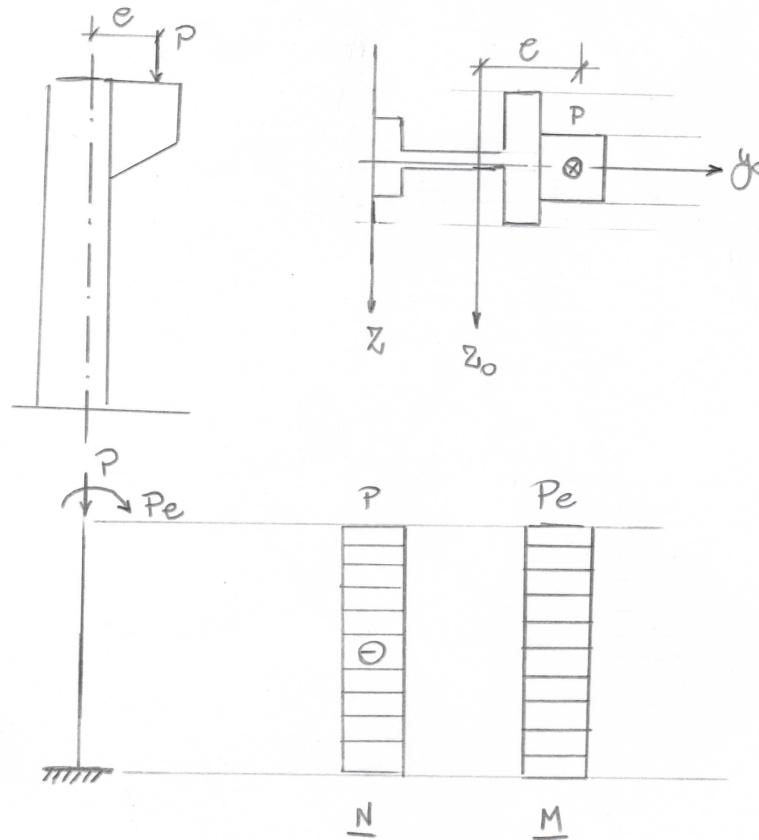
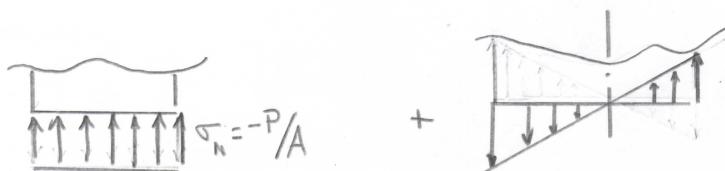


Flexão Normal Composta

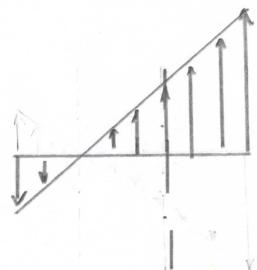
$$\begin{cases} N \neq 0 \\ V \neq 0 \\ M \neq 0 \end{cases}$$



Fazendo um corte na base do Pilar.



Deveremos somar as contribuições



Agora, a linha neutra não é mais bárcântrica!!

(Lembrar que para flexões simples elas coincidem)

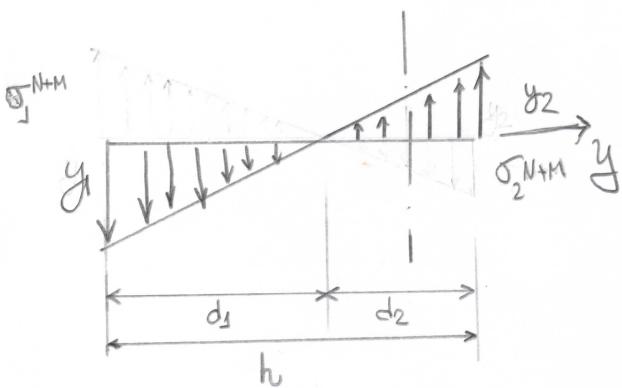
A tensão é:

$$\sigma^{NM} = \sigma^N + \sigma^M$$

$$\boxed{\sigma = \frac{N}{A} + \frac{M}{I_{Z0}} y}$$

Para nosso exemplo:

$$\sigma_1^{NM} = -\frac{P}{A} + \frac{(Pe)y_1}{I_{Z0}}$$



$$\sigma_2^{NM} = -\frac{P}{A} - \frac{(Pe)y_2}{I_{Z0}}$$

Para acharmos a nova posição da linha neutra:

$$\frac{\sigma_2 - \sigma_1}{h} = \frac{\sigma_1}{d_1} = \frac{\sigma_2}{d_2} \quad (\text{semelhança de triângulos})$$

Condições de projeto e/ou verificação

$$\begin{cases} \bar{\sigma}_c \leq \sigma_1 \leq \bar{\sigma}_T \\ \bar{\sigma}_c \leq \sigma_2 \leq \bar{\sigma}_T \end{cases}$$

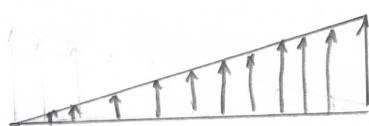
* importando-se com os sinais

\oplus TRAÇÃO
 \ominus COMPRESSÃO

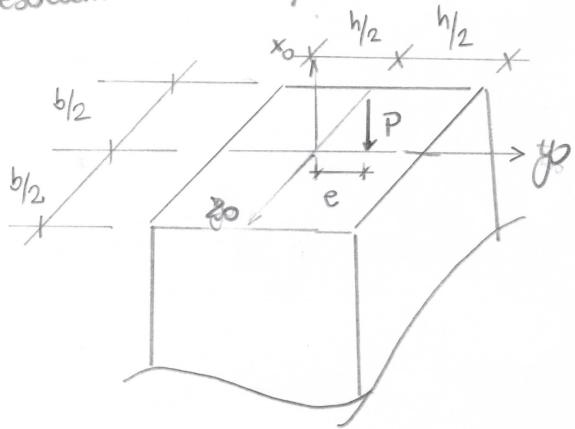
Caso particular interessante ($\bar{\sigma}_T = 0$):

$$\sigma_1 \leq \bar{\sigma}_T \Rightarrow -\frac{P}{A} + \frac{(Pe)y_1}{I_{Z0}} \leq 0$$

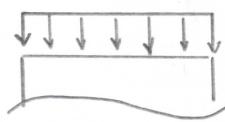
$$e \leq \frac{I_{Z0}}{A y_1} \quad (\text{neste caso, } P \text{ não provoca trago})$$



Resolvendo o exemplo para uma seção retangular:

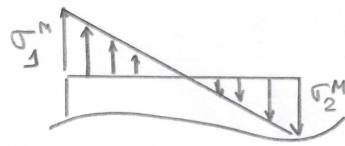


Normal



$$\sigma = \frac{N}{A} = -\frac{P}{bh}$$

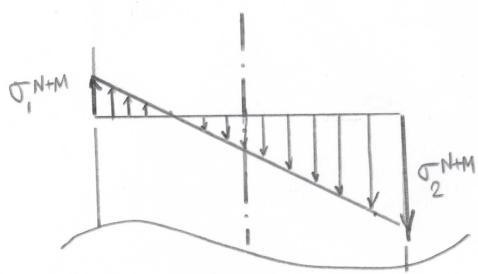
Momento



$$\sigma_1^M = \frac{(Pe) \cdot \frac{h}{2}}{\frac{bh^3}{12}} = \frac{6Pe}{bh^2}$$

$$\sigma_2^M = \frac{-(Pe) \cdot \frac{h}{2}}{\frac{bh^3}{12}} = -\frac{6Pe}{bh^2}$$

Somando:



$$\sigma_1^{N+M} = -\frac{P}{bh} + \frac{6Pe}{bh^2}$$

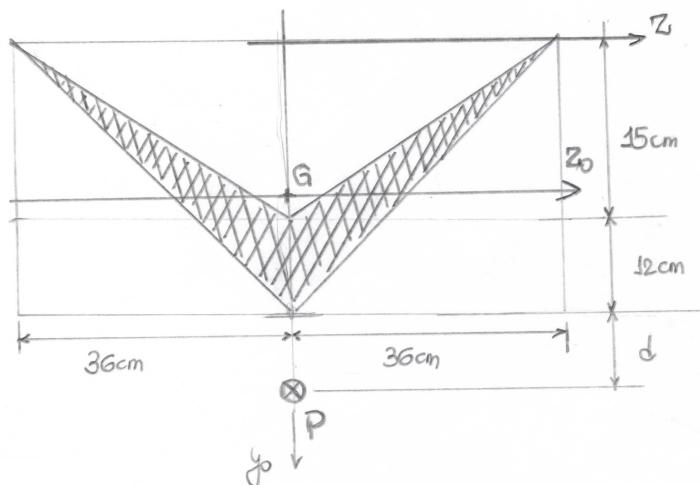
$$\sigma_2^{N+M} = -\frac{P}{bh} - \frac{6Pe}{bh^2}$$

Para que não haja traçado:

$$\sigma_1^{N+M} \leq 0 \Rightarrow -\frac{P}{bh} + \frac{6Pe}{bh^2} \leq 0 \quad -1 + \frac{6e}{h} \leq 0$$

$$\frac{6e}{h} \leq 1 \Rightarrow \boxed{e \leq \frac{h}{6}}$$

Exemplo: Para um pilar com o seção transversal a seguir, encontre d de tal forma que $|\sigma_{\max}^T| = |\sigma_{\max}^C|$



$$y_1 = \frac{27}{3} = 9 \text{ cm}$$

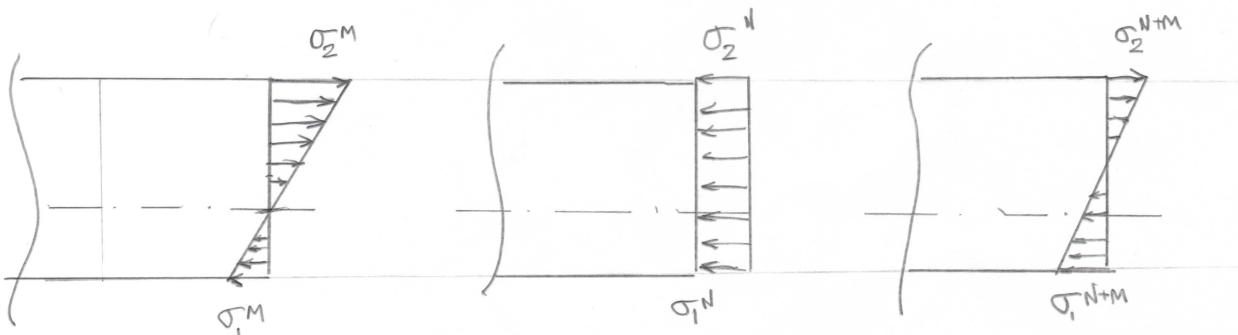
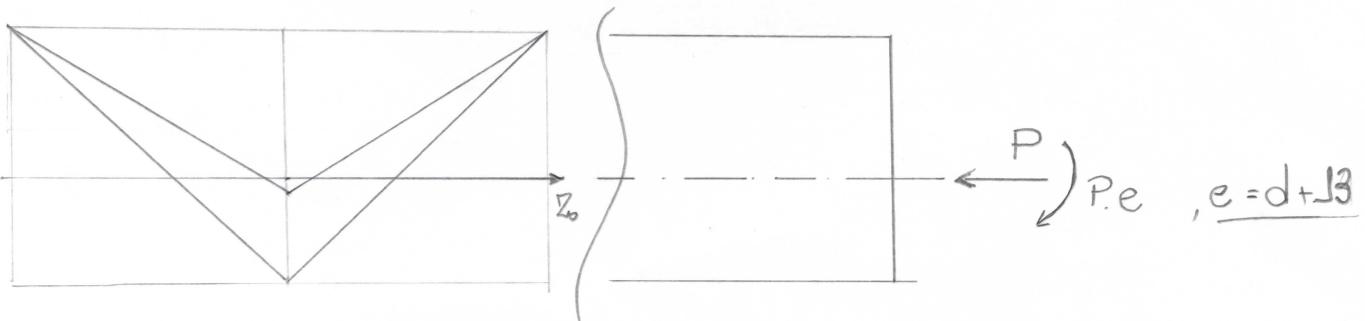
$$y_G = \frac{972.9 - 540.5}{432} \Rightarrow y_G = \frac{8748 - 2700}{432} \Rightarrow y_G = 14 \text{ cm}$$

$$y_2 = \frac{15}{3} = 5 \text{ cm}$$

$$I_{Z0} = I_{Z0}^1 - I_{Z0}^2 = (I_z + d_1^2 A_1) - (I_z + d_2^2 A_2), \quad d_1 = y_1 - y_0; \quad d_2 = y_2 - y_0$$

$$I_{Z0} = \left[\frac{72.27^3}{36} + 972(9-14)^2 \right] - \left[\frac{72.15^3}{36} + 540(5-14)^2 \right]$$

$$I_{Z0} = [39366 + 24300] - [6750 + 43740] = 63666 - 50490 = 13176 \text{ cm}^4$$



Calculate as tensors:

$$G_1^{\text{NM}} = \frac{-P}{A} - \frac{(Pe)h_1}{I_{Z0}}$$

$$G_2^{\text{NM}} = \frac{-P}{A} + \frac{(Pe)h_2}{I_{Z0}} > 0 \quad (\sigma_{\max}^r)$$

$$\sigma_{\max}^r = |\sigma_{\max}^e|$$

$$-\frac{P}{A} + \frac{(Pe)h_2}{I_{Z0}} = \frac{P}{A} + \frac{(Pe)h_1}{I_{Z0}}$$

$$\frac{2P}{A} = \frac{Pe}{I_{Z0}} (h_2 - h_1) \quad e = \frac{2I_{Z0}}{A(h_2 - h_1)}$$

$$h_1 = 13 \text{ cm}, h_2 = 14 \text{ cm}$$

$$e = \frac{2 \cdot 13176}{432(14-13)} \Rightarrow e = 61 \text{ cm}$$

$$d = e - 13 = 61 - 13$$

$$= 48$$

$$\boxed{d = 48 \text{ cm}}$$