

Universidade de São Paulo  
Escola Superior de Agricultura "Luiz de Queiroz"  
Departamento de Ciências Exatas  
LCE 0220 - Cálculo II

Professoras: Renata Alcarde Sermarini e Cristiane Mariana Rodrigues da Silva

Lista de Exercício: Integral por Frações Parciais

1. Nos exercícios a seguir, calcular a integral indefinida:

(a)  $\int \frac{2x^3}{x^2+x} dx$

(b)  $\int \frac{2x+1}{2x^2+3x-2} dx$

(c)  $\int \frac{x-1}{x^3+x^2-4x-4} dx$

(d)  $\int \frac{3x^2}{2x^3-x^2-2x+1} dx$

(e)  $\int \frac{x^2+5x+4}{x^2-2x+1} dx$

(f)  $\int \frac{x-1}{(x-2)^2(x-3)^2} dx$

(g)  $\int \frac{(x^2+1)}{x^4-7x^3+18x^2-20x+8} dx$

(h)  $\int \frac{1}{x^3-4x^2} dx$

(i)  $\int \frac{x^3+2x^2+4}{2x^2+2} dx$

(j)  $\int \frac{5}{x^3+4x} dx$

(k)  $\int \frac{3x-1}{x^2-x+1} dx$

(l)  $\int \frac{1}{x^3+8} dx$

(m)  $\int \frac{x-1}{(x^2+2x+3)^2} dx$

(n)  $\int \frac{1}{x(x^2-x+1)^2} dx$

(o)  $\int \frac{4x^4}{x^4-x^3-6x^2+4x+8} dx$

(p)  $\int \frac{x^2}{3x^2-\frac{1}{2}x-\frac{1}{2}} dx$

(q)  $\int \frac{1}{x^3+9x} dx$

(r)  $\int \frac{1}{(x^2+1)(x^2+4)} dx$

(s)  $\int \frac{x^3+x^2+2x+1}{x^3-1} dx$

(t)  $\int \frac{x^3}{(x^2+2)^2} dx$

(u)  $\int \frac{1}{x^4-3x^3+3x^2-x} dx$

(v)  $\int \frac{x}{(x-1)^2(x+1)^2} dx$

(w)  $\int \frac{x^2+2x-1}{(x-1)^2(x^2+1)} dx$

2. Calcular a área da região sob o gráfico de  $y = \frac{1}{x^2+2x+5}$ , de  $x = -2$  até  $x = 2$ .

Respostas:

1. Nos exercícios a seguir, calcular a integral indefinida:

(a)  $x^2 - 2x + 2 \ln|x+1| + c$

(b)  $\frac{2}{5} \ln|x-1/2| + \frac{3}{5} \ln|x+2| + c$

(c)  $\frac{1}{12} \ln|x-2| + \frac{2}{3} \ln|x+1| - \frac{3}{4} \ln|x+2| + c$

(d)  $\frac{3}{2} \ln|x-1| + \frac{1}{2} \ln|x+1| - \frac{1}{2} \ln|x-1/2| + c$

(e)  $x + 7 \ln|x-1| - \frac{10}{x-1} + c$

(f)  $3 \ln \left| \frac{x-2}{x-3} \right| - \frac{1}{x-2} - \frac{2}{x-3} + c$

(g)  $\ln \left( \frac{x-2}{x-1} \right)^2 + \frac{1}{x-2} - \frac{5}{2(x-2)^2} + c$

(h)  $\frac{1}{16} \ln \left| \frac{x-4}{x} \right| + \frac{1}{4x} + c$

$$(i) \frac{x^2}{9} + x - \frac{1}{4} \ln(x^2 + 1) + \operatorname{arctg}(x) + c$$

$$(j) \frac{5}{4} \left[ \ln|x| - \frac{1}{2} \ln(x^2 + 4) \right] + c$$

$$(k) \frac{3}{2} \ln|x^2 - x + 1| + \frac{1}{\sqrt{3}} \operatorname{arctg} \left( \frac{2x-1}{\sqrt{3}} \right) + c$$

$$(l) \frac{1}{12} \ln|x+2| - \frac{1}{24} \ln|x^2 - 2x + 4| + \frac{1}{4\sqrt{3}} \operatorname{arctg} \left( \frac{x-1}{\sqrt{3}} \right) + c$$

$$(m) \frac{-x-2}{2(x^2+2x+3)} - \frac{1}{2\sqrt{2}} \operatorname{arctg} \left( \frac{x+1}{\sqrt{2}} \right) + c$$

$$(n) \ln|x| - \frac{1}{2} \ln|x^2 - x + 1| + \frac{5\sqrt{3}}{9} \operatorname{arctg} \left( \frac{2x-1}{\sqrt{3}} \right) + \frac{x+1}{3(x^2-x+1)} + c$$

$$(o) 4x + \frac{4}{9} \ln|x+1| - 4 \ln|x+2| + \frac{68}{9} \ln|x-2| - \frac{16}{3(x-2)} + c$$

$$(p) \frac{1}{3}x + \frac{1}{10} \ln|x-1/2| - \frac{2}{45} \ln|x+1/3| + c$$

$$(q) \frac{1}{9} \left[ \ln|x| - \frac{1}{2} \ln(x^2 + 9) \right] + c$$

$$(r) \frac{1}{3} \operatorname{arctg}(x) - \frac{1}{6} \operatorname{arctg}(x/2) + c$$

$$(s) x + \frac{5}{3} \ln|x-1| - \frac{1}{3} \ln|x^2 + x + 1| + c$$

$$(t) \frac{1}{2} \ln(x^2 + 2) + \frac{1}{x^2 + 2} + c$$

$$(u) \ln \left| \frac{x-1}{x} \right| + \frac{1}{x-1} - \frac{1}{2(x-1)^2} + c$$

$$(v) \frac{1}{4} \left( \frac{1}{x+1} - \frac{1}{x-1} \right) + c$$

$$(w) \ln|x-1| - \frac{1}{x-1} - \frac{1}{2} \ln(x^2 + 1) - \operatorname{arctg}(x) + c$$

$$2. \frac{1}{2} [\operatorname{arctg}(3/2) - \operatorname{arctg}(-1/2)] u.a.$$

FLEMMING, D.M.; GONÇALVES, M.B. **Cálculo A: Funções, limites, derivação e integração**. 6<sup>a</sup> ed. São Paulo. Pearson, 2012. 448p.