IFSC USP
Instituto de Física de São Carlos

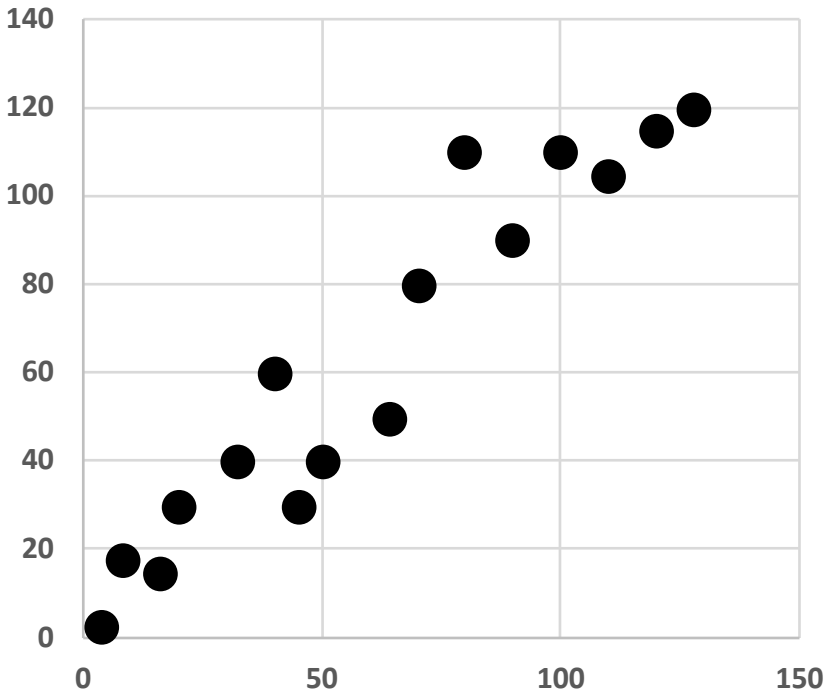
Movimento Unidimensional e Método dos Mínimos Quadrados

Prof. Rafael Guido

rvcguido@ifsc.usp.br

Relações lineares entre dados experimentais

- *Muitas vezes, a relação encontrada experimentalmente entre duas grandezas físicas é linear ou pode ser linearizada.*



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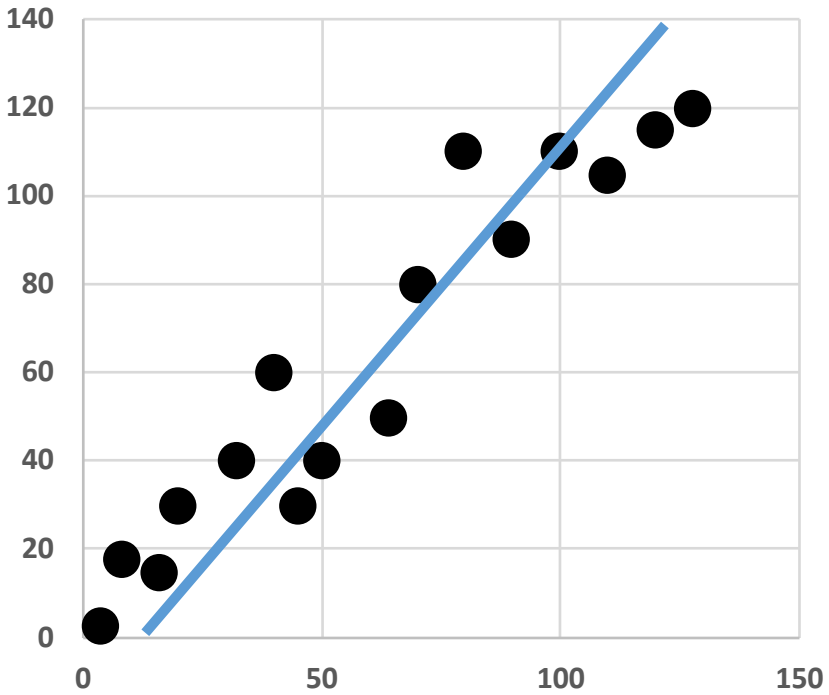

Método gráfico

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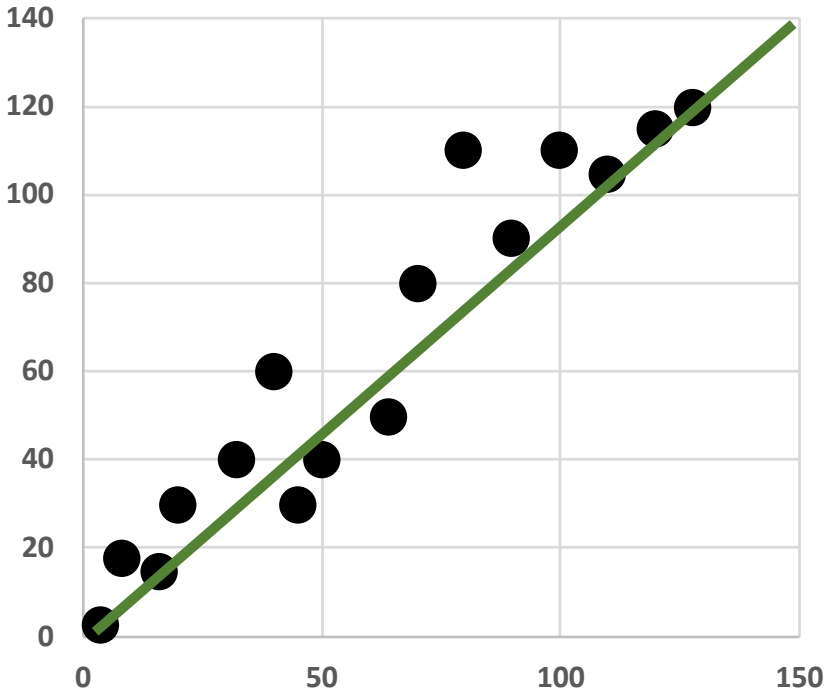

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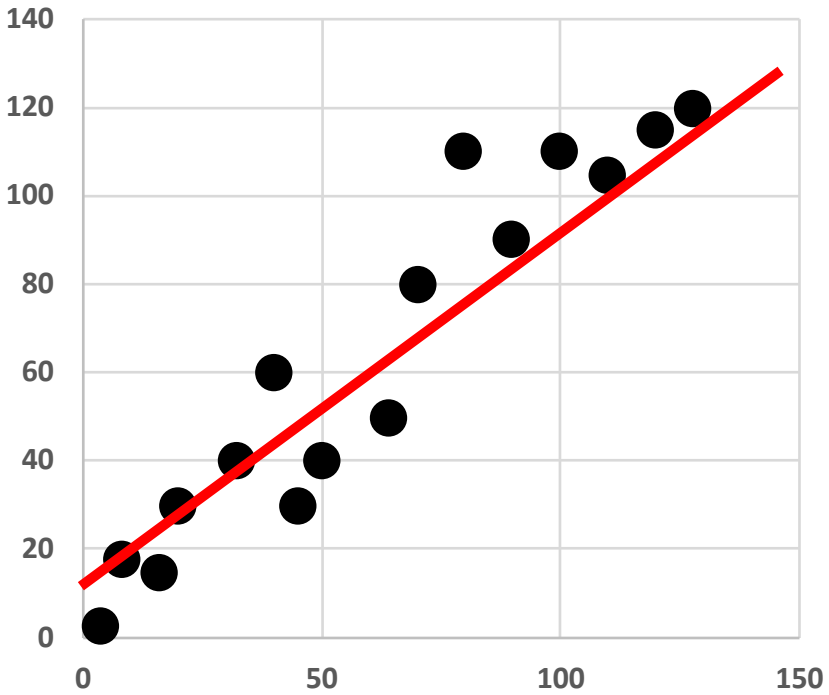

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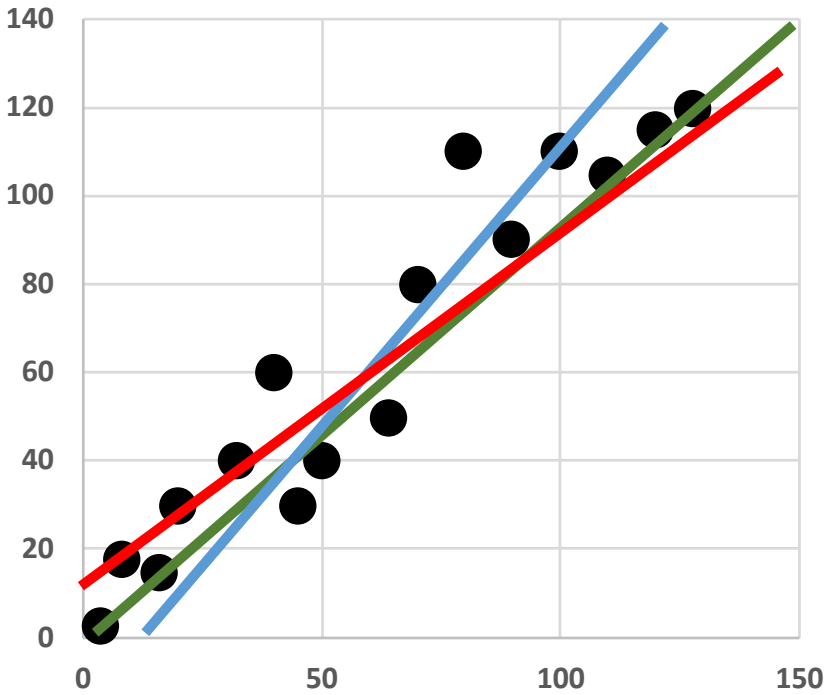

Método gráfico

Qual a melhor?

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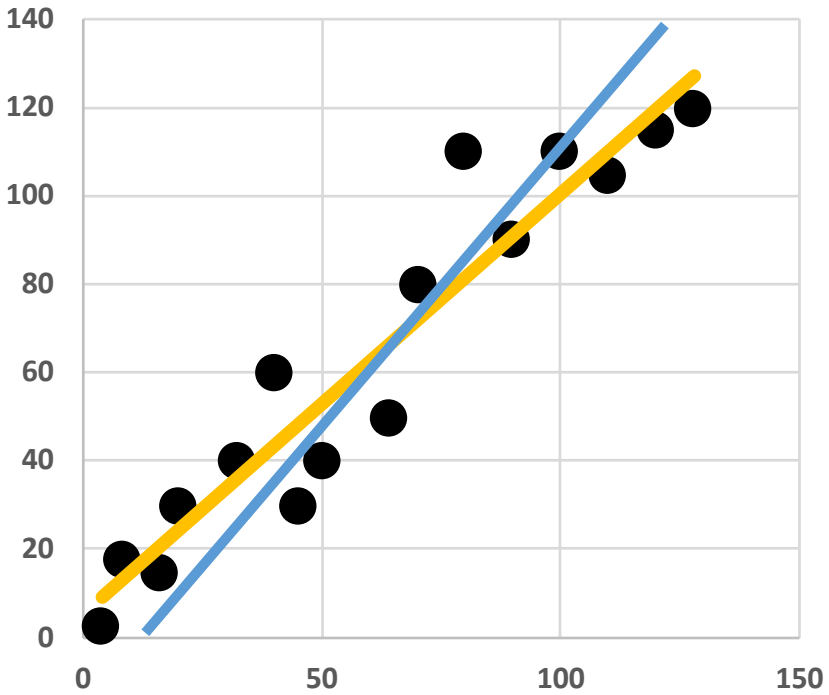

Método gráfico (a olho)

Método analítico (mínimos quadrados)

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coeficiente angular: $a = \frac{\sum(x_i - \bar{x})y_i}{\sum(x_i - \bar{x})^2}$

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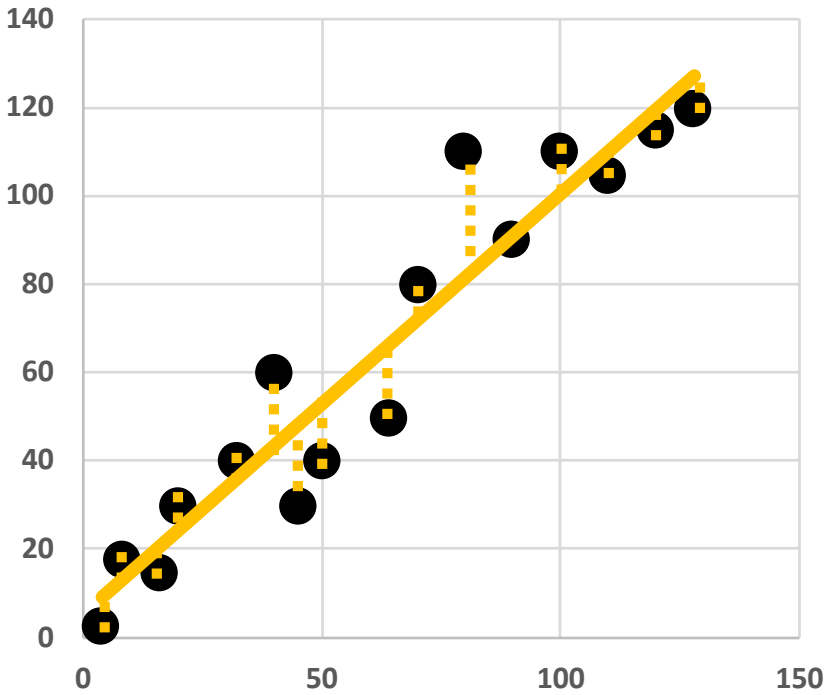

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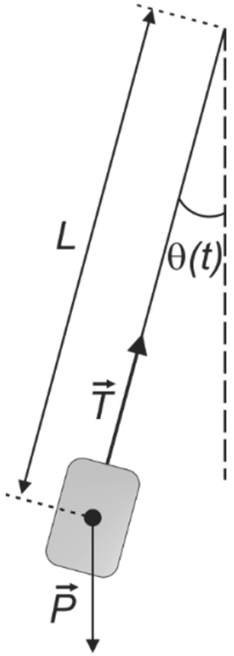
dispersão média do ajuste: $\Delta y = \sqrt{\frac{\sum(ax_i + b - y_i)^2}{N-2}}$

incerteza do coeficiente angular: $\Delta a = \frac{\Delta y}{\sqrt{\sum(x_i - \bar{x})^2}}$

incerteza do coeficiente linear: $\Delta b = \sqrt{\frac{\sum x_i^2}{N\sum(x_i - \bar{x})^2}}\Delta y$

Experimental: Pêndulo simples



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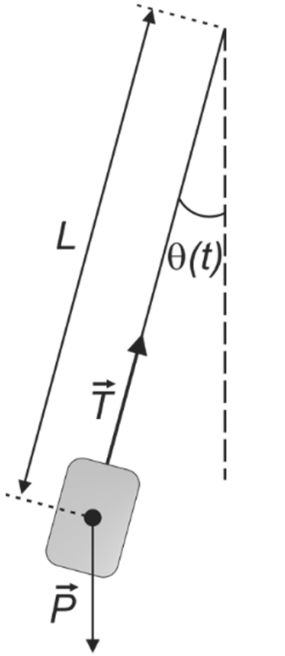
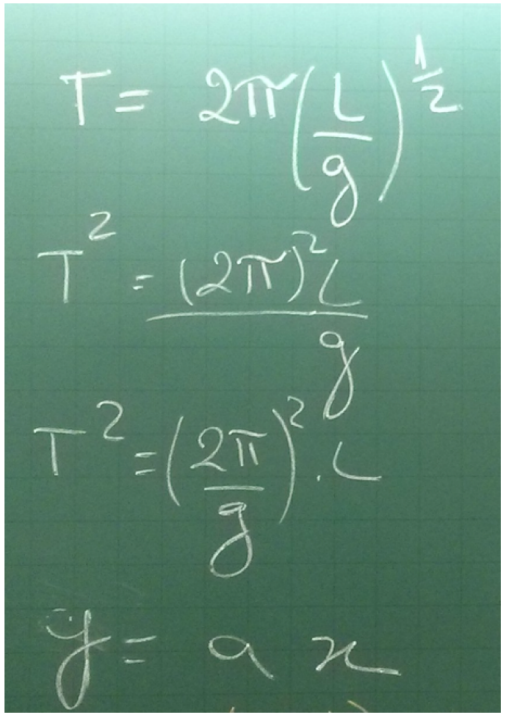


$$T = 2\pi\sqrt{\frac{L}{g}}$$



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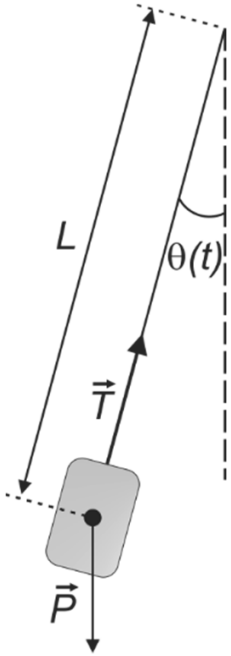

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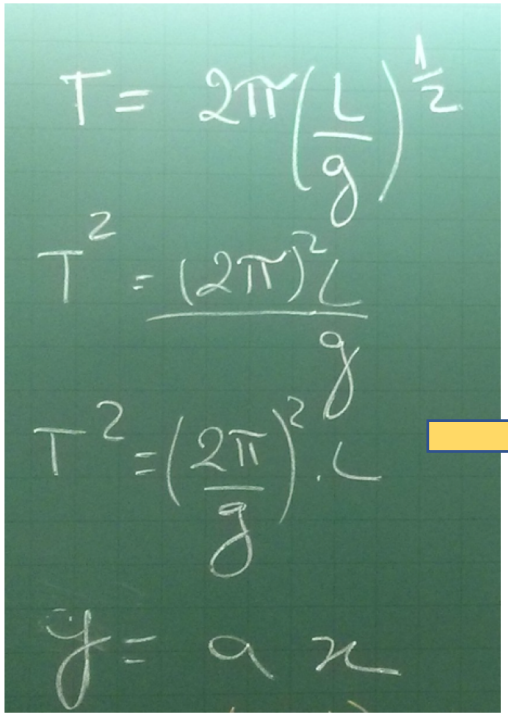

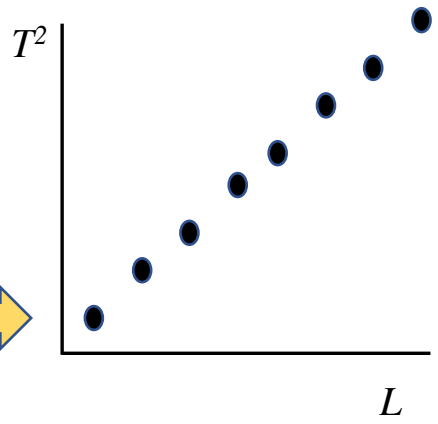

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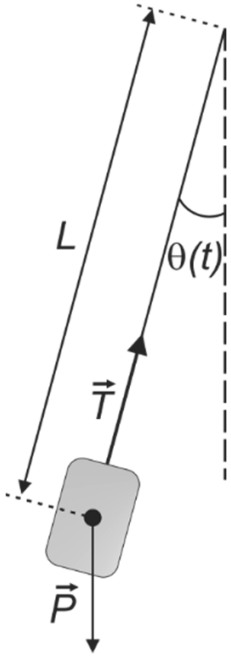

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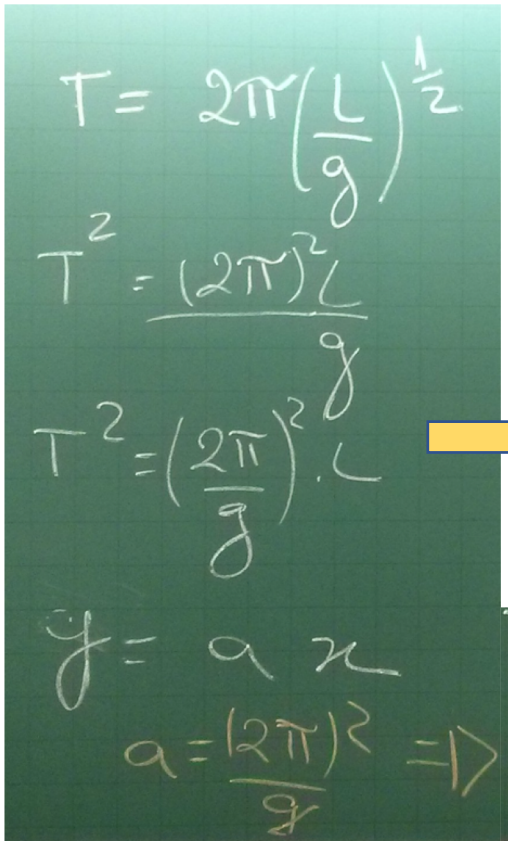
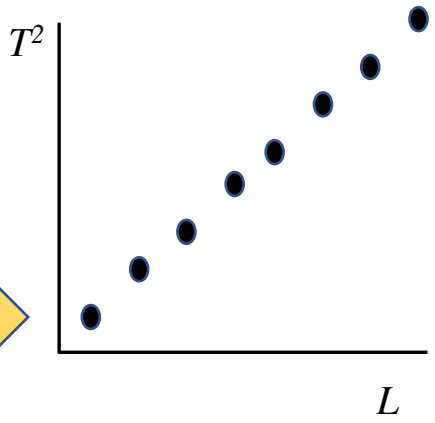
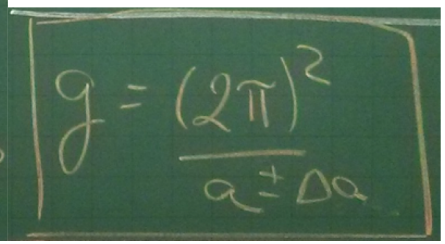




Método dos mínimos quadrados



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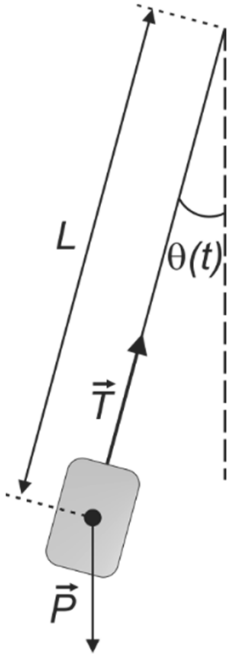

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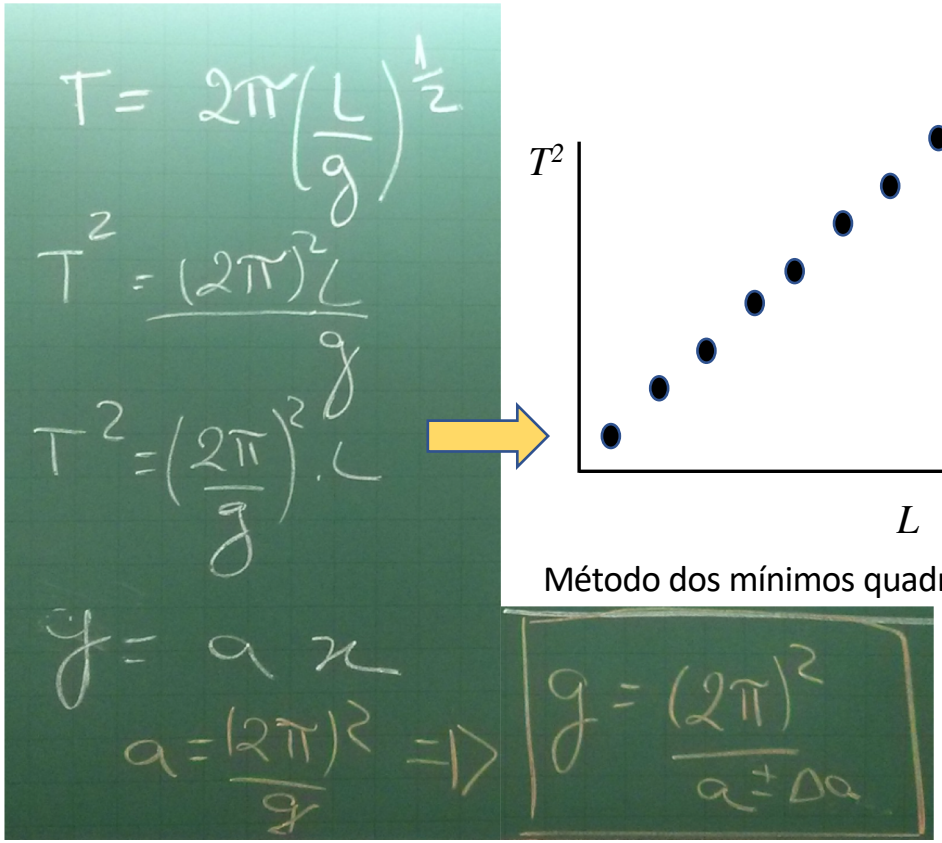
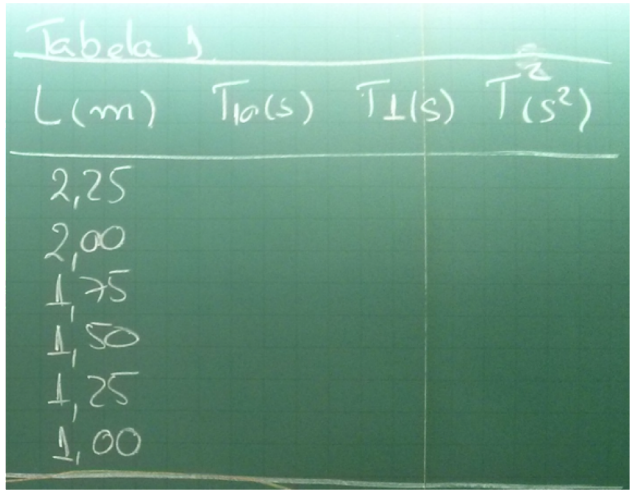


Método dos mínimos quadrados

Exemplo



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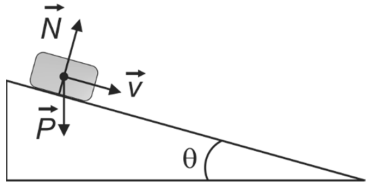
Experimental: Plano inclinado



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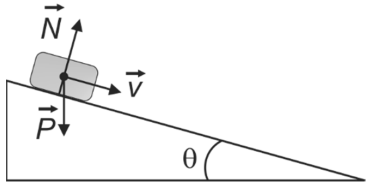

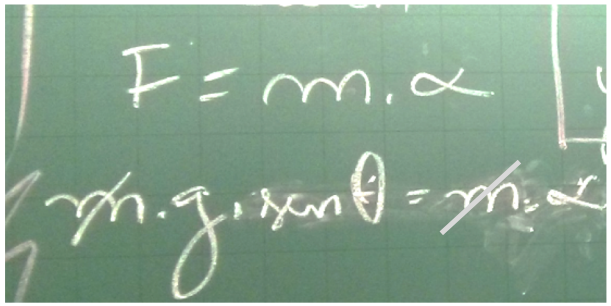

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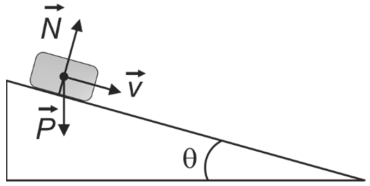

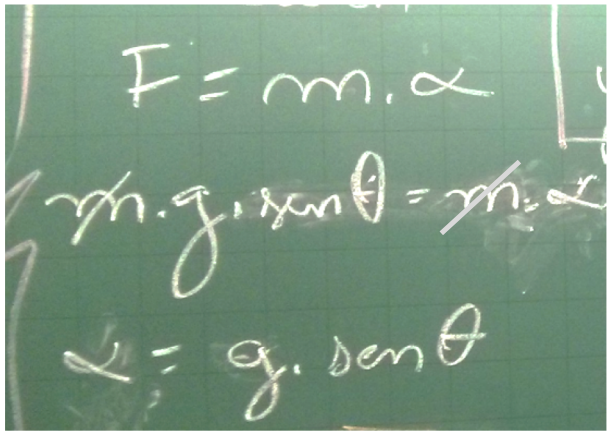

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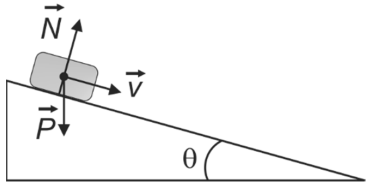

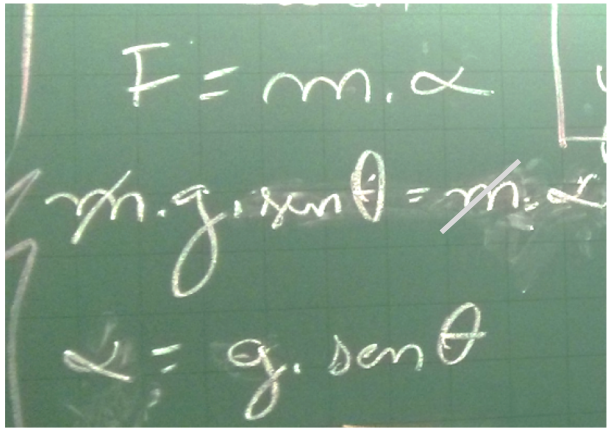

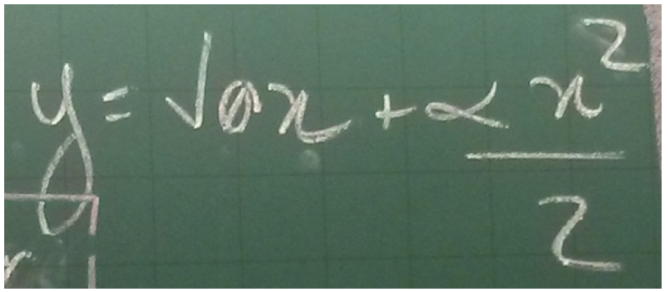

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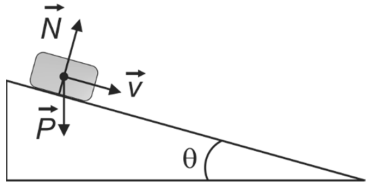

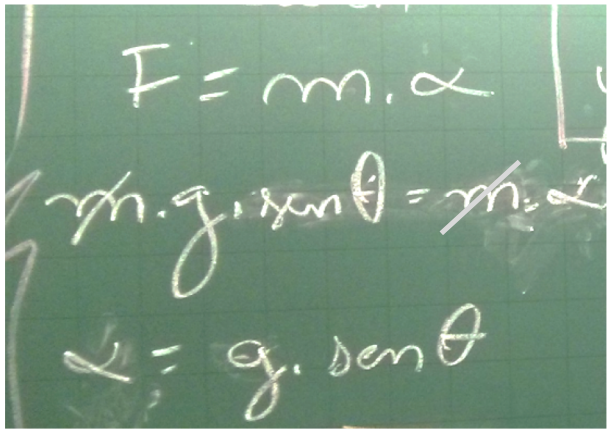

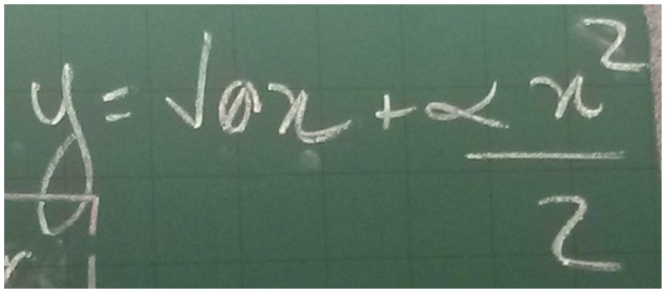

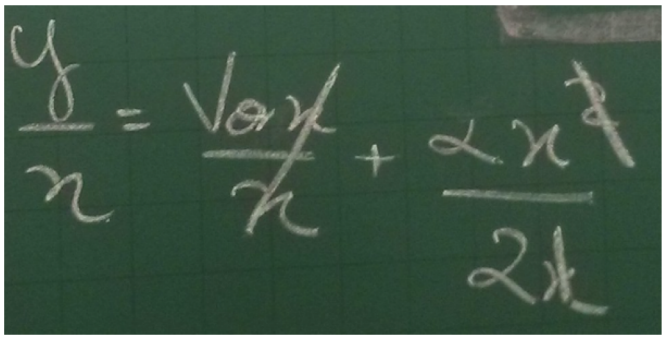

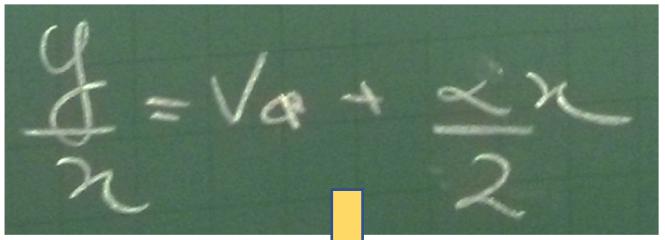

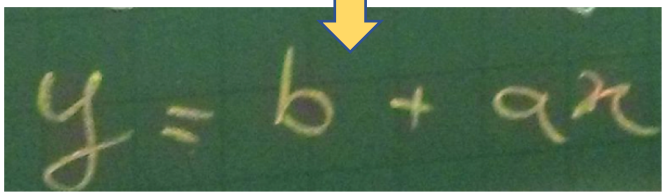

x

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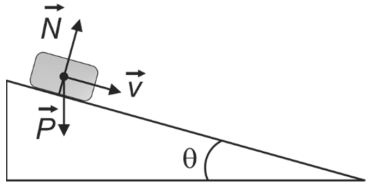

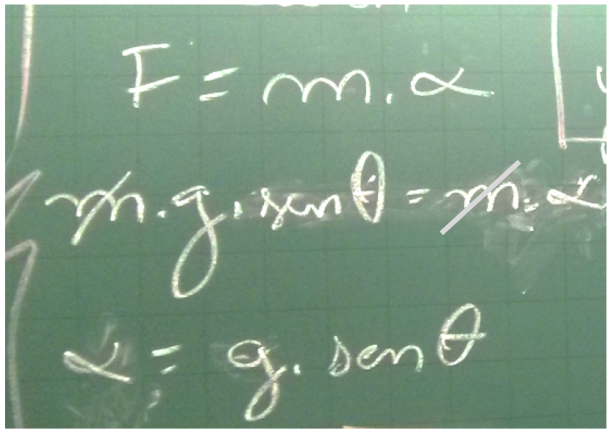

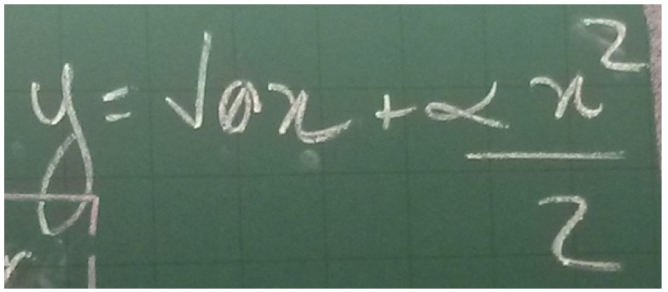

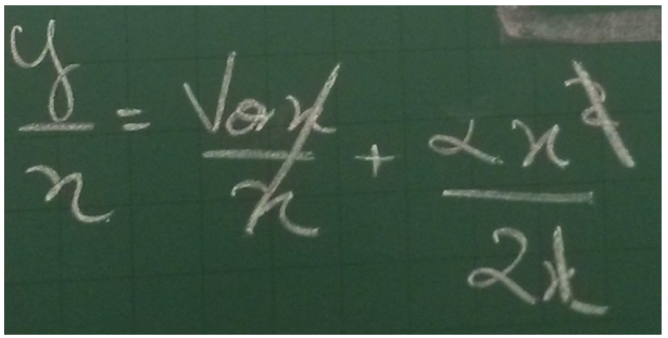

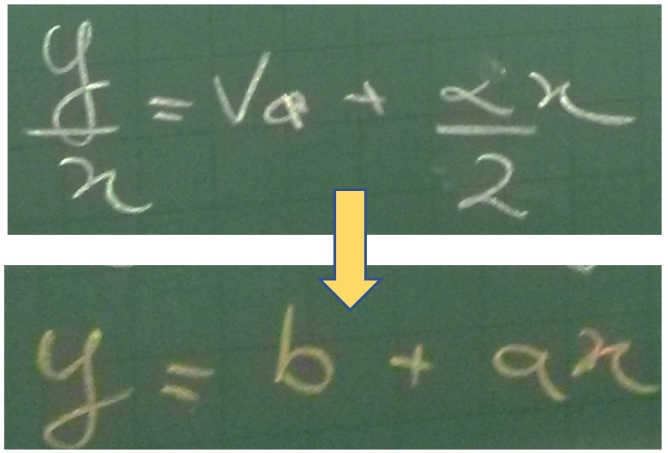

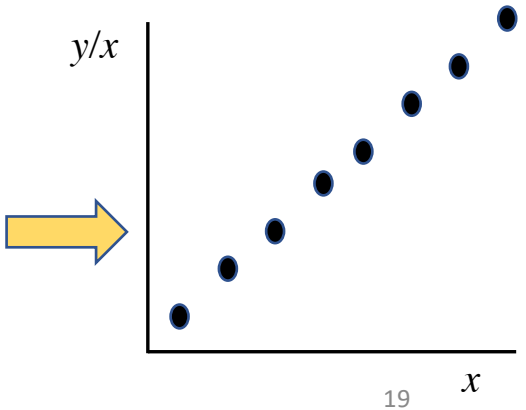

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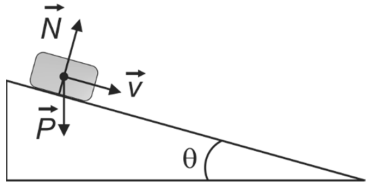

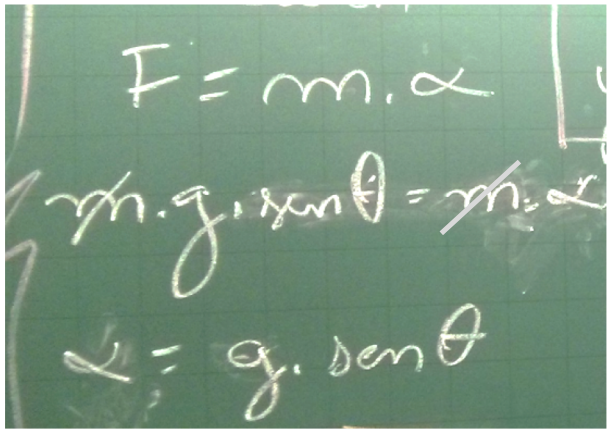

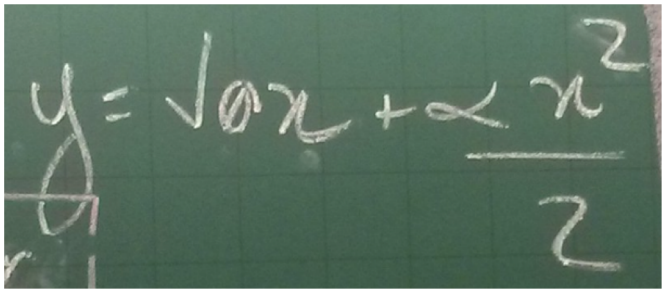

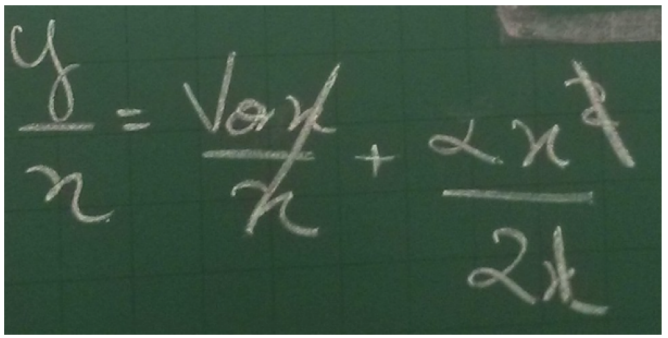

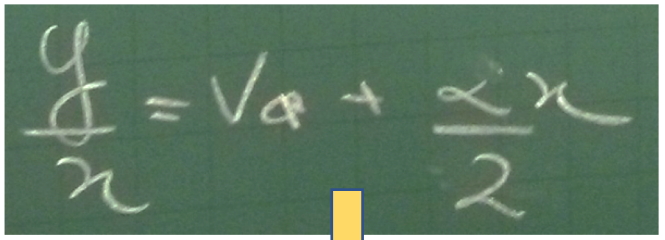

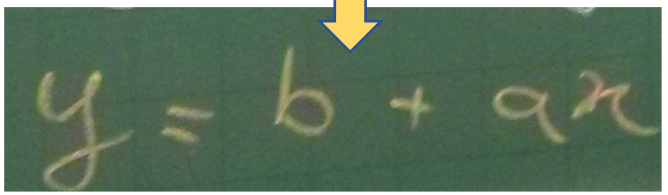

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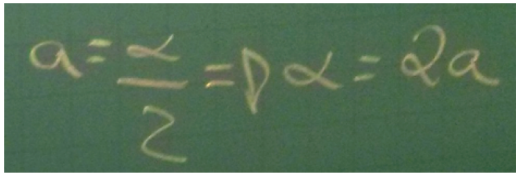

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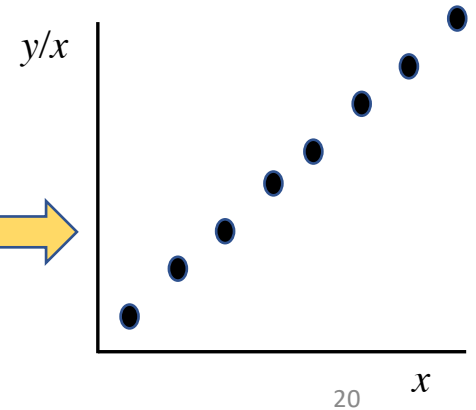

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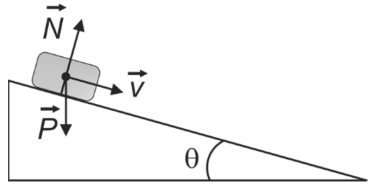

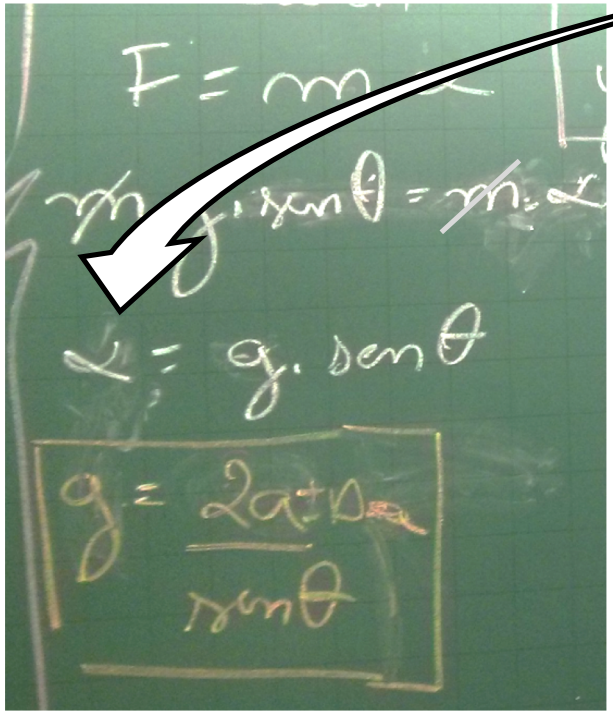

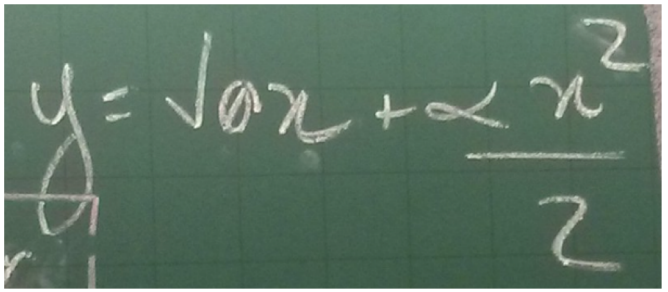

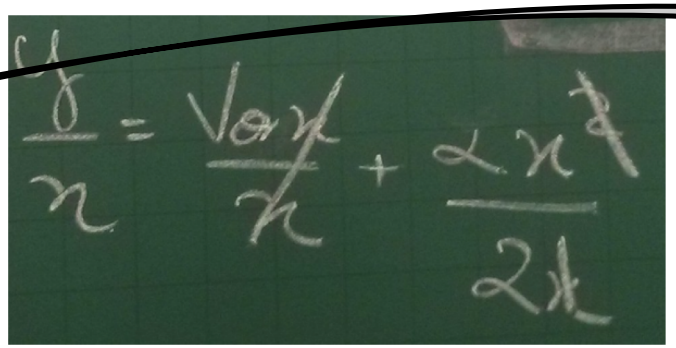

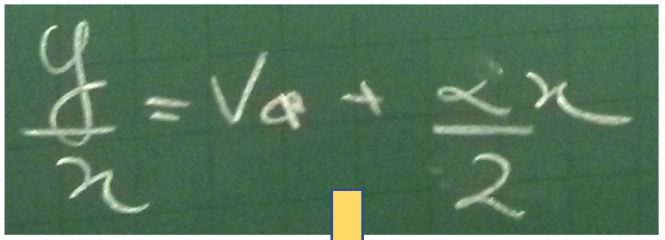

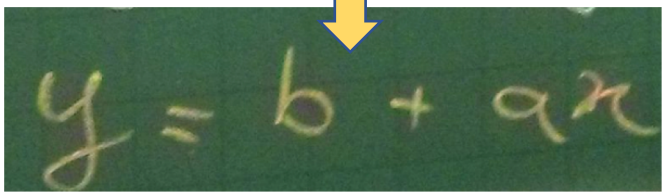

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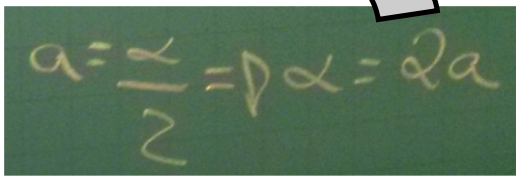
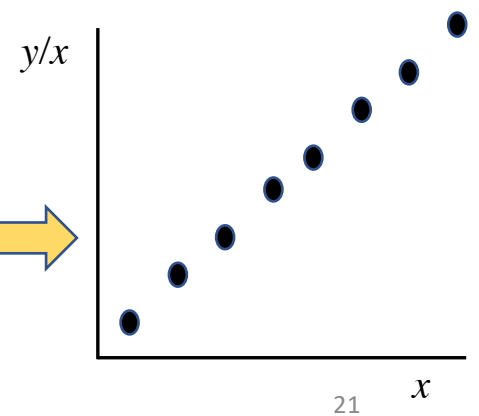
Instituto de Física de São Carlos

coeficiente angular: $a = \frac{\sum(x_i - \bar{x})y_i}{\sum(x_i - \bar{x})^2}$

incerteza do coeficiente angular: $\Delta a = \frac{\Delta y}{\sqrt{\sum(x_i - \bar{x})^2}}$



Método dos mínimos quadrados



Exemplo de aplicação MMQ

coeficiente angular: $a = \frac{\sum(x_i - \bar{x})y_i}{\sum(x_i - \bar{x})^2}$

i	x_i	y_i
1	139	122
2	126	114
3	90	86
4	144	134
5	163	146

$x_{\text{médio}} = 132,4$
$y_{\text{médio}} = 120,4$

(xi - x)*y	(xi - x)^2
805,2	43,56
-729,6	40,96
-3646,4	1797,76
1554,4	134,56
4467,6	936,36

$$a = \frac{(139-132,4)\times 122 + (126-132,4)\times 114 + (90-132,4)\times 86 + (144-132,4)\times 134 + (163-132,4)\times 146}{(139-132,4)^2 + (126-132,4)^2 + (90-132,4)^2 + (144-132,4)^2 + (163-132,4)^2}$$

$$a = \frac{(805,2) + (-729,6) + (-3646,4) + (1554,4) + (4467,6)}{(43,56) + (40,96) + (1797,76) + (134,56) + (936,36)}$$

$$a = \frac{2451,2}{2953,2}$$

$$a = 0,83$$

Exemplo de aplicação MMQ

i	x_i	y_i
1	139	122
2	126	114
3	90	86
4	144	134
5	163	146

$x_{\text{médio}} = 132{,}4$
$y_{\text{médio}} = 120{,}4$
$a = 0{,}83$

coeficiente linear: $b = \bar{y} - a\bar{x}$

$b = 120{,}4 - (0{,}83 \text{x} 132{,}4)$

$b = 120{,}4 - 109{,}892$

$b = 10{,}508$

Exemplo de aplicação MMQ

dispersão média do ajuste: $\Delta y = \sqrt{\frac{\sum(ax_i+b-y_i)^2}{N-2}}$

i	x_i	y_i
1	139	122
2	126	114
3	90	86
4	144	134
5	163	146

$x_{\text{médio}} = 132{,}4$
$y_{\text{médio}} = 120{,}4$
$a = 0{,}83$
$b = 10{,}508$

$$\Delta y = \sqrt{\frac{[(0{,}83\text{x}\mathbf{139})+(10{,}508-\mathbf{122})]^2 + [(0{,}83\text{x}\mathbf{126})+(10{,}508-\mathbf{114})]^2 + [(0{,}83\text{x}\mathbf{90})+(10{,}508-\mathbf{86})]^2 + [(0{,}83\text{x}\mathbf{144})+(10{,}508-\mathbf{144})]^2 + [(0{,}83\text{x}\mathbf{163})+(10{,}508-\mathbf{146})]^2}{5 - 2}}$$

$$\Delta y = \sqrt{\frac{15{,}039 + 1{,}184 + 0{,}627 + 15{,}777 + 0{,}041}{3}}$$

$$\sqrt{\frac{32{,}667}{3}}$$

$$\Delta y = \sqrt{10{,}889} = 3{,}299$$