# Matter

## **1.1** Atoms and Nuclei

Matter is made of *atoms*. Atoms are made up of electrons and *nuclei*. Each atomic nucleus has four important physical properties: *mass, electric charge, magnetism* and *spin*.

The *mass* of bulk matter is largely due to the mass of the nuclei. A large number of other physical properties, such as heat capacity and viscosity, are strongly dependent on the nuclear mass.

The *electric charge* of atomic nuclei is supremely important. Atoms and molecules are bound together by strong electrostatic interactions between the positively charged nuclei and the negatively charged electrons. The chemical properties of each element are determined by the electric charge on the atomic nuclei.

The other two properties, nuclear magnetism and nuclear spin, are much less evident. The magnetism of a nucleus implies that it interacts with magnetic fields, like a small bar magnet. However, nuclear magnetism is very weak and is of little consequence for atomic or molecular structure. The bulk magnetism of some materials, such as iron, is due to the electrons, not to the nuclei.

The *spin* of the nucleus is even less tangible. The spin of a nucleus indicates that, very loosely speaking, the atomic nucleus behaves as if it is spinning around, rotating in space like a tiny planet.

Nuclear magnetism and nuclear spin have almost no effect on the normal chemical and physical behaviour of substances. Nevertheless, these two properties provide scientists with a wonderful tool for spying on the microscopic and internal structure of objects without disturbing them.

Magnetic nuclei interact with magnetic fields. These magnetic fields may come from the molecular environment, e.g. the surrounding electrons, or from other nuclear spins in the same molecule. Magnetic fields may also originate from sources outside the sample, such as an external apparatus. This book tells a small part of a long, complicated, and rather unlikely story: How the extremely weak magnetic interactions of atomic nuclei with the molecular environment on one hand, and with the spectrometer apparatus on the other hand, give access to detailed molecular information which is inaccessible by any other current method.

## **1.2** Spin

The concept of spin is difficult. It was forced upon scientists by the experimental evidence.<sup>1</sup> Spin is a highly abstract concept, which may never be entirely 'grasped' beyond knowing how to manipulate the quantum mechanical equations.

Nevertheless, it is worth trying. NMR involves detailed manipulations of nuclear spins. The field has developed to a high level of sophistication, in part because of the possibility of thinking 'physically' and 'geometrically' about spins without being entirely wrong. Geometrical arguments can never tell the whole truth, because the human mind is probably incapable of grasping the entire content of quantum mechanics.

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Nevertheless, it is possible to acquire a feel for spin beyond a purely technical proficiency in the equations. In this book, I will try to communicate how I think one should think about nuclear spins, as well as presenting the technical mathematics.

#### 1.2.1 Classical angular momentum

A rotating object possesses a quantity called *angular momentum*. This may be visualized as a vector pointing along the axis about which the object rotates; your right hand may be used to figure out which way the arrow points. If your thumb points along the rotation axis, then the right-hand fingers 'wrap around' in the direction of the rotation:



**Figure 1.1** Macroscopic angular momentum.

#### 1.2.2 Quantum angular momentum

In quantum mechanics, angular momentum is *quantized*. Consider, for example, a diatomic molecule:



As described in many texts (see *Further Reading*), and discussed further in Chapter 7, a rotating diatomic molecule possesses a set of stable rotational states, in which the total angular momentum  $L_{tot}$  has one of the values<sup>2</sup>

$$L_{\rm tot} = [J(J+1)]^{1/2}\hbar \tag{1.1}$$

where *J* takes *integer* values J = 0, 1, 2... and  $\hbar \approx 1.054 \times 10^{-34}$  J s is Planck's constant divided by  $2\pi$ . This equation implies the *quantization of total angular momentum*.

The rotational energy of a molecule is proportional to the square of the total angular momentum, so the energy is also quantized. For a rigid molecule, the energies of the stable rotational states are

$$E_J = BJ(J+1) \tag{1.2}$$

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where *B* is called the rotational constant for the molecule. *B* is small for a heavy molecule and is large for a light one.

The molecule may be in a stable state with zero total angular momentum, or with total angular momentum  $\sqrt{2}\hbar$ , or with total angular momentum  $\sqrt{6}\hbar$ , etc. The actual rotational state of a molecule depends on its history and its environment.

The total angular momentum of the molecule determines how fast it is rotating, but conveys no information on the axis of the rotation.

More detail about the rotation of the molecule is given by specifying a second quantum number,  $M_J$ . This quantum number  $M_J$  takes one of the 2J + 1 integer values  $M_J = -J, -J + 1 \dots + J$ , and says something about the direction of the rotation. The quantum number  $M_J$  is sometimes referred to as the *azimuthal quantum number*. The physical significance of  $M_J$  is examined more closely in Chapters 7 and 10.

In the absence of an external field, each of the 2J + 1 states with the same value of *J* but different values of  $M_J$  are *degenerate*, meaning that they have the same energy.

The application of a magnetic field breaks the degeneracy, causing each of the (2J + 1) sublevels to have a slightly different energy. This is called the *Zeeman effect*. The energy separation between the  $M_J$  sublevels in a magnetic field is called the *Zeeman splitting*.

The basic features of this phenomenon are displayed by any physical system that is able to rotate. Whatever the system is, there is always the same structure of (2J + 1)-fold degenerate energy levels. The stable physical states of a rotating quantum system are always specified by a quantum number J for the total angular momentum and an azimuthal quantum number  $M_J$  that carries information on the direction of the rotation. The total angular momentum is always given by  $[J(J + 1)]^{1/2}\hbar$ , and the azimuthal quantum number  $M_J$  always takes one of the values  $M_J = -J, -J + 1... + J$ . The degeneracy of the  $M_J$  sublevels may be broken by applying an electric or magnetic field.

#### **1.2.3** Spin angular momentum

*Spin* is also a form of angular momentum. However, it is not produced by a rotation of the particle, but is an *intrinsic* property of the particle itself.

The total angular momentum of particles with spin takes values of the form  $[S(S + 1)]^{1/2}\hbar$  (the symbol *S* is used instead of *J* to mark a distinction between spin angular momentum and rotational angular momentum). Particles with spin *S* have (2S + 1) sublevels, which are degenerate in the absence of external fields, but which may have a different energy if a magnetic or electric field is applied.

Each elementary particle has a particular value for the spin quantum number *S*. For some particles, *S* is given by an integer, i.e. one of 0, 1, 2... For other particles, *S* is given by a half integer, i.e. one of 1/2, 3/2, 5/2...

Particles with integer spin are called *bosons*. Particles with half-integer spin are called *fermions*.

The spin of an elementary particle, such as an electron, is *intrinsic* and is independent of its history. Elementary particles simply *have* spin; molecules *acquire* rotational angular momentum by energetic collisions. At the absolute zero of the temperature scale, all rotational motion ceases (J = 0). A particle such as an electron, on the other hand, always has spin, even at absolute zero.

Half-integer spin posed severe problems for the physicists of the 1920s and 1930s. It may be shown that half-integer spin cannot arise from 'something rotating', and, at that time, no other way of producing angular momentum could be imagined. The concept of half-integer spin was resisted until the pressure of experimental evidence became overwhelming. One of the greatest triumphs of theoretical physics was Dirac's derivation of electron spin-1/2 from relativistic quantum mechanics. Nowadays, spin is a central concept in our theoretical understanding of the world.

A particle like an electron may, therefore, have two kinds of angular momentum: (i) A 'conventional' angular momentum arising from its *motion*. For example, an electron in an atom may have *orbital angular momentum* due to its circulating motion around the nucleus. Such motion is associated with an *integer* 

angular momentum quantum number and behaves just like the angular momentum of a rotating molecule. (ii) 'Intrinsic' or *spin angular momentum*, which arises from nothing, being simply a feature of the electron's 'nature', and which is always the same, namely spin = 1/2.

There is no such concept as the rotation of the electron around its own axis; there is only spin.

The concept of intrinsic angular momentum is very difficult to grasp. Why should this be so? Why is the intrinsic angular momentum of a particle more difficult to understand than intrinsic mass and intrinsic electric charge?

The level of difficulty of a concept tends to be inversely proportional to its familiarity in the macroscopic world. The concept of intrinsic mass is relatively easy to accept because mass has familiar everyday manifestations. This is because the mass of two particles is the sum of the masses of the individual particles. The mass of a book is therefore the sum of the masses of all the electrons, quarks, etc. of which the book is composed (minus a relativistic correction – but let's forget about that!). So the concept of mass 'makes it' to the macroscopic world. We can 'feel' mass and can imagine that fundamental particles 'have a mass'.

Electric charge is a little more difficult, because there are negative and positive charges, and in almost all cases they cancel out for macroscopic objects. However, by performing simple experiments, like rubbing a balloon on a woolly jumper, it is possible to separate some of the charges and achieve obvious macroscopic effects, such as sticking a balloon to the ceiling. Through such experiences it is possible to get a feel for charge, and become comfortable with the idea that fundamental particles 'have a charge'.

Similarly, magnetism acquires familiarity through the existence of ferromagnetic objects that possess macroscopic magnetism.

Spin is more difficult, because there is no such thing as macroscopic spin. Matter is built up in such a way that the spins of the different particles cancel out in any large object. Spin doesn't 'make it' to the macroscopic world.<sup>2</sup>

This is not to say that spin is unimportant. In fact electron spin has a very profound effect on the everyday world, because the stability of molecules and their chemical behaviour rely on it (as will be discussed shortly, in the context of the Pauli principle). However, this effect is not *obviously* a consequence of electron spin, and there are no large objects that have angular momentum 'by themselves', without rotating.<sup>3</sup>

Probably no-one really understands spin on a level above the technical mathematical rules. Fortunately it doesn't matter so much. We know the rules for spin and that's enough to be able to exploit the phenomenon.

#### **1.2.4** Combining angular momenta

Consider a system with two parts, each one being a source of angular momentum, with quantum numbers  $J_1$  and  $J_2$ . The angular momenta may be due to rotational motion or to spin. The total angular momentum of the entire system is given by  $[J_3(J_3 + 1)]^{1/2}\hbar$ , where  $J_3$  takes one of the following possible values:

$J_{3} = \begin{cases}  J_{1} - J_{2}  \\  J_{1} - J_{2}  + 1 \\ \vdots \\  J_{1} + J_{2}  \end{cases} $ (1.3)	.3)
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Expressed in words, this means that the complete system has a total angular momentum quantum number given either by the sum of the two individual angular momentum quantum numbers, or by the difference in the two individual angular moment quantum numbers, or any of the values in between, in integer steps.

In general, each of the possible total angular momentum states has a different energy. In many cases that state itself behaves like a new object with angular momentum quantum number  $J_3$ .

An important example of this combination rule involves two particles of spin-1/2, i.e.  $S_1 = S_2 = 1/2$ . In this case, we have  $|S_1 - S_2| = 0$  and  $|S_1 + S_2| = 1$ . There are, therefore, only two possibilities for the total

Nuclei

angular momentum quantum number, namely  $S_3 = 0$  and  $S_3 = 1$ . In the state  $S_3 = 0$ , the spins of the two particles cancel each out. This idea is often expressed by saying that the spins are 'antiparallel' in the  $S_3 = 0$  state:

#### Figure 1.3

The combination of two spins-1/2 leads to a singlet state with spin S = 0 and a triplet state with spin S = 1.



In the 'parallel' spin state  $S_3 = 1$ , on the other hand, the spins of the two particles reinforce each other. In general, the  $S_3 = 0$  and  $S_3 = 1$  states have different energy. Note that it is not possible to make a general statement as to which state has the *lowest* energy; this depends on the details of the interactions in the system.

The  $S_3 = 1$  energy level has three substates, with azimuthal quantum number  $M_S = \{-1, 0, 1\}$ . If the environment is isotropic (the same in all directions of space), the three substates have the same energy. States with total angular momentum  $S_3 = 1$  are often called *triplet states*, to stress this threefold degeneracy. The degeneracy of the  $S_3 = 1$  level may be broken by applying an external field (magnetic or electric).

The  $S_3 = 0$  level, on the other hand, is not degenerate. The only state in this level has quantum number  $M_S = 0$ . States with total angular momentum  $S_3 = 0$  are often called *singlet states*.

#### 1.2.5 The Pauli Principle

The spin of particles has profound consequences. The Pauli principle<sup>4</sup> states

two fermions may not have identical quantum states.

Since the electron is a fermion, this has major consequences for atomic and molecular structure. For example, the periodic system, the stability of the chemical bond, and the conductivity of metals may all be explained by allowing electrons to fill up available quantum states, at each stage pairing up electrons with opposite spin before proceeding to the next level. This is called the *Aufbau principle* of matter, and is explained in standard textbooks on atomic and molecular structure (see *Further Reading*).

The everyday fact that one's body does not collapse spontaneously into a black hole, therefore, depends on the spin-1/2 of the electron.

# 1.3 Nuclei

The next sections discuss briefly how the energy level structures of molecules, atoms, nuclei, and even the elementary particles within the nuclei, fit into the angular momentum hierarchy of nature, according to the rule given in Equation 1.3.

#### **1.3.1** The fundamental particles

According to modern physics, everything in the universe is made up of three types of particle: *leptons*, *quarks* and *force particles*.

*Leptons* are low-mass particles. Six varieties of lepton have currently been identified, but only one is familiar to non-specialists. This is the *electron*, a lepton with electric charge -e and spin-1/2. The unit of electric charge e is defined as *minus* the electron charge and is equal to  $1.602 \times 10^{-19}$  C.

Figure 1.4

A neutron.

*Quarks* are relatively heavy particles. At the time of writing (2007), it is believed that there are six 'flavours' of quarks in nature, all of which have spin-1/2. Three of the quarks have electric charge +2e/3. The other three quarks have electric charge -e/3. Apart from their charge, the quarks are distinguished by additional quantum numbers called 'strangeness', 'charm', 'top' and 'bottom', but there is no need to discuss these topics here. It is speculated that quarks are themselves built up of extended objects, which have received the dull name 'superstrings'.

*Force particles* are responsible for mediating the action of the different particles on each other. The most important force particle is the *photon*, which is the particle manifestation of the electromagnetic field. Light, which consists of electromagnetic waves, can be viewed as a stream of photons. The photon has no mass and no electric charge, and has spin = 1. There are also force particles called *gluons* and *vector bosons*. Gluons are manifestations of the so-called *strong nuclear force*, which holds the atomic nucleus and its constituent particles together. Vector bosons are manifestations of the *weak nuclear force*, which is responsible for radioactive  $\beta$ -decay.

#### **1.3.2** Neutrons and protons

The *neutron* and the *proton* both consist of three quarks, stuck together by gluons.<sup>4</sup>

The *neutron* is composed of three quarks: two with charge -e/3 and one with charge +2e/3. Therefore, the total electric charge of the neutron is zero; hence its name. The neutron has spin-1/2. The neutron spin is due to combinations of quark spins.<sup>5</sup> For example, if two of the quark spins are antiparallel, we get an S = 0 state. Addition of the third quark spin gives a total neutron spin S = 1/2:



The *proton* is also composed of three quarks, but this time two of the quarks have charge +2e/3 and the other one has charge -e/3. Therefore, the total electric charge of the proton is +e. Just as for the neutron, an antiparallel spin configuration for two of the quarks gives rise to a net spin-1/2 state for the proton:<sup>4</sup>



What happens if the quark spins inside the neutron and proton have different configurations? For example, if the three quark spins were all parallel, the total spin would be 3/2. Such particles do in fact exist, but they are only known in high-energy physics experiments (for example, the spin-3/2 state of the proton is known as the  $\Delta^+$  particle). High-spin neutrons and protons have enormously higher energies than ordinary neutrons and protons. Under ordinary circumstances, these exotic states may be ignored. The

Particle	Rest mass/kg	Charge	Spin
e	$\begin{array}{c} 9.109 \times 10^{-31} \\ 1.675 \times 10^{-26} \\ 1.673 \times 10^{-26} \\ 0 \end{array}$	-e	1/2
n		0	1/2
p		+e	1/2
photon		0	1

**Table 1.1** Some properties of the most important elementary particles.

neutron and proton may, therefore, be treated as distinct and independent particles, both with well-defined spin-1/2.

We also ignore the numerous other particles formed by combinations of different sets of quarks. From now on, the only particles to be considered are the electron, the neutron, the proton and the photon, whose relevant properties are summarized in Table 1.1.

#### 1.3.3 Isotopes

The atomic nucleus consists of neutrons and protons.<sup>6</sup> Neutrons and protons are known collectively as *nucleons*.

An atomic nucleus is specified by three numbers: the *atomic number*, the *mass number*, and the *spin quantum number*.

The *atomic number Z* specifies the number of protons inside the nucleus. The electric charge of the nucleus is *Ze*. The electric charge of the nucleus determines the chemical properties of the atom of which the nucleus is a part. The atomic number is traditionally denoted by a chemical symbol, for example H for Z = 1, He for Z = 2, C for Z = 6, N for Z = 7, O for Z = 8, etc. The periodic table of the elements lists the atomic nuclei in order of increasing atomic number.

The *mass number* specifies the number of nucleons in the nucleus, i.e. the total number of protons and neutrons. Nuclei with the same atomic number but different mass numbers are called *isotopes*. Most isotopes in existence are *stable*, meaning that the nucleus in question has no measurable tendency to explode or disintegrate. Several isotopes are *unstable*, or *radioactive*, meaning that the nucleus tends to disintegrate spontaneously, ejecting energetic particles, which are often dangerous. NMR is mainly concerned with stable isotopes. Stable nuclei are usually formed from approximately equal numbers of protons and neutrons.

Some common examples of stable isotopes are:

$${}^{1}H = p$$
  ${}^{12}C = 6p + 6n$   
 ${}^{2}H = p + n$   ${}^{13}C = 6p + 7n$ 

and so on.

Some examples of unstable (radioactive) isotopes are:

$${}^{3}H = p + 2n$$
  
 ${}^{14}C = 6p + 8n$ 

Lighter atomic nuclei were formed by the primal condensation of nucleons as the heat of the big bang dissipated. Heavier nuclei (beyond and including iron) were synthesized later by nuclear fusion processes inside stars. These complex nuclear processes led to a mix of isotopes with varying *isotopic distributions*. These distributions are almost uniform over the surface of the Earth. For example, ~98.9% of carbon nuclei

Isotope	Ground-state spin	Natural abundance/%	Gyromagnetic ratio $\gamma/10^6$ rad s <sup>-1</sup> T <sup>-1</sup>	NMR frequency at 11.74 T $(\omega^0/2\pi)/MHz$
<sup>1</sup> H	1/2	$\sim 100$	267.522	-500.000
<sup>2</sup> H	1	0.015	41.066	-76.753
<sup>3</sup> H	1/2	0	285.349	-533.320
$^{10}B$	3	19.9	28.747	-53.718
$^{11}B$	3/2	80.1	85.847	-160.420
<sup>13</sup> C	1/2	1.1	67.283	-125.725
$^{14}N$	1	99.6	19.338	-36.132
$^{15}N$	1/2	0.37	-27.126	+50.684
<sup>17</sup> O	5/2	0.04	-36.281	+67.782
<sup>19</sup> F	1/2	$\sim 100$	251.815	-470.470
<sup>23</sup> Na	3/2	$\sim 100$	70.808	-132.259
<sup>27</sup> Al	5/2	$\sim 100$	69.763	-130.285
<sup>29</sup> Si	1/2	4.7	-53.190	+99.336
<sup>31</sup> P	1/2	$\sim 100$	108.394	-202.606
<sup>35</sup> Cl	3/2	75.77	10.610	-48.990
<sup>37</sup> Cl	3/2	24.23	8.832	-40.779
<sup>63</sup> Cu	3/2	69.17	71.118	-132.577
<sup>65</sup> Cu	3/2	30.83	76.044	-142.018
$^{107}$ Ag	1/2	51.84	-10.889	+20.239
<sup>109</sup> Ag	1/2	48.16	-12.518	+23.268
<sup>129</sup> Xe	1/2	24.4	-74.521	+139.045
<sup>207</sup> Pb	1/2	22.1	55.805	-104.603
<sup>12</sup> C	0	98.9		
<sup>16</sup> O	0	$\sim 100$		

**Table 1.2** A selection of nuclear isotopes and their properties. A complete listing of nuclear spins, gyromagnetic ratios and Larmor frequencies (omitting the sign) may be found on the website www.webelements.com.

have six neutrons (<sup>12</sup>C), and  $\sim$ 1.1% have seven neutrons (<sup>13</sup>C). The small local variations in nuclear isotopic distributions are useful for locating the origin and the age of objects. Some natural isotopic abundances are shown in Table 1.2.

## **1.4** Nuclear Spin

#### **1.4.1** Nuclear spin states

Most atomic nuclei possess spin. The nuclear spin quantum number is conventionally denoted I.

The nucleus of the main isotope of hydrogen, <sup>1</sup>H, contains a single proton and has I = 1/2. The spins of other nuclei are formed by combining together the spins of the protons and the neutrons according to the usual rule (Equation 1.3).

Consider, for example, the <sup>2</sup>H nucleus, which contains one proton and one neutron. The proton and neutron spins may be combined in a parallel configuration, leading to a nuclear spin I = 1, or in an antiparallel configuration, leading to a nuclear spin I = 0:



These two nuclear spin states have a large energy difference of  $\sim 10^{11}$  kJ mol<sup>-1</sup>. This greatly exceeds the energies available to ordinary chemical reactions or usual electromagnetic fields (for comparison, the available thermal energy at room temperature is around  $\sim 2.5$  kJ mol<sup>-1</sup>). The nuclear excited states may, therefore, be ignored, except in exotic circumstances.<sup>7</sup> The value of *I* in the lowest energy nuclear state is called the *ground state nuclear spin*. For deuterium (symbol D or <sup>2</sup>H), the ground state nuclear spin is *I* = 1.

For higher mass nuclei, the ground state is one of a large number of possible spin configurations of the protons and neutrons:





In general, there are no simple rules for which of the many possible states is the ground state. For our purposes, the ground state nuclear spin is best regarded as an empirical property of each isotope.

Nevertheless, one property may be stated with certainty: from Equation 1.3, isotopes with even mass numbers have integer spin and isotopes with odd mass numbers have half-integer spin.

Two further guidelines apply to isotopes with even mass numbers:

1. If the numbers of protons and neutrons are both even, the ground state nuclear spin is given by I = 0. Some examples are: the nucleus <sup>12</sup>C, which contains six protons and six neutrons; the nucleus <sup>16</sup>O, which contains eight protons and eight neutrons; and the nucleus <sup>56</sup>Fe, which has 26 protons and 30 neutrons. All of these have a ground state spin I = 0.



2. If the numbers of protons and neutrons are both odd, the ground state nuclear spin is an integer larger than zero. Some examples are the nuclei <sup>2</sup>H (1p + 1n, ground state spin I = 1), <sup>10</sup>B (5p + 5n, ground state spin I = 3), <sup>14</sup>N (7p + 7n, ground state spin I = 1) and <sup>40</sup>K (19p + 21n, ground state spin I = 4).

These rules may be understood using models of nuclear structure, a subject that will not be discussed further here.

From now on, the ground state nuclear spin is simply called the 'nuclear spin', for the sake of simplicity. Table 1.2 shows some of the nuclear isotopes of importance in NMR, together with their natural abundances. An overview of all nuclear spins is given in the inside cover as Plates A, B and C.

#### 1.4.2 Nuclear Zeeman splitting

A nuclear state with spin I is (2I + 1)-fold degenerate. If a magnetic field is applied, the degeneracy is broken, just as in the case of ordinary angular momentum (see Figure 1.2). The splitting between the nuclear spin levels is called the *nuclear Zeeman splitting*. NMR is the spectroscopy of the nuclear Zeeman sublevels.

Figure 1.8 sketches the nuclear Zeeman levels of a <sup>1</sup>H nucleus, a <sup>15</sup>N nucleus, and a <sup>27</sup>Al nucleus as a function of the applied magnetic field. The <sup>1</sup>H and <sup>15</sup>N nuclei are both spin-1/2, and so the nuclear ground state splits into two sublevels in the applied magnetic field (since  $2 \times (1/2) + 1 = 2$ ). The <sup>27</sup>Al nucleus is spin-5/2 and, hence, the nuclear ground state splits into six levels in the applied magnetic field (since  $2 \times (5/2) + 1 = 6$ ).



The Zeeman splitting of a proton nucleus is about 10 times larger than the Zeeman splitting of a <sup>15</sup>N nucleus, in the same magnetic field. This is because a proton is about 10 times more magnetic than a <sup>15</sup>N nucleus. This will be discussed in Chapter 2.

The Zeeman splitting *within* the nuclear ground state must not be confused with the enormously larger splitting between the nuclear ground state and the nuclear excited states. The Zeeman splittings are far smaller than thermal energies and are the subject of NMR spectroscopy. The splitting between the nuclear excited state and the nuclear ground state is so large that the nuclear excited states may be completely ignored in ordinary chemistry and spectroscopy.<sup>6</sup>

#### **1.4.3** Zero-spin nuclei

A minority of nuclear isotopes have zero nuclear spin in the ground state and display no nuclear Zeeman effect. By a quirk of fate, organic substances contain many of these spinless isotopes. The most common isotopes of carbon, oxygen and sulfur, namely <sup>12</sup>C, <sup>16</sup>O and <sup>32</sup>S, all have zero nuclear spin and are NMR silent.

#### **1.4.4** Spin-1/2 nuclei

Nuclei with spin I = 1/2 are of major importance in NMR. As discussed in Chapter 8, such nuclei are spherical in shape and have convenient magnetic properties. Plate A shows the distribution of spin-1/2 nuclei in the periodic table of the elements. They are mainly scattered around the right-hand side of the periodic table.

Most chemical elements have no spin-1/2 isotope. The alkali and alkaline earth metals possess no spin-1/2 isotopes at all. In contrast, spin-1/2 isotopes are well represented in organic materials. The most common isotopes of hydrogen and phosphorus have spin-1/2 (<sup>1</sup>H and <sup>31</sup>P), and carbon and nitrogen possess the rare spin-1/2 isotopes <sup>13</sup>C and <sup>15</sup>N. The abundant fluorine isotope <sup>19</sup>F is also of importance.

Spin-1/2 isotopes are well represented in the precious and heavy metals, and the noble gases contain a possess two spin-1/2 isotopes, namely  ${}^{3}$ He and  ${}^{129}$ Xe.

#### **1.4.5** Quadrupolar nuclei with integer spin

Nuclei with spin I > 1/2 are known as 'quadrupolar nuclei', for reasons discussed in Chapter 8. The NMR of such nuclei is a rich but relatively difficult field, which is increasing in popularity, especially in the context of solid-state NMR.

Quadrupolar nuclei with integer values of *I* are uncommon. Plate B shows their distribution in the periodic table. By far the most abundant nucleus of this type is <sup>14</sup>N, which occurs in almost 100% natural abundance. Deuterium (<sup>2</sup>H) is of great importance despite its low natural abundance, since it is relatively easy to separate from <sup>1</sup>H by physical methods and to prepare <sup>2</sup>H-enriched substances.

The nuclear spins I = 3, 4, 5, 6 and 7 are represented by only one isotope each, some of which are obscure. There are no nuclei at all with ground state spin I = 2. The I = 7 isotope <sup>176</sup>Lu has the highest nuclear spin in the entire periodic table.

#### **1.4.6** Quadrupolar nuclei with half-integer spin

Quadrupolar nuclei with I = 3/2, 5/2, 7/2 or 9/2 are common (see Plate C). They are well represented throughout the entire periodic table, but are particularly prominent for the alkali metals, the boron-to-thallium group, and the halogens. There is a striking alternation across many parts of the periodic table, with every other element possessing an abundant isotope with a half-integer quadrupolar nucleus. I do not know the reason for this alternation, which must reflect some feature of nuclear structure and energetics.

The importance of the isotopes is not necessarily reflected by their abundance. For example, the isotope <sup>17</sup>O has a very low natural abundance but is, nevertheless, of great importance since it is the only stable oxygen isotope with nuclear spin.

## **1.5** Atomic and Molecular Structure

#### 1.5.1 Atoms

The atomic nucleus has a positive electric charge +Ze. An *atom* is composed of a nucleus surrounded by *Z* electrons, each with charge -e. For example, an atom of <sup>4</sup>He consists of a nucleus, containing two neutrons and two protons, surrounded by a cloud of two electrons. The simplest atom is hydrogen, which contains a nucleus of one proton and a single orbiting electron.

An atom is electrically charged if the total charge on the nucleus does not balance out exactly the charge on the electron cloud. Such species are called *ions*. For example, a nucleus containing 11 protons and 12 neutrons, surrounded by a cloud of 10 electrons, is called a  ${}^{23}Na^+$  ion. A nucleus containing 17 protons and 18 neutrons, surrounded by a cloud of 18 electrons, is called a  ${}^{35}Cl^-$  ion.

Atomic structure does not really concern us here, but it is worth seeing how the angular momentum of the atom works out (see *Further Reading* for a more detailed discussion). There are three sources of angular momentum in a hydrogen atom: the electron spin, the proton spin, and the electron orbital angular momentum, which is associated with the motion of the electron around the nucleus. The orbital angular momentum of the electron is characterized by a quantum number, usually called *l*. Since this type of angular momentum is associated with motion, the quantum number, usually called *l*. Since this type of electron orbital angular momentum are called *s-orbitals*. Quantum states with electron orbital angular momentum l = 1 are called *p-orbitals*, and so on. In addition, the energy levels of the H atom display *fine structure*, which is due to the coupling of the electron orbital angular momentum to the *electron spin*. Even closer observation of the energy levels reveals *hyperfine structure*, which is due to the participation of the proton spin.

Atoms containing more than one electron have many more possibilities, because each electron is a source of orbital angular momentum as well as spin angular momentum. In such systems, the *Pauli principle* comes into play. Electrons may only occupy identical orbital states if their spins are antiparallel. In practice this means that the lowest energy states in an atom usually have small values of total electron spin.

#### 1.5.2 Molecules

An electron cloud containing more than one nucleus is called a *molecule*. For example, a molecule of water  $({}^{1}\text{H}_{2}{}^{16}\text{O})$  consists of a cloud of 10 electrons surrounding three nuclei: one with eight protons and eight neutrons (the oxygen nucleus), and two consisting of a single proton (the hydrogen nuclei).

In a molecule, the quantum mechanical motion of the electrons constrains the nuclei to a particular geometric configuration. In the case of the water molecule, the three nuclei are geometrically stable only when the three nuclei form a triangular configuration with an H–O–H angle of around 105°.

A typical molecule contains many potential sources of angular momentum: the motion of the electrons around the nuclei, the motion of the nuclear framework around the centre of mass of the molecule, in some cases the rotation of internal molecular groups, the electron spins, and the nuclear spins. However, because of the Pauli principle, and the quantum rules for chemical bonding, the electron orbital angular momenta and the electron spin angular momenta almost always cancel out in the lowest energy state of a chemically stable molecule (there are a few exceptions, such as  $O_2$ , which has total electron spin S = 1 in the ground state, and NO, which has finite orbital angular momentum in the ground state). In most cases, the only sources of angular momentum in the molecular ground state are the molecular rotation and the nuclear spins.

The motion of the nuclear framework can very often be treated 'classically': the molecule is treated as an ordinary object rotating in space. Inside this rotating molecule are trapped the nuclear spins. This ignores the quantum nature of the molecular motion. Nevertheless, it almost always gives reasonable answers, and we will use it from now on.

Molecules differing only in the mass numbers of the nuclei are called *isotopomers*. Isotopomers usually have almost identical chemical and physical properties, since these are determined almost completely by the charges on the nuclei and the number of surrounding electrons. All substances are mixtures of isotopomers, even when chemically pure. For example, pure water is composed mainly of the predominant isotopomer  ${}^{1}\text{H}_{2}{}^{16}\text{O}$ , but there are also small amounts of the minor isotopomers  ${}^{1}\text{H}^{2}\text{H}{}^{17}\text{O}$ ,  ${}^{1}\text{H}_{2}{}^{17}\text{O}$ , etc. The relative abundance of the isotopomers is governed by the natural statistical distributions of the various isotopes.

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There are slight differences in the chemical and physical properties of isotopomers, due to the different nuclear masses. Molecules with different nuclear masses have different vibrational energy levels, altering slightly the rates of certain chemical reactions. Physical properties, such as diffusive mobilities, are also influenced by the molecular mass. This allows isotopomers to be separated by techniques such as gas chromatography. For example, it is possible to separate <sup>13</sup>CO molecules from the abundant <sup>12</sup>CO isotopomers by passing the gas through very long chromatographic columns. The <sup>13</sup>C-labelled carbon monoxide is used as a starting material for total organic synthesis of other <sup>13</sup>C-labelled substances. Many modern NMR experiments use isotopically enriched substances prepared in this way.

NMR is unusual in that different isotopomers behave completely differently. In many cases, one isotopomer gives a large signal while another isotopomer gives none, even though the substances are physically and chemically almost indistinguishable. NMR achieves this distinction because it is based upon the nuclear spin, rather than the nuclear mass.

The mass of a molecule is roughly equal to the sum of the nuclei in the molecule. In the biological sciences, it is popular to specify molecular masses in units of daltons (abbreviation Da). A <sup>12</sup>C atom has a mass of exactly 12 Da. A medium-sized protein molecule has a molecular mass of around  $30 \times 10^3$  Da = 30 kDa (30 kilodaltons). Outside the biological sciences, molecular masses are usually specified in units of grams per mole (g mol<sup>-1</sup>).

## **1.6** States of Matter

The states of matter are assembled from the basic building blocks of atoms, molecules and ions. We will now review the special features of the different material states, since the motions of the molecules in these states have a large effect on NMR experiments.

Traditionally, the material states are classified in terms of their bulk mechanical properties (i.e. hard or soft, rigid or flowing). These bulk mechanical properties often reflect the mobility of the constituent molecules or atoms.

#### 1.6.1 Gases

*Gases* are defined as low-density material phases that fill the volume of any container to which they are confined, independent of its shape. Gases are characterized by very high mobility of the molecules or atoms. It is possible to perform NMR experiments on gases, although this is not done very often.<sup>8</sup>

## 1.6.2 Liquids

*Liquids* are relatively dense material phases characterized by their flow under shear forces. There is high molecular mobility in liquid phases. This molecular mobility has two aspects: translation mobility and rotational mobility (Figure 1.9). *Translation molecular mobility* means that the molecules or atoms slide past each other rather freely and wander over appreciable distances in a relatively short time. For example, a water molecule at room temperature typically wanders over a distance of  $\sim$ 7 µm over a time interval of 10 ms. *Rotational molecular mobility* means that the individual molecules rotate around their own centres of gravity in a more or less random fashion. For example, a medium-sized protein molecule in water solution typically rotates through an angle of around 1 rad after a time of around 10 ns. Both of these types of mobility are very important for NMR, because they tend to average out many of the nuclear spin interactions, simplifying the behaviour of the nuclear spins.

Matter



#### **Figure 1.9** In a liquid, molecules have both translation mobility (a) and rotational mobility (b). If the liquid is isotropic, the mobilities are the same in all directions.

A further distinction must be made between *isotropic* and *anisotropic* liquid phases.

In isotropic liquids, the translation and rotational mobilities of the molecules are the same in all directions. This is true, for example, in ordinary water or for molecules dissolved at low concentration in common solvents. As discussed in Chapter 8, the molecular motion in an isotropic liquid effectively removes many of the nuclear spin interactions, leading to rather simple NMR spectra.

There are also anisotropic liquids (also called liquid crystals), in which the molecules adopt a non-isotropic spatial configuration. In some cases the molecules are arranged in layers, in other cases like coins stacked up on top of each other, and in still other cases the molecules adopt a sort of loose helical structure. Despite these loose spatial configurations, the molecules are still very mobile and the substance flows under shear forces, which distinguishes a liquid crystal from a solid. An everyday example of a liquid crystal is a soap film, in which the soap molecules are arranged in layers. As far as NMR is concerned, anisotropic liquids behave very differently from isotropic liquids because the translation and rotational mobilities of the molecules depend on the direction (Figure 1.10). Consider, for example, a case in which the molecules are shaped like long rods and all the rods are, on average, aligned along a particular direction in space (called the *director* in liquid-crystal science). It is easier for the molecules to spin around an axis that is parallel with the director than around an axis that is perpendicular to the director. In the former case they have a better chance of completing a full rotation without hitting a neighbouring molecule, as opposed to the latter case, where collisions are almost inevitable. This motional anisotropy leads to incomplete averaging of the nuclear spin interactions and, hence, more complicated NMR spectra. Nevertheless, the NMR spectra of liquid crystals, and of molecules dissolved in liquid crystals, can be very informative. One way in which anisotropic liquids are used to enhance the study of biological molecules is sketched in Section 16.4.



#### Figure 1.10

In an anisotropic liquid, the molecular mobilities depend on the direction in space.

## 1.6.3 Solids

*Solids* are high-density material phases that resist shear forces without flowing. Solid materials have an enormous variety of atomic structures. For example, there are *molecular solids*, which are composed of distinct molecules held together by rather weak intermolecular forces, and *non-molecular solids*, in which the atomic nuclei are gripped in an extended electron cloud network, which may extend to the boundaries of the material. There are *crystals*, in which the nuclei are arranged on a repeating lattice, as well as *glasses* and *amorphous solids*, in which long-range repeating patterns are difficult to identify.

The distinction between 'solid' and 'liquid' is not always unambiguous. The behaviour of matter under shear forces depends strongly on the time-scale involved. For example, an ordinary liquid resists deformations and appears as a hard solid on a time-scale of microseconds – as may be tested by diving stomach first into a swimming pool. On the other hand, hard solids, such as rocks, may behave as liquids under long time-scales.<sup>9</sup>

There is a great variety of electronic behaviour in solids. The majority of materials are *electrical insulators*, with very restricted electron mobility. However, in some cases the electron mobility is high, even though the atomic nuclei are more or less fixed in place. Such substances are *metals*. In special circumstances, the electrons may pair up to form bosons that form macroscopic coherent quantum waves. This happens in *superconductors*.

The motion of atoms and molecules is usually greatly restricted in solids. It is this feature that distinguishes solids from liquids on the molecular level. Nevertheless, there can be substantial local motion. For example, some molecular solids display considerable rotational motion of the molecules around their own lattice positions. This often happens for solids composed of near-spherical molecules, e.g. the football-like fullerene molecule  $C_{60}$ . At room temperature, the  $C_{60}$  'footballs' rotate randomly and isotropically around their own centres in the solid:



**Figure 1.11** Rotation of  $C_{60}$ molecules in a solid.

Even in molecules that do not jump or rotate as entire units, there are often local groups which have considerable local mobility. For example, methyl groups  $-CH_3$  usually rotate rapidly at room temperature, even in rigid solids. Often, solids display a variety of phases at different temperatures and pressures, with different modes of atomic or molecular mobility.

Generally speaking, the NMR spectra of solids are generally broader and more complex than in liquids. Nevertheless, there has been much recent technical progress in the NMR of solids. The development of experimental techniques has made it possible to obtain solid-state NMR spectra with a resolution approaching that obtained in isotropic liquids in many cases. This is very useful, because there are many substances which cannot, or should not, be dissolved or melted.

# Notes

- 1. Electron spin was first postulated by the Dutch graduate students Uhlenbeck and Goudsmidt in 1925. Their supervisor stated that they could risk publication since they didn't yet have a reputation to destroy. The influential physicist Pauli initially poured scorn on the idea but later became one of its chief proponents.
- 2. Throughout this book, a box drawn around an equation indicates that it is important in its own right.
- 3. Some large objects do in fact 'have angular momentum "by themselves", without rotating', although the consequences are subtle and require fine experimental observations. For example, the *Einstein-de Haas effect* shows that the act of magnetizing a substance imparts angular momentum to it (for example, an object suspended by a fine thread starts rotating slowly as it is magnetized). A magnetized substance, like a lump of iron, does have angular momentum, without rotating! There is even a converse phenomenon called the Barnett effect: rotating an object magnetizes it! See E. T. Jaynes, *Rev. Mod. Phys.* 34, 143 (1962).
- 4. This is a simplified version of the Pauli principle. The full version is as follows:

The quantum mechanical state of a system containing two identical bosons is invariant under exchange of the two bosons. The quantum mechanical state of a system containing two identical fermions changes sign under exchange of the two fermions.

The theorem that two fermions may not occupy the same state follows as a consequence of the full principle. As a matter of fact, the 'Pauli principle' was first given for fermions by Heisenberg, and for bosons by Bose. It is a little mysterious why the principle has become attached to the name of Pauli.

- 5. The discussion of neutron spin, proton spin, and nuclear spin given in this chapter is oversimplified. In fact, it is known that the orbital motions of the quarks and gluons also contribute to the spin of the proton and neutron (see, e.g., S. D. Bass, Science 315, 1672-1673 (2007)). I deliberately give a naive picture of neutron and proton spin here, because the details are not so important anyway for NMR and because it does make a good story.
- 6. The neutrons and protons are bound together by an interchange of particles known as mesons. Each meson is made up of two quarks.
- 7. In *Mössbauer spectroscopy*, transitions between the nuclear energy levels are excited using energetic *γ*-rays.
- 8. There are some interesting applications of NMR in gases, although the low density of matter in a gas leads to relatively weak NMR signals. For example, NMR of gaseous <sup>129</sup>Xe is used to make images of cavities inside hollow objects, including lungs. In this case, a special technique called *optical pumping* is used to prepare <sup>129</sup>Xe gas with a very high nuclear spin polarization, so as to enhance the NMR signals. See, for example, B. M. Goodson, *J. Magn. Reson.* **155**, 157–216 (2002). Some conventional gas-phase NMR spectra, obtained without the help of optical pumping, are shown in Figure 19.21.
- 9. The first edition of this book propagated the urban myth that old glass windows are thicker at the bottom because the glass creeps slowly under gravity. This is not true. The thickening at the bottom is simply due to the manufacturing process. Thanks to the copy editor for pointing this out.

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# **Further Reading**

- For the fundamentals of quantum mechanics and atomic structure, see J. J. Sakurai, *Modern Quantum Mechanics*, Addison-Wesley, 1994, and C. Cohen-Tannoudji, B. Diu, F. Laloë, *Quantum Mechanics* Wiley, London, 1977.
- For atomic structure and orbitals, see P. W. Atkins, *Molecular Quantum Mechanics*, Oxford University Press, Oxford, 1983.
- For the consituents of matter, see F. Close, 'The quark structure of matter' in *The New Physics*, P. Davies, (ed.), Cambridge University Press, Cambridge, 1989.

## **Exercises**

- **1.1** Which of the following statements *must* be correct, which *might possibly* be correct, and which *cannot* be correct?
  - (i) The nucleus <sup>89</sup>Y (atomic number = 39) has a ground state spin I = 1/2.
  - (ii) The nucleus  ${}^{90}$ Zr (atomic number = 40) has a ground state spin I = 1.
  - (iii) The nucleus  ${}^{91}$ Zr (atomic number = 40) has a ground state spin I = 1/2.
  - (iv) The nucleus  ${}^{92}$ Mo (atomic number = 42) has a ground state spin I = 0.
  - (v) The nucleus <sup>138</sup>La (atomic number = 57) has a ground state spin I = 0.
- **1.2** If a particle with spin S = 5/2 couples to a particle with spin S = 3/2, what are the possible values for the spin of the resulting particle?