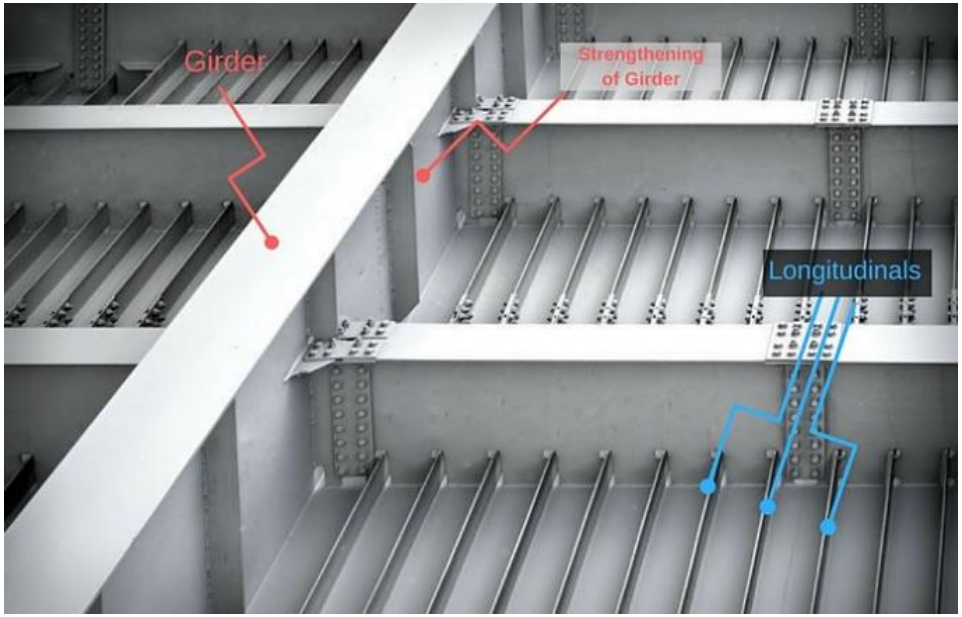
DEPARTAMENTO DE ENGEHARIA NAVAL E OCEÂNICA ESCOLA POLITÉCNICA DA USP

Análise de Vigas : $\delta\,(mm)$

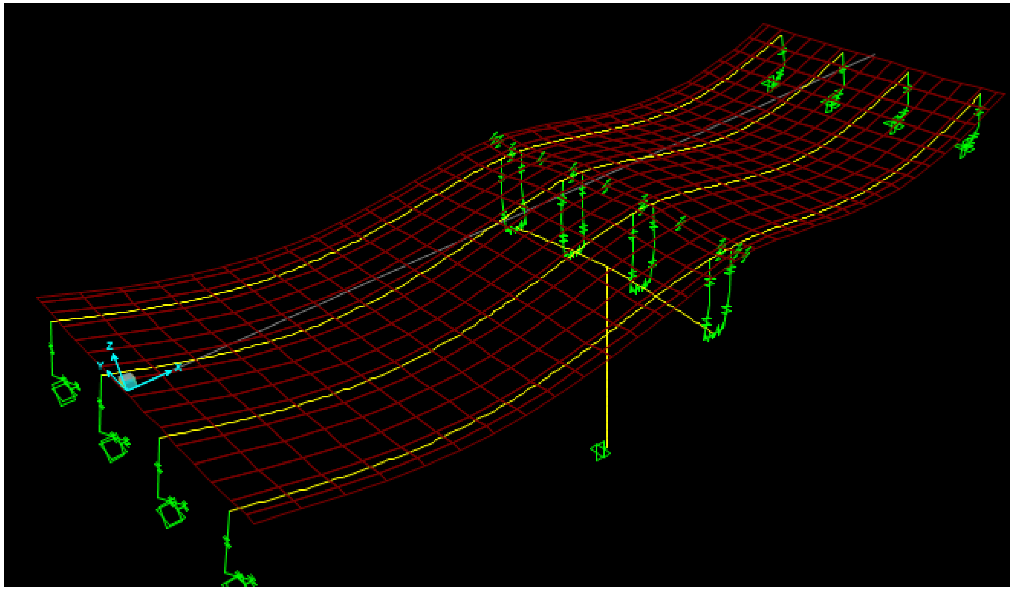


PNV 3212 – Mecânica Dos Sólidos I

2020

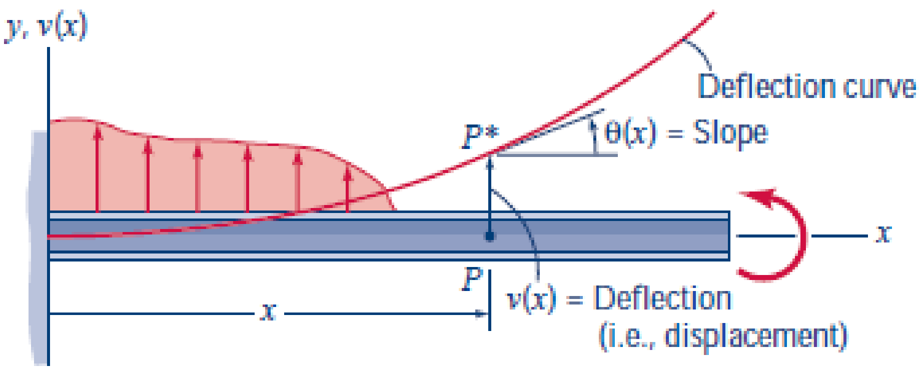
Deflection of beams

$$\delta \ll t$$



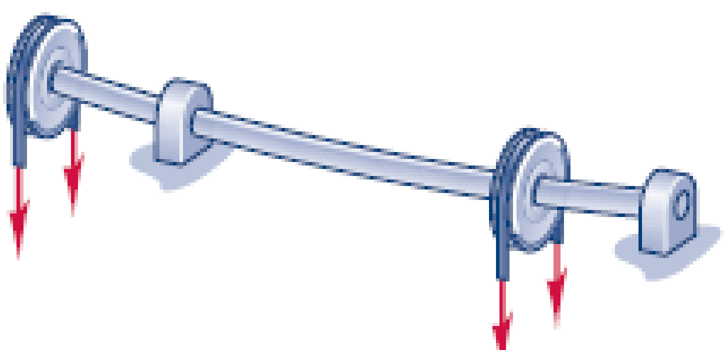
Deflection of beams

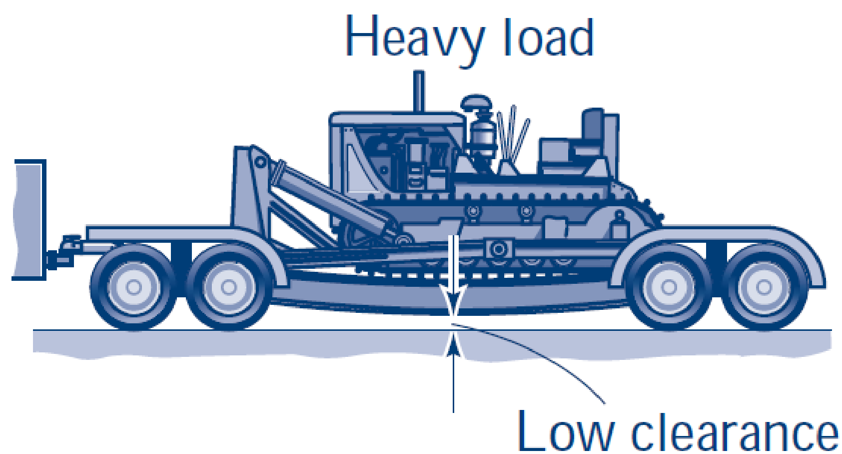
Deslocamento transversal (vertical)



$$\delta \ll t$$

$$\delta(x) \equiv v(x)$$





Deflexões em vigas

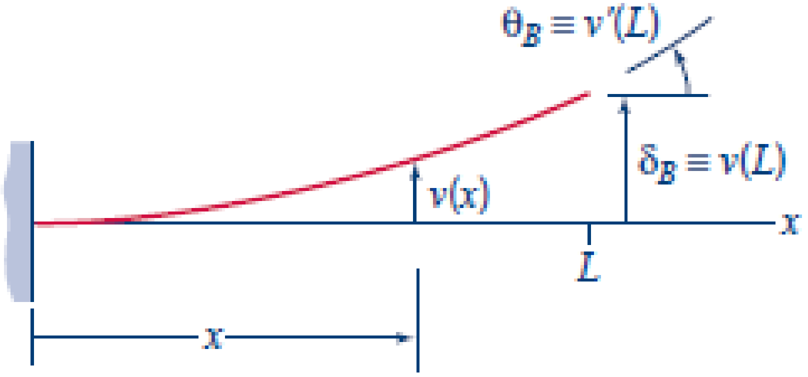
- **Hipóteses**
  - Problema é independente do tempo.
  - O formato da viga é um prisma reto, cujo comprimento é muito maior que as outras dimensões (**Esbelta**).
  - A viga é constituída de um material **linearmente elástico**.
  - O efeito Poisson é negligenciável.
  - A seção transversal é simétrica em relação ao plano vertical.
  - Planos perpendiculares à linha neutra permanecem quase planos e perpendiculares ao eixo deformado depois da deformação (**Navier**).
  - O ângulo de rotação da seção transversal é muito pequeno.
  - O efeitos de momento de inércia da rotação é desprezado.
  - A viga é constituída de material homogêneo .
  - The distribution of flexural stress on a given cross section is not affected by the deformation due to shear.
  - Distorção da seção transversal é pequena o suficiente para ser desprezada!

Deflexões em vigas

- **Caminho**
  1. Relação Momento- curvatura
  2. Equação diferencial relacionando v(x) – M(x)
  3. Condições de contorno

Deflexões em vigas

- Fórmula

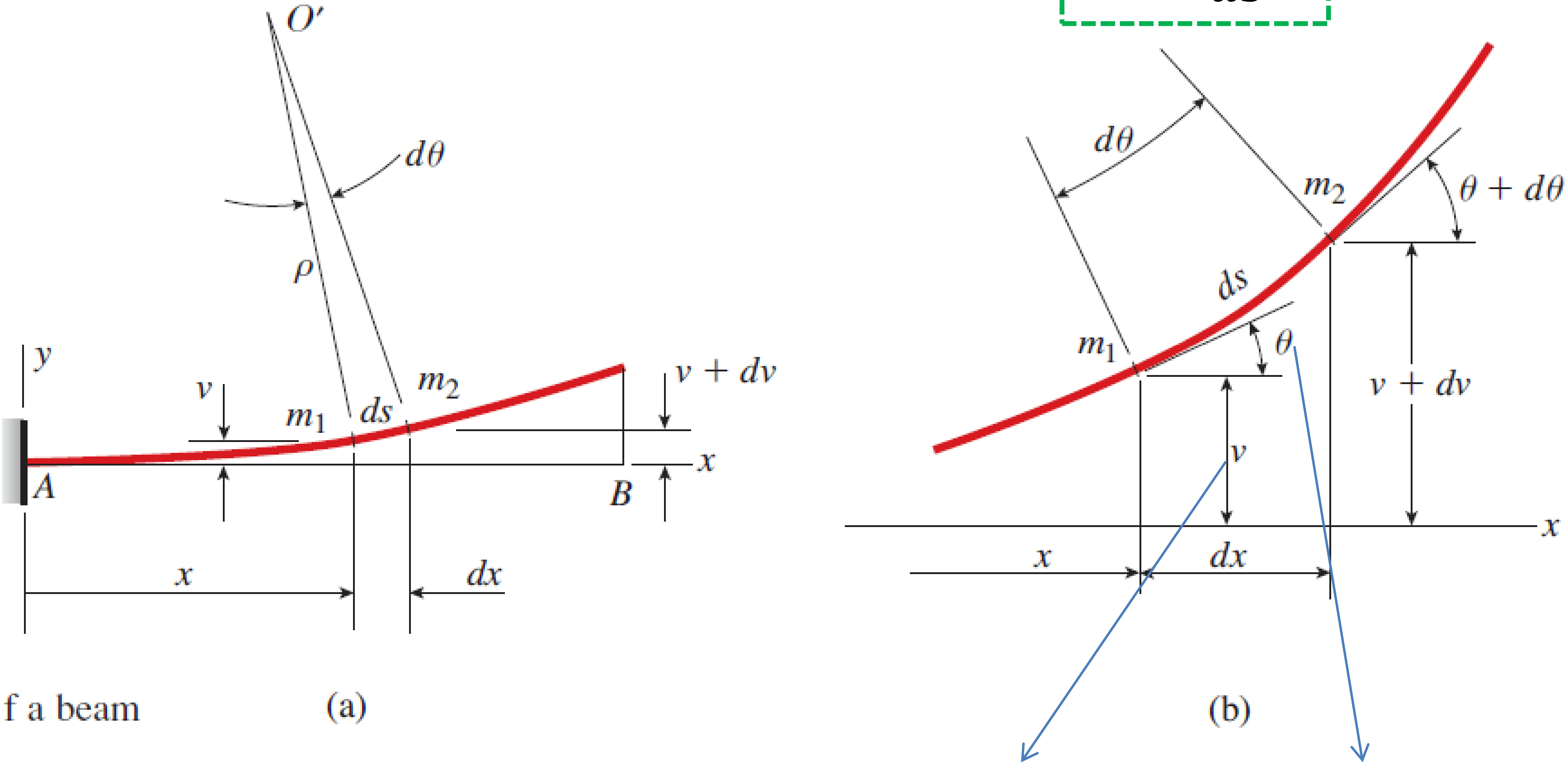


$$\mathrm{EI}\,\frac{d^2v(x)}{dx^2} = M(x)$$

Deflexões em vigas

- Curvatura $\kappa(x) = \dfrac{1}{\rho(x)}$

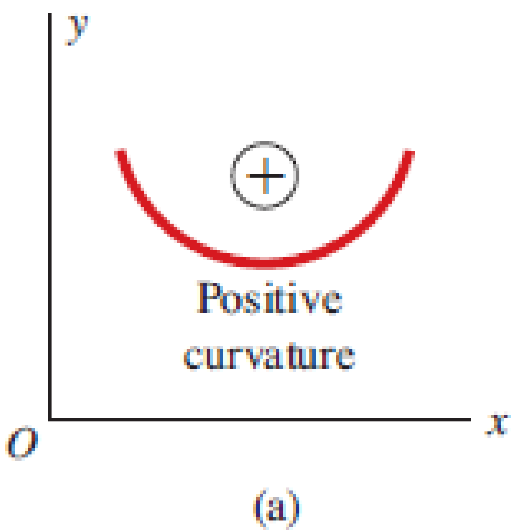
$$ds = \rho d\theta \rightarrow \kappa = \frac{d\theta}{ds}$$



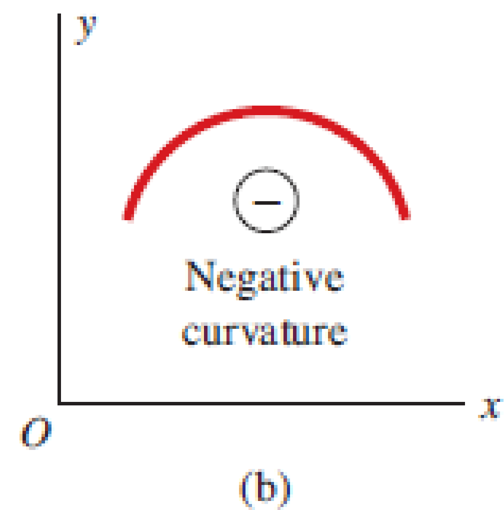
Flexão de uma viga → deflexão vertical + **rotação**

Deflexões em vigas

- Curvatura $\kappa(x) = \dfrac{1}{\rho(x)}$



$$\kappa = \frac{d\theta}{ds}$$



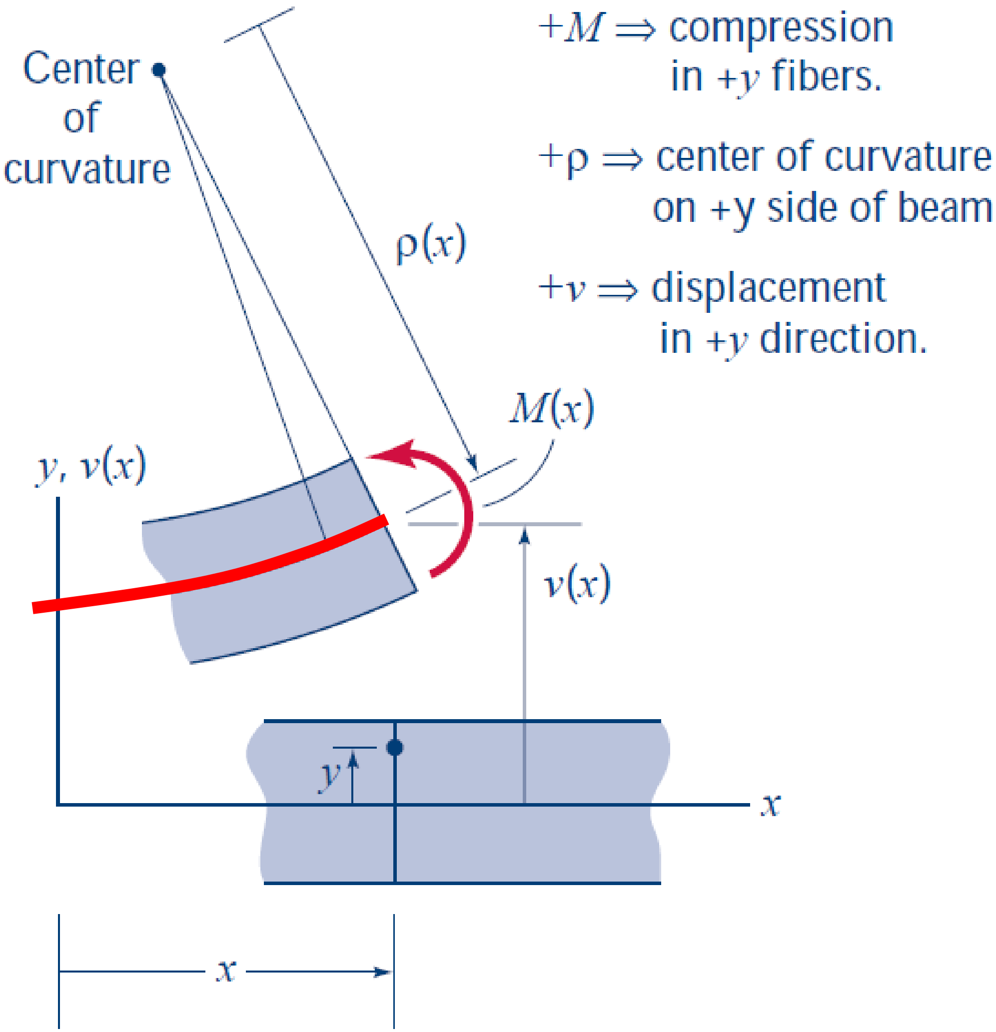
M+

M-

Flexão de uma viga → deflexão vertical + **rotação**

Deflexões em vigas

- Momento-curvatura



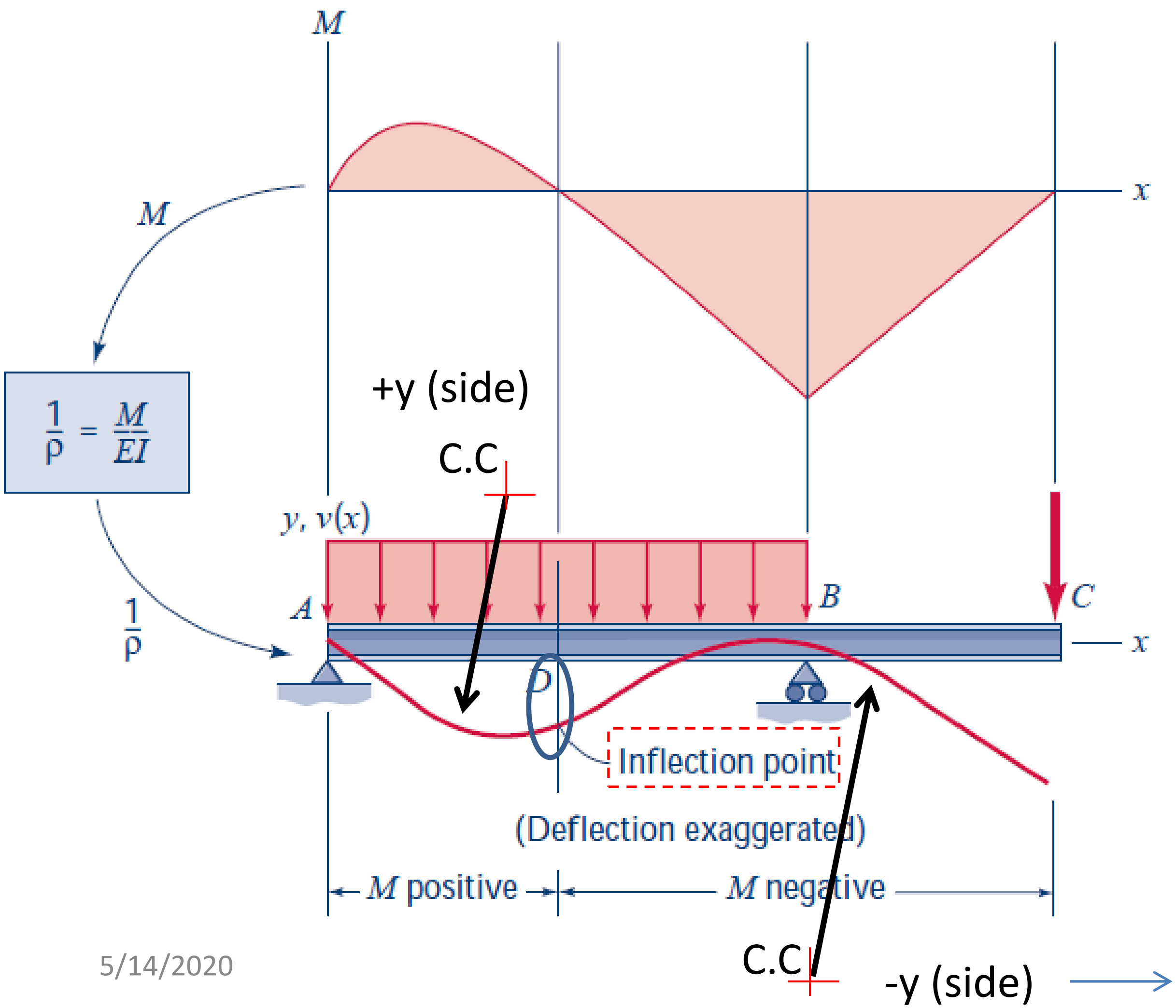
$$\frac{M(x)}{\mathrm{EI}} = \frac{1}{\rho(x)}$$

$$\frac{M(x)}{\mathrm{EI}} = \kappa(x)$$

$$\boldsymbol{\kappa(x) = \frac{1}{\rho(x)}}$$

Deflexões em vigas

- Momento-curvatura



$$\frac{M(x)}{\mathrm{EI}} = \frac{1}{\rho(x)}$$

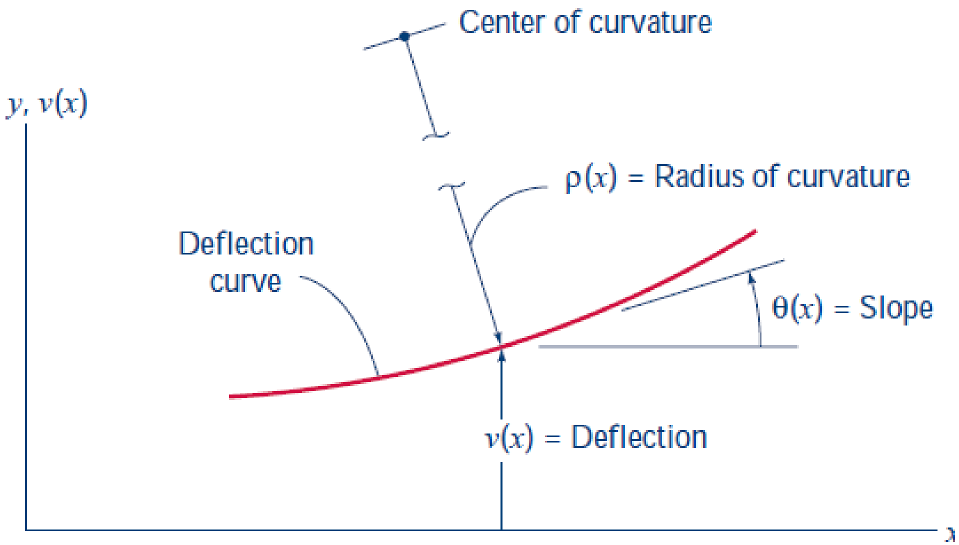
$$\boldsymbol{\kappa(x) = \frac{1}{\rho(x)}}$$

Concavidade (+/-)!!!

Deflexões em vigas

$$\kappa(x) = \frac{1}{\rho(x)}$$

- Inclinação θ

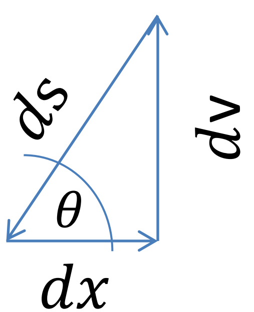


$$\frac{dv(x)}{dx} = \tan\theta$$

Buildings, automobiles, aircraft, and ships, undergo relatively small changes in shape while in service

Pequenas deflexões

very small angles of rotation, very small deflections, and very small curvatures.



$$\frac{dv(x)}{dx} \cong \theta$$

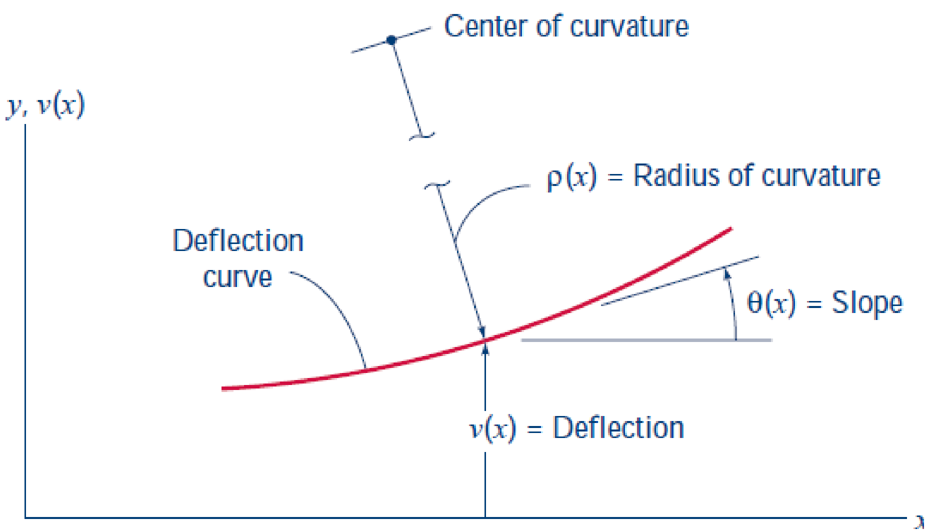
$$\frac{dv(x)}{dx} \ll 1$$

$$\frac{dv}{ds} = \sin\theta \qquad \frac{dx}{ds} = \cos\theta$$

Deflexões em vigas

$$\kappa(x) = \frac{1}{\rho(x)}$$

- Inclinação θ



very small angles of rotation, very small deflections, and very small curvatures.

$$\theta < 15°$$

θ °	θ [rad]	tan θ	cos θ
5	0.087	0.087	0.996
10	0.175	0.176	0.985
15	0.262	0.268	0.966

$$\frac{M(x)}{\mathrm{EI}} = \frac{d\theta}{dx}$$

$$\frac{dx}{ds} = \cos\theta$$

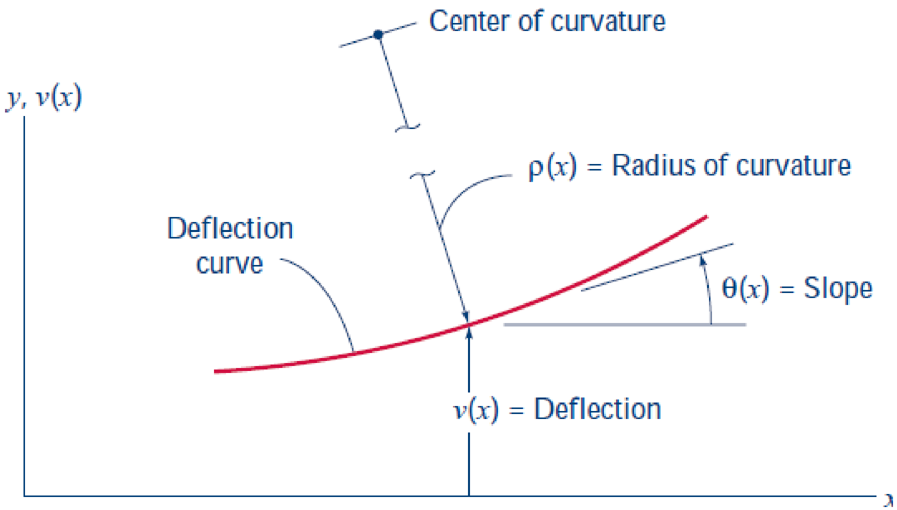
$$dx \approx ds$$

$$\kappa = \frac{d\theta}{dx}$$

Deflexões em vigas

$$\boldsymbol{\kappa(x) = \frac{1}{\rho(x)}}$$

- Inclinação θ



very small angles of rotation, very small deflections, and very small curvatures.

$$\frac{M(x)}{\mathrm{EI}} = \frac{d\theta}{dx}$$

Moment-deflection equation

$$\frac{dv}{dx} \approx \theta \rightarrow \frac{d^2v}{dx^2} \approx \frac{d\theta}{dx} \rightarrow \frac{M(x)}{\mathrm{EI}} = \frac{d^2v(x)}{dx^2}$$

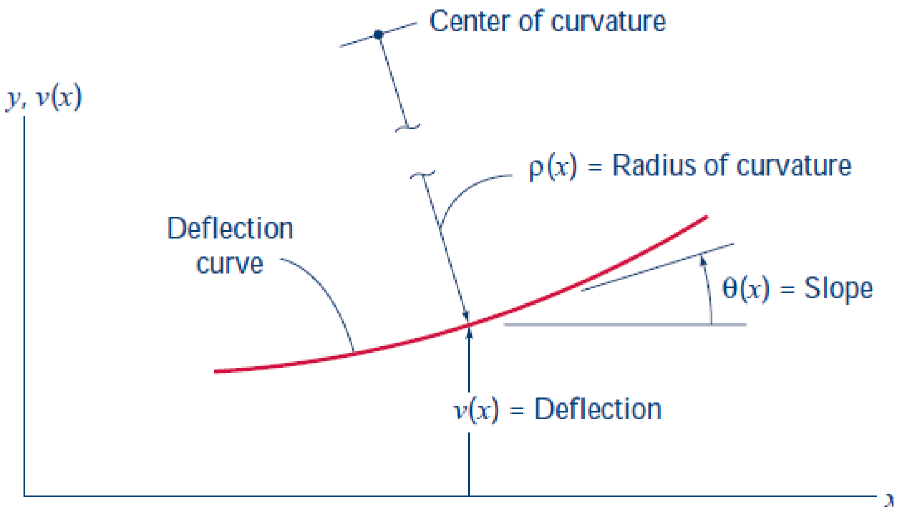
$$\frac{dV}{dx} = -q(x)$$

$$\frac{dM}{dx} = -V(x)$$

Deflexões em vigas

$$\boldsymbol{\kappa(x)} = \frac{\mathbf{1}}{\boldsymbol{\rho(x)}}$$

- Inclinação θ



very small angles of rotation, very small deflections, and very small curvatures.

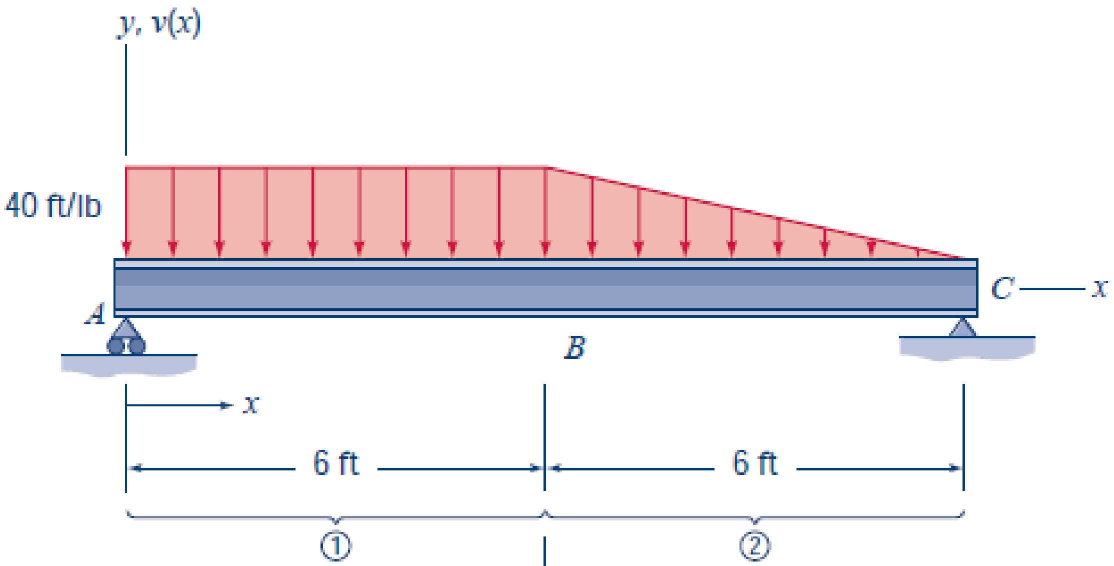
Load-deflection equation

$$\mathbf{E}\boldsymbol{I}\,\frac{\boldsymbol{d^2v(x)}}{\boldsymbol{dx^2}} = \boldsymbol{M(x)} \rightarrow \mathbf{E}\boldsymbol{I}\,\frac{\boldsymbol{d^3v(x)}}{\boldsymbol{dx^3}} = -\boldsymbol{V(x)} \rightarrow \mathbf{E}\boldsymbol{I}\,\frac{\boldsymbol{d^4v(x)}}{\boldsymbol{dx^4}} = \mathbf{q}\boldsymbol{(x)}$$

$$\frac{dM}{dx} = -V(x)$$

$$\frac{dV}{dx} = -q(x)$$

Deflexões em vigas



$$EI\,\frac{d^4v(x)}{dx^4}=q(x)$$

$(0 < x \leq 6\text{ft})$
$p_1(x) = -40$ lb/ft
$V_1(x) = (220 - 40x)$ lb
$M_1(x) = (220x - 20x^2)$ lb · ft
$v_1(x)$ = deflection in interval 1

$(6\text{ft} \leq x < 12\text{ft})$
$p_2(x) = -40\left(\frac{12-x}{6}\right)$ lb/ft
$V_2(x) = \left[-140 + \frac{10}{3}(12-x)^2\right]$ lb
$M_2(x) = \left[140(12-x) - \frac{10}{9}(12-x)^3\right]$ lb · ft
$v_2(x)$ = deflection in interval 2

Boundary Conditions and Continuity Conditions

$$v_1(0) = v_2(12) = 0$$

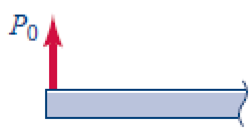
$$v_1(6) = v_2(6)$$

$$\theta_1(6) = \theta_2(6)$$

Deflexões em vigas

Boundary Conditions and Continuity Conditions

TABLE 7.1 Boundary Conditions

	Type	Symbol*	2nd Order	4th Order
BC	Fixed end		$v = 0$ $v' = 0$	$v = 0$ $v' = 0$
	Simple support		$v = 0$	$v = 0$ $M = 0$
	Free end		No BC	$V = 0$ $M = 0$
	Concentrated force		No BC	$V = P_0$ $M = 0$
	Concentrated couple		No BC	$V = 0$ $M = -M_0$

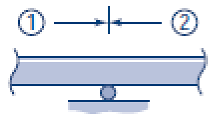
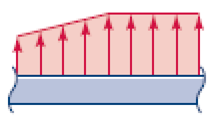
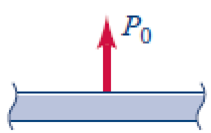
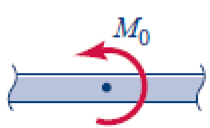
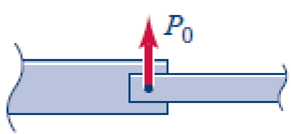
*These boundary conditions also apply if the boundary under consideration is the other end of the beam (i.e., $x = L$).

$$EI\,\frac{d^4 v(x)}{dx^4} = q(x)$$

Deflexões em vigas

Boundary Conditions and Continuity Conditions

TABLE 7.2 Continuity Conditions*

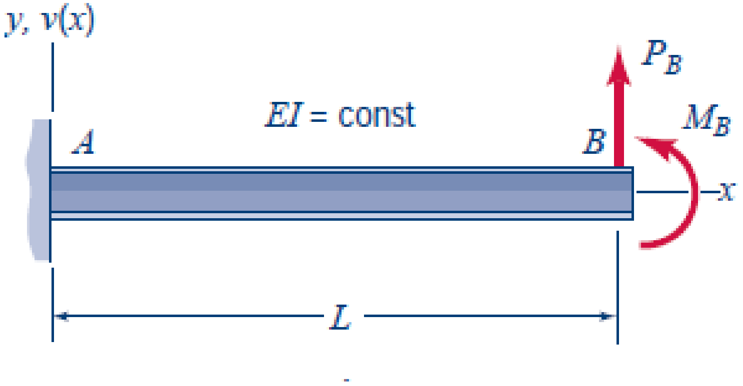
	Type	Symbol	2nd Order	4th Order
CC	Roller		$v_1 = v_2 = 0$ $v'_1 = v'_2$	$v_1 = v_2 = 0$ $v'_1 = v'_2$ $M_1 = M_2$
	Discontinuity in load function		$v_1 = v_2$ $v'_1 = v'_2$	$v_1 = v_2$ $v'_1 = v'_2$ $V_1 = V_2$ $M_1 = M_2$
	Concentrated force		$v_1 = v_2$ $v'_1 = v'_2$	$v_1 = v_2, \quad v'_1 = v'_2$ $V_2 - V_1 = P_0$ $M_1 = M_2$
	Concentrated couple		$v_1 = v_2$ $v'_1 = v'_2$	$v_1 = v_2, \quad v'_1 - v'_2$ $V_1 = V_2$ $M_2 - M_1 = -M_0$
	Pin, with force		$v_1 = v_2$	$v_1 = v_2$ $V_2 - V_1 = P_0$ $M_1 = M_2 = 0$

*The displacement (v) and slope (v') continuity conditions that are listed in Table 7.2 are obtained by inspection, that is, by simply looking at the figures in the "Symbol" column. The continuity conditions on shear force (V) and bending moment (M) are obtained by taking a local free-body diagram of the "joint" that is common to beam segments (1) and (2).

$$\mathbf{E}\boldsymbol{I}\,\frac{d^4 v(x)}{dx^4} = \mathbf{q}(x)$$

Deflexões em vigas

Exemplo 1



$$\frac{M(x)}{EI} = \frac{d^2v(x)}{dx^2}$$