

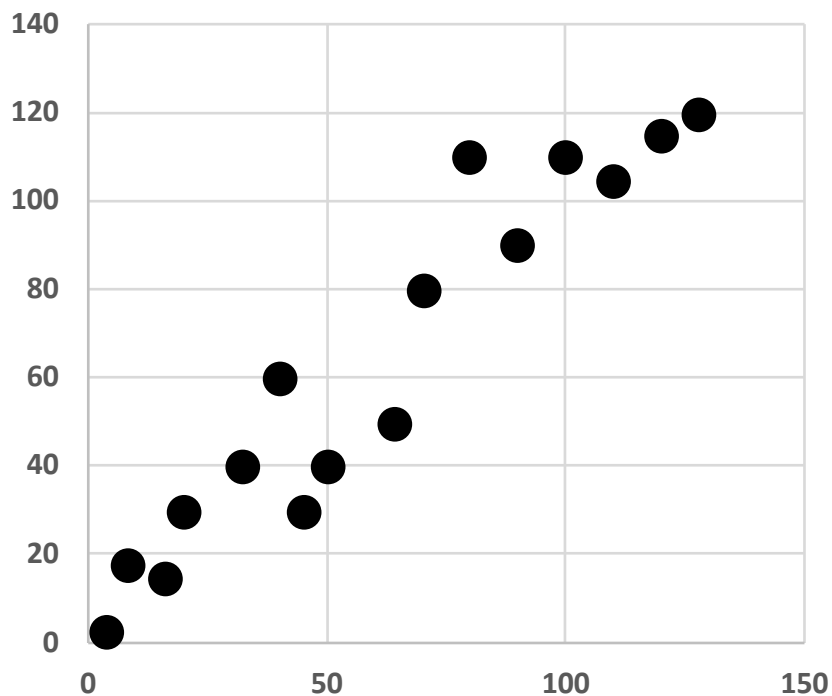
Movimento Unidimensional e Método dos Mínimos Quadrados

Prof. Rafael Guido

rvcguido@ifsc.usp.br

Relações lineares entre dados experimentais

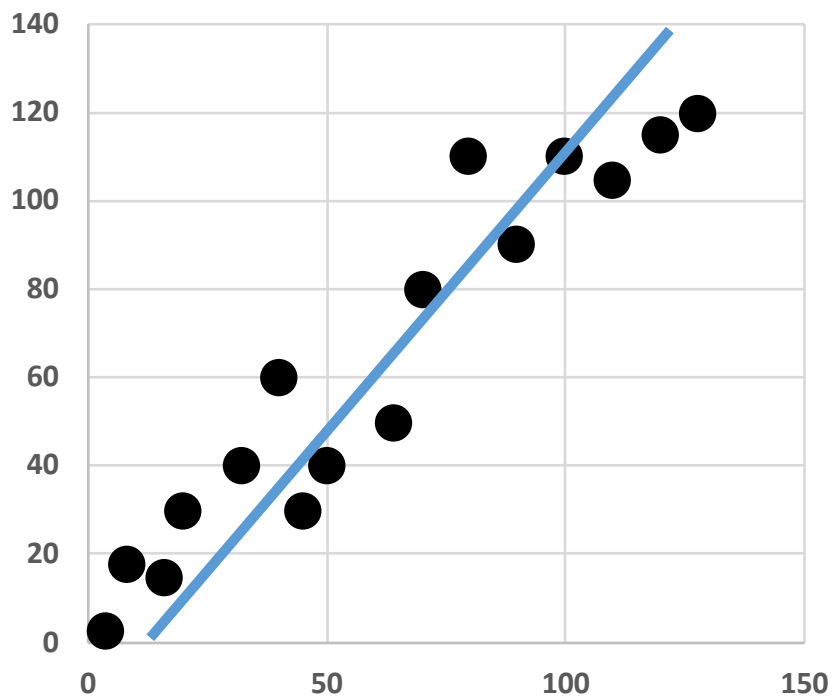
- *Muitas vezes, a relação encontrada experimentalmente entre duas grandezas físicas é linear ou pode ser linearizada.*



$$y = ax + b$$

Relações lineares entre dados experimentais

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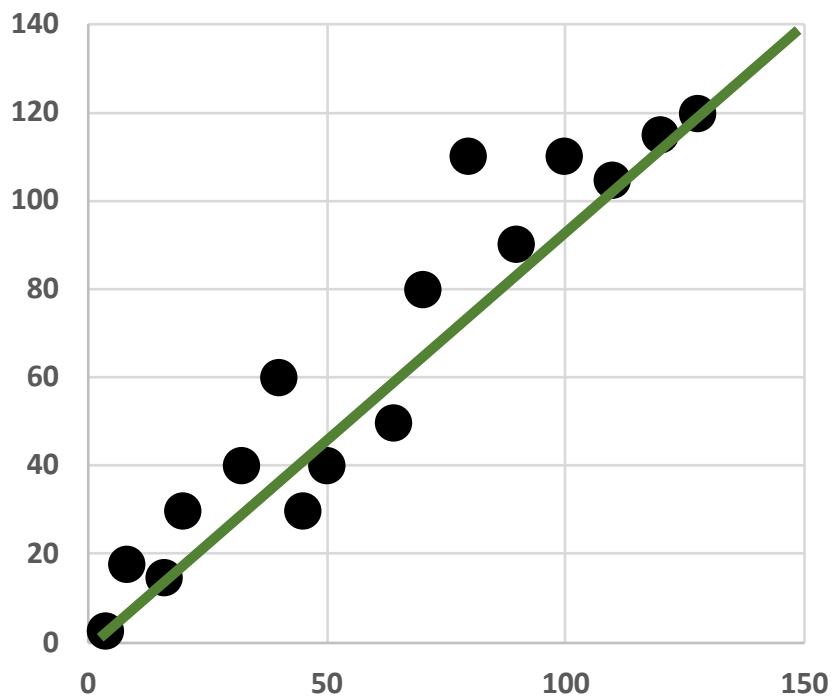


Método gráfico

$$y = ax + b$$

Relações lineares entre dados experimentais

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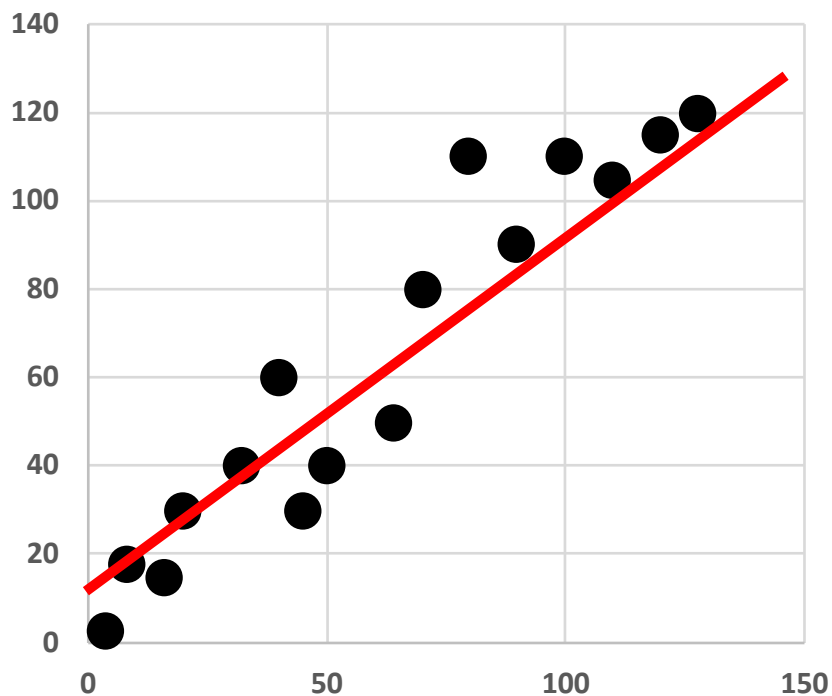


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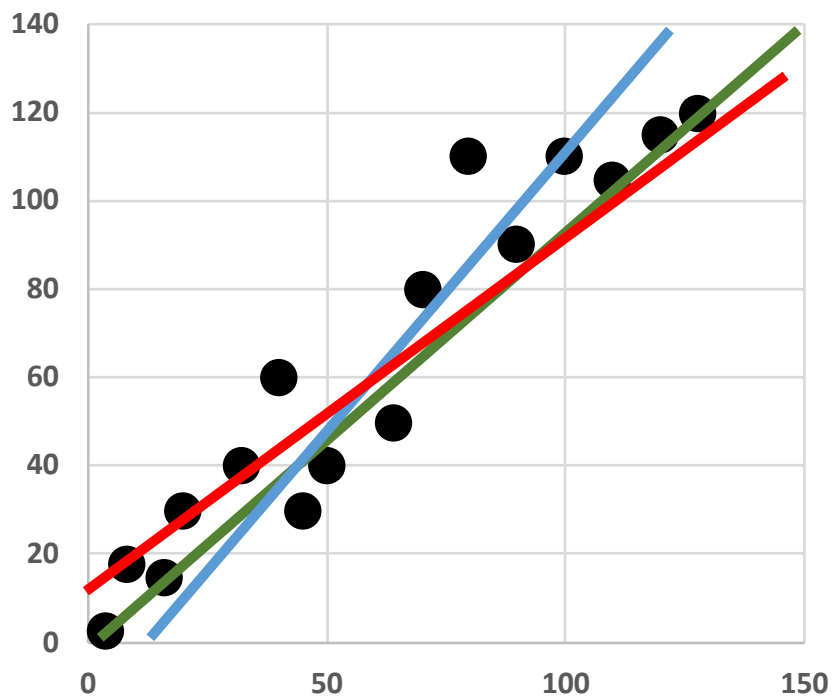


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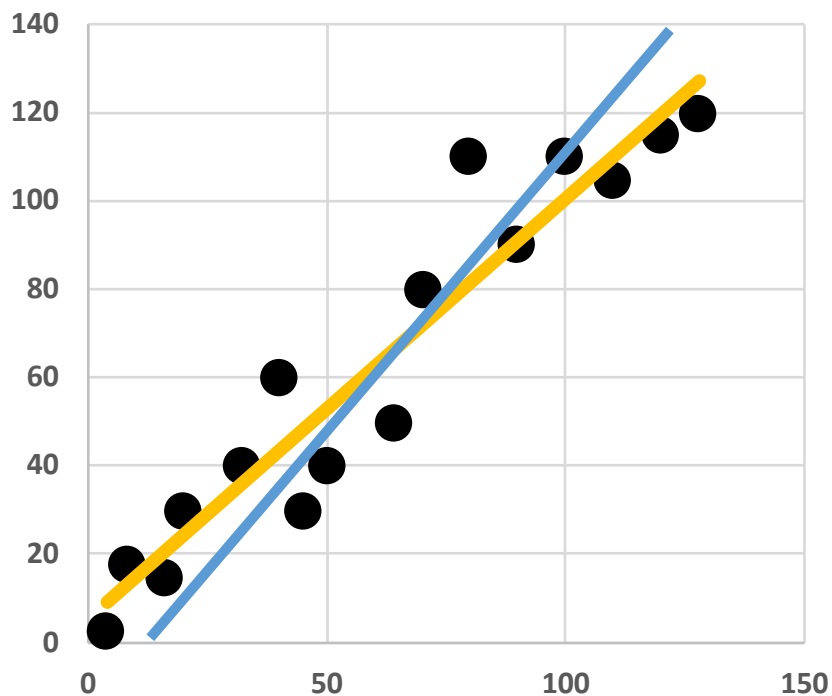
Método gráfico

Qual a melhor?

$$y = ax + b$$

Relações lineares entre dados experimentais

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Método gráfico (a olho)

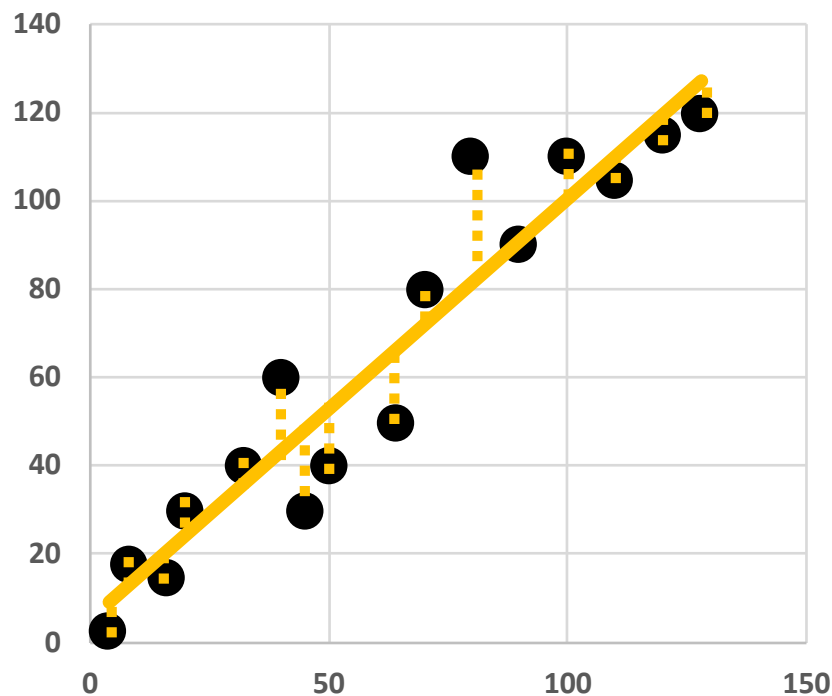
Método analítico (mínimos quadrados)

$$y = ax + b$$

$$\text{coeficiente angular: } a = \frac{\sum(x_i - \bar{x})y_i}{\sum(x_i - \bar{x})^2}$$

Relações lineares entre dados experimentais

- Muitas vezes, a relação encontrada experimentalmente entre duas grandezas físicas é linear ou pode ser linearizada.



Método gráfico (a olho)

Método analítico (mínimos quadrados)

$$y = ax + b$$

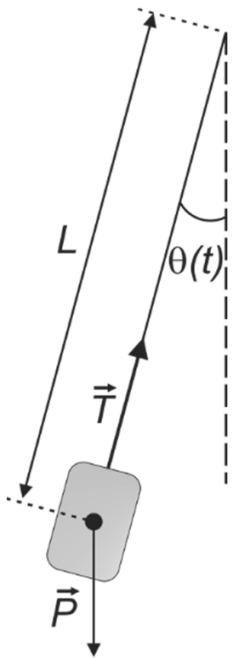
coeficiente angular: $a = \frac{\sum(x_i - \bar{x})y_i}{\sum(x_i - \bar{x})^2}$

dispersão média do ajuste: $\Delta y = \sqrt{\frac{\sum(ax_i + b - y_i)^2}{N-2}}$

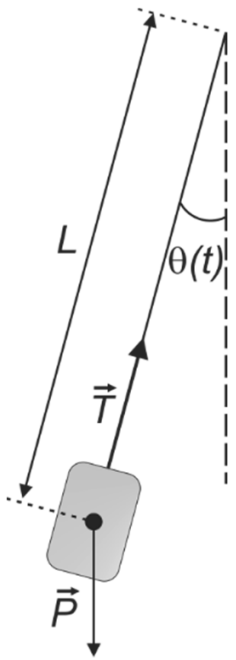
incerteza do coeficiente angular: $\Delta a = \frac{\Delta y}{\sqrt{\sum(x_i - \bar{x})^2}}$

incerteza do coeficiente linear: $\Delta b = \sqrt{\frac{\sum x_i^2}{N \sum(x_i - \bar{x})^2}} \Delta y$

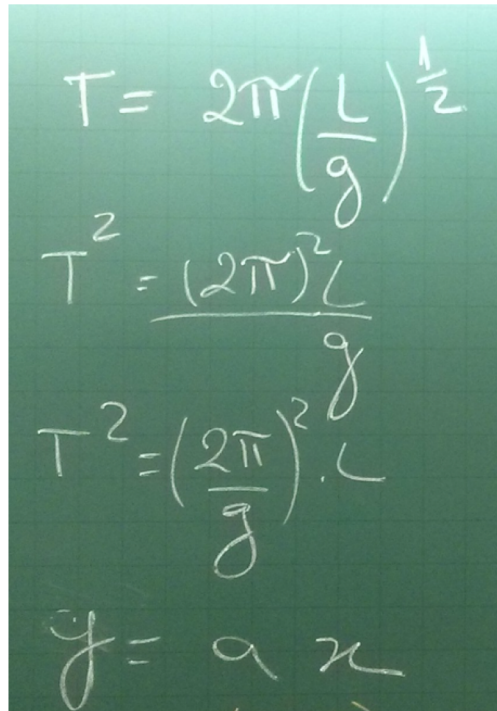
Experimental: Pêndulo simples



Experimental: Pêndulo simples



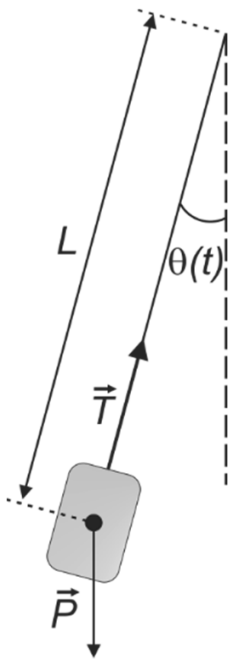
$$T = 2\pi \sqrt{\frac{L}{g}}$$



Handwritten equations on a green chalkboard background:

$$T = 2\pi \left(\frac{L}{g}\right)^{\frac{1}{2}}$$
$$T^2 = \frac{(2\pi)^2 L}{g}$$
$$T^2 = \left(\frac{2\pi}{g}\right)^2 \cdot L$$
$$g = a \cdot \pi$$

Experimental: Pêndulo simples



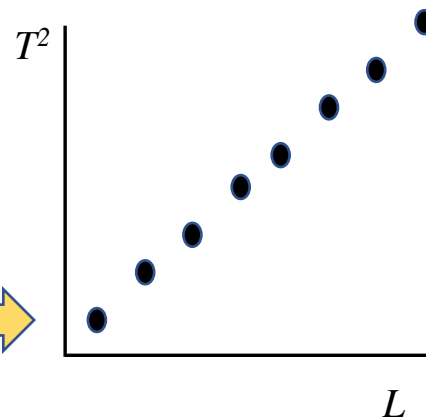
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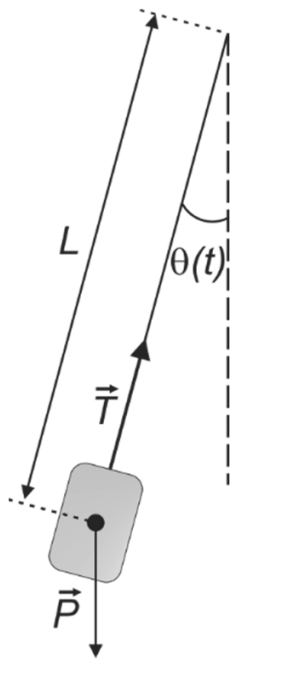
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Experimental: Pêndulo simples



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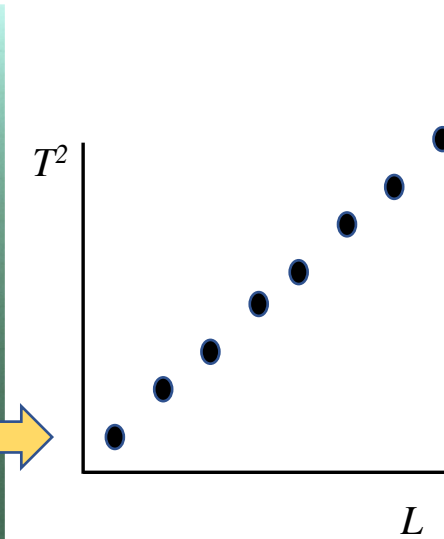
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$$y = a x$$

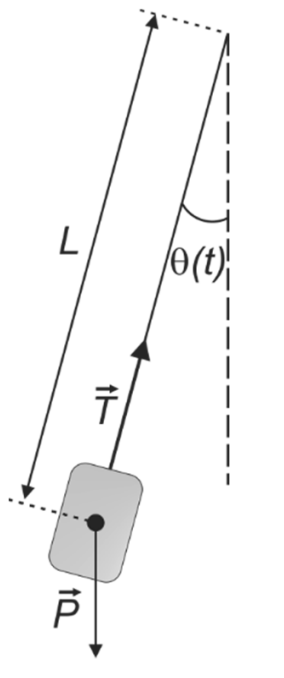
$$a = \frac{(2\pi)^2}{g} \Rightarrow$$



Método dos mínimos quadrados

$$g = \frac{(2\pi)^2}{a \pm \Delta a}$$

Experimental: Pêndulo simples



$$T = 2\pi \sqrt{\frac{L}{g}}$$

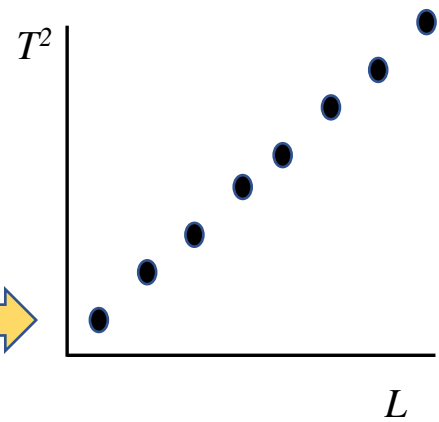
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$$g = a \cdot x$$

$$a = \frac{(2\pi)^2}{g} \Rightarrow$$



Método dos mínimos quadrados

$$g = \frac{(2\pi)^2}{a \pm \Delta a}$$

Exemplo

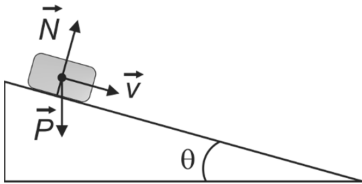
Tabela 1

L (m)	T_0 (s)	T_{10} (s)	T^2 (s ²)
2,25			
2,00			
1,75			
1,50			
1,25			
1,00			

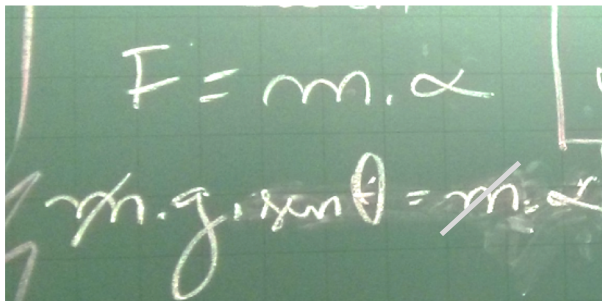
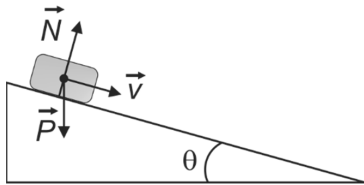
coeficiente angular: $a = \frac{\sum(x_i - \bar{x})y_i}{\sum(x_i - \bar{x})^2}$

incerteza do coeficiente angular: $\Delta a = \frac{\Delta y}{\sqrt{\sum(x_i - \bar{x})^2}}$

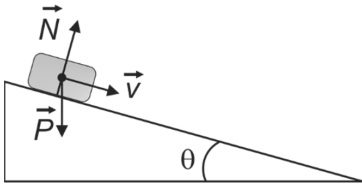
Experimental: Plano inclinado

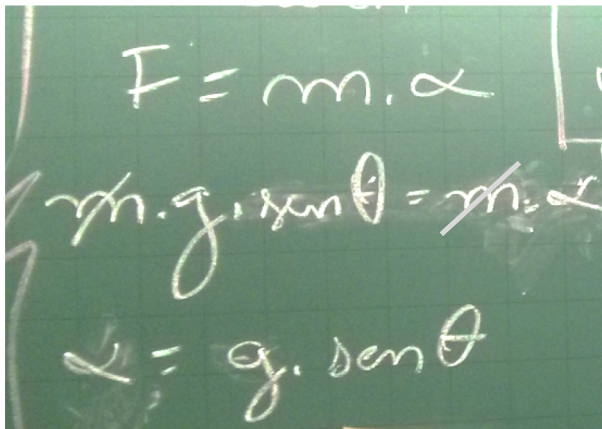


Experimental: Plano inclinado



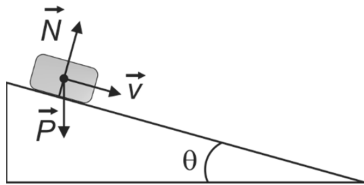
Experimental: Plano inclinado





$F = m \cdot a$
 $m \cdot g \cdot \sin \theta = m \cdot a$
 $a = g \cdot \sin \theta$

Plano inclinado

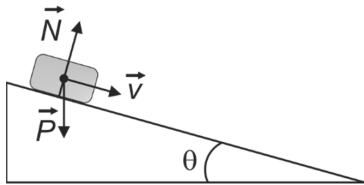


Handwritten equation on a chalkboard: $y = \sqrt{a}x + \frac{\alpha x^2}{2}$

Handwritten equations on a chalkboard:

$$F = m \cdot \alpha$$
$$m \cdot g \cdot \sin \theta = m \cdot \alpha$$
$$\alpha = g \cdot \sin \theta$$

Plano inclinado



$$F = m \cdot a$$
$$m \cdot g \cdot \sin \theta = m \cdot a$$
$$a = g \cdot \sin \theta$$

$$y = v_0 x + \frac{a x^2}{2}$$

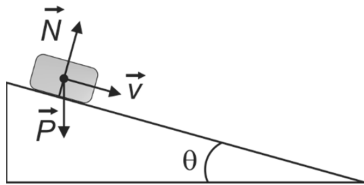
$$\frac{y}{x} = \frac{v_0 x}{x} + \frac{a x^2}{2x}$$

$$\frac{y}{x} = v_0 + \frac{a x}{2}$$



$$y = b + a x$$

Plano inclinado



$$F = m \cdot a$$

$$m \cdot g \cdot \sin \theta = m \cdot a$$

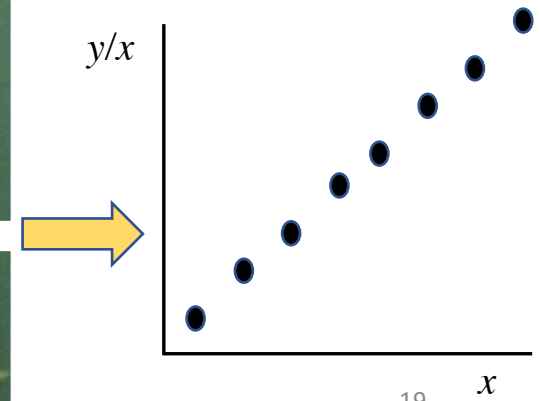
$$a = g \cdot \sin \theta$$

$$y = \sqrt{a} x + \frac{a}{2} x^2$$

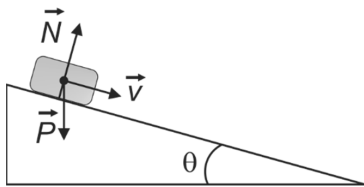
$$\frac{y}{x} = \frac{\sqrt{a} x}{x} + \frac{a x^2}{2x}$$

$$\frac{y}{x} = \sqrt{a} + \frac{a}{2} x$$

$$y = b + ax$$



Plano inclinado



$$F = m \cdot a$$

$$m \cdot g \cdot \sin \theta = m \cdot a$$

$$a = g \cdot \sin \theta$$

$$y = \sqrt{a} x + \frac{a}{2} x^2$$

$$\frac{y}{x} = \frac{\sqrt{a} x}{x} + \frac{a x^2}{2x}$$

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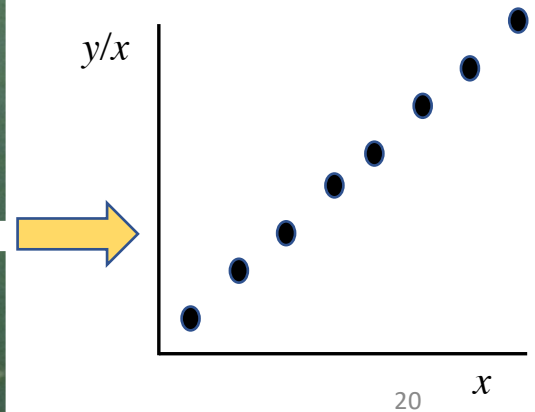
$$y = b + ax$$

coeficiente angular: $a = \frac{\sum(x_i - \bar{x})y_i}{\sum(x_i - \bar{x})^2}$

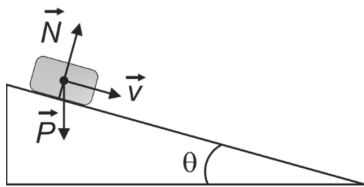
incerteza do coeficiente angular: $\Delta a = \frac{\Delta y}{\sqrt{\sum(x_i - \bar{x})^2}}$

$$a = \frac{a}{2} = \frac{a}{2} = 2a$$

Método dos mínimos quadrados



Plano inclinado



$F = m \cdot a$
 $m \cdot g \cdot \sin \theta = m \cdot a$
 $a = g \cdot \sin \theta$
 $g = \frac{2a + v_0^2}{v_0^2 \sin^2 \theta}$

$$y = \sqrt{a}x + \frac{a}{2}x^2$$

$$\frac{y}{x} = \frac{\sqrt{a}x}{x} + \frac{a}{2}x$$

$$\frac{y}{x} = \sqrt{a} + \frac{a}{2}x$$

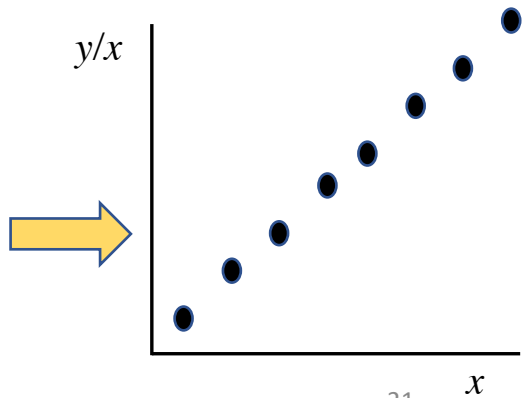
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$$a = \frac{v}{x} = \sqrt{a} = 2a$$

Método dos mínimos quadrados



Exemplo de aplicação MMQ

$$\text{coeficiente angular: } a = \frac{\sum(x_i - \bar{x})y_i}{\sum(x_i - \bar{x})^2}$$

i	x_i	y_i
1	139	122
2	126	114
3	90	86
4	144	134
5	163	146

$$a = \frac{(139-132,4) \times 122 + (126-132,4) \times 114 + (90-132,4) \times 86 + (144-132,4) \times 134 + (163-132,4) \times 146}{(139-132,4)^2 + (126-132,4)^2 + (90-132,4)^2 + (144-132,4)^2 + (163-132,4)^2}$$

$$a = \frac{(805,2) + (-729,6) + (-3646,4) + (1554,4) + (4467,6)}{(43,56) + (40,96) + (1797,76) + (134,56) + (936,36)}$$

$$a = \frac{2451,2}{2953,2}$$

$$a = 0,83$$

$$x_{\text{médio}} = 132,4$$

$$y_{\text{médio}} = 120,4$$

$(x_i - \bar{x}) \cdot y_i$	$(x_i - \bar{x})^2$
805,2	43,56
-729,6	40,96
-3646,4	1797,76
1554,4	134,56
4467,6	936,36

Exemplo de aplicação MMQ

$$\text{coeficiente linear: } b = \bar{y} - a\bar{x}$$

i	x_i	y_i
1	139	122
2	126	114
3	90	86
4	144	134
5	163	146

$$b = 120,4 - (0,83 \times 132,4)$$

$$b = 120,4 - 109,892$$

$$b = 10,508$$

$$\begin{aligned}x_{\text{m\u00e9dio}} &= 132,4 \\y_{\text{m\u00e9dio}} &= 120,4 \\a &= 0,83\end{aligned}$$

Exemplo de aplicação MMQ

$$\text{dispersão média do ajuste: } \Delta y = \sqrt{\frac{\sum(ax_i + b - y_i)^2}{N-2}}$$

i	x_i	y_i
1	139	122
2	126	114
3	90	86
4	144	134
5	163	146

$$\begin{aligned} x_{\text{médio}} &= 132,4 \\ y_{\text{médio}} &= 120,4 \\ a &= 0,83 \\ b &= 10,508 \end{aligned}$$

$$\Delta y = \sqrt{\frac{[(0,83 \times 139) + (10,508 - 122)]^2 + [(0,83 \times 126) + (10,508 - 114)]^2 + [(0,83 \times 90) + (10,508 - 86)]^2 + [(0,83 \times 144) + (10,508 - 144)]^2 + [(0,83 \times 163) + (10,508 - 146)]^2}{5 - 2}}$$

$$\Delta y = \sqrt{\frac{15,039 + 1,184 + 0,627 + 15,777 + 0,041}{3}}$$

$$\sqrt{\frac{32,667}{3}}$$

$$\Delta y = \sqrt{10,889} = 3,299$$