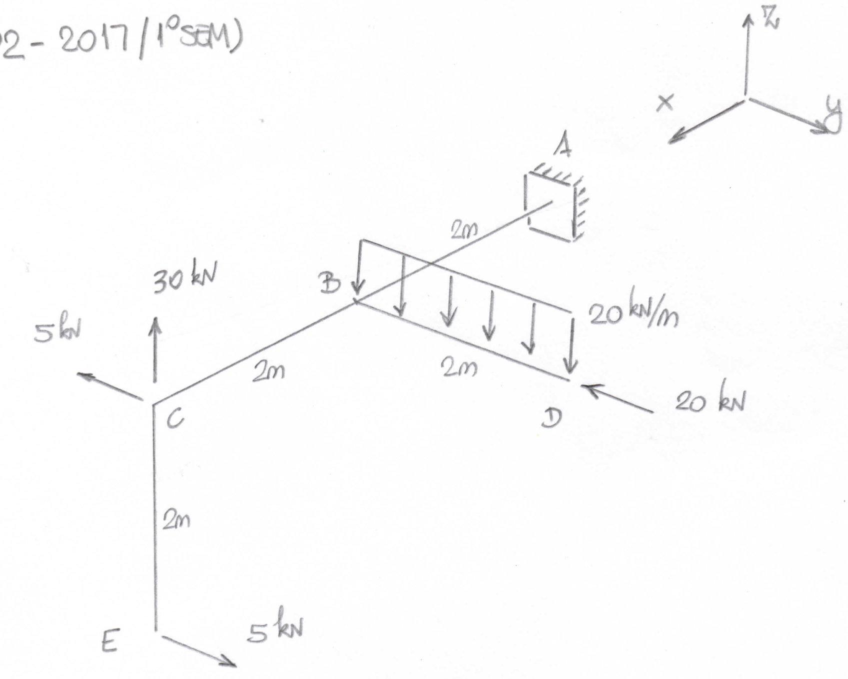
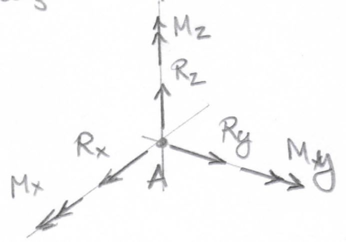
Q1 (P2 - 2017/1°SEM)



Reações do apoio:



$\sum F_x = 0: \; R_x = 0$

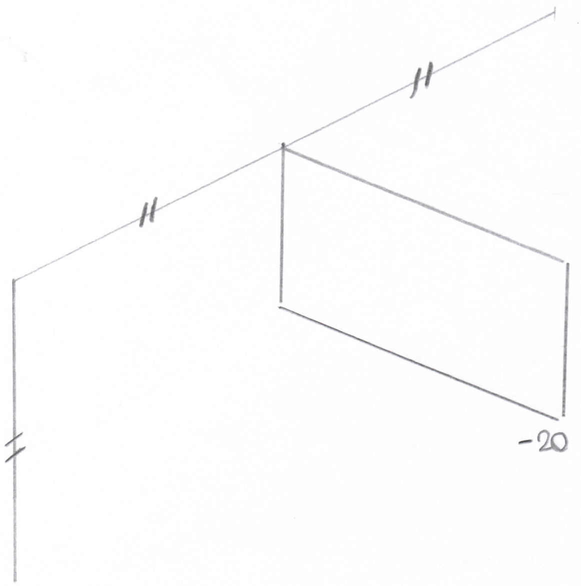
$\sum F_y = 0: \; R_y - 20 - 5 + 5 = 0 \Rightarrow R_y = 20\,kN$

$\sum F_z = 0: \; R_z + 30 - 20.2 = 0 \Rightarrow R_z = 10\,kN$

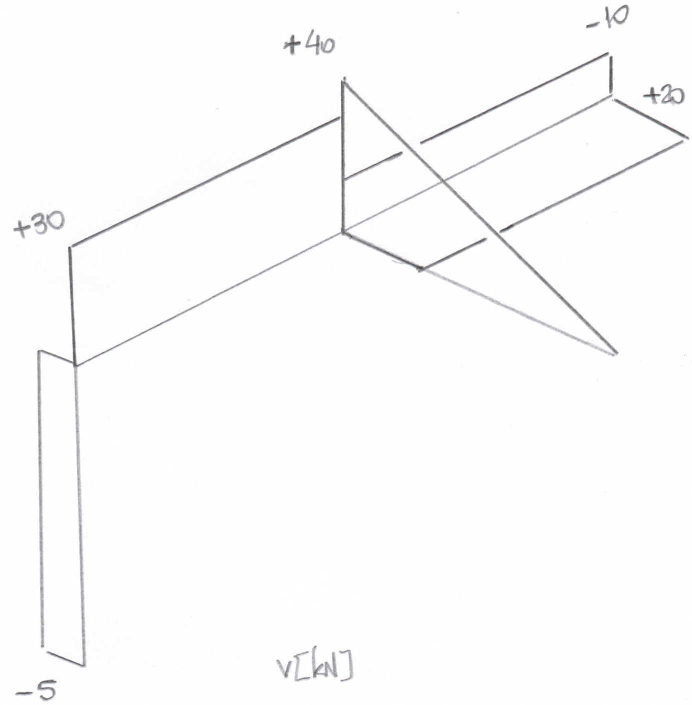
$\sum M_{x,A} = 0: \; M_x - 20.2.1 + 5.2 = 0 \Rightarrow M_x = 30\,kNm$

$\sum M_{y,A} = 0: \; M_y + 20.2.2 - 30.4 = 0 \Rightarrow M_y = 40\,kN$

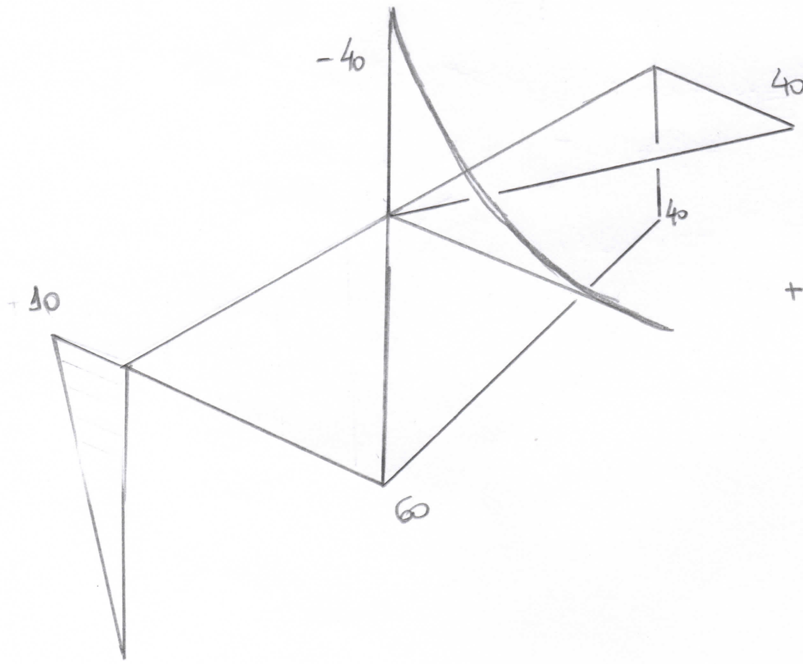
$\sum M_{z,A} = 0: \; M_z - 20.2 - 5.4 + 5.4 = 0 \Rightarrow M_z = 40\,kN$



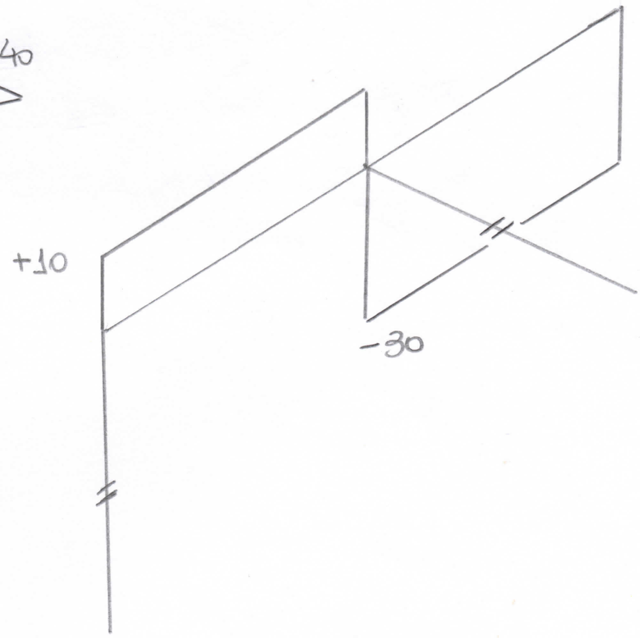
N[kN]



V[kN]



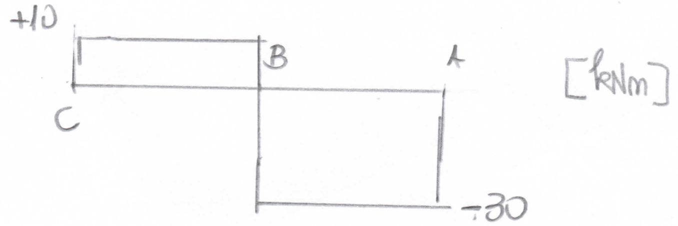
M[kNm]



T[kNm]

Dimensionar a barra AC considerando: (cheia)

- $G = 70\,GPa$
- $\tau_R = 160\,MPa$
- $S = 2$
- $\theta_{máx,c} = 0{,}03\,rad$.



$l_{AB} = l_{BC} = l = 2m$

* 1º critério:

Como $|T_{AB}| > |T_{BC}| \Rightarrow |\tau_{AB}| > |\tau_{BC}|$

$$|\tau_{AB}| \leq \bar{\tau} = \frac{\tau_R}{S} \Rightarrow \frac{|T_{AB}| \cdot R}{\frac{\pi R^4}{2}} \leq \frac{\tau_R}{S} \Rightarrow \frac{2\,|T_{AB}|}{\pi R^3} \leq \frac{\tau_R}{S}$$

$$R \geq \sqrt[3]{\frac{2|T_{AB}|S}{\pi \tau_R}} \Rightarrow R \geq \sqrt[3]{\frac{2 \cdot 30 \cdot 10^3 \cdot 2}{\pi\, 160 \cdot 10^6}}$$

$\therefore R \geq 0{,}062m$

* 2º critério:

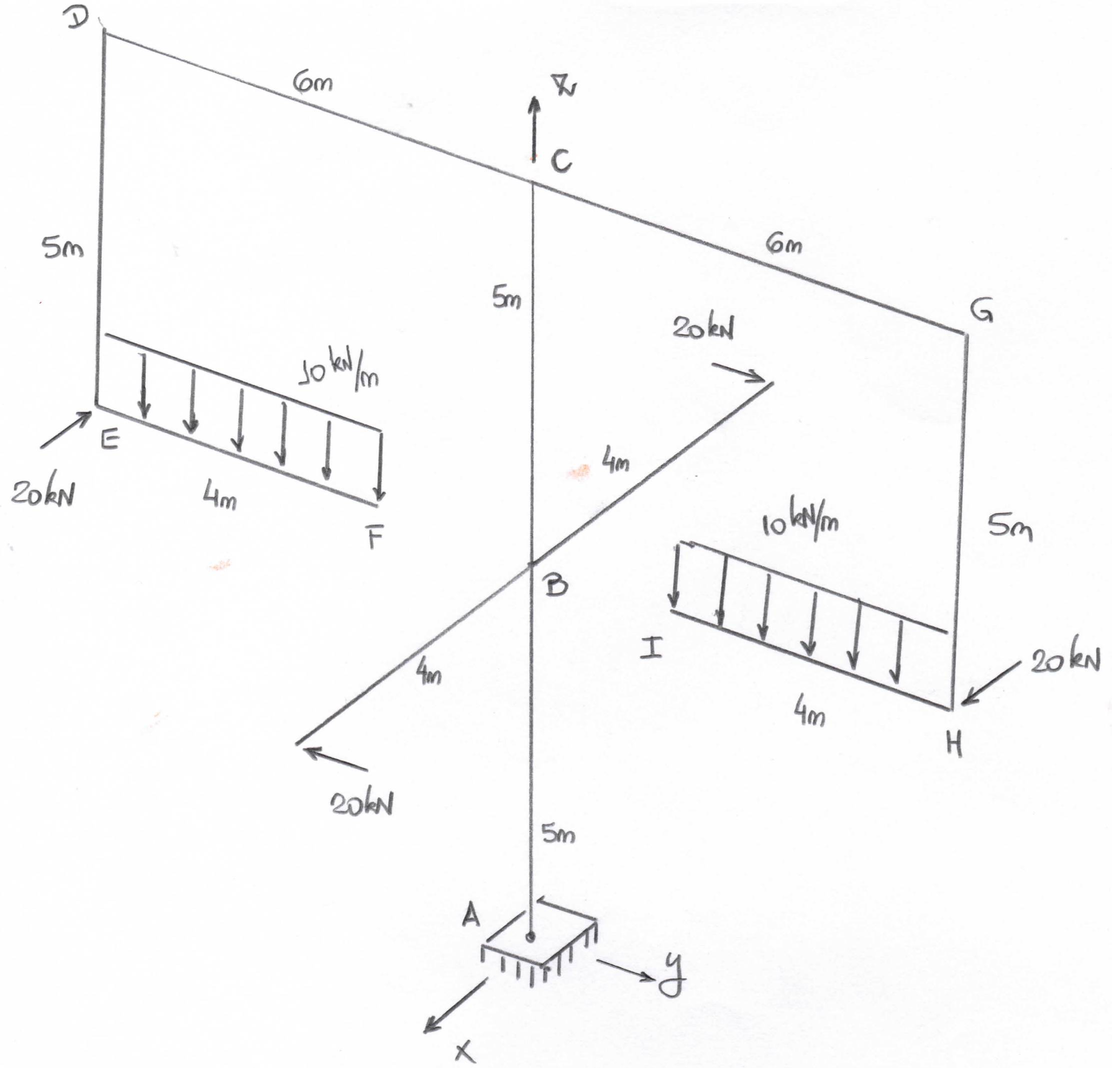
$$\theta_c \leq \theta_{máx,c} \Rightarrow |\theta_{AB} + \theta_{BC}| \leq \theta_{máx,c}$$

$$\left| \frac{T_{AB}\, l_{AB}}{GJ} + \frac{T_{BC}\, l_{BC}}{GJ} \right| \leq \theta_{máx,c} \Rightarrow \frac{l}{G} \cdot \frac{2}{\pi R^4} |T_{AB} + T_{BC}| \leq \theta_{máx,c}$$

$$R \geq \sqrt[4]{\frac{2l\,|T_{AB} + T_{BC}|}{\pi G \theta_{máx,c}}} \Rightarrow R \geq \sqrt[4]{\frac{2 \cdot l \cdot |-30 \cdot 10^3 + 10 \cdot 10^3|}{\pi\, 70 \cdot 10^9 \cdot 0{,}03}}$$

$\therefore R \geq 0{,}059m$

Logo $R \geq 0{,}062m$ (ou $6{,}2cm$).

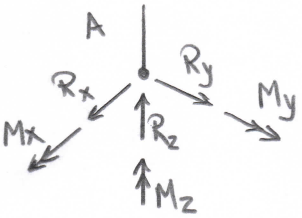


Reações de apoio:

$$\sum F_x = 0:\ R_x + 20 - 20 = 0 \Rightarrow \boxed{R_x = 0}$$

$$\sum F_y = 0:\ R_y - 20 + 20 = 0 \Rightarrow \boxed{R_y = 0}$$

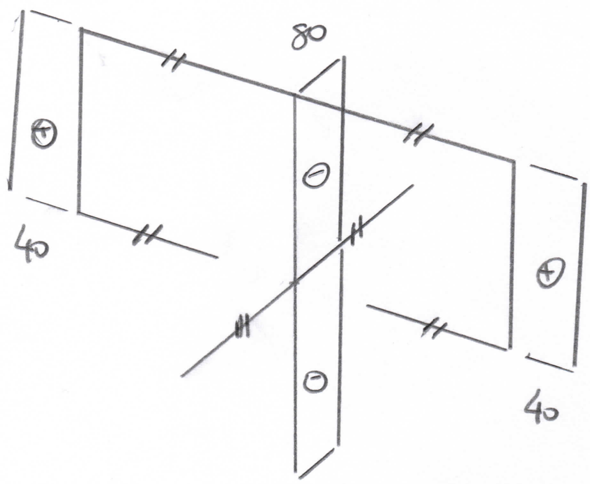
$$\sum F_z = 0:\ R_z - 10 \cdot 4 - 10 \cdot 4 = 0 \Rightarrow \boxed{R_z = 80\,kN}$$



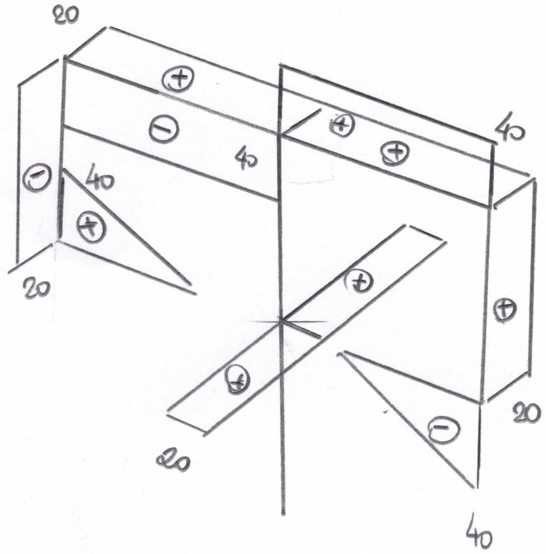
$$\sum M_{x,A} = 0:\ M_x + 20 \cdot 5 - 20 \cdot 5 + 40 \cdot 4 - 40 \cdot 4 = 0 \Rightarrow \boxed{M_x = 0}$$

$$\sum M_{y,A} = 0:\ M_y + 20 \cdot 5 - 20 \cdot 5 = 0 \Rightarrow \boxed{M_y = 0}$$

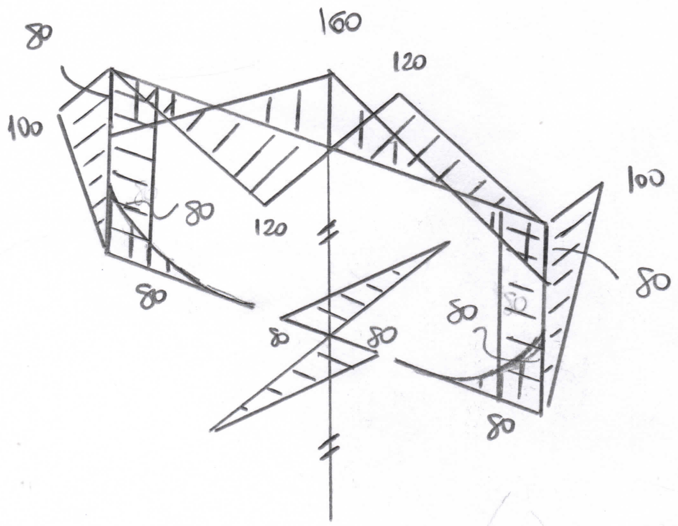
$$\sum M_{z,A} = 0:\ M_z - 20 \cdot 6 - 20 \cdot 4 - 20 \cdot 6 - 20 \cdot 4 = 0 \Rightarrow \boxed{M_z = 400\,kNm}$$



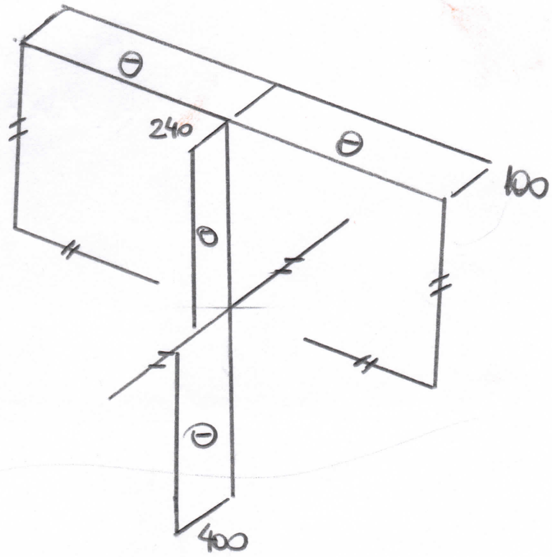
N [kN]



V [kN]



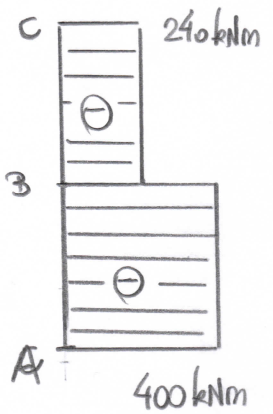
M_f [kNm]



T [kNm]

① Dimensionamento da barra AC: Determinar o diâmetro da barra AC (circular, cheia) sendo $G = 70 GPa$, $\tau_R = 150 MPa$ e $\theta_{máx}(C) = 0{,}05 rad$, com $s = 1{,}5$.

Diagrama de torção:



critérios:

① $\tau_{máx} \leq \bar{\tau}$ $\quad \tau_{máx} = máx(|\tau_{AB}|, |\tau_{BC}|)$

$$\frac{|T_{AB}|}{J} \cdot R \leq \frac{\tau_R}{s} \quad , \quad J = \frac{\pi R^4}{2}$$

$$\frac{2|T_{AB}|}{\pi R^3} \leq \frac{\tau_R}{s} \Rightarrow R \geqslant \sqrt[3]{\frac{2|T_{AB}|s}{\pi \tau_R}}$$

$$R \geqslant \sqrt[3]{\frac{2 \cdot 400 \cdot 10^3 \cdot 1{,}5}{\pi \cdot 150 \cdot 10^6}} \Rightarrow \underline{R \geqslant 0{,}137 m}$$

② $\theta_C \leq \theta_{máx}(C)$

$\theta_C = \theta_{AB} + \theta_{BC}$ (mesmo sinal, tratarei em módulo)

$$\frac{T_{AB}\, l_{AB}}{GJ} + \frac{T_{BC}\, l_{BC}}{GJ} \leq \theta_{máx} \quad , \quad l_{AB} = l_{BC} = 5m$$

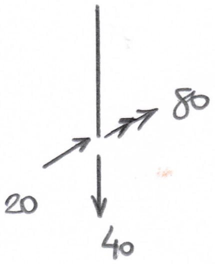
$$\frac{2|T_{AB} + T_{BC}| \cdot l}{\pi G R^4} \leq \theta_{máx}$$

$$R \geqslant \sqrt[4]{\frac{2|T_{AB} + T_{BC}| \cdot l}{\pi G \theta_{máx}}}$$

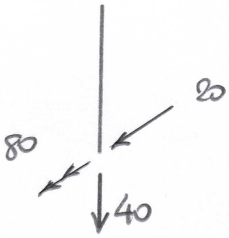
$$R \geqslant \sqrt[4]{\frac{2 \cdot 640 \cdot 10^3 \cdot 5}{\pi\, 70 \cdot 10^9 \cdot 0{,}05}} \Rightarrow \underline{R \geqslant 0{,}155 m}$$

Logo: $\underline{R_{min} = 15{,}5 cm}$ //

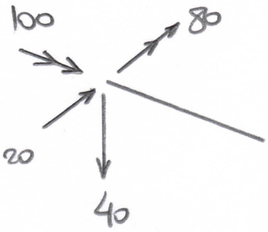
Barra DE



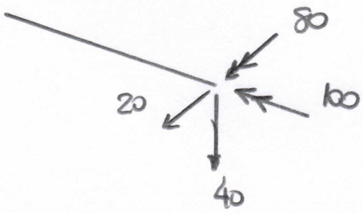
Barra GH



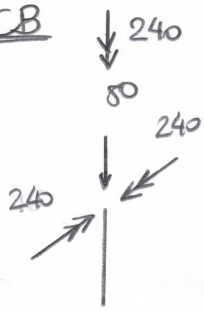
Barra CD



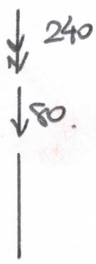
Barra CG



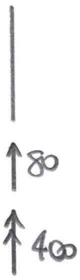
Barra CB



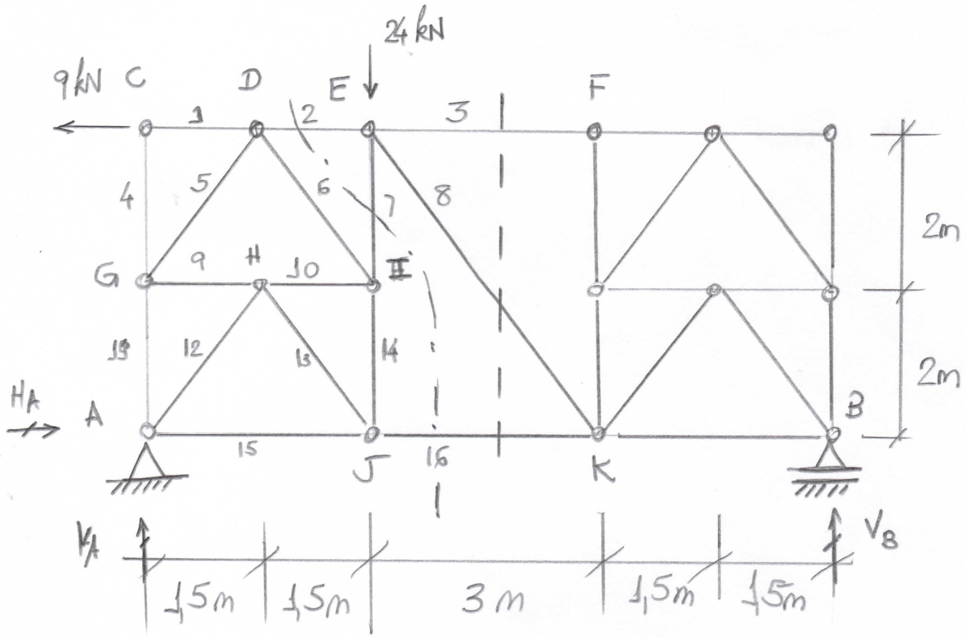
(cancela 20, 80, 100 acima)



Barra AB



Prova 1°sem/2016



Barra	N (kN)
2	-6
3	-3
7	-20
8	-5

① Reações de apoio

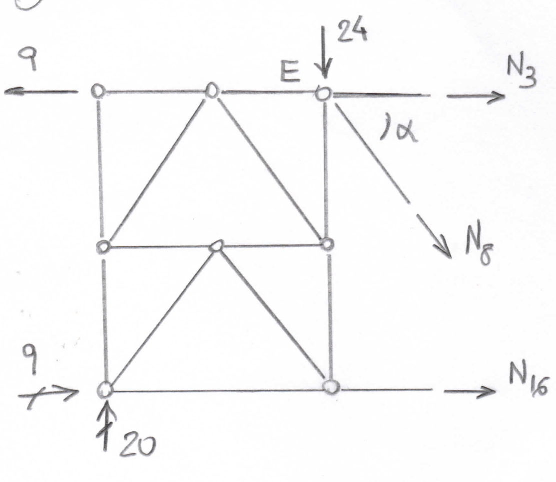
$\sum F_H = 0: \; H_A - 9 = 0 \Rightarrow \boxed{H_A = 9\,kN}$

$\sum F_V = 0: \; V_A + V_B = 24$

$\sum M_A = 0: \; 9.4 - 24.3 + V_B . 9 = 0 \Rightarrow \; 9V_B = 72 - 36 = 36 \Rightarrow \boxed{V_B = 4\,kN}$

$V_A = 24 - V_B \Rightarrow \boxed{V_A = 20\,kN}$

② corte:



$\sum F_H = 0: \; \cancel{9} - \cancel{9} + N_3 + N_8 \cos\alpha + N_{16} = 0$

$\sum F_V = 0: \; 20 - 24 - N_8 \operatorname{sen}\alpha = 0$

$N_8 = \frac{-4}{\operatorname{sen}\alpha} = \frac{-4}{4/5} \Rightarrow \boxed{N_8 = -5\,kN}$

$\sum M_E = 0: \; 4.N_{16} + 9.4 - 20.3 = 0$

$4N_{16} = 60 - 36 = 24$

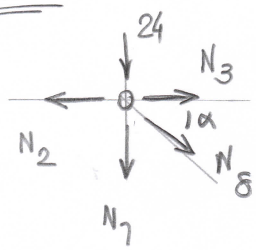
$N_{16} = 6\,kN$

$\operatorname{sen}\alpha = 4/5$

$\cos\alpha = 3/5$

$N_3 = -N_{16} - N_8 \cos\alpha = -6 - (-\cancel{5}) . \frac{3}{\cancel{5}} \Rightarrow \boxed{N_B = -3\,kN}$

Nó E



$$\sum F_H = 0: \quad -N_2 + N_3 + N_8 \cos\alpha = 0$$

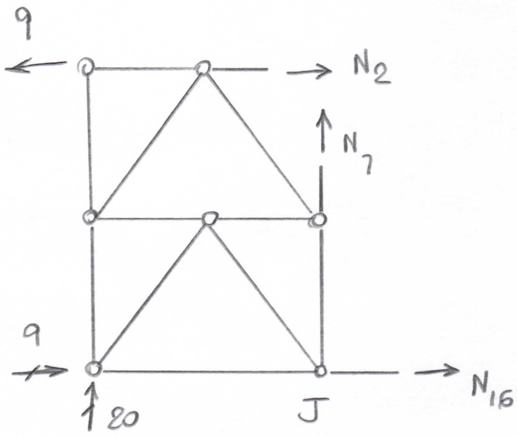
$$N_2 = (-3) + (-\cancel{5}) \cdot 3/\cancel{5} \Rightarrow \boxed{N_2 = -6\,kN}$$

$$\sum F_V = 0: \quad -24 - N_7 - N_8 \operatorname{sen}\alpha = 0$$

$$N_7 = -24 - (-5) \cdot 4/5 = -24 + 4$$

$$\boxed{N_7 = -20\,kN}$$

③ corte alternativo:



$$\sum F_H = 0: \quad -\cancel{9} + \cancel{9} + N_2 + N_{16} = 0$$

$$N_2 = -N_{16}$$

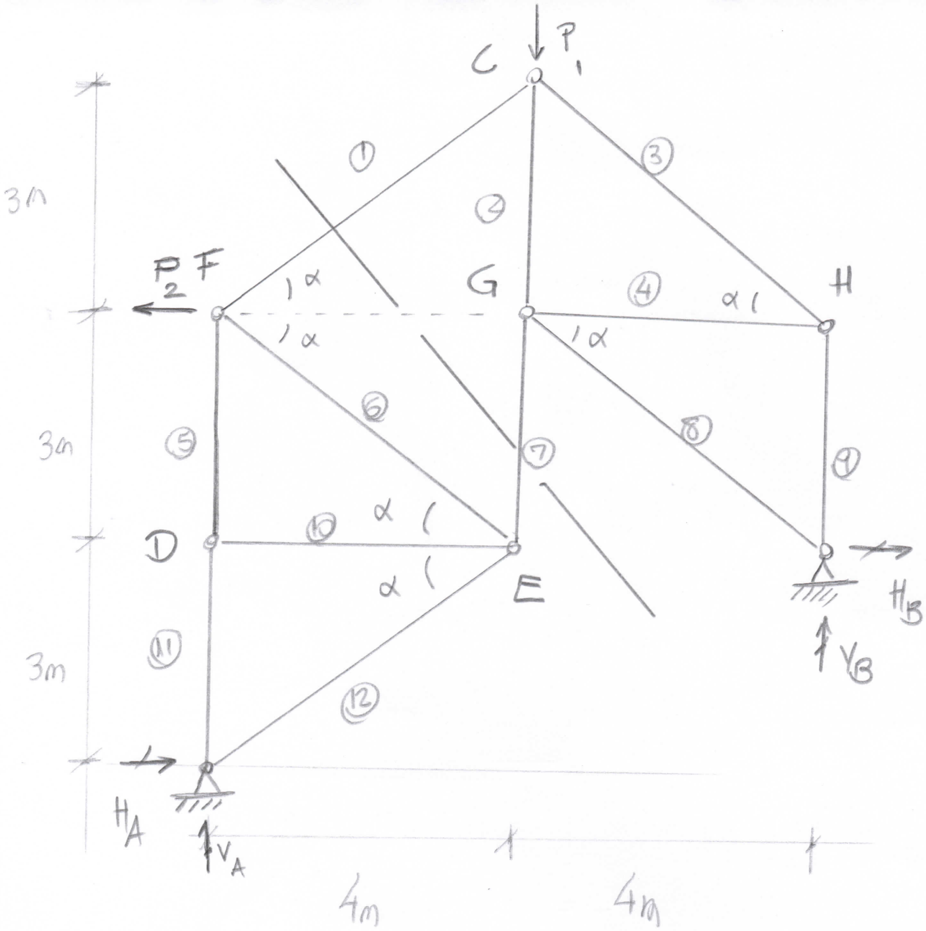
$$\sum F_V = 0: \quad 20 + N_7 = 0 \Rightarrow \boxed{N_7 = -20\,kN}$$

$$\sum M_J = 0: \quad -N_2 \cdot 4 + 9 \cdot 4 - 20 \cdot 3 = 0$$

$$4N_2 = -60 + 36 = -24$$

$$\boxed{N_2 = -6\,kN}$$

31



$P_2 = 3000$ kgf
$P_1 = 6000$ kgf

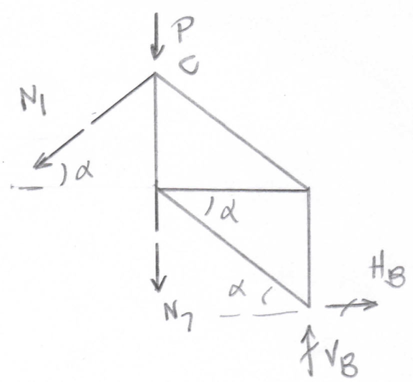
$sen\alpha = 3/5$
$\cos\alpha = 4/5$

Equilíbrio

$\sum F_H = 0: \; H_A + H_B = P_2$

$\sum F_V = 0: \; V_A + V_B = P_1$

$\sum M_A = 0: \; -P_1 \cdot 4 + V_B \cdot 8 - H_B \cdot 3 + P_2 \cdot 6 = 0$



$\sum F_H = 0: \; -N_1 \cos\alpha + H_B = 0$

$\therefore N_1 = H_B / \cos\alpha$

$\sum F_V = 0: \; -P_1 - N_7 + V_B - N_1 sen\alpha = 0$

$N_7 = V_B - P_1 - N_1 sen\alpha$

$\sum M_C = 0: \; V_B \cdot 4 + H_B \cdot 6 = 0$

$$\begin{cases} 6H_B + 4V_B = 0 \\ -3H_B + 8V_B = 4P_1 - 6P_2 \quad (\times 2) \end{cases}$$

$20V_B = 8P_1 - 12P_2 \quad V_B = 600$ kgf

$H_B = -\frac{4}{6} V_B \Rightarrow H_B = -400$ kgf

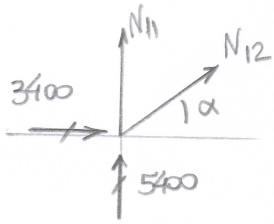
$H_A = P_2 - H_B \Rightarrow H_A = 3400$ kgf

$V_A = P_1 - V_B \Rightarrow V_A = 5400$ kgf

$N_1 = -500 \text{ kgf}$

$N_7 = -5100 \text{ kgf}$

Nó A:



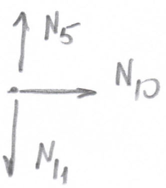
$\sum F_H = 0: \ 3400 + N_{12}\cos\alpha = 0$

$N_{12} = -4250 \text{ kgf}$

$\sum F_V = 0: \ 5400 + N_{11} + N_{12}\,\text{sen}\,\alpha = 0$

$N_{11} = -2850 \text{ kgf}$

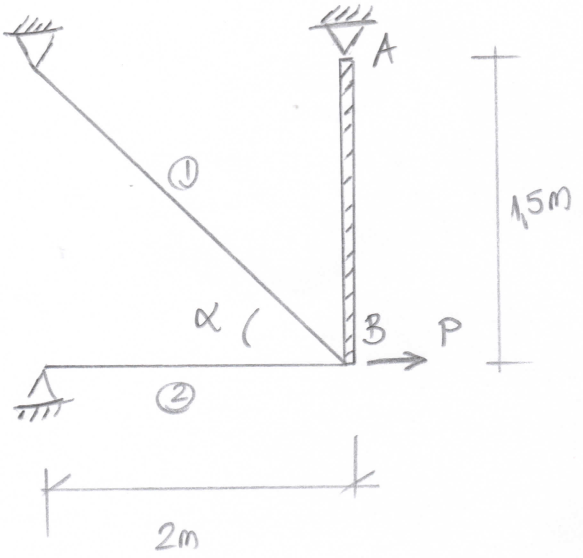
Nó D:



$N_{10} = 0$

$N_5 = N_{11} = -2850 \text{ kgf}$

7



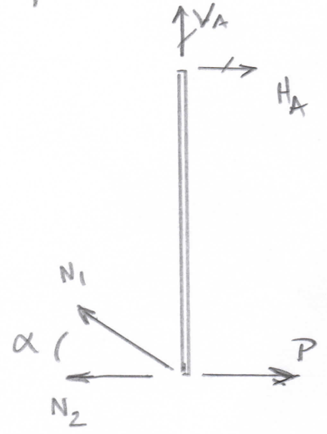
$P = 1890\ kgf$

$EA = 10^4\ kgf.$

$sen\,\alpha = \frac{1,5}{2,5} = 0,6$

$\cos\alpha = 0,8$

Equilíbrio



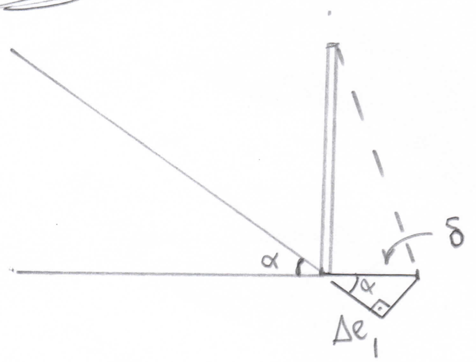
$\sum F_H = 0: \ H_A - N_1\cos\alpha - N_2 + P = 0$

$\sum F_V = 0: \ V_A + N_1\,sen\,\alpha = 0$

$\sum M_A = 0: \ P.1,5 - N_2.1,5 - N_1\cos\alpha.1,5 = 0$

$N_1\cos\alpha + N_2 = P \Rightarrow \underline{0,8N_1 + N_2 = P}$

Williot



$\cos\alpha = \frac{\Delta l_1}{\delta} \Rightarrow \delta = \frac{\Delta l_1}{\cos\alpha}$

$\Delta l_2 = \delta$

Logo: $\frac{\Delta l_1}{\cos\alpha} = \Delta l_2$. $\frac{N_1 l_1}{\cancel{EA}} \cdot \frac{1}{\cos\alpha} = \frac{N_2 l_2}{\cancel{EA}}$

$l_1 = 2,5m$

$l_2 = 2m$

$\frac{N_1 \cdot 2,5}{0,8} = 2N_2 \Rightarrow N_1 = 0,64N_2$

Assim: $0,8 \cdot 0,64N_2 + N_2 = P$

$N_2 = 1250\ kgf$ e $N = 800\ kgf.$