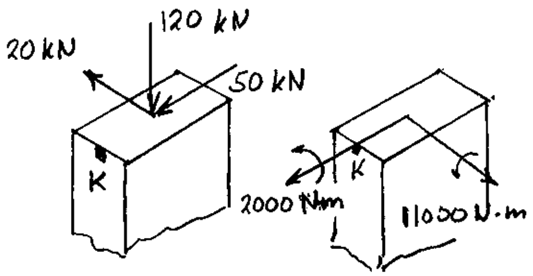
$P = 120\ \text{kN}$ (compression)

$V_x = -20\ \text{kN}$

$V_z = 50\ \text{kN}$

$M_z = (20\times10^3)(100\times10^{-3}) = 2000\ \text{N}\cdot\text{m}$

$M_x = (120\times10^3)(50\times10^{-3}) + (50\times10^3)(100\times10^{-3}) = 11000\ \text{N}\cdot\text{m}$

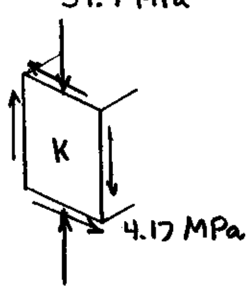


Stresses at point K

$$\sigma_K = -\frac{P}{A} - \frac{M_x z}{I_x} + \frac{M_z x}{I_z} = -\frac{120\times10^3}{7.2\times10^{-3}} - \frac{(11000)(60\times10^{-3})}{8.64\times10^{-6}} + 0$$

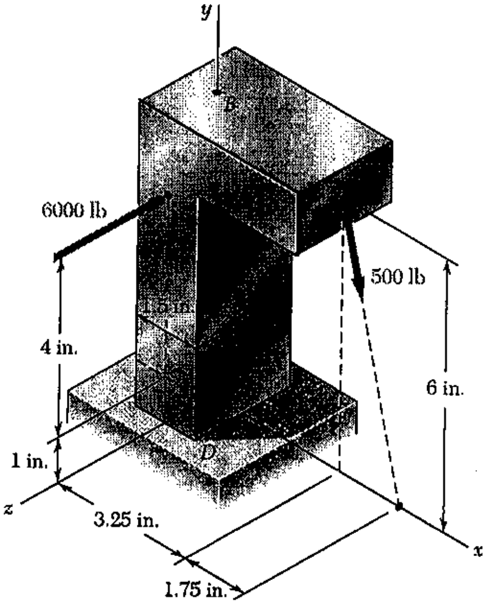
$$= -16.67\ \text{MPa} - 76.39\ \text{MPa} + 0 = -93.1\ \text{MPa}$$

$$\tau_K = \frac{3}{2}\frac{|V_x|}{A} = \frac{3}{2}\,\frac{20\times10^3}{7.2\times10^{-3}} = 4.17\ \text{MPa}$$



PROBLEM 8.55

8.55 Two forces are applied to the small post *BD* as shown. Knowing that the vertical portion of the post has a cross section of 1.5 × 2.4 in., determine the principal stresses, principal planes, and maximum shearing stress at point *H*.



SOLUTION

Components of 500 lb force.

$$F_x = \frac{(500)(1.75)}{6.25} = 140 \text{ lb.}$$

$$F_y = -\frac{(500)(6)}{6.25} = -480 \text{ lb.}$$

Moment arm of 500 lb force

$$\vec{r} = 3.25\,\vec{i} + (6-1)\,\vec{j}$$

Moment of 500 lb force

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.25 & 5 & 0 \\ 140 & -480 & 0 \end{vmatrix} = -2260\,\vec{k} \text{ lb·in}$$

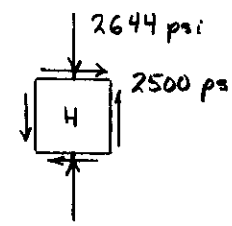
At the section containing point H: $P = -480$ lb. $V_x = 140$ lb.

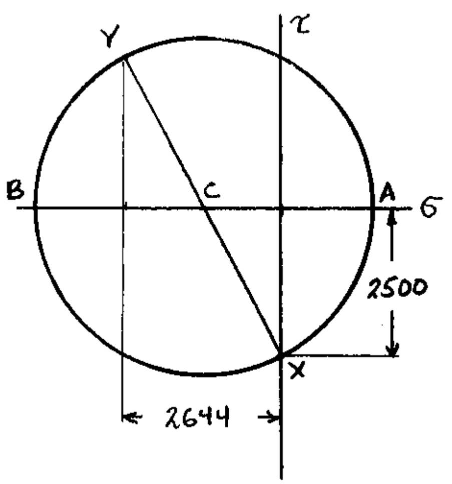
$V_z = -6000$ lb., $M_z = -2260$ lb·in, $M_x = -(4)(6000) = -24000$ lb·in.

$A = (1.5)(2.4) = 3.6 \text{ in}^2$ $\quad I_z = \frac{1}{12}(2.4)(1.5)^3 = 0.675 \text{ in}^4$

$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{480}{3.6} + \frac{(-2260)(0.75)}{0.675} = -2644 \text{ psi}$$

$$\tau_H = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{6000}{3.6} = 2500 \text{ psi}$$





$$\sigma_c = -\frac{2644}{2} = -1322 \text{ psi}$$

$$R = \sqrt{\left(\frac{2644}{2}\right)^2 + (2500)^2} = 2828 \text{ psi}$$

$$\sigma_a = \sigma_c + R = 1506 \text{ psi}$$

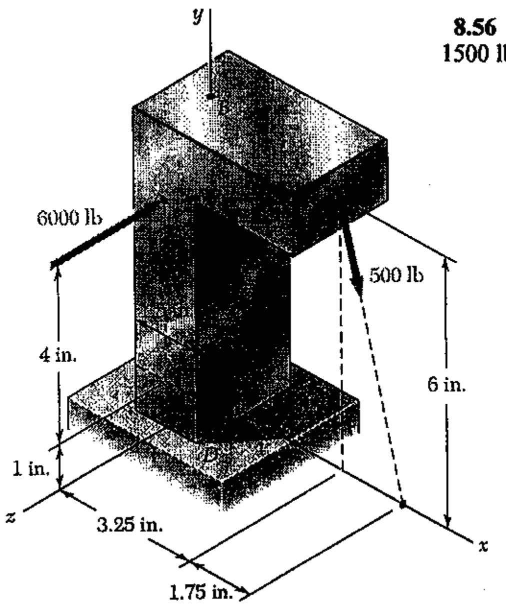
$$\sigma_b = \sigma_c - R = -4150 \text{ psi}$$

$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = \frac{(2)(2500)}{2644} = 1.891$$

$$\theta_a = 31.1°, \quad \theta_b = 121.1°$$

$$\tau_{max} = R = 2828 \text{ psi}$$

PROBLEM 8.56



8.55 Two forces are applied to the small post *BD* as shown. Knowing that the vertical portion of the post has a cross section of 1.5 × 2.4 in., determine the principal stresses, principal planes, and maximum shearing stress at point *H*.

8.56 Solve Prob 8.55, assuming that the magnitude of the 6000-lb force is reduced to 1500 lb.

SOLUTION

Components of 500 lb. force

$$F_x = \frac{(500)(1.75)}{6.25} = 140 \text{ lb}$$

$$F_y = -\frac{(500)(6)}{6.25} = -480 \text{ lb.}$$

Moment arm of 500 lb. force

$$\vec{r} = 3.25\,\vec{i} + (6-1)\,\vec{j}$$

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.25 & 5 & 0 \\ 140 & -480 & 0 \end{vmatrix} = -2260\,\vec{k} \text{ lb}\cdot\text{in}$$

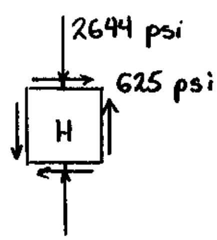
At the section containing point H: $P = -480$ lb $\quad V_x = 140$ lb.

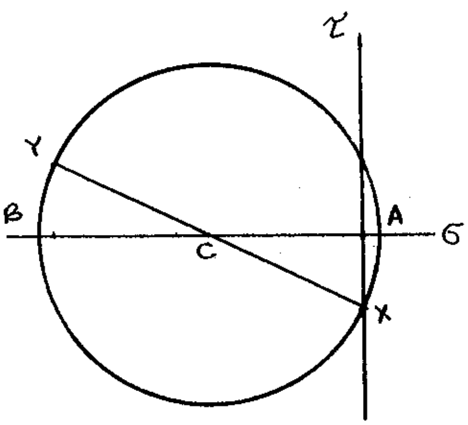
$V_z = -1500$ lb, $\quad M_z = -2260$ lb·in, $\quad M_x = -(4)(1500) = -6000$ lb·in.

$A = (1.5)(2.4) = 3.6 \text{ in}^2 \qquad I_z = \frac{1}{12}(2.4)(1.5)^3 = 0.675 \text{ in}^4$

$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{480}{3.6} + \frac{(-2260)(0.75)}{0.675} = -2644 \text{ psi}$$

$$\tau_H = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{1500}{3.6} = 625 \text{ psi}$$





$\sigma_c = \frac{1}{2}\sigma_H = -1322$ psi

$R = \sqrt{\left(\frac{2644}{2}\right)^2 + (625)^2} = 1462$ psi

$\sigma_a = \sigma_c + R = 140$ psi

$\sigma_b = \sigma_c - R = -2784$ psi

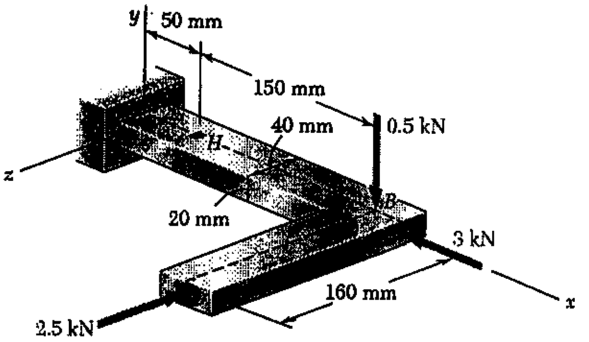
$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = \frac{(2)(625)}{2644} = 0.4728$

$\theta_a = 12.7° \qquad \theta_b = 102.7°$

$\tau_{max} = R = 1462$ psi

PROBLEM 8.57

8.57 Three forces are applied to the machine component *ABD* as shown. Knowing that the cross section containing point *H* is a 20 × 40-mm rectangle, determine the principal stresses and the maximum shearing stress at point *H*.



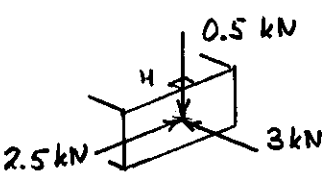
SOLUTION

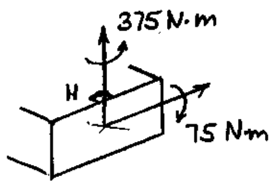
Equivalent force-couple system at section containing point H

$F_x = -3\text{kN}, \quad F_y = -0.5\text{ kN}, \quad F_z = -2.5\text{ kN}$

$M_x = 0, \quad M_y = (0.150)(2500) = 375\text{ N}\cdot\text{m}$

$M_z = -(0.150)(500) = -75\text{ N}\cdot\text{m}$



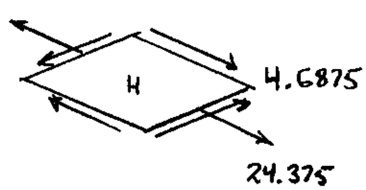


$A = (20)(40) = 800\text{ mm}^2 = 800 \times 10^{-6}\text{ m}^2$

$I_z = \frac{1}{12}(40)(20)^3 = 26.667 \times 10^3\text{ mm}^4 = 26.667 \times 10^{-9}\text{ m}^4$

$$\sigma_H = \frac{P}{A} - \frac{M_z y}{I_z} = \frac{-3000}{800 \times 10^{-6}} - \frac{(-75)(10 \times 10^{-3})}{26.667 \times 10^{-9}} = 24.375\text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{|V_z|}{A} = \frac{3}{2} \cdot \frac{2500}{800 \times 10^{-6}} = 4.6875\text{ MPa}$$



$\sigma_c = \frac{1}{2}\sigma_H = 12.1875\text{ MPa}$

$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (4.6875)^2} = 13.0579\text{ MPa}$

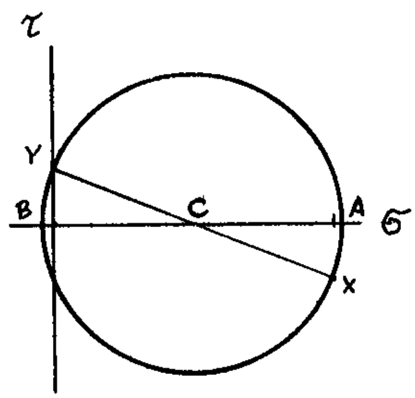
$\sigma_a = \sigma_c + R = 25.2\text{ MPa}$ ◄

$\sigma_b = \sigma_c - R = -0.87\text{ MPa}$ ◄

$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(4.6875)}{24.375} = 0.3846$

$\theta_a = 10.5^\circ, \quad \theta_b = 100.5^\circ$

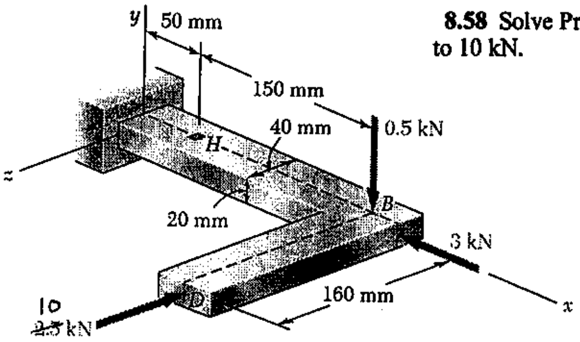
$\tau_{max} = R = 13.06\text{ MPa}$ ◄



PROBLEM 8.58

8.57 Three forces are applied to the machine component *ABD* as shown. Knowing that the cross section containing point *H* is a 20 × 40-mm rectangle, determine the principal stresses and the maximum shearing stress at point *H*.

8.58 Solve Prob. 8.57, assuming that the magnitude of the 2.5-kN force is increased to 10 kN.



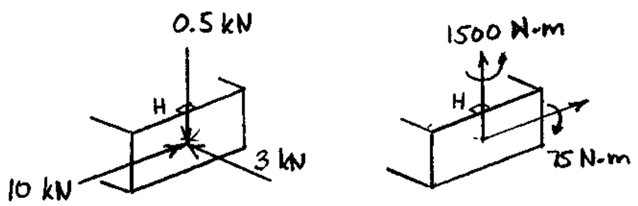
SOLUTION

Equivalent force-couple system at section containing point H.

$F_x = -3 \text{ kN}, \quad F_y = -0.5 \text{ kN}, \quad F_z = -10 \text{ kN}$

$M_x = 0, \quad M_y = (0.150)(10000) = 1500 \text{ N}\cdot\text{m}$

$M_z = -(0.150)(500) = -75 \text{ N}\cdot\text{m}$

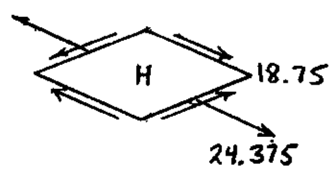


$A = (20)(40) = 800 \text{ mm}^2 = 800 \times 10^{-6} \text{ m}^2$

$I_z = \frac{1}{12}(40)(20)^3 = 26.667 \times 10^3 \text{ mm}^4 = 26.667 \times 10^{-9} \text{ m}^4$

$$\sigma_H = \frac{P}{A} - \frac{M_z y}{I_z} = \frac{-3000}{800 \times 10^{-6}} - \frac{(-75)(10 \times 10^{-3})}{26.667 \times 10^{-9}} = 24.375 \text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{|V_z|}{A} = \frac{3}{2} \cdot \frac{10000}{800 \times 10^{-6}} = 18.75 \text{ MPa}$$



$\sigma_c = \frac{1}{2}\sigma_H = 12.1875 \text{ MPa}$

$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (18.75)^2} = 22.363 \text{ MPa}$

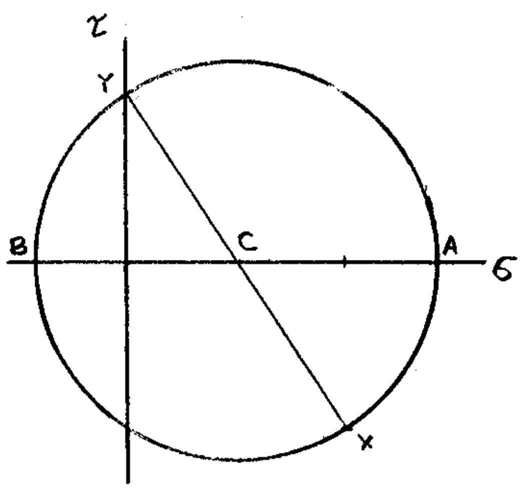
$\sigma_a = \sigma_c + R = 34.6 \text{ MPa}$ ◄

$\sigma_b = \sigma_c - R = -10.18 \text{ MPa}$ ◄

$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(18.75)}{24.375} = 1.5385$

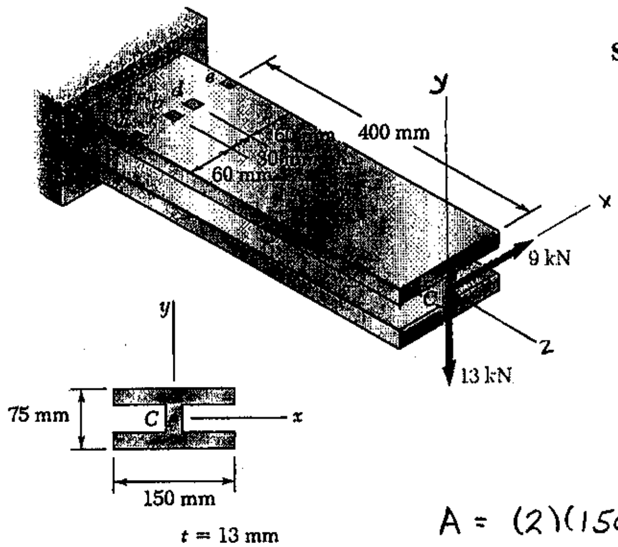
$\theta_a = 28.5°, \quad \theta_b = 118.5°$

$\tau_{max} = R = 22.4 \text{ MPa}$ ◄



PROBLEM 8.59

8.59 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *a* and *b*.



SOLUTION

Equivalent force-couple system at section containing points a and b.

$F_x = 9 \text{ kN}, \quad F_y = -13 \text{ kN}, \quad F_z = 0$

$M_x = (0.400)(13\times10^3) = 5200 \text{ N}\cdot\text{m}$

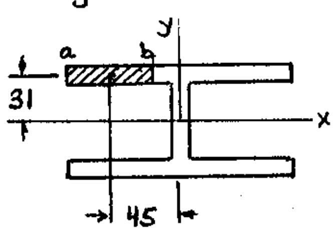
$M_y = 0.400)(9\times10^3) = 3600 \text{ N}\cdot\text{m}$

$M_z = 0$

$A = (2)(150)(13) + (13)(75-26) = 4537 \text{ mm}^2 = 4537\times10^{-6} \text{ m}^2$

$I_x = 2\left[\frac{1}{12}(150)(13)^3 + (150)(13)(37.5-6.5)^2\right] + \frac{1}{12}(13)(75-26)^3 = 3.9303\times10^6 \text{ mm}^4 = 3.9303\times10^{-6} \text{ m}^4$

$I_y = 2\cdot\frac{1}{12}(13)(150)^3 + \frac{1}{12}(75-26)(13)^3 = 7.3215\times10^6 \text{ mm}^4 = 7.3215\times10^{-6} \text{ m}^4$



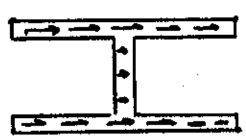
For point a $\quad Q_x = 0 \qquad Q_y = 0$

For point b $\quad A^* = (60)(13) = 780 \text{ mm}^2$

$\bar{x} = -45 \text{ mm} \qquad \bar{y} = 31 \text{ mm}$

$Q_x = A^*\bar{y} = 24.18\times10^3 \text{ mm}^3 = 24.18\times10^{-6} \text{ m}^3$

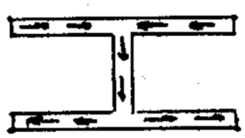
$Q_y = A^*\bar{x} = -35.1\times10^{-3} \text{ mm}^3 = -35.1\times10^{-6} \text{ m}^3$



At point $\underline{a}$ $\quad \sigma_a = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$

$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(-75\times10^{-3})}{7.3215\times10^{-6}} = 86.5 \text{ MPa}$

$\tau_a = 0$



At point $\underline{b}$ $\quad \sigma_b = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$

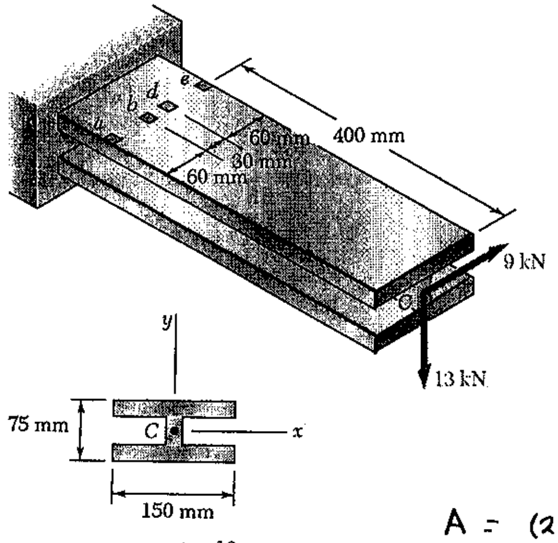
$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(-15\times10^{-3})}{7.3215\times10^{-6}} = 57.0 \text{ MPa}$

$\tau_b = \frac{|V_x||Q_y|}{I_y t} + \frac{|V_y||Q_x|}{I_x t} = \frac{(9\times10^3)(35.1\times10^{-6})}{(7.3215\times10^{-6})(13\times10^{-3})} + \frac{(13\times10^3)(24.18\times10^{-6})}{(3.9303\times10^{-6})(13\times10^{-3})}$

$= 3.32 \text{ MPa} + 6.15 \text{ MPa} = 9.47 \text{ MPa}$

PROBLEM 8.60

8.60 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *d* and *e*.



SOLUTION

Equivalent force-couple system at section containing points a and b.

$F_x = 9\text{ kN}, \quad F_y = -13\text{ kN}, \quad F_z = 0$

$M_x = (0.400)(13\times10^3) = 5200\text{ N}\cdot\text{m}$

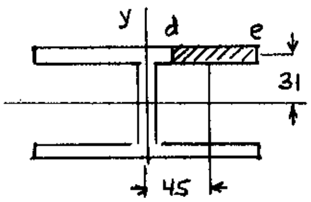
$M_y = (0.400)(9\times10^3) = 3600\text{ N}\cdot\text{m}$

$M_z = 0$

$A = (2)(150)(13) + (13)(75-26) = 4537\text{ mm}^2$
$= 4537\times10^{-6}\text{ m}^2$

$I_x = 2\left[\frac{1}{12}(150)(13)^3 + (150)(13)(37.5-6.5)^2\right] + \frac{1}{12}(13)(75-26)^3 = 3.9303\times10^6\text{ mm}^4$
$= 3.9303\times10^{-6}\text{ m}^4$

$I_y = 2\left[\frac{1}{12}(13)(150)^3\right] + \frac{1}{12}(75-26)(13)^3 = 7.3215\times10^6\text{ mm}^4 = 7.3215\times10^{-6}\text{ m}^4$

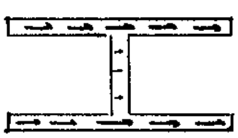


For point d $\quad A^* = (60)(13) = 780\text{ mm}^2$
$\bar{x} = 45\text{ mm} \qquad \bar{y} = 31\text{ mm}$

$Q_x = A^*\bar{y} = 24.18\times10^3\text{ mm}^3 = 24.18\times10^{-6}\text{ m}^3$

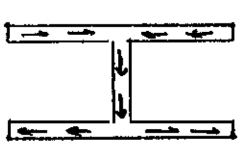
$Q_y = A^*\bar{x} = 35.1\times10^3\text{ mm}^3 = 35.1\times10^{-6}\text{ m}^3$

For point e $\quad Q_x = 0, \quad Q_y = 0$



At point d $\quad \sigma_d = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$

$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(15\times10^{-3})}{7.3215\times10^{-6}} = 42.2\text{ MPa}$



Due to V_x $\quad \tau_d = \frac{|V_x||Q_y|}{I_y t} = \frac{(9000)(35.1\times10^{-6})}{(7.3215\times10^{-6})(13\times10^{-3})} = 3.32\text{ MPa} \rightarrow$

Due to V_y $\quad \tau_d = \frac{|V_y||Q_x|}{I_x t} = \frac{(13000)(24.18\times10^{-6})}{(3.9303\times10^{-6})(13\times10^{-3})} = 6.15\text{ MPa} \leftarrow$

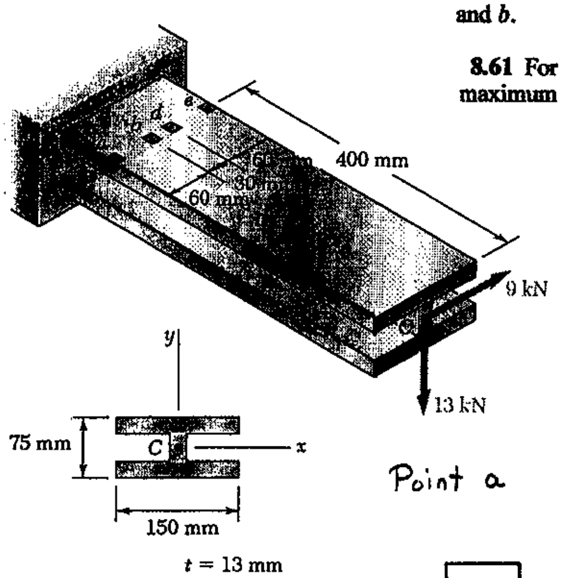
Net $\quad \tau_d = 2.83\text{ MPa}$

At point e $\quad \sigma_e = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(75\times10^{-3})}{7.3215\times10^{-6}}$

$= 12.74\text{ MPa}$

$\tau_e = 0$

PROBLEM 8.61



8.59 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *a* and *b*.

8.61 For the beam and loading of Prob. 8.59, determine the principal stresses and the maximum shearing stress at points *a* and *b*.

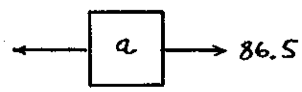
SOLUTION

From the solution of Prob. 8.59

$\sigma_a = 86.5$ MPa $\qquad \tau_a = 0$

$\sigma_b = 57.0$ MPa $\qquad \tau_b = 9.47$ MPa

Point a



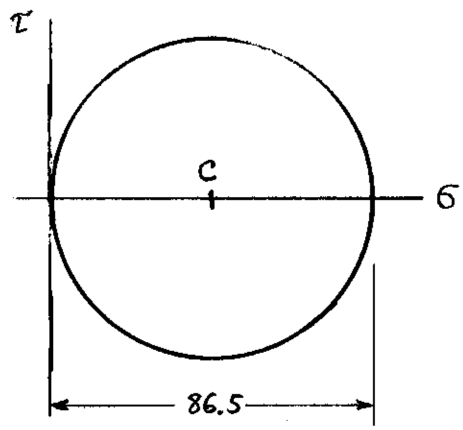
$\sigma_c = \frac{86.5}{2}$ MPa $= 43.25$ MPa

$R = \frac{86.5}{2}$ MPa $= 43.25$ MPa

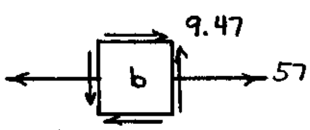
$\sigma_{max} = \sigma_c + R = 86.5$ MPa ◄

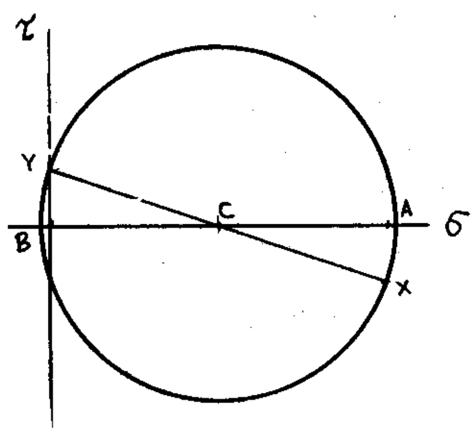
$\sigma_{min} = \sigma_c - R = 0$

$\tau_{max} = R = 43.3$ MPa ◄



Point b





$\sigma_c = \frac{57.0}{2} = 28.5$ MPa

$R = \sqrt{\left(\frac{57.0}{2}\right)^2 + (9.47)^2} = 30.03$ MPa

$\sigma_{max} = \sigma_c + R = 58.5$ MPa ◄

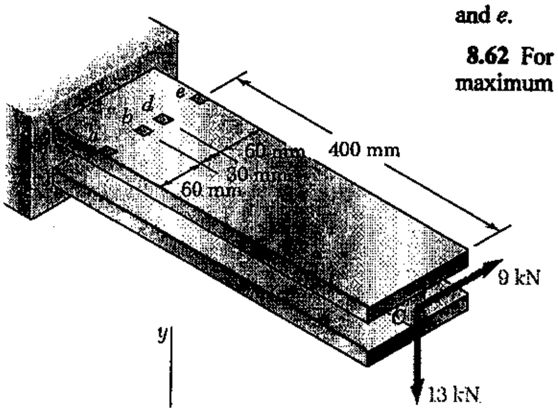
$\sigma_{min} = \sigma_c - R = -1.53$ MPa ◄

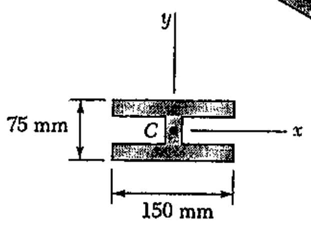
$\tau_{max} = R = 30.0$ MPa ◄

PROBLEM 8.62

8.60 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points d and e.

8.62 For the beam and loading of Prob. 8.60, determine the principal stresses and the maximum shearing stress at points d and e.





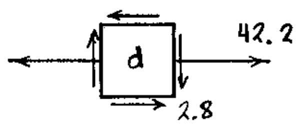
SOLUTION

From the solution of Prob 8.60

$\sigma_d = 42.2$ MPa $\quad \tau_d = 2.83$ MPa

$\sigma_e = 12.74$ MPa $\quad \tau_e = 0$

Point d



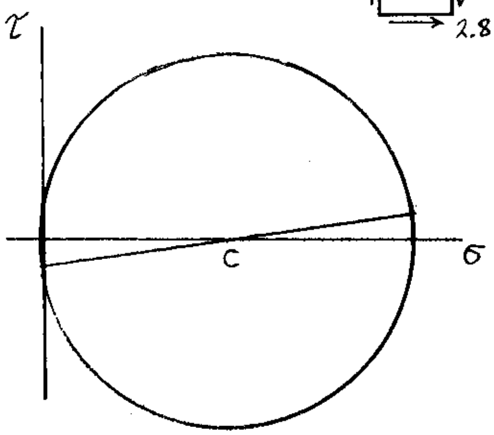
$$\sigma_c = \frac{42.2}{2} = 42.2 \text{ MPa}$$

$$R = \sqrt{\left(\frac{42.2}{2}\right)^2 + (2.83)^2} = 21.29 \text{ MPa}$$

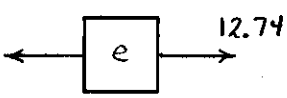
$\sigma_{max} = \sigma_c + R = 42.4$ MPa ◄

$\sigma_{min} = \sigma_c - R = -0.19$ MPa ◄

$\tau_{max} = R = 21.3$ MPa ◄



Point e



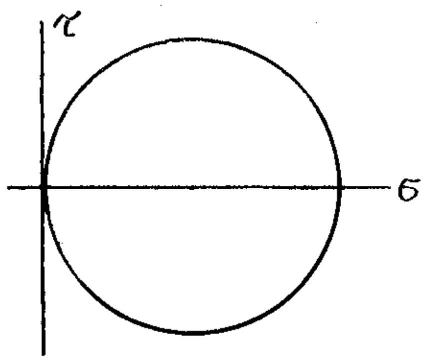
$$\sigma_c = \frac{12.74}{2} = 6.37 \text{ MPa}$$

$$R = \frac{12.74}{2} = 6.37 \text{ MPa}$$

$\sigma_{max} = \sigma_c + R = 12.74$ MPa ◄

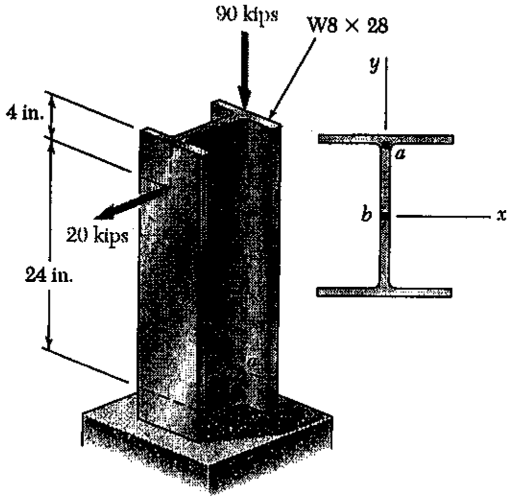
$\sigma_{min} = \sigma_c - R = 0$ ◄

$\tau_{max} = R = 6.37$ MPa ◄



PROBLEM 8.63

8.63 Two forces are applied to a W8 × 28 rolled-steel beam as shown. Determine the principal stresses, principal planes, and maximum shearing stress at point *a*.



SOLUTION

For W8 × 28 rolled steel section

$A = 8.25\ \text{in}^2,\quad d = 8.06\ \text{in},\quad b_f = 6.535\ \text{in}$

$t_f = 0.465\ \text{in},\quad t_w = 0.285\ \text{in},\quad I_x = 98.0\ \text{in}^4$

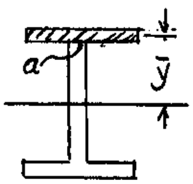
At the section containing points a and b.

$P = -90\ \text{kips},\qquad V = 20\ \text{kips}$

$M = (20)(24) - (4.03)(90) = 117.3\ \text{kip}\cdot\text{in}$

At point a $\qquad y = \frac{1}{2}d - t_f = 4.03 - 0.465 = 3.565\ \text{in}$

$$\sigma = \frac{P}{A} + \frac{My}{I} = -\frac{90}{8.25} - \frac{(-117.3)(3.565)}{98.0} = -6.642\ \text{ksi}$$

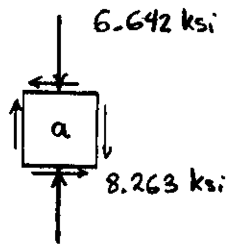


$\bar{y} = \frac{1}{2}d - \frac{1}{2}t_f = 4.03 - 0.2325 = 3.7975\ \text{in}$

$A_f = b_f t_f = (6.535)(0.465) = 3.0388\ \text{in}^2$

$Q_a = A_f \bar{y} = 11.540\ \text{in}^3$

$$\tau = \frac{VQ_a}{It_w} = \frac{(20)(11.540)}{(98.0)(0.285)} = 8.263\ \text{ksi}$$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-8.263)}{0 + 6.642} = -2.4881$$

$\theta_a = -34.1°,\quad \theta_b = 55.9°$

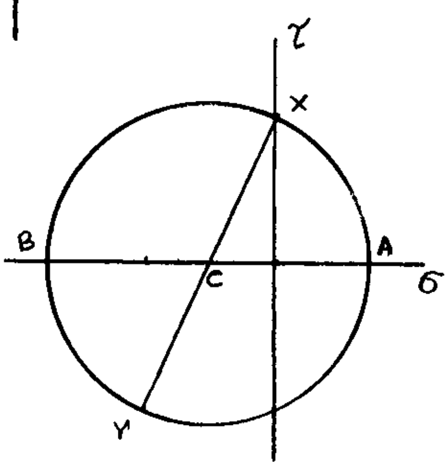
$\sigma_c = -\frac{6.642}{2} = -3.321\ \text{ksi}$

$R = \sqrt{\left(\frac{6.642}{2}\right)^2 + (8.263)^2} = 8.905\ \text{ksi}$

$\sigma_a = \sigma_c + R = 5.58\ \text{ksi}$

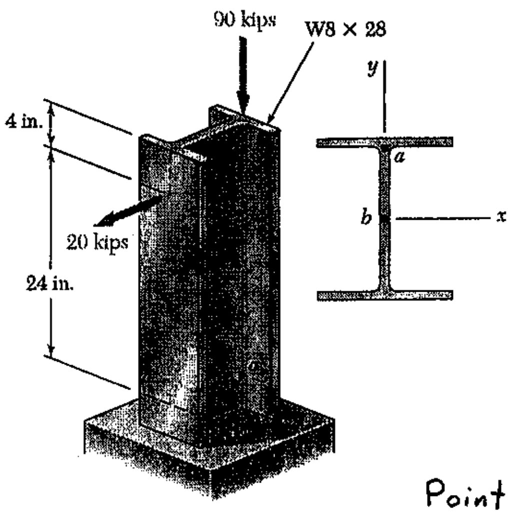
$\sigma_b = \sigma_c - R = -12.23\ \text{ksi}$

$\tau_{max} = R = 8.91\ \text{ksi}$



PROBLEM 8.64

8.64 Two forces are applied to a W8 × 28 rolled-steel beam as shown. Determine the principal stresses, principal planes, and maximum shearing stress at point *b*.



SOLUTION

For W 8 × 28 rolled steel section

$A = 8.25\ in^2$, $d = 8.06$ in, $b_f = 6.535$ in

$t_f = 0.465$ in, $t_w = 0.285$ in, $I_x = 98.0\ in^4$

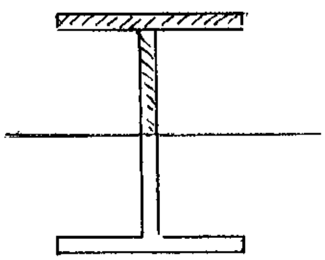
At the section containing points a and b.

$P = -90$ kips, $\quad V = 20$ kips

$M = (20)(24) - (4.03)(90) = -117.3$ kip·in.

Point b lies on the neutral axis of bending

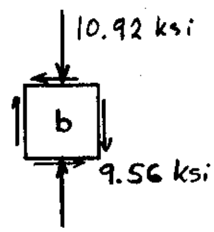
At point b $\quad \sigma = \dfrac{P}{A} = \dfrac{-90}{8.25} = -10.92$ ksi



Part	A (in^2)	$\bar{y}$ (in)	$A\bar{y}$ (in^3)
Flange	3.0388	3.7975	11.540
Half-web	1.0161	1.7825	1.811
Σ			13.351

$Q_b = 13.351\ in^3$

$\tau = \dfrac{VQ_b}{It_w} = \dfrac{(20)(13.351)}{(98.0)(0.285)} = 9.56$ ksi



$\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(-9.56)}{0 + 10.92} = -1.7509$

$\theta_a = -30.1°$ $\quad \theta_b = 59.9°$ ◄

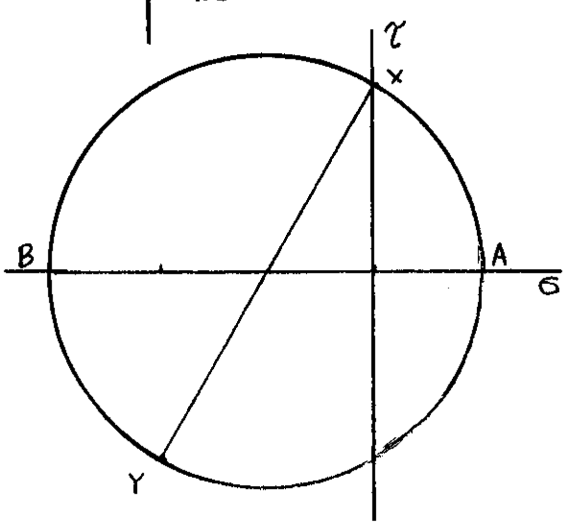
$\sigma_c = -\dfrac{10.92}{2} = -5.46$ ksi

$R = \sqrt{\left(\dfrac{10.92}{2}\right)^2 + (9.56)^2} = 11.01$ ksi

$\sigma_{max} = \sigma_c + R = 5.55$ ksi ◄

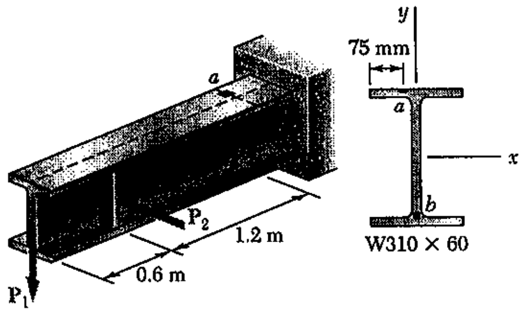
$\sigma_{min} = \sigma_c - R = -16.47$ ksi ◄

$\tau_{max} = R = 11.01$ ksi ◄



PROBLEM 8.65

8.65 Two forces $\mathbf{P}_1$ and $\mathbf{P}_2$ are applied as shown in directions perpendicular to the longitudinal axis of a W310 × 60 beam. Knowing that $P_1 = 25$ kN and $P_2 = 24$ kN, determine the principal stresses and the maximum shearing stress at point a.



SOLUTION

At the section containing points a and b

$M_x = (1.8)(25) = 45$ kN·m

$M_y = -(1.2)(24) = -28.8$ kN·m

$V_x = -24$ kN $\qquad V_y = -25$ kN

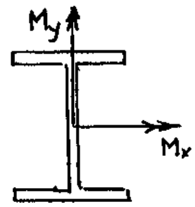
For W 310 × 60 rolled steel section

$d = 303$ mm, $\quad b_f = 203$ mm, $\quad t_f = 13.1$ mm, $\quad t_w = 7.5$ mm

$I_x = 129 \times 10^6\ \text{mm}^4 = 129 \times 10^{-6}\ \text{m}^4, \qquad I_y = 18.3 \times 10^6\ \text{mm}^4 = 18.3 \times 10^{-6}\ \text{m}^4$

Normal stress at point a $\qquad x = -\frac{b_f}{2} + 75 = -26.5$ mm

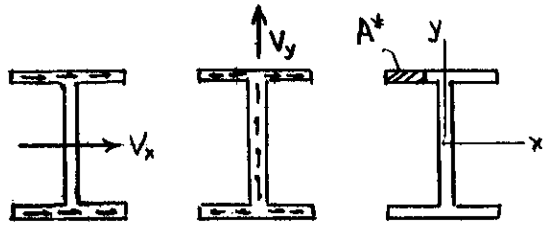
$y = \frac{1}{2}d = 151.5$ mm



$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(45\times10^3)(151.5\times10^{-3})}{129\times10^{-6}} - \frac{(-28.8\times10^3)(-26.5\times10^{-3})}{18.3\times10^{-6}}$$

$$= 52.849\ \text{MPa} - 41.705\ \text{MPa} = 11.144\ \text{MPa}$$

Shearing stress at point a



$$\tau_{xz} = -\frac{V_x A^* \bar{x}}{I_y t_f} - \frac{V_y A^* \bar{y}}{I_x t_f}$$

$A^* = (75\times10^{-3})(13.1\times10^{-3}) = 982.5\times10^{-6}\ \text{m}^2$

$\bar{x} = -\frac{b_f}{2} + \frac{75}{2} = -64$ mm

$\bar{y} = \frac{d}{2} - \frac{t_f}{2} = 144.95$ mm

$$\tau_{xz} = -\frac{(-24\times10^3)(982.5\times10^{-6})(-64\times10^{-3})}{(18.3\times10^{-6})(13.1\times10^{-3})} - \frac{(-25\times10^3)(982.5\times10^{-6})(144.95\times10^{-3})}{(129\times10^{-6})(13.1\times10^{-3})}$$

$$= -6.295\ \text{MPa} + 2.107\ \text{MPa} = -4.188\ \text{MPa}$$

$\sigma_{ave} = \frac{11.144}{2} = 5.572$ MPa

$R = \sqrt{\left(\frac{11.144}{2}\right)^2 + (4.188)^2} = 6.970$ MPa

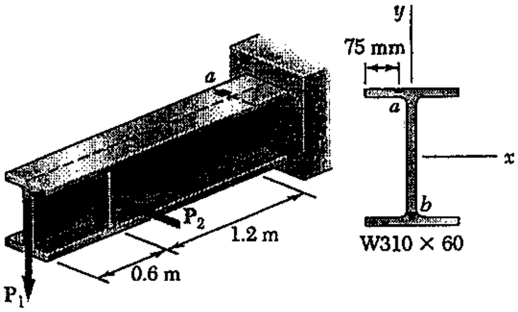
$\sigma_{max} = \sigma_{ave} + R = 12.54$ MPa ◄

$\sigma_{min} = \sigma_{ave} - R = -1.40$ MPa ◄

$\tau_{max} = R = 6.97$ MPa ◄

PROBLEM 8.66

8.66 **Two forces $\mathbf{P}_1$ and $\mathbf{P}_2$ are applied as shown in directions perpendicular to the longitudinal axis of a W310 × 60 beam. Knowing that $P_1 = 25$ kN and $P_2 = 24$ kN, determine the principal stresses and the maximum shearing stress at point *b*.**



SOLUTION

At the section containing points a and b

$M_x = (1.8)(25) = 45$ kN·m

$M_y = -(1.2)(24) = -28.8$ kN·m

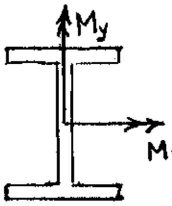
$V_x = -24$ kN, $\quad V_y = -25$ kN

For W310×60 rolled steel section

$d = 303$ mm, $b_f = 203$ mm, $t_f = 13.1$ mm, $t_w = 7.5$ mm

$I_x = 129 \times 10^6$ mm^4 $= 129 \times 10^{-6}$ m^4, $\quad I_y = 18.3 \times 10^6$ mm^4 $= 18.3 \times 10^{-6}$ m^4

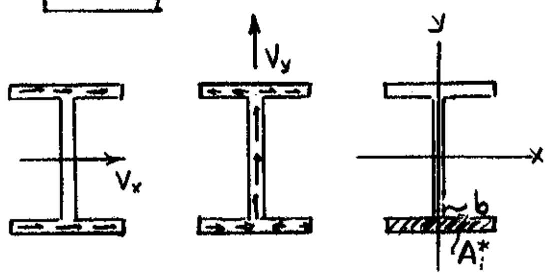
Normal stress at point b $\qquad x \approx 0, \quad y = -\frac{1}{2}d + t_f = -138.4$ mm.



$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(45\times10^3)(-138.4\times10^{-3})}{129\times10^{-6}} - 0$$

$$= -48.28 \text{ MPa}$$

Shearing stress at point b.

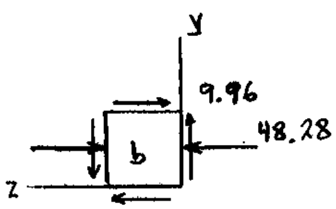


$$\tau_{yz} = -\frac{V_y A^* \bar{y}}{I_x t_w}$$

$A^* = A_f = b_f t_f = 2659$ mm^2
$= 2659 \times 10^{-6}$ m^2

$\bar{x} = 0, \quad \bar{y} = -\frac{1}{2}d + \frac{1}{2}t_f = -144.95$ mm

$$\tau_{yz} = -\frac{(-25\times10^3)(2659\times10^{-6})(-144.95\times10^{-3})}{(129\times10^{-6})(7.5\times10^{-3})} = -9.96 \text{ MPa}$$



$\sigma_{ave} = -\dfrac{48.28}{2} = -24.14$ MPa

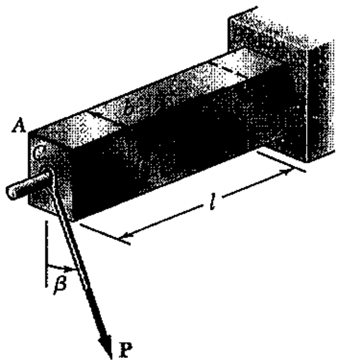
$R = \sqrt{\left(\dfrac{48.28}{2}\right)^2 + (9.96)^2} = 26.11$

$\sigma_{max} = \sigma_{ave} + R = 1.97$ MPa ◄

$\sigma_{min} = \sigma_{ave} - R = -50.3$ MPa ◄

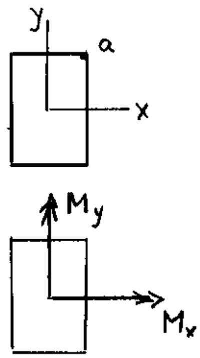
$\tau_{max} = R = 26.1$ MPa ◄

PROBLEM 8.67



8.67 A force **P** is applied to a cantilever beam by means of a cable attached to a bolt located at the center of the free end of the beam. Knowing that **P** acts in a direction perpendicular to the longitudinal axis of the beam, determine (*a*) the normal stress at point *a* in terms of *P*, *b*, *h*, *l*, and *β*, (*b*) the values of *β* for which the normal stress at *a* is zero.

SOLUTION



$$I_x = \frac{1}{12} b h^3 \qquad I_y = \frac{1}{12} h b^3$$

$$\sigma = \frac{M_x (h/2)}{I_x} - \frac{M_y (b/2)}{I_y}$$

$$= \frac{6 M_x}{b h^2} - \frac{6 M_y}{h b^2}$$

$$\vec{P} = P \sin\beta \, \vec{i} - P \cos\beta \, \vec{j} \qquad \vec{r} = l \, \vec{k}$$

$$\vec{M} = \vec{r} \times \vec{P} = l \vec{k} \times (P \sin\beta \, \vec{i} - P \cos\beta \, \vec{j}) = Pl \cos\beta \, \vec{i} + Pl \sin\beta \, \vec{j}$$

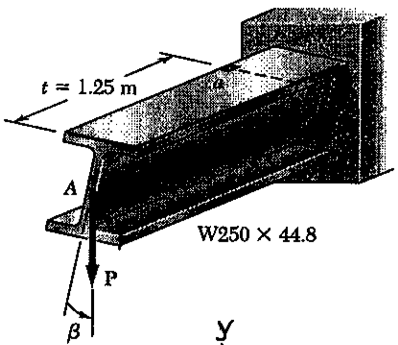
$$M_x = Pl \cos\beta \qquad M_y = Pl \sin\beta$$

(a) $$\sigma = \frac{6 Pl \cos\beta}{b h^2} - \frac{6 Pl \sin\beta}{h b^2} = \frac{6Pl}{bh}\left[\frac{\cos\beta}{h} - \frac{\sin\beta}{b} \right]$$ ◄

(b) $$\sigma = 0 \qquad \frac{\cos\beta}{h} - \frac{\sin\beta}{b} = 0 \qquad \tan\beta = \frac{b}{h}$$

$$\beta = \tan^{-1}\left(\frac{b}{h}\right)$$ ◄

PROBLEM 8.68



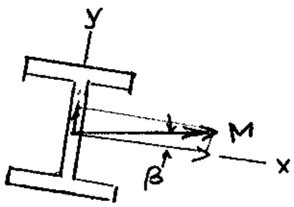
8.68 A vertical force **P** is applied at the center of the free end of cantilever beam *AB*. (*a*) If the beam is installed with the web vertical ($\beta = 0$) and with its longitudinal axis *AB* horizontal, determine the magnitude of the force **P** for which the normal stress at point *a* is +120 MPa. (*b*) Solve part *a*, assuming that the beam is installed with $\beta = 3°$.

SOLUTION

For W 250 × 44.8 rolled steel section

$S_x = 535 \times 10^3\ mm^3 = 535 \times 10^{-6}\ m^3$

$S_y = 95.0 \times 10^3\ mm = 95.0 \times 10^{-6}\ m^3$



At the section containing point a

$M_x = Pl\cos\beta \quad , \quad M_y = Pl\sin\beta$

Stress at a

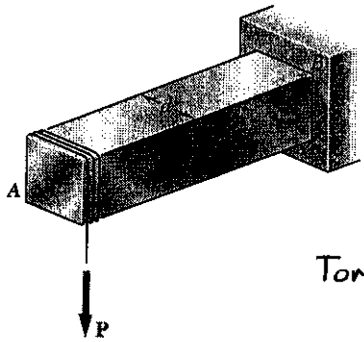
$$\sigma = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{Pl\cos\beta}{S_x} + \frac{Pl\sin\beta}{S_y}$$

Allowable load $\quad P_{all} = \frac{\sigma_{all}}{l}\left[\frac{\cos\beta}{S_x} + \frac{\sin\beta}{S_y}\right]^{-1}$

(a) $\beta = 0 \quad P_{all} = \frac{120 \times 10^6}{1.25}\left[\frac{1}{535 \times 10^{-6}} + 0\right]^{-1} = 51.4 \times 10^3\ N = 51.4\ kN$

(b) $\beta = 3° \quad P_{all} = \frac{120 \times 10^6}{1.25}\left[\frac{\cos 3°}{535 \times 10^{-6}} + \frac{\sin 3°}{95.0 \times 10^{-6}}\right]^{-1} = 39.7\ kN$

PROBLEM 8.69



***8.69** A 500-lb force **P** is applied to a wire that is wrapped around the bar AB as shown. Knowing that the cross section of the bar is a square of side $d = 0.75$ in., determine the principal stresses and the maximum shearing stress at point a.

SOLUTION

Bending: Point a lies on the neutral axis.

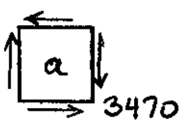
$$\sigma = 0$$

Torsion: $\tau = \dfrac{T}{c_1 a b^2}$ where $a = b = d$ and $c_1 = 0.208$ for a square section.

Since $T = \dfrac{Pd}{2}$ $\quad \tau = \dfrac{P}{0.416\, d^2} = 2.404 \dfrac{P}{d^2}$

Transverse shear: $V = P$ $\quad \tau = \dfrac{3}{2}\dfrac{V}{A} = 1.5 \dfrac{P}{d^2}$

Using superposition: $\tau = 3.904 \dfrac{P}{d^2} = 3.904 \dfrac{500}{(0.75)^2} = 3470$ psi

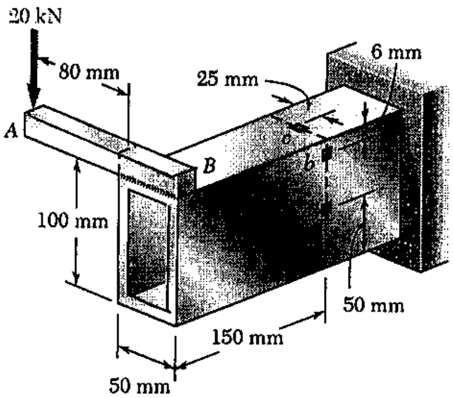


$\sigma_{max} = 3470$ psi ◀

$\sigma_{min} = -3470$ psi ◀

$\tau_{max} = 3470$ psi ◀

PROBLEM 8.70



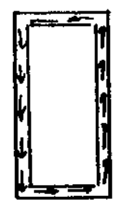
***8.70** A vertical 20-kN force is applied to end A of the bar AB, which is welded to an extruded aluminum tube. Knowing that the tube has a uniform wall thickness of 6 mm, determine the shearing stress at points a, b, and c.

SOLUTION

$$I = \frac{1}{12}(50)(100)^3 - \frac{1}{12}(38)(88)^3 = 2.0087\times10^{6}\ \text{mm}^4 = 2.0087\times10^{-6}\ \text{m}^4$$

Torsion: $T = (20\times10^3)(80+25)(10^{-3}) = 2100\ \text{N}\cdot\text{m}$

$$a = (44)(94) = 4.136\times10^3\ \text{mm}^2 = 4.136\times10^{-3}\ \text{m}^2$$

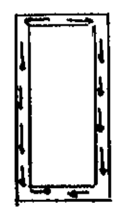


For points a, b, and c

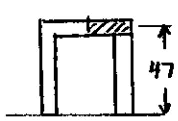
$$\tau = \frac{T}{2ta} = \frac{2100}{(2)(6\times10^{-3})(4.136\times10^{-3})} = 42.31\ \text{MPa}$$

Transverse shear: $V = 20\times10^3\ \text{N}$

Point c − on symmetry axis $\tau = 0$



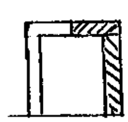
Point b



$$Q_b = (25)(6)(47) = 7.05\times10^3\ \text{mm}^3 = 7.05\times10^{-6}\ \text{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(20\times10^3)(7.05\times10^{-6})}{(2.0087\times10^{-6})(6\times10^{-3})} = 11.70\ \text{MPa}$$

Point a



$$Q_a = Q_b + (6)(44)(22) = 12.858\times10^3\ \text{mm}^3 = 12.858\times10^{-6}\ \text{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(20\times10^3)(12.858\times10^{-6})}{(2.0087\times10^{-6})(6\times10^{-3})} = 21.34\ \text{MPa}$$

Net shearing stress:

Point a $\quad \tau = 42.31 - 0 = 42.3\ \text{MPa}$ ◄

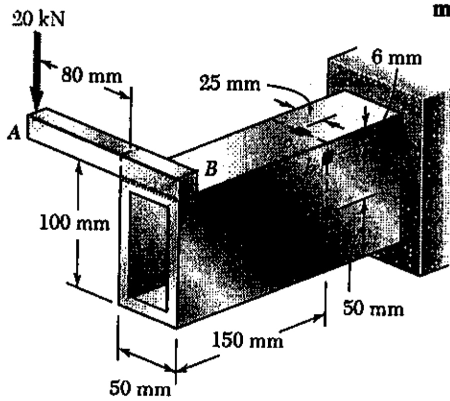
Point b $\quad \tau = 42.31 - 11.70 = 30.6\ \text{MPa}$ ◄

Point c $\quad \tau = 42.31 - 21.34 = 21.0\ \text{MPa}$ ◄

PROBLEM 8.71

***8.70** A vertical 20-kN force is applied to end *A* of the bar *AB*, which is welded to an extruded aluminum tube. Knowing that the tube has a uniform wall thickness of 6 mm, determine the shearing stress at points *a*, *b*, and *c*.

***8.71** For the tube and loading of Prob. 8.70, determine the principal stresses and the maximum shearing stress at point *b*.



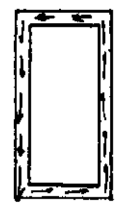
SOLUTION

Bending: $M = (20\times10^3)(150\times10^{-3}) = 3000\ \mathrm{N\cdot m}$

$$I = \tfrac{1}{12}(50)(100)^3 - \tfrac{1}{12}(38)(88)^3 = 2.0087\times10^6\ \mathrm{mm}^4 = 2.0087\times10^{-6}\ \mathrm{m}^4$$

$$\sigma = \frac{My}{I} = \frac{(3000)(44\times10^{-3})}{2.0087\times10^{-6}} = 65.7\ \mathrm{MPa}$$

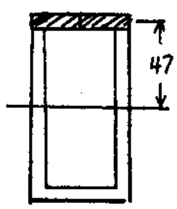
Torsion: $T = (20\times10^3)(80+25)(10^{-3}) = 2100\ \mathrm{N\cdot m}$

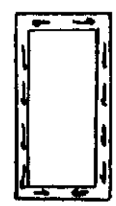


$$a = (44)(94) = 4.136\times10^3\ \mathrm{mm}^2 = 4.136\times10^{-3}\ \mathrm{m}^2$$

$$\tau = \frac{T}{2ta} = \frac{2100}{(2)(6\times10^{-3})(4.136\times10^{-3})} = 42.31\ \mathrm{MPa}$$

Transverse shear: $V = 20\times10^3\ \mathrm{N}$

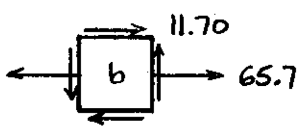


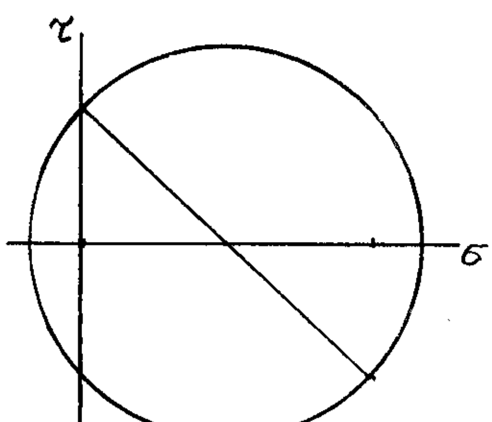


$$Q = (50)(6)(47) = 14.1\times10^3\ \mathrm{mm}^3 = 14.1\times10^{-6}\ \mathrm{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(20\times10^3)(14.1\times10^{-6})}{(2.0087\times10^{-6})(12\times10^{-3})} = 11.70\ \mathrm{MPa}$$

Net shearing stress $\tau = 42.31 - 11.70 = 30.6\ \mathrm{MPa}$





$\sigma_c = \frac{1}{2}\sigma = 32.85\ \mathrm{MPa}$

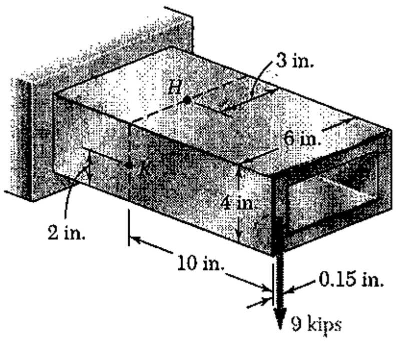
$R = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{65.7}{2}\right)^2 + (30.6)^2} = 44.89\ \mathrm{MPa}$

$\sigma_{max} = \sigma_c + R = 77.7\ \mathrm{MPa}$ ◄

$\sigma_{min} = \sigma_c - R = -12.04\ \mathrm{MPa}$ ◄

$\tau_{max} = R = 44.9\ \mathrm{MPa}$ ◄

PROBLEM 8.72



*8.72 Knowing that the structural tube shown has a uniform wall thickness of 0.3 in., determine the principal stresses, principal planes, and maximum shearing stress at (*a*) point *H*, (*b*) point *K*.

SOLUTION

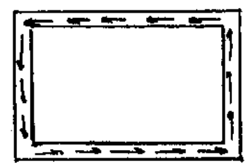
At the section containing points H and K

$V = 9$ kips $\qquad M = (9)(10) = 90$ kip·in.

$T = (9)(3 - 0.15) = 25.65$ kip·in

Torsion:

$a = (5.7)(3.7) = 21.09 \text{ in}^2$

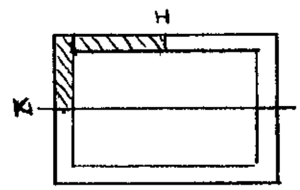


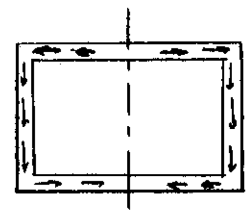
$$\tau = \frac{T}{2ta} = \frac{25.65}{(2)(0.3)(21.09)} = 2.027 \text{ ksi}$$

Transverse shear:

$Q_H = 0$

$Q_K = (3)(2)(1) - (2.7)(1.7)(0.85)$
$\quad = 2.0985 \text{ in}^3$



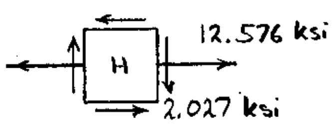


$$I = \tfrac{1}{12}(6)(4)^3 - \tfrac{1}{12}(5.4)(3.4)^3 = 14.3132 \text{ in}^4$$

$$\tau_H = 0 \qquad \tau_K = \frac{VQ_a}{It} = \frac{(9)(2.0985)}{(14.3132)(0.3)} = 4.398 \text{ ksi}$$

Bending: $\quad \sigma_H = \dfrac{Mc}{I} = \dfrac{(90)(2)}{14.3132} = 12.576$ ksi , $\qquad \sigma_K = 0$

(a) Point H :



$\sigma_c = \dfrac{12.576}{2} = 6.288$ ksi

$R = \sqrt{\left(\dfrac{12.576}{2}\right)^2 + (2.027)^2} = 6.607$ ksi

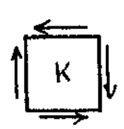
$\sigma_{max} = \sigma_c + R = 12.90$ ksi ◀

$\sigma_{min} = \sigma_c - R = -0.32$ ksi ◀

$\tan 2\theta_p = \dfrac{2\tau}{\sigma} = -0.3224 \qquad \theta_p = -8.9^\circ, 81.1^\circ$ ◀

$\tau_{max} = R = 6.61$ ksi ◀

(b) Point K: $\qquad \sigma = 0 \qquad \tau = 2.027 + 4.398 = 6.425$ ksi



$\sigma_{max} = 6.43$ ksi ◀

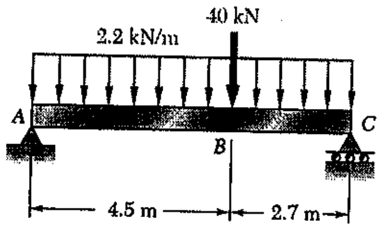
$\sigma_{min} = -6.43$ ksi ◀

$\theta_p = \pm 45^\circ$ ◀

$\tau_{max} = 6.43$ ksi ◀

PROBLEM 8.73

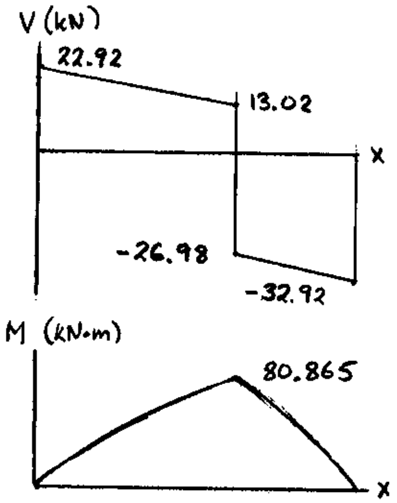
8.73 (*a*) Knowing that $\sigma_{all} = 165$ MPa and $\tau_{all} = 100$ MPa, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.



$$+\Sigma M_C = 0$$

$$-7.2 R_A + (2.2)(7.2)(3.6) + (40)(2.7) = 0$$

$$R_A = 22.92 \text{ kN}$$



$$V_A = R_A = 22.92 \text{ kN}$$

$$V_B^- = 22.92 - (2.2)(4.5) = 13.02 \text{ kN}$$

$$V_B^+ = 13.02 - 40 = -26.98 \text{ kN}$$

$$V_C = -26.98 - (2.2)(2.7) = -32.92 \text{ kN}$$

$$M_A = 0$$

$$M_B = 0 + \tfrac{1}{2}(22.92 + 13.02)(4.5) = 80.865 \text{ kN·m}$$

$$M_C = 0$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{80.865 \times 10^3}{165 \times 10^6} = 490 \times 10^{-6} \text{ m}^3 = 490 \times 10^3 \text{ mm}^3$$

Shape	S (10^3 mm^3)
W 360 × 39	578
W 310 × 38.7	549 ←
W 250 × 44.8	535
W 200 × 52	512

Try W 310 × 38.7

$d = 310$ mm $\quad t_f = 9.7$ mm

$t_w = 5.8$ mm

$$\sigma_m = \frac{M_B}{S} = \frac{80.865 \times 10^3}{549 \times 10^{-6}} = 147.3 \text{ MPa}$$

$$\tau_m = \frac{|V|_{max}}{d\,t_w} = \frac{32.92 \times 10^3}{(310 \times 10^{-3})(5.8 \times 10^{-3})} = 18.31 \text{ MPa}$$

$c = \tfrac{1}{2} d = 155$ mm $\qquad y_b = c - t_f = 155 - 9.7 = 145.3$ mm

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{145.3}{155}\right)(147.3) = 138.1 \text{ MPa}$$

At point B $\qquad \tau_w = \dfrac{V}{d\,t_w} = \dfrac{(26.98 \times 10^3)}{(310 \times 10^{-3})(5.8 \times 10^{-3})} = 15.0$ MPa

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_w^2} = \sqrt{(69.05)^2 + (15.0)^2} = 70.66 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 69.05 + 70.66 = 139.7 \text{ MPa}$$

PROBLEM 8.74

8.74 Knowing that the shear and bending moment in a given section of a W21 × 101 rolled-steel beam are, respectively, 120 kips and 300 kip · ft, determine the values in that section of (*a*) the maximum normal stress σ_m, (*b*) the principal stress σ_{max} at the junction of a flange and the web.

SOLUTION

$M = 300 \text{ kip}\cdot\text{ft} = 3600 \text{ kip}\cdot\text{in} \qquad V = 120 \text{ kips.}$

For W21×101 shape $\qquad d = 21.36 \text{ in} \qquad b_f = 12.290 \text{ in} \qquad t_f = 0.800 \text{ in.}$

$t_w = 0.500 \text{ in}, \quad I_z = 2420 \text{ in}^4, \quad S_z = 227 \text{ in}^3, \quad c = \frac{1}{2}d = 10.68 \text{ in.}$

(a) $\sigma_m = \dfrac{M}{S} = \dfrac{3600}{227} = 15.86 \text{ ksi}$ ◄

(b) $y_b = c - t_f = 9.88 \text{ in}$

$\sigma_b = \dfrac{y_b}{c}\sigma_m = 14.67 \text{ ksi} \qquad 7.336$

$A_f = b_f t_f = 9.832 \text{ in}^2 \qquad \bar{y} = \frac{1}{2}(c + y_b) = 10.28 \text{ in.}$

$Q = A_f \bar{y} = 101.07 \text{ in}^3$

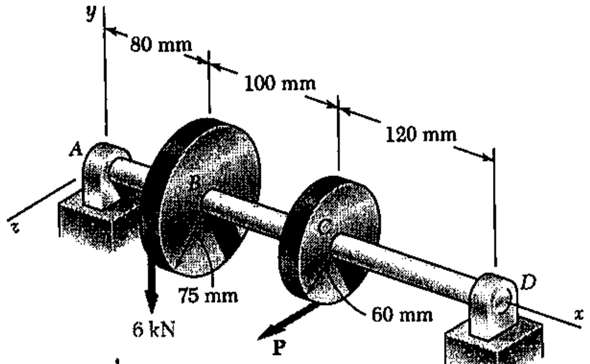
$\tau_b = \dfrac{VQ}{I_z t_w} = \dfrac{(120)(101.07)}{(2420)(0.500)} = 10.024 \text{ ksi}$

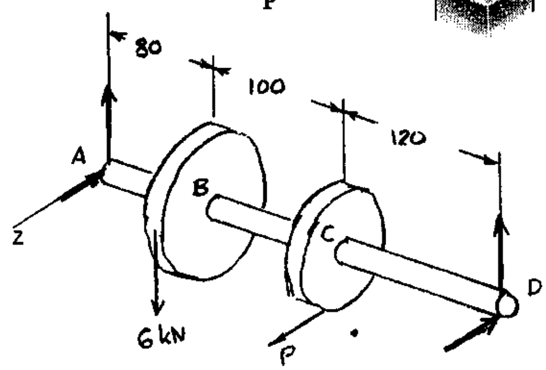
$R = \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{7.336^2 + 10.024^2} = 12.421 \text{ ksi}$

$\sigma_{max} = \dfrac{\sigma_b}{2} + R = 7.336 + 10.421 = 19.76 \text{ ksi}$ ◄

PROBLEM 8.75

8.75 The 6-kN force is vertical and the force **P** is parallel to the z axis. Knowing that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of the solid shaft AD.





SOLUTION

$\Sigma M_x = 0$

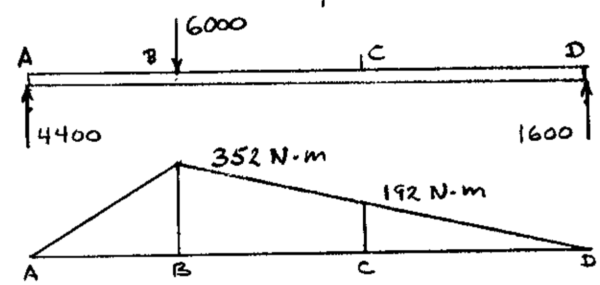
$(6\times10^3)(75\times10^{-3}) - (60\times10^{-3})P = 0$

$P = 7.5\times10^3$ N

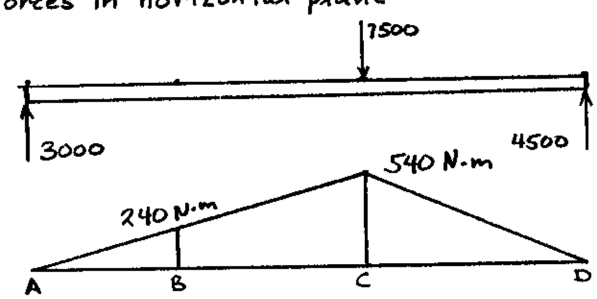
Over portion BC

$T = (6\times10^3)(75\times10^{-3}) = 450$ N·m

Forces in vertical plane



Forces in horizontal plane



Bending moments

At B $\quad M = \sqrt{352^2 + 240^2}$

$= 426.0$ N·m

At C $\quad M = \sqrt{540^2 + 192^2}$

$= 573.1$ N·m

Critical section is just to the left of gear C

$M = 573.1$ N·m $\qquad T = 450$ N·m $\qquad \sqrt{M^2 + T^2} = 728.67$ N·m

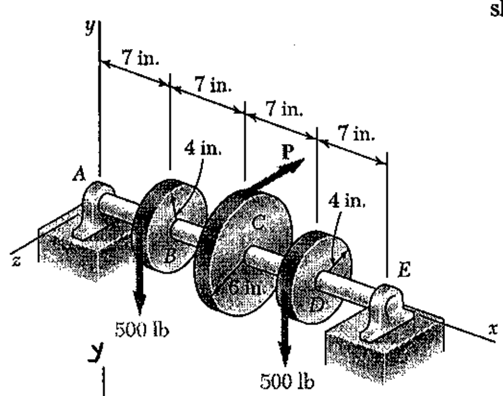
$$\tau_{all} = \frac{c}{J}\left(\sqrt{M^2+T^2}\right)_{max}$$

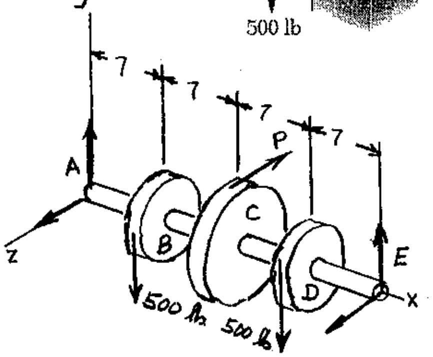
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M^2+T^2}\right)_{max}}{\tau_{all}} = \frac{728.67}{60\times10^6} = 12.145\times10^{-6}\ \text{m}^3$$

$c = 19.77\times10^{-3}$ m $\qquad d = 2c = 39.5\times10^{-3}$ m $= 39.5$ mm

PROBLEM 8.76

8.76 The two 500-lb forces are vertical and the force **P** is parallel to the z axis. Knowing that $\tau_{all} = 8$ ksi, determine the smallest permissible diameter of the solid shaft AE.





SOLUTION

$\Sigma M_x = 0 \qquad (4)(500) - 6P + (4)(500) = 0$

$P = 666.67$ lb.

Torques:

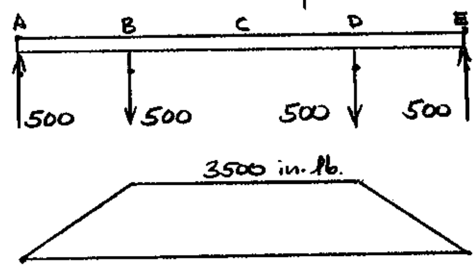
AB: $T = 0$

BC: $T = -(4)(500) = -2000$ in·lb

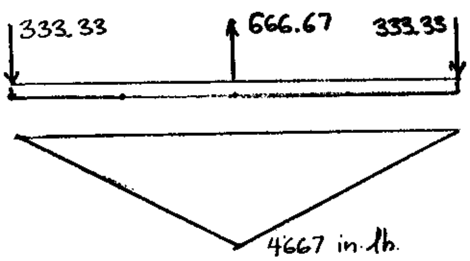
CD: $T = (4)(500) = 2000$ in·lb

DE: $T = 0$

Forces in vertical plane



Forces in horizontal plane



Critical sections are either side of disk C

$T = 2000$ in·lb $\qquad M_z = 3500$ in·lb

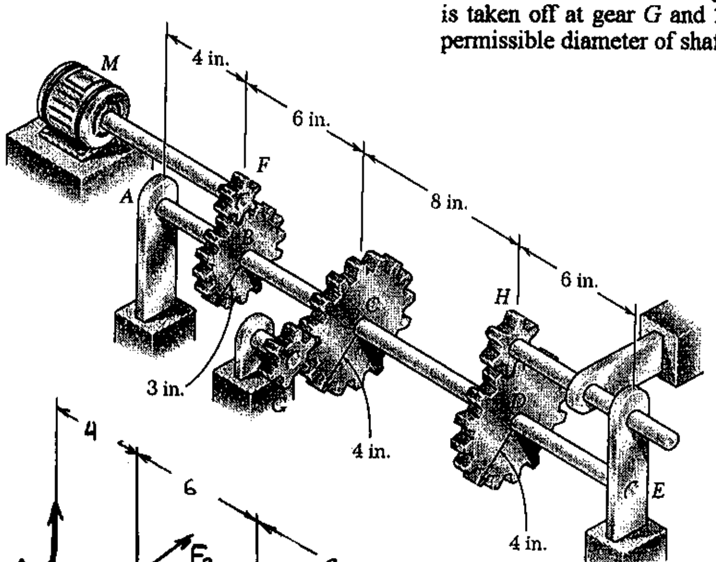
$M_y = 4667$ in lb

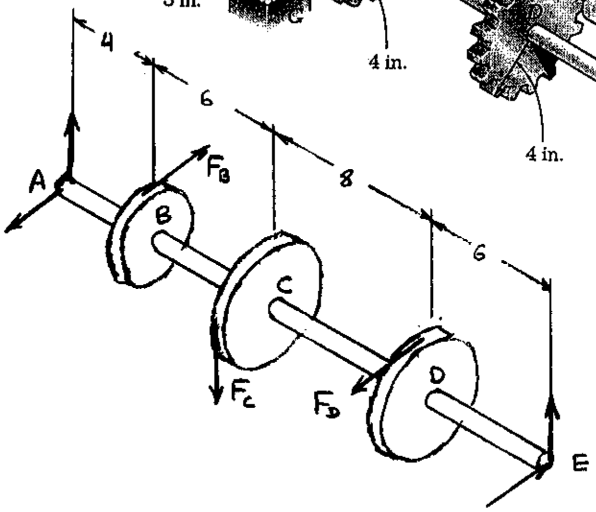
$$\tau_{all} = \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{all}} = \frac{\sqrt{4667^2 + 3500^2 + 2000^2}}{8 \times 10^3} = 0.77083 \text{ in}^3$$

$c = 0.789$ in. $\qquad d = 2c = 1.578$ in.

PROBLEM 8.77





8.77 The solid shaft *AE* rotates at 600 rpm and transmits 60 hp from the motor *M* to machine tools connected to gears *G* and *H*. Knowing that $\tau_{all} = 8$ ksi and that 40 hp is taken off at gear *G* and 20 hp is taken off at gear *H*, determine the smallest permissible diameter of shaft *AE*.

SOLUTION

$$60 \text{ hp} = (60)(6600) = 396 \times 10^3 \text{ in}\cdot\text{lb/sec}$$

$$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$$

Torque on gear B

$$T_B = \frac{P}{2\pi f} = \frac{396 \times 10^3}{2\pi(10)} = 6302.5 \text{ in}\cdot\text{lb.}$$

Torques on gears C and D

$$T_C = \frac{40}{60} T_B = 4201.7 \text{ in}\cdot\text{lb.}$$

$$T_D = \frac{20}{60} T_B = 2100.8 \text{ in}\cdot\text{lb.}$$

Shaft torques

AB: $T_{AB} = 0$

BC: $T_{BC} = 6302.5$ in·lb

CD: $T_{CD} = 2100.8$ in·lb

DE: $T_{DE} = 0$

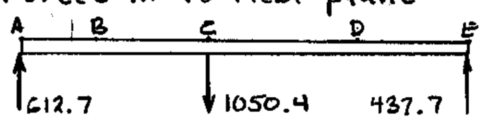
Gear forces

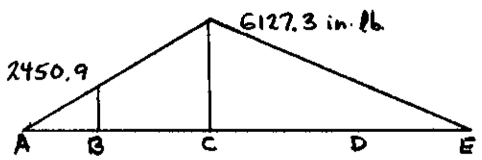
$$F_B = \frac{T_B}{r_B} = \frac{6302.5}{3} = 2100.8 \text{ lb.}$$

$$F_C = \frac{T_C}{r_C} = \frac{4201.7}{4} = 1050.4 \text{ lb.}$$

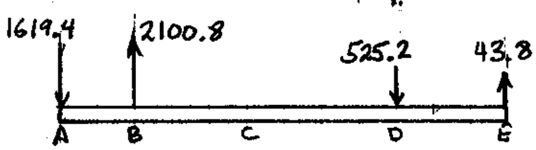
$$F_D = \frac{T_D}{r_D} = \frac{2100.8}{4} = 525.2 \text{ lb.}$$

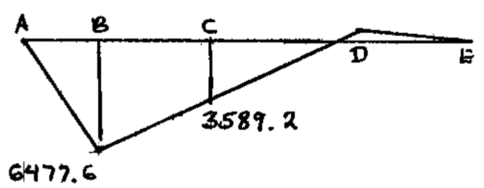
Forces in vertical plane





Forces in horizontal plane





At B^+ $\sqrt{M_z^2 + M_y^2 + T^2}$

$$= \sqrt{2450.9^2 + 6477.6^2 + 6302.5^2}$$

$$= 9364 \text{ in}\cdot\text{lb}$$

At C^- $\sqrt{M_z^2 + M_y^2 + T^2}$

$$= \sqrt{6127.3^2 + 3589.2^2 + 6302.5^2}$$

$$= 9495 \text{ in}\cdot\text{lb.} \quad \text{(maximum)}$$

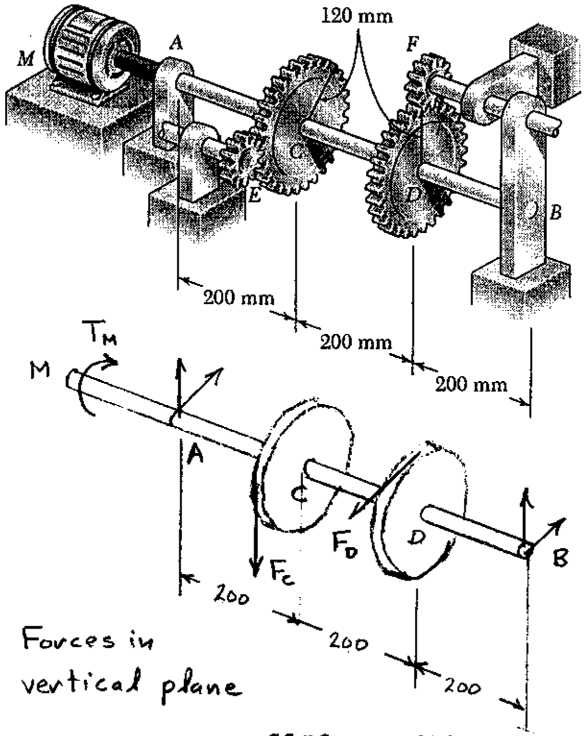
$$\tau_{all} = \frac{c}{J}\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)_{max}$$

$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)}{\tau_{all}} = \frac{9495}{8 \times 10^3} = 1.1868 \text{ in}^3$$

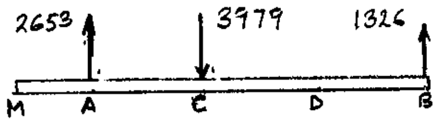
$c = 0.911$ in $\qquad d = 2c = 1.822$ in. ◄

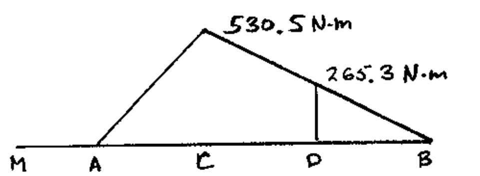
PROBLEM 8.78

8.78 The motor M rotates at 300 rpm and transmits 30 kW to the solid shaft AB through a flexible connection. Half of this power is transferred to a machine tool connected to gear E and the other half to a machine tool connected to gear F. Knowing that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of shaft AB.

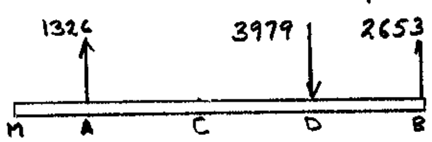


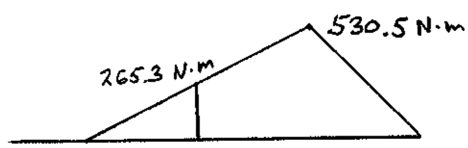
Forces in vertical plane





Forces in horizontal plane





SOLUTION

$$300 \text{ rpm} = \frac{300}{60} = 5 \text{ Hz}$$

$$T_M = \frac{P}{2\pi f} = \frac{30 \times 10^3}{(2\pi)(5)} = 954.9 \text{ N}\cdot\text{m}$$

Torques on gears C and T

$$T_c = T_D = \tfrac{1}{2} T_m = 477.5 \text{ N}\cdot\text{m}$$

Shaft torques.

MA: $T_{MA} = 954.9$ N·m

AC: $T_{AC} = 954.9$ N·m

CD: $T_{CD} = 477.5$ N·m

DB: $T_{DB} = 0$

Gear forces

$$F_c = \frac{T_c}{r_c} = \frac{477.5}{120 \times 10^{-3}} = 3979 \text{ N}$$

$$F_D = \frac{T_D}{r_D} = \frac{477.5}{120 \times 10^{-3}} = 3979 \text{ N}$$

Critical point is just to the left of gear C

$T_{AC} = 954.9$ N·m

$M_{cz} = 530.5$ N·m

$M_{cy} = 265.3$ N·m

$$\sqrt{M_z^2 + M_y^2 + T^2} = 1124.1 \text{ N}\cdot\text{m}$$

$$\tau_{all} = \frac{c}{J}\sqrt{M_z^2 + M_y^2 + T^2}$$

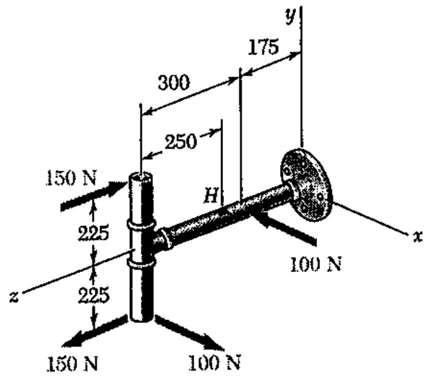
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M_z^2 + M_y^2 + T^2}}{\tau_{all}}$$

$$= \frac{1124.1}{60 \times 10^6} = 18.735 \times 10^{-6} \text{ m}^3$$

$$c = 22.85 \times 10^{-3} \text{ m}$$

$$d = 2c = 45.7 \times 10^{-3} \text{ m} = 45.7 \text{ mm}$$ ◄

PROBLEM 8.79



Dimensions in mm

8.79 Several forces are applied to the pipe assembly shown. Knowing that each section of pipe has inner and outer diameters respectively equal to 36 mm and 42 mm, determine the normal and shearing stresses at point H located at the top of the outer surface of the pipe.

SOLUTION

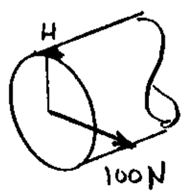
At the section containing point H

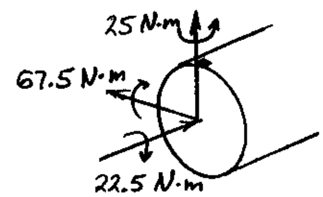
$P = 0, \quad V_x = 100\text{ N}, \quad V_y = 0$

$M_x = -(0.450)(150) = -67.5\text{ N}\cdot\text{m}$

$M_y = (0.250)(100) = 25\text{ N}\cdot\text{m}$

$M_z = -(0.225)(100) = -22.5\text{ N}\cdot\text{m}$





$d_o = 42\text{ mm} \qquad d_i = 32\text{ mm}$

$c_o = 21\text{ mm} \qquad c_i = 18\text{ mm}$

$t = c_o - c_i = 3\text{ mm}$

$A = \pi(c_o^2 - c_i^2) = 367.57\text{ mm}^2 = 367.57\times10^{-6}\text{ m}^2$

$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 70.30\times10^3\text{ mm}^4 = 70.30\times10^{-9}\text{ m}^4, \quad J = 2I = 140.59\times10^{-9}\text{ m}^4$

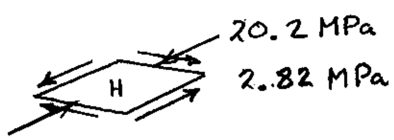
For half-pipe $Q = \frac{2}{3}(c_o^3 - c_i^3) = 2.286\times10^3\text{ mm}^3 = 2.286\times10^{-6}\text{ m}^3$

$\sigma_H = \frac{M_x y}{I_x} = \frac{(-67.5)(21\times10^{-3})}{70.30\times10^{-9}} = -20.2\text{ MPa}$

Due to torque $\quad (\tau_H)_T = \frac{Tc}{J} = \frac{(22.5)(21\times10^{-3})}{140.59\times10^{-9}} = 3.36\text{ MPa}$

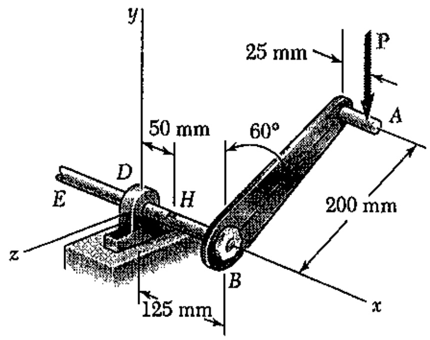
Due to shear $\quad (\tau_H)_V = \frac{VQ}{It} = \frac{(100)(2.286\times10^{-6})}{(70.30\times10^{-9})(6\times10^{-3})} = 0.54\text{ MPa}$

Net $\quad \tau_H = 3.36 - 0.54 = 2.82\text{ MPa}$



PROBLEM 8.80

8.80 A vertical force **P** of magnitude 250 N is applied to the crank at point *A*. Knowing that the shaft *BDE* has a diameter of 18 mm, determine the principal stresses and the maximum shearing stress at point *H* located at the top of the shaft, 50 mm to the right of support *D*



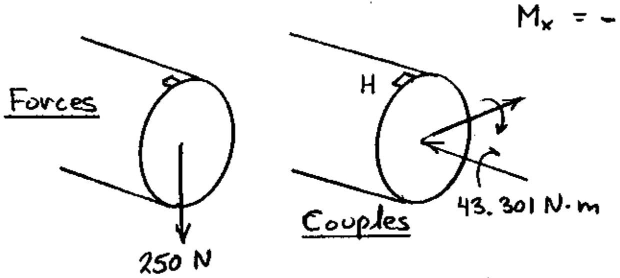
SOLUTION

Force-couple system at the centroid of the section containing point H.

$F_x = 0, \quad V_y = -250\text{ N}, \quad V_z = 0$

$M_z = -(125 - 50 + 25)(10^{-3})(250) = -25\text{ N}\cdot\text{m}$

$M_x = -(200 \sin 60°)(10^{-3})(250) = -43.301\text{ N}\cdot\text{m}$



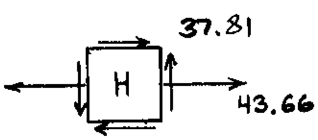
$d = 18\text{ mm} \qquad c = \frac{1}{2}d = 9\text{ mm}$

$I = \frac{\pi}{4}c^4 = 5.153 \times 10^3\text{ mm}^4$
$= 5.153 \times 10^{-9}\text{ m}^4$

$J = 2I = 10.306 \times 10^{-9}\text{ m}^4$

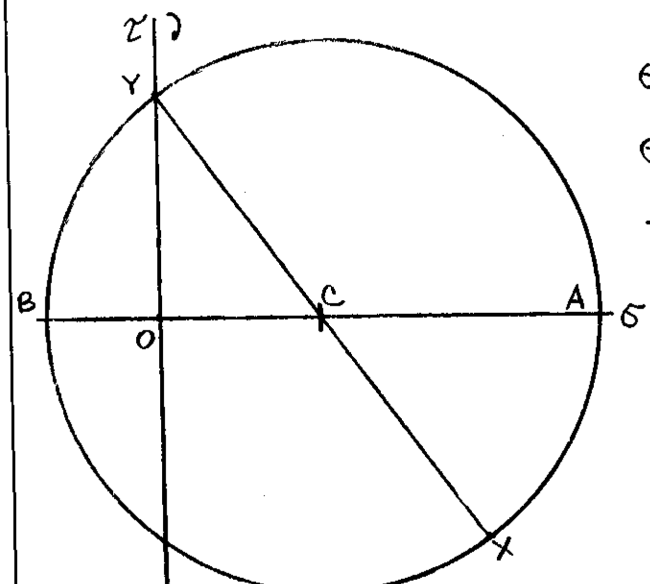
At point H $\qquad \sigma_H = -\frac{M_z y}{I_x} = -\frac{(-25)(9 \times 10^{-3})}{5.153 \times 10^{-9}} = 43.66\text{ MPa}$

$\tau_H = \frac{Tc}{J} = \frac{(43.301)(9 \times 10^{-3})}{10.306 \times 10^{-9}} = 37.81\text{ MPa}$



$\sigma_c = \frac{1}{2}\sigma_H = 21.83\text{ MPa}$

$R = \sqrt{\left(\frac{43.66}{2}\right)^2 + (37.81)^2} = 43.66\text{ MPa}$



$\sigma_a = \sigma_c + R = 65.5\text{ MPa}$

$\sigma_b = \sigma_c - R = -21.8\text{ MPa}$

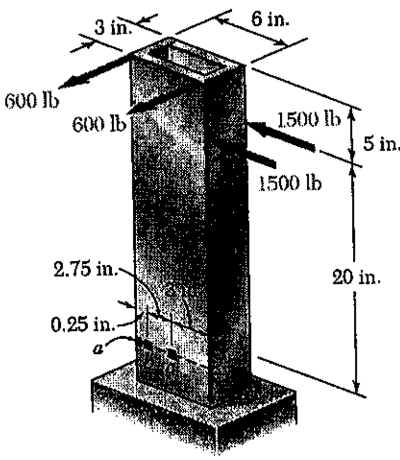
$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{75.62}{43.66} = 1.7320$

$\theta_a = 30°, \qquad \theta_b = 120°$

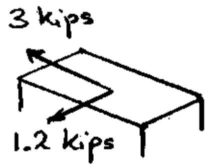
$\tau_{max} = R = 43.7\text{ MPa}$

PROBLEM 8.81

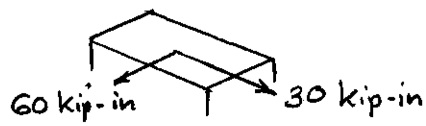
8.81 Knowing that the structural tube shown has a uniform wall thickness of 0.25 in., determine the normal and shearing stresses at the three points indicated.



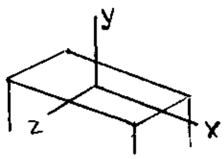
SOLUTION



Shear



Bending moment



$b_o = 6 \text{ in.} \quad b_i = b_o - 2t = 5.5 \text{ in.}$

$h_o = 3 \text{ in.} \quad h_i = h_o - 2t = 2.5 \text{ in.}$

$I_x = \frac{1}{12}(b_o h_o^3 - b_i h_i^3) = 6.3385 \text{ in}^4 \qquad I_z = \frac{1}{12}(h_o b_o^3 - h_i b_i^3) = 19.3385 \text{ in}^4$

Normal stresses

$$\sigma = \frac{M_z x}{I_z} - \frac{M_x z}{I_x}$$

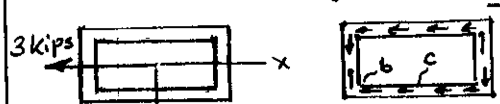
(a) $\frac{(60)(-3)}{19.3385} - \frac{(30)(1.5)}{6.3385} = -16.41 \text{ ksi}$

(b) $\frac{(60)(-2.75)}{19.3385} - \frac{(30)(1.5)}{6.3385} = -15.63 \text{ ksi}$

(c) $\frac{(60)(0)}{19.3385} - \frac{(30)(1.5)}{6.3385} = -7.10 \text{ ksi}$

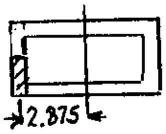
Shearing stresses (a) Point $\underline{a}$ is an outside corner; $\tau_a = 0$

Direction of shearing stresses



Due to V_x

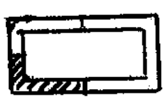
At point $\underline{b}$



$Q_{zb} = (1.5)(0.25)(2.875) = 1.0781 \text{ in}^3$

$\tau_{b,V_x} = \frac{V_x Q_z}{I_z t} = \frac{(3)(1.0781)}{(19.3385)(0.25)} = 0.669 \text{ ksi}$

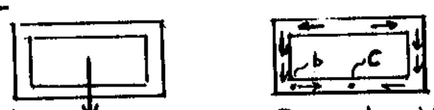
At point $\underline{c}$



$Q_{zc} = Q_{zb} + (2.75)(0.25)(\frac{2.75}{2})$

$= 2.0234 \text{ in}^3$

$\tau_{c,V_x} = \frac{V_x Q_z}{I_z t} = \frac{(3)(2.0234)}{(19.3385)(0.25)} = 1.256 \text{ ksi}$



Due to V_z

At point $\underline{b}$

$Q_{xb} = (2.75)(0.25)(1.375) = 0.9453 \text{ in}^3$

$\tau_{b,V_z} = \frac{V_z Q_x}{I_x t} = \frac{(1.2)(0.9453)}{(6.3385)(0.25)} = 0.716 \text{ ksi}$

At point $\underline{c}$ (symmetry axis)

$\tau_{c,V_z} = 0$

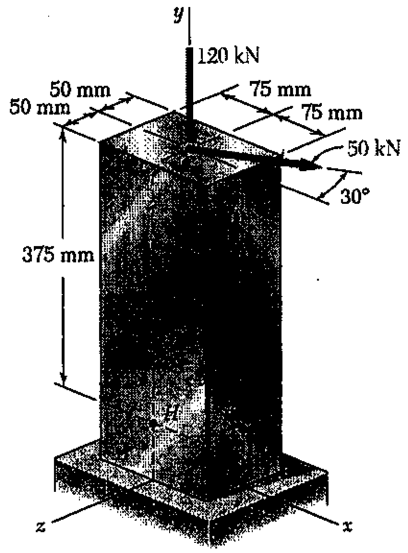
Net shearing stress at points $\underline{b}$ and $\underline{c}$

$\tau_b = 0.716 - 0.669 = 0.047 \text{ ksi}$

$\tau_c = 1.256 \text{ ksi}$

PROBLEM 8.82

8.82 For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point *H*.



SOLUTION

Components of force at point C.

$F_x = 50 \cos 30° = 43.301$ kN

$F_z = -50 \sin 30° = -25$ kN, $\quad F_y = -120$ kN

Section forces and couples at the section containing points H and K.

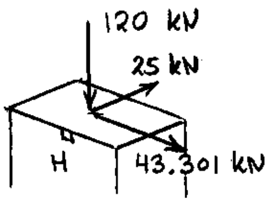
$P = 120$ kN (compression)

$V_x = 43.301$ kN, $\quad V_z = -25$ kN

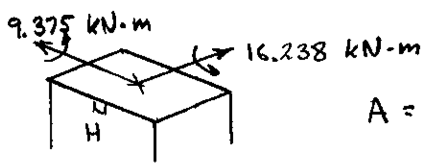
$M_x = -(25)(0.375) = -9.375$ kN·m

$M_y = 0$

$M_z = -(43.301)(0.375) = -16.238$ kN·m



Forces



Couples

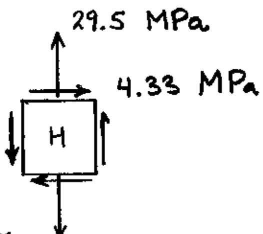
$A = (100)(150) = 15 \times 10^3$ mm^2
$= 15 \times 10^{-3}$ m^2

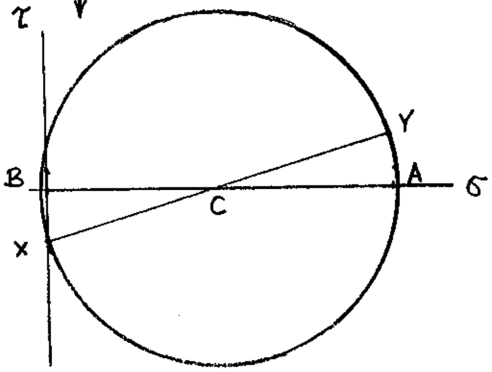
$I_x = \frac{1}{12}(150)(100)^3 = 12.5 \times 10^6$ mm^4
$= 12.5 \times 10^{-6}$ m^4

Stresses at point H

$$\sigma_H = -\frac{P}{A} - \frac{M_x z}{I_x} = -\frac{(120 \times 10^3)}{15 \times 10^{-3}} - \frac{(-9.375 \times 10^3)(50 \times 10^{-3})}{12.5 \times 10^{-6}} = 29.5 \text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{V_x}{A} = \frac{3}{2}\frac{43.301 \times 10^3}{15 \times 10^{-3}} = 4.33 \text{ MPa}$$





$\sigma_c = \frac{1}{2}\sigma_H = 14.75$ MPa

$R = \sqrt{\left(\frac{29.5}{2}\right)^2 + 4.33^2} = 15.37$ MPa

$\sigma_a = \sigma_c + R = 30.1$ MPa ◄

$\sigma_b = \sigma_c - R = -0.62$ MPa ◄

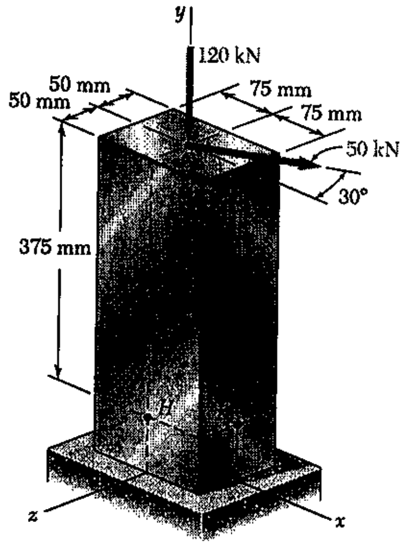
$\tan 2\theta_p = \frac{2\tau_H}{-\sigma_H} = -0.2936$

$\theta_a = -8.2° \quad \theta_b = 81.8°$ ◄

$\tau_{max} = R = 15.37$ MPa ◄

PROBLEM 8.83

8.83 For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point K.



SOLUTION

Components of force at point C

$F_x = 50 \cos 30° = 43.301$ kN

$F_z = -50 \sin 30° = -25$ kN $\qquad F_y = -120$ kN

Section forces and couples at the section containing points H and K.

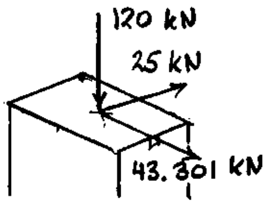
$P = 120$ kN (compression)

$V_x = 43.301$ kN, $\qquad V_z = -25$ kN

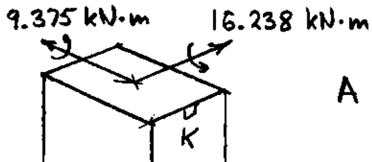
$M_x = -(25)(0.375) = -9.375$ kN·m

$M_y = 0$

$M_z = -(43.301)(0.375) = -16.238$ kN·m



Forces



Couples

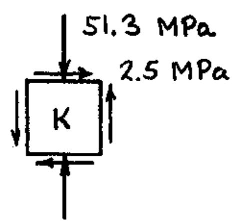
$A = (100)(150) = 15 \times 10^3$ mm^2 $= 15 \times 10^{-3}$ m^2

$I_z = \frac{1}{12}(100)(150)^3 = 28.125 \times 10^6$ mm^4 $= 28.125 \times 10^{-6}$ m^4

Stresses at point K

$$\sigma_K = -\frac{P}{A} + \frac{M_z x}{I_z} = -\frac{120 \times 10^3}{15 \times 10^{-3}} + \frac{(-16.238 \times 10^3)(75 \times 10^{-3})}{28.125 \times 10^{-6}} = -51.3 \text{ MPa}$$

$$\tau_K = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{25 \times 10^3}{15 \times 10^{-3}} = 2.5 \text{ MPa}$$



$\sigma_c = \frac{1}{2}\sigma_K = -25.65$ MPa

$R = \sqrt{\left(\frac{51.3}{2}\right)^2 + (2.5)^2} = 25.77$ MPa

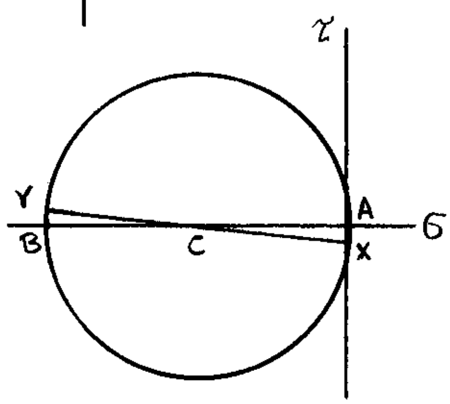
$\sigma_a = \sigma_c + R = 0.12$ MPa ◄

$\sigma_b = \sigma_c - R = -51.4$ MPa ◄

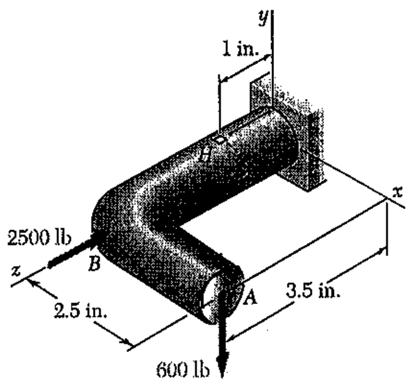
$\tan 2\theta_p = \frac{2\tau_K}{-\sigma_K} = 0.09747$

$\theta_a = 2.8°$ $\qquad \theta_b = 92.8°$ ◄

$\tau_{max} = R = 25.8$ MPa ◄



PROBLEM 8.84



8.84 Forces are applied at points A and B of the solid cast-iron bracket shown. Knowing that the bracket has a diameter of 0.8 in., determine the principal stresses and the maximum shearing stress (*a*) at point H, (*b*) at point K.

SOLUTION

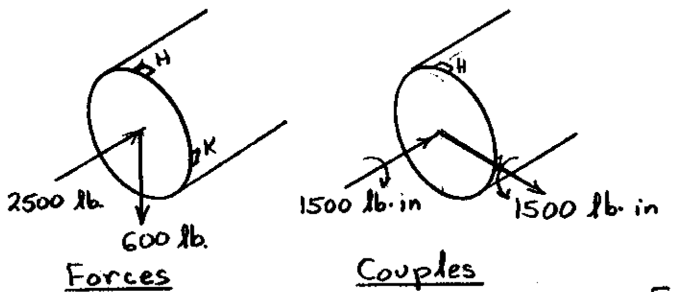
At the section containing points H and K

$P = 2500$ lb. (compression)

$V_y = -600$ lb $\qquad V_x = 0$

$M_x = (3.5 - 1)(600) = 1500$ lb·in

$M_y = 0 \qquad M_z = -(2.5)(600) = -1500$ lb·in



$c = \frac{1}{2}d = 0.4$ in

$A = \pi c^2 = 0.50265$ in^2

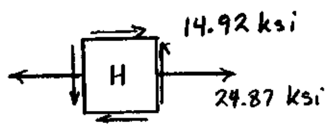
$I = \frac{\pi}{4}c^4 = 20.106 \times 10^{-3}$ in^4

$J = 2I = 40.212 \times 10^{-3}$ in^4

For semi circle $\quad Q = \frac{2}{3}c^3 = 42.667 \times 10^{-3}$ in^3

(a) At point H: $\quad \sigma_H = \frac{P}{A} + \frac{Mc}{I} = -\frac{2500}{0.50265} + \frac{(1500)(0.4)}{20.106 \times 10^{-3}} = 24.87 \times 10^3$ psi

$\tau_H = \frac{Tc}{J} = \frac{(1500)(0.4)}{40.212 \times 10^{-3}} = 14.92 \times 10^3$ psi



$\sigma_{ave} = \frac{24.87}{2} = 12.435$ ksi

$R = \sqrt{\left(\frac{24.87}{2}\right)^2 + (14.92)^2} = 19.423$ ksi

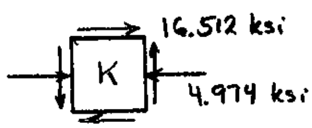
$\sigma_{max} = \sigma_{ave} + R = 31.9$ ksi ◀

$\sigma_{min} = \sigma_{ave} - R = -6.99$ ksi ◀

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 19.42$ ksi ◀

(b) At point K: $\quad \sigma_K = \frac{P}{A} = -\frac{2500}{0.50265} = -4.974 \times 10^3$ psi

$\tau_K = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(1500)(0.4)}{40.212 \times 10^{-3}} + \frac{(600)(42.667 \times 10^{-3})}{(20.106 \times 10^{-3})(0.8)} = 16.512 \times 10^3$ psi



$\sigma_{ave} = -\frac{4.974}{2} = -2.487$ ksi

$R = \sqrt{\left(-\frac{4.974}{2}\right)^2 + (16.512)^2} = 16.698$ ksi

$\sigma_{max} = \sigma_{ave} + R = 14.21$ ksi ◀

$\sigma_{min} = \sigma_{ave} - R = -19.18$ ksi ◀

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 16.70$ ksi ◀

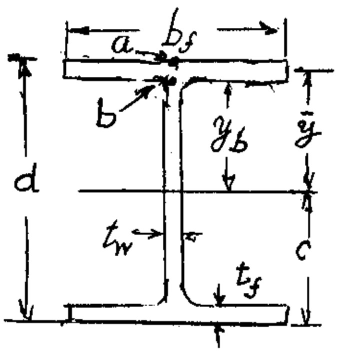
PROBLEM 8.C1

8.C1 Let us assume that the shear V and the bending moment M have been determined in a given section of a rolled-steel beam. Write a computer program to calculate in that section, from the data available in Appendix C, (*a*) the maximum normal stress σ_m, (*b*) the principal stress σ_{max} at the junction of a flange and the web. Use this program to solve parts *a* and *b* of the following problems:

(1) Prob. 8.1 (Use $V = 400$ kN and $M = 100$ kN · m)
(2) Prob. 8.2 (Use $V = 200$ kN and $M = 100$ kN · m)
(3) Prob. 8.3 (Use $V = 320$ kips and $M = 32 \times 10^3$ kip · in.)
(4) Prob. 8.74.

SOLUTION

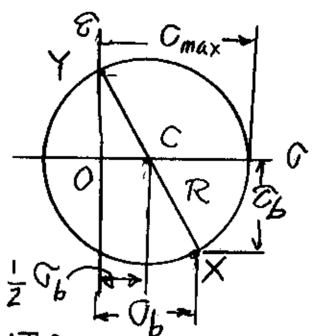
We enter the given values of V and M obtain from Appendix C the values of d, b_f, t_f, t_w, I, and S for the given WF shape.



We compute $c = d/2$, $y_b = c - t_f$

$\bar{y} = c - \frac{1}{2}t_f$, $\sigma_a = M/S$, $\sigma_b = \sigma_a(y_b/c)$

$Q = b_f t_f \bar{y}$, $\tau_b = \dfrac{VQ}{I t_w}$



From Mohr's circle:

$$\sigma_{max} = \frac{1}{2}\sigma_b + R$$

$$\sigma_{max} = \frac{1}{2}\sigma_b + \sqrt{\left(\frac{1}{2}\sigma_b\right)^2 + \tau_b^2}$$

PROGRAM OUTPUTS

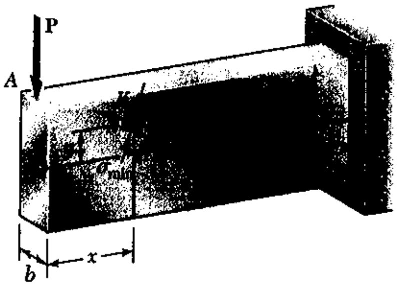
```
Prob. 8.1

Given Data:
V = 400 kN, M = 100 kN.m
d = 252 mm, bf = 203 mm
tf = 13.5 mm, tw =  8.6 mm
I =  87.30 (10^6 mm^4)
S =  693.0 (10^3 mm^3)
Answers:
(a) SIGA = 144.3 MPa
(b) SIGM = 250.1 MPa
```

```
Prob. 8.2

Given Data:
V = 200 kN, M = 100 kN.m
d = 252 mm, bf = 203 mm
tf = 13.5 mm, tw =  8.6 mm
I =  87.30 (10^6 mm^4)
S =  693.0 (10^3 mm^3)
Answers:
(a) SIGA = 144.3 MPa
(b) SIGM = 172.7 MPa
```

```
Prob. 8.3

Given Data:
V = 320 kips, M = 32000 kip.in.
d = 36.74 in., bf = 16.655 in.
tf = 1.680 in., tw = 0.945 in.
I = 20300 in^4, S = 1110 in^3
Answers:
(a) SIGA = 28.8 ksi
(b) SIGM = 28.5 ksi
```

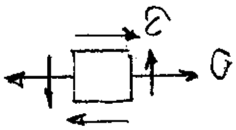
```
Prob. 8.74

Given Data:
V = 120 kips, M =  3600 kip.in.
d = 21.36 in., bf = 12.290 in.
tf = 0.800 in., tw = 0.500 in.
I =  2420 in^4, S =  227 in^3
Answers:
(a) SIGA = 15.86 ksi
(b) SIGM = 19.76 ksi
```

PROBLEM 8.C2



8.C2 A cantilever beam AB with a rectangular cross section of width b and depth $2c$ supports a single concentrated load **P** at its end A. Write a computer program to calculate, for any values of x/c and y/c, (*a*) the ratios σ_{max}/σ_m and σ_{min}/σ_m, where σ_{max} and σ_{min} are the principal stresses at point $K(x, y)$ and σ_m the maximum normal stress in the same transverse section, (*b*) the angle θ_p that the principal planes at K form with a transverse and a horizontal plane through K. Use this program to check the values shown in Fig. 8.8 and to verify that σ_{max} exceeds σ_m if $x \leq 0.544c$, as indicated in the second footnote on page 499.

SOLUTION



Since the distribution of the normal stresses is linear, we have $\sigma = \sigma_m (y/c)$ (1)

where $\sigma_m = \dfrac{Mc}{I} = \dfrac{Pxc}{I}$ (2)

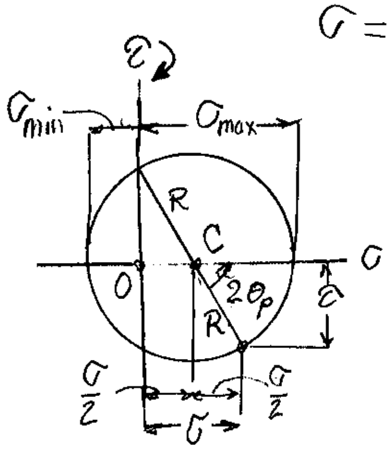
We use Eq. (8.4), page 498: $\tau = \dfrac{3}{2}\dfrac{P}{A}\left(1 - \dfrac{y^2}{c^2}\right)$ (3)

Dividing (3) by (2): $\dfrac{\tau}{\sigma_m} = \dfrac{3}{2}\dfrac{I}{A}\;\dfrac{1-(y/c)^2}{xc}$

or, since $\dfrac{I}{A} = \dfrac{\frac{1}{12}b(2c)^3}{b(2c)} = \dfrac{1}{3}c^2$; $\dfrac{\tau}{\sigma_m} = \dfrac{1}{2}\;\dfrac{1-(y/c)^2}{x/c}$ (4)

Letting $X = x/c$ and $Y = y/c$, Eqs. (1) and (4) yield

$$\sigma = \sigma_m Y \qquad \tau = \sigma_m \frac{1-Y^2}{2X}$$



Using Mohr's circle, we calculate

$$R = \sqrt{\left(\tfrac{1}{2}\sigma\right)^2 + \tau^2}$$

$$= \frac{1}{2}\sigma_m\sqrt{Y^2 + \left(\frac{1-Y^2}{X}\right)^2}$$

$$\frac{\sigma_{max}}{\sigma_m} = \frac{1}{2}Y + R \qquad \frac{\sigma_{min}}{\sigma_m} = \frac{1}{2}Y - R$$

$$\tan 2\theta_p = \frac{\tau}{\sigma/2} = \frac{1-Y^2}{2X(Y/2)} = \frac{1-Y^2}{XY} \qquad \theta_p = \frac{1}{2}\tan^{-1}\left(\frac{1-Y^2}{XY}\right)$$

NOTE

For $y > 0$, the angle θ_p is ↺, which is opposite to what was arbitrarily assumed in Fig. P8.C2.

(CONTINUED)

PROGRAM OUTPUTS

For x/c = 2 :

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta
1.0	0.000	1.000	0.00
0.8	-0.010	0.810	6.34
0.6	-0.040	0.640	14.04
0.4	-0.090	0.490	23.20
0.2	-0.160	0.360	33.69
0.0	-0.250	0.250	45.00
-0.2	-0.360	0.160	-33.69
-0.4	-0.490	0.090	-23.20
-0.6	-0.640	0.040	-14.04
-0.8	-0.810	0.010	-6.34
-1.0	-1.000	0.000	-0.00

For x/c = 8 :

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta
1.0	0.000	1.000	0.00
0.8	-0.001	0.801	1.61
0.6	-0.003	0.603	3.80
0.4	-0.007	0.407	7.35
0.2	-0.017	0.217	15.48
0.0	-0.062	0.063	45.00
-0.2	-0.217	0.017	-15.48
-0.4	-0.407	0.007	-7.35
-0.6	-0.603	0.003	-3.80
-0.8	-0.801	0.001	-1.61
-1.0	-1.000	0.000	-0.00

To check that $\sigma_{max} > \sigma_m$ if $x \leq 0.544c$, we run the program for $x/c = 0.544$ and for $x/c = 0.545$ and observe that σ_{max}/σ_m exceeds 1 for several values of y/c in the first case, but does not exceed 1 in the second case.

For x/c = 0.544 :

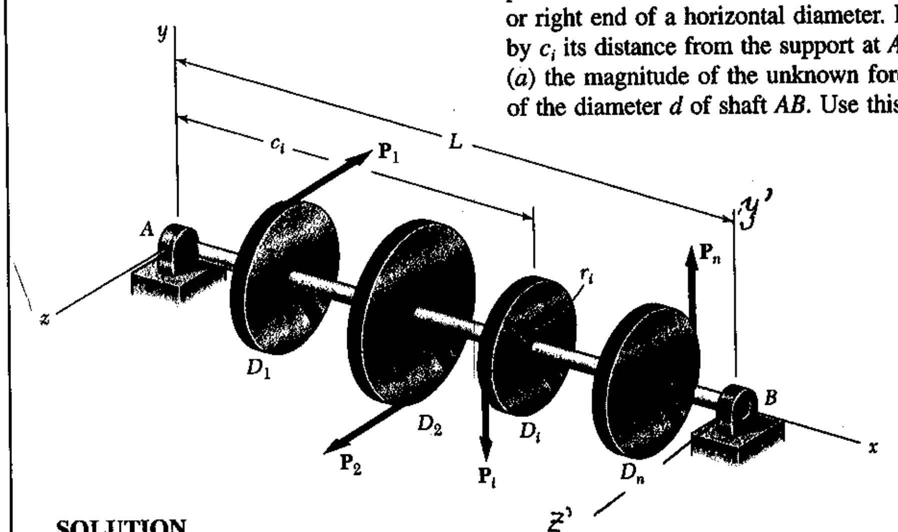
y/c	Sigmin/Sigm	Sigmax/Sigm	Theta
0.30	-0.700	0.9997	39.92
0.31	-0.690	1.0001	39.72
0.32	-0.680	1.0004	39.51
0.33	-0.670	1.0005	39.30
0.34	-0.660	1.0005	39.09
0.35	-0.650	1.0003	38.88
0.36	-0.640	1.0000	38.66
0.37	-0.630	0.9996	38.44
0.38	-0.619	0.9990	38.21
0.39	-0.608	0.9983	37.98
0.40	-0.598	0.9975	37.74

For x/c = 0.545 :

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta
0.30	-0.698	0.9982	39.91
0.31	-0.689	0.9986	39.71
0.32	-0.679	0.9989	39.50
0.33	-0.669	0.9990	39.29
0.34	-0.659	0.9990	39.08
0.35	-0.649	0.9988	38.87
0.36	-0.639	0.9986	38.65
0.37	-0.628	0.9982	38.42
0.38	-0.618	0.9976	38.20
0.39	-0.607	0.9970	37.96
0.40	-0.596	0.9962	37.73

PROBLEM 8.C3

8.C3 Disks $D_1, D_2, \ldots, D_n$ are attached as shown in Fig. P8.C3 to the solid shaft AB of length L, uniform diameter d, and allowable shearing stress τ_{all}. Forces $\mathbf{P}_1, \mathbf{P}_2, \ldots, \mathbf{P}_n$ of known magnitude (except for one of them) are applied to the disks, either at the top or bottom of a vertical diameter, or at the left or right end of a horizontal diameter. Denoting by r_i the radius of disk D_i and by c_i its distance from the support at A, write a computer program to calculate (*a*) the magnitude of the unknown force $\mathbf{P}_i$, (*b*) the smallest permissible value of the diameter d of shaft AB. Use this program to solve Probs. 8.75 and 8.76.



SOLUTION

1. Determine the unknown force P_i by equating to zero the sum of their torques T_i about the x axis.
2. Determine the components $(F_y)_i$ and $(F_z)_i$ of all forces.
3. Determine the components A_y and A_z of reaction at A by summing moments about axes $Bz' \parallel z$ and $By' \parallel y$:

$$\Sigma M_{z'} = 0: \quad -A_y L - \Sigma (F_y)_i (L - c_i) = 0, \quad A_y = -\frac{1}{L} \Sigma (F_y)_i (L - c_i)$$

$$\Sigma M_{y'} = 0: \quad A_z L + \Sigma (F_z)_i (L - c_i) = 0, \quad A_z = -\frac{1}{L} \Sigma (F_z)_i (L - c_i)$$

4. Determine $(M_y)_i$, $(M_z)_i$, and torque T_i just to the left of disk D_i:

$$(M_y)_i = A_z c_i + \sum_k (F_z)_k \langle c_i - c_k \rangle^1$$

$$(M_z)_i = -A_y c_i - \sum_k (F_y)_k \langle c_i - c_k \rangle^1$$

$$T_i = \sum_k T_k \langle c_i - c_k \rangle^0$$

where $\langle \; \rangle$ indicates a singularity function.

5. The minimum diameter d required to the left of D_i is obtained by first computing $(J/c)_i$ from Eq. (8.7):

$$\left(\frac{J}{c}\right)_i = \frac{\sqrt{(M_y)_i^2 + (M_z)_i^2 + T_i^2}}{\tau_{all}}$$

(CONTINUED)

PROBLEM 8.C3 CONTINUED

6 Recalling that $J = \frac{1}{2}\pi c^4$ and, thus, that $\left(\frac{J}{c}\right)_i = \frac{1}{2}\pi c_i^3$, we have $c_i = \frac{2}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$ and $d_i = \frac{4}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$

This is the required diameter just to the left of disk D_i

7. The required diameter just to the right of disk D_i is obtained by replacing T_i with T_{i+1} in the above computation.

8. The smallest permissible value of the diameter of the shaft is the <u>largest</u> of the values obtained for d_i

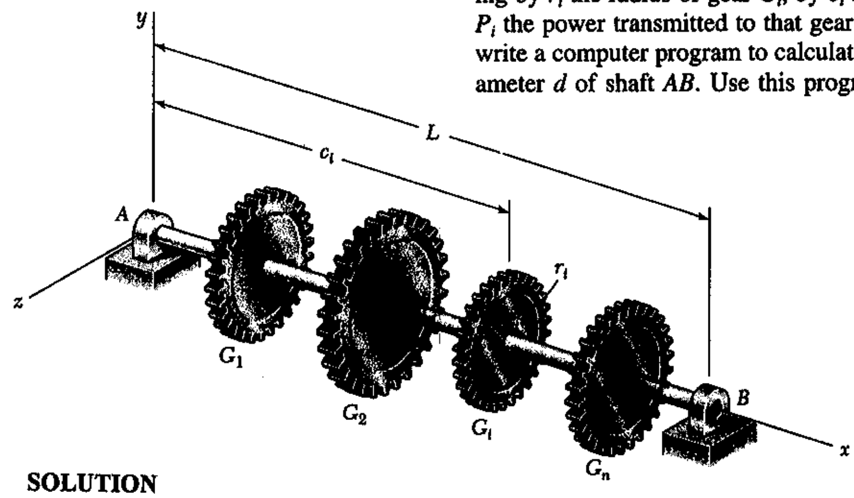
PROGRAM OUTPUTS

```
Prob. 8.75
Length of shaft =  300 mm
TAU = 60 MPa
For Disk 1
Force= 6.000 kN
Radius of disk =  75 mm
Distance from A in mm =  80
For Disk 2
Force= 0.000 kN
Radius of disk =  60 mm
Distance from A in mm = 180
Unknown force=  -7.500 kN
AY=  4.400 kN, AZ= -3.000 kN
BY=  1.600 kN, BZ= -4.500 kN
Just to the left of Disk 1
MY=-240.00 Nm
MZ=-352.00 Nm
T=   0.00 Nm
Diameter must be at least  33.07 mm
Just to the right of Disk 1
T= 450.00 Nm
Diameter must be at least  37.47 mm
Just to the left of Disk 2
MY=-540.00 Nm
MZ=-192.00 Nm
T= 450.00 Nm
Diameter must be at least  39.55 mm
Just to the right of Disk 2
T=   0.00 Nm
Diameter must be at least  36.51 mm
```

```
Prob. 8.76
Length of shaft=   28 in.
TAU (ksi)=      8
For Disk 1
Force =  0.500 kips
Radius of disk =   4.0 in.
Distance from A=   7.0 in.
For Disk 2
Force =  0.000 kips
Radius of disk =   6.0 in.
Distance from A=  14.0 in.
For Disk 3
Force =  0.500 kips
Radius of disk =   4.0 in.
Distance from A=  21.0 in.
Unknown force= -0.667 kips
AY=  0.500 kips, AZ=  0.333 kips
BY=  0.500 kips, BZ=  0.333 kips
Just to the left of Disk 1
MY=  2.3333 kip.in.
MZ=  -3.5000 kip.in.
T=   0.0000 kip.in.
Diameter must be at least  1.389 in.
Just to the right of Disk 1
T=   2.00 kip.in.
Diameter must be at least  1.437 in.
Just to the left of Disk 2
MY=  4.6667 kip.in.
MZ=  -3.5000 kip.in.
T=   2.0000 kip.in.
Diameter must be at least  1.578 in.
Just to the right of Disk 2
T=  -2.00 kip.in.
Diameter must be at least  1.578 in.
Just to the left of Disk 3
MY=  2.3333 kip.in.
MZ=  -3.5000 kip.in.
T=  -2.0000 kip.in.
Diameter must be at least  1.437 in.
Just to the right of Disk 3
T=   0.00 kip.in.
Diameter must be at least  1.389 in.
```

PROBLEM 8.C4

8.C4 The solid shaft AB of length L, uniform diameter d, and allowable shearing stress τ_{all} rotates at a given speed expressed in rpm (Fig. P8.C4). Gears $G_1, G_2, \ldots, G_n$ are attached to the shaft and each of these gears meshes with another gear (not shown), either at the top or bottom of its vertical diameter, or at the left or right end of its horizontal diameter. One of these other gears is connected to a motor and the rest of them to various machine tools. Denoting by r_i the radius of gear G_i, by c_i its distance from the support at A, and by P_i the power transmitted to that gear (+ sign) or taken off that gear (− sign), write a computer program to calculate the smallest permissible value of the diameter d of shaft AB. Use this program to solve Probs. 8.25, 8.29, and 8.77.



SOLUTION

1. Enter ω in rpm and determine frequency $f = \omega/60$.
2. For each gear, determine the torque $T_i = P_i/2\pi f$, where P_i is the power input (+) or output (−) at the gear.
3. For each gear, determine the force $F_i = T_i/r_i$ exerted on the gear and its components $(F_y)_i$ and $(F_z)_i$.
4. Determine the components A_y and A_z of reaction at A by summing moments about axes $Bz' \parallel z$ and $By' \parallel y$:

$$\Sigma M_{z'} = 0: \ -A_y L - \Sigma (F_y)_i (L - c_i) = 0, \quad A_y = -\frac{1}{L} \Sigma (F_y)_i (L - c_i)$$

$$\Sigma M_{y'} = 0: \ A_z L + \Sigma (F_z)_i (L - c_i) = 0, \quad A_z = -\frac{1}{L} \Sigma (F_z)_i (L - c_i)$$

5. Determine $(M_y)_i$, $(M_z)_i$, and torque T_i just to the left of gear G_i:

$$(M_y)_i = A_z c_i + \sum_k (F_z)_k \langle c_i - c_k \rangle^1$$

$$(M_z)_i = -A_y c_i - \sum_k (F_y)_k \langle c_i - c_k \rangle^1$$

$$T_i = \sum_k T_k \langle c_i - c_k \rangle^0$$

Where $\langle\ \rangle$ indicates a singularity function.

(CONTINUED)

PROBLEM 8.C4 CONTINUED

6. The minimum diameter d required to the left of G_i is obtained by first computing $(J/c)_i$ from Eq. (8.7):

$$\left(\frac{J}{c}\right)_i = \frac{\sqrt{(M_y)_i^2 + (M_z)_i^2 + T_i^2}}{\tau_{all}}$$

7. Recalling that $J = \frac{1}{2}\pi c^4$ and, thus, that $\left(\frac{J}{c}\right)_i = \frac{1}{2}\pi c_i^3$ we have $c_i = \frac{2}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$ and $d_i = \frac{4}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$

This is the required diameter just to the left of gear G_i.

8. The required diameter just to the right of gear G_i is obtained by replacing T_i with T_{i+1} in the above computation.

9. The smallest permissible value of the diameter of the shaft is the largest of the values obtained for d_i.

PROGRAM OUTPUTS

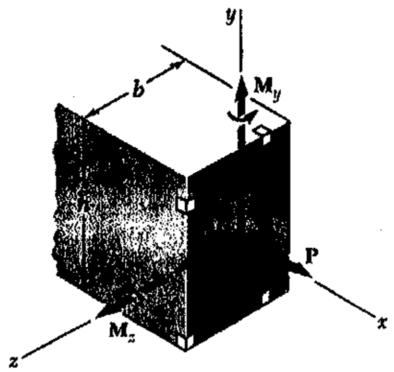
```
Prob. 8.25
Omega = 600 rpm
Number of Gears: 2
Length of shaft =  400 mm
Tau =  60 MPa
For Gear 1
Power input =  80.00 kW
Radius of gear= 80 mm
Distance from A in mm = 120
For Gear 2
Power input = -80.00 kW
Radius of gear= 60 mm
Distance from A in mm = 280
AY= 11.141 kN, AZ=   6.366
BY=  4.775 kN, BZ=  14.854
Just to the left of Gear 1
MY= 763.94 Nm
MZ= -1336.90 Nm
T=   0.00 Nm
Diameter must be at least  50.75 mm
Just to the right of Gear 1
T=1273.24 Nm
Diameter must be at least  55.35 mm
Just to the left of Gear 2
MY=1782.54 Nm
MZ=-572.96 Nm
T=1273.24 Nm
Diameter must be at least  57.71 mm
Just to the right of Gear 2
T=   0.00 Nm
Diameter must be at least  54.17 mm
```

(CONTINUED)

```
Prob. 8.29
Omega = 450 rpm
Number of Gears: 3
Length of shaft =  750 mm
Tau =  55 MPa
For Gear 1
Power input =  -8.00 kW
Radius of gear= 60 mm
Distance from A in mm = 150
For Gear 2
Power input =  20.00 kW
Radius of gear=100 mm
Distance from A in mm = 375
For Gear 3
Power input = -12.00 kW
Radius of gear= 60 mm
Distance from A in mm = 600
AY= -0.849 kN, AZ=   4.386
BY= -3.395 kN, BZ=   2.688
Just to the left of Gear 1
MY= 657.84 Nm
MZ= 127.32 Nm
T=   0.00 Nm
Diameter must be at least  39.59 mm
Just to the right of Gear 1
T=-169.77 Nm
Diameter must be at least  40.00 mm
Just to the left of Gear 2
MY=1007.98 Nm
MZ= 318.31 Nm
T=-169.77 Nm
Diameter must be at least  46.28 mm
Just to the right of Gear 2
T= 254.65 Nm
Diameter must be at least  46.52 mm
Just to the left of Gear 3
MY= 403.19 Nm
MZ= 509.30 Nm
T= 254.65 Nm
Diameter must be at least  40.13 mm
Just to the right of Gear 3
T=   0.00 Nm
Diameter must be at least  39.18 mm
```

```
Prob. 8.77
Omega = 600 rpm
Number of Gears: 3
Length of shaft =   24 in.
Tau =   8 ksi
For Gear 1
Power input =  60.00 hp
Radius of gear= 3.00 in.
Distance from A in inches =   4.0
FY=            0
FZ =           2.100845
For Gear 2
Power input = -40.00 hp
Radius of gear= 4.00 in.
Distance from A in inches =  10.0
FY=            1.050423
FZ =           0
For Gear 3
Power input = -20.00 hp
Radius of gear= 4.00 in.
Distance from A in inches =  18.0
FY=            0
FZ =          -.5252113
AY=-0.6127 kips, AZ=-1.6194 kips
BY=-0.4377 kips, BZ= 0.0438 kips
Just to the left of Gear 1
MY= -6.478 kip.in.
MZ=  2.451 kip.in.
T=  0.000 kip.in.
Diameter must be at least  1.640 in.
Just to the right of Gear 1
T=  6.3025 kip.in.
Diameter must be at least  1.813 in.
Just to the left of Gear 2
MY= -3.589 kip.in.
MZ=  6.127 kip.in.
T=  6.303 kip.in.
Diameter must be at least  1.822 in.
Just to the right of Gear 2
T=  2.1008 kip.in.
Diameter must be at least  1.677 in.
Just to the left of Gear 3
MY=  0.263 kip.in.
MZ=  2.626 kip.in.
T=  2.101 kip.in.
Diameter must be at least  1.290 in.
Just to the right of Gear 3
T=  0.0000 kip.in.
Diameter must be at least  1.189 in.
```

PROBLEM 8.C5



8.C5 Write a computer program that can be used to calculate the normal and shearing stresses at points with given coordinates y and z located on the surface of a machine part having a rectangular cross section. The internal forces are known to be equivalent to the force-couple system shown. Write the program so that the loads and dimensions can be expressed in either SI or U.S. customary units. Use this program to solve (*a*) Prob. 8.50, (*b*) Prob. 8.53.

SOLUTION

ENTER: b AND h

PROGRAM: $A = bh \quad I_y = b^3h/12 \quad I_z = bh^3/12$

FOR POINT ON SURFACE, ENTER y AND z

NOTE y AND z MUST SATISFY ONE OF FOLLOWING:

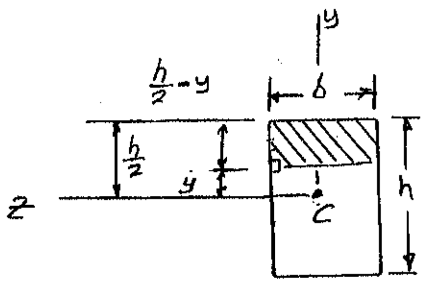
$$y^2 = h^2/4 \text{ AND } z^2 \le b^2/4 \qquad (1)$$

OR

$$z^2 = b^2/4 \text{ AND } y^2 \le h^2/4 \qquad (2)$$

IF EITHER (1) OR (2) ARE SATISFIED, COMPUTE

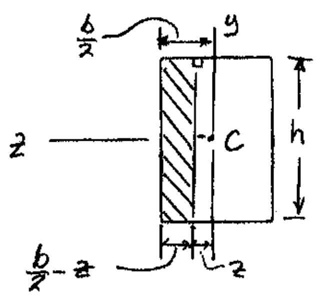
$$\sigma = \frac{P}{A} + \frac{M_y z}{I_y} - \frac{M_z y}{I_z}$$



IF $z^2 = b^2/4$, THEN POINT IS ON VERTICAL SURFACE AND

$$Q_z = b\left(\frac{h}{2} - y\right)\left(\frac{h}{2} + y\right)\frac{1}{2} = b\left(\frac{h^2}{8} - \frac{y^2}{2}\right)$$

$$\tau = \frac{V_y Q_z}{I_z b}$$



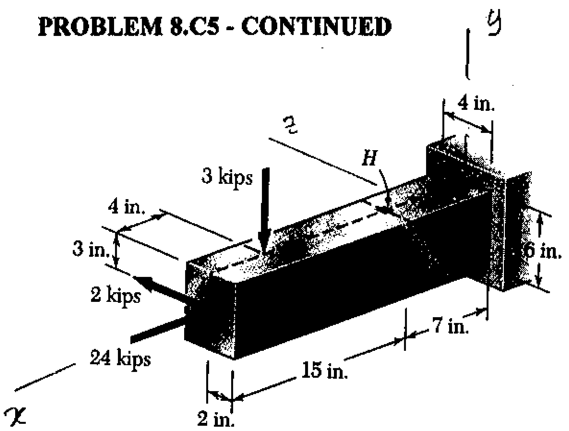
IF $y^2 = h^2/4$, THE POINT IS ON HORIZONTAL SURFACE, AND

$$Q_y = h\left(\frac{b}{2} - z\right)\left(\frac{b}{2} + z\right)\frac{1}{2} = h\left(\frac{b^2}{8} - \frac{z^2}{2}\right)$$

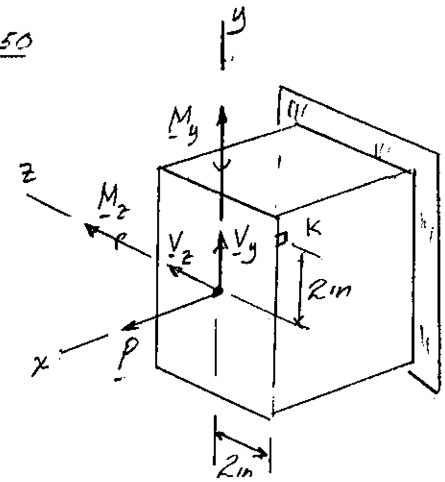
$$\tau = \frac{V_z Q_y}{I_y h}$$

CONTINUED

PROBLEM 8.C5 - CONTINUED



PROBLEM 8.50
POINT K



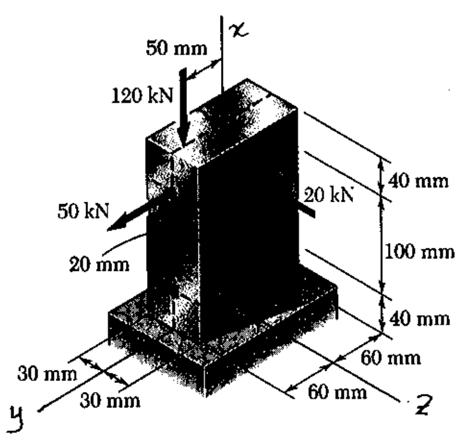
FORCE-COUPLE SYSTEM

$P = 24$ kips $\quad V_y = -3$ kips $\quad V_z = 2$ kips

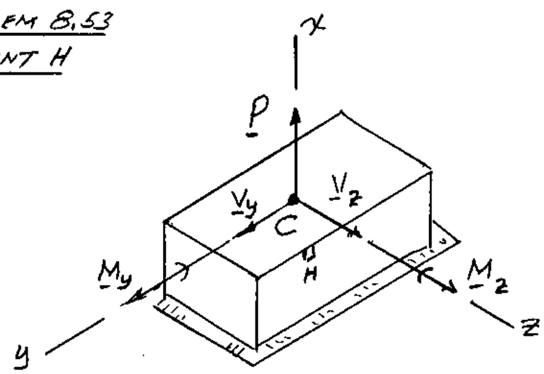
$M_y = -(2\text{ kips})(15\text{ in.}) = -30$ kip·in. $\quad M_z = -(3\text{ kips})(15\text{ in.} - 4\text{ in.}) = -33$ kip·in.

POINT K $\quad y = 2$ in. $\quad z = -2$ in.

```
Problem 8.50          Force-Couple at Centroid
               P =   -24.000 kips
              MY =   -30.000 kip·in.      MZ =   -33.000 kip·in.
              VY =     3.000 kips         VZ =     2.000 kips
             + + + + + + + + + + + + + + + + + + + + + + + + + + +
             At point of coordinates:      y  =   2.000 in.   z =  -2.000 in.
                   sigma =   1.792 ksi
                     tau =   0.104 ksi
```



PROBLEM 8.53
POINT H



FORCE-COUPLE SYSTEM $\quad P = -120$ kN $\quad V_y = 50$ kN $\quad V_z = -20$ kN

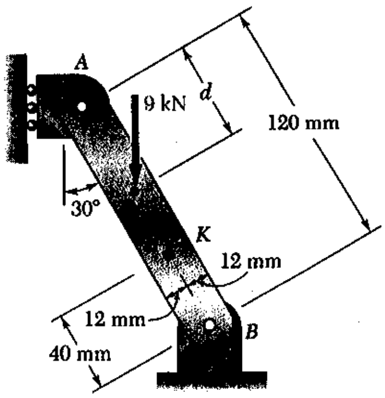
$M_y = (20\text{ kN})(0.1\text{ m}) = 2$ kN·m

$M_z = (120\text{ kN})(0.05\text{ m}) + (50\text{ kN})(0.1\text{ m}) = 11$ kN·m

POINT H $\quad y = 20$ mm $\quad z = 30$ mm

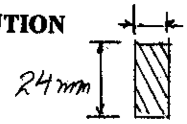
```
Problem 8.53           Force-Couple at Centroid
                 P = -120000.00 N
                MY =    2000.00 N·m     MZ =  11000.00 N·m
                VY =   50000.00 N       VZ = -20000.00 N
              + + + + + + + + + + + + + + + + + + + + + + + + + + +
              At point of coordinates:      y  =  20.00 mm   z =  30.00 mm
                     sigma = -14.352 MPa
                       tau =   9.259 MPa
```

PROBLEM 8.C6



8.C6 Member *AB* has a rectangular cross section of 10 × 24 mm. For the loading shown, write a computer program that can be used to determine the normal and shearing stresses at points *H* and *K* for values of *d* from 0 to 120 mm, using 15-mm increments. Use this program to solve Prob. 8.35.

SOLUTION



CROSS SECTION

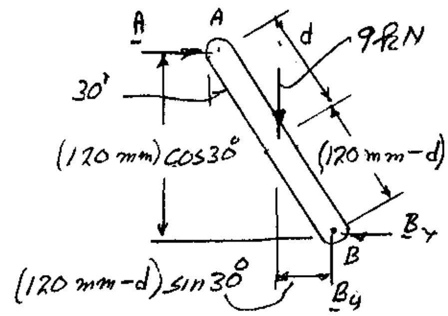
ENTER

$A = (0.010\ m)(0.024\ m) = 240 \times 10^{-6}\ m^2$

$I = (0.010\ m)(0.024\ m)^3/12 = 138.24 \times 10^{-9}\ m^4$

$c = 0.5(0.024\ m) = 12 \times 10^{-3}\ m$

COMPUTE REACTION AT A.

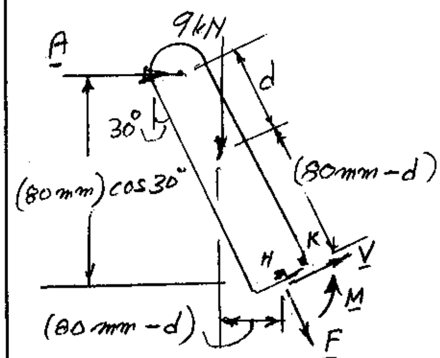


$+\circlearrowleft \Sigma M_B = 0:$

$(9\ kN)(120 - d)\sin 30° - A(120)\cos 30° = 0$

$A = (9\ kN)\dfrac{(120\ mm - d)}{120\ mm}\tan 30°$

FREE BODY FROM A TO SECTION CONTAINING POINTS H AND K.



DEFINE: IF $d < 80$ mm THEN STP = 1 ELSE STP = 0

PROGRAM FORCE-COUPLE SYSTEM

$F = -A\sin 30° - (9\ kN)\cos 30°(STP)$

$V = -A\cos 30° + (9\ kN)\sin 30°(STP)$

$M = A(80\ mm)\cos 30° - (9\ kN)(80\ mm - d)\sin 30°(STP)$

AT POINT H:

$\sigma_H = +F/A \qquad \tau_H = \frac{3}{2}V/A$

AT POINT K:

$\sigma_K = +F/A - Mc/I \qquad \tau_K = 0$

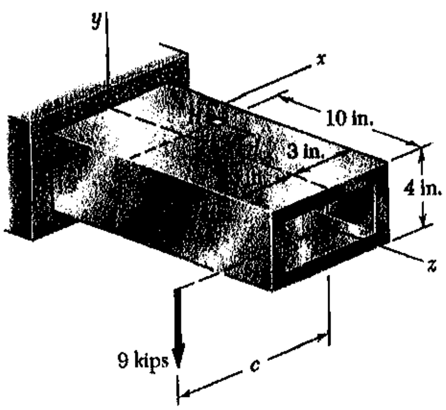
PROGRAM OUTPUT

Problem 8.35

Stresses in MPa

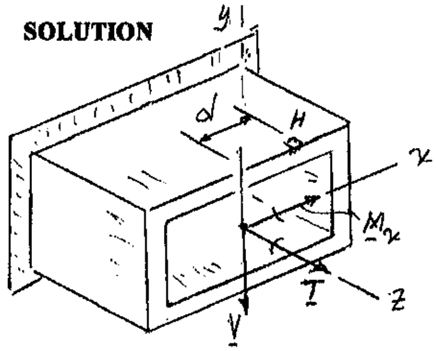
d mm	SigmaH	TauH	SigmaK	TauK
0.0	-43.30	0.00	-43.30	0.00
15.0	-41.95	3.52	-65.39	0.00
30.0	-40.59	7.03	-87.47	0.00
45.0	-39.24	10.55	-109.55	0.00
60.0	-37.89	14.06	-131.64	0.00
75.0	-36.54	17.58	-153.72	0.00
90.0	-2.71	-7.03	-96.46	0.00
105.0	-1.35	-3.52	-48.23	0.00
120.0	0.00	0.00	0.00	0.00

PROBLEM 8.C7



***8.C7** The structural tube shown has a uniform wall thickness of 0.3 in. A 9-kip force is applied to a bar (not shown) that is welded to the end of the tube. Write a computer program that can be used to determine, for any given value of *c*, the principal stresses, principal planes, and maximum shearing stress at point *H* for values of *d* from −3 in. to 3 in., using one-inch increments. Use this program to solve Prob. 8.72*a*.

SOLUTION



FORCE-COUPLE SYSTEM

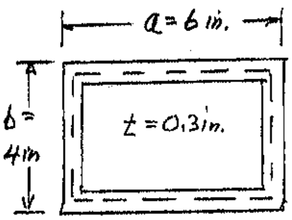
ENTER:

$V = 9\text{ kips}\downarrow$

$M_x = (9\text{ kips})(10\text{ in.})$

$= 90\text{ kip}\cdot\text{in.}$

$T = (9\text{ kips})\,c$

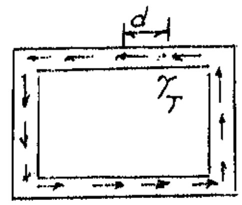


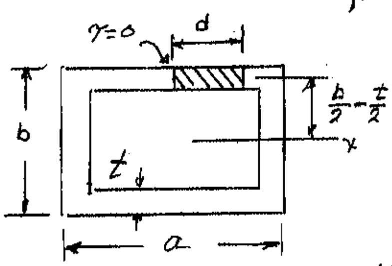
AREA ENCLOSED

$\mathcal{a} = (a-t)(b-t)$

$$\tau_T = \frac{T}{2t\mathcal{a}} = \frac{9c}{2t\mathcal{a}}$$

τ_T = SHEARING STRESS DUE TO TORSION



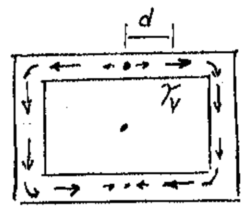


$Q = dt\left(\frac{b}{2}-\frac{t}{2}\right)$

$I = ab^3/12 - (a-2t)(b-2t)^3/12$

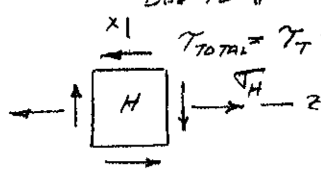
$$\tau_V = \frac{VQ}{It}$$

τ_V = SHEARING STRESS DUE TO V



$\tau_{TOTAL} = \tau_T - \tau_V$

BENDING: $\sigma_H = \dfrac{M_x\left(\frac{b}{2}\right)}{I}$



PRINCIPAL STRESSES

$\sigma_{ave} = \frac{1}{2}\sigma_H$; $R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_{TOTAL}^2}$

$\sigma_{max} = \sigma_{ave} + R$; $\sigma_{min} = \sigma_{ave} - R$; $\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{\tau_{TOTAL}}{\sigma_{ave}}\right)$; $\tau_{max} = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_{TOTAL}^2}$

```
Rectangular tube of uniform thickness t = 0.3 in,
Outside dimensions
  Horizontal width a = 6 in.
  Vertical depth   b = 4 in.
Vertical load    P = 9 kips; line of action at x = -c
Find normal and shearing streses at
     Point H (x = d, y = b/2)
```

Problem 8.72   Program Output for Value of c = 2.85 in.

d in.	sigma ksi	tauV ksi	tauT ksi	tauTotal ksi	sigmaMax ksi	sigmaMin ksi	tauMax ksi	theta p degrees
-3.00	12.58	-3.49	-2.03	-5.52	14.65	-2.08	8.36	-18.49
-2.00	12.58	-2.33	-2.03	-4.35	13.94	-1.36	7.65	-16.00
-1.00	12.58	-1.16	-2.03	-3.19	13.34	-0.76	7.05	-12.78
→ 0.00	12.58	0.00	-2.03	-2.03	12.89	-0.32	6.61	-8.73 ←
1.00	12.58	1.16	-2.03	-0.86	12.63	-0.06	6.35	-3.89
2.00	12.58	2.33	-2.03	0.30	12.58	-0.01	6.30	1.36
3.00	12.58	3.49	-2.03	1.46	12.74	-0.17	6.46	6.46