Formulário de opinião!

Já preencheu? Não esqueça de preencher!

Formulário de opinião! Até agora...

- ✓ Concentração: Possibilidades para resolvermos isso. Intervalo?
- ✓ Gravação das aulas;
- ✓ Avisos e Fórum de Discussão;
- ✓ Seminários;
- ✓ Exercício valendo nota, simulado da prova, opções...
- ✓ Listas fragmentadas → exercícios sugeridos.

Formulário de opinião! Até agora...

Obrigada!

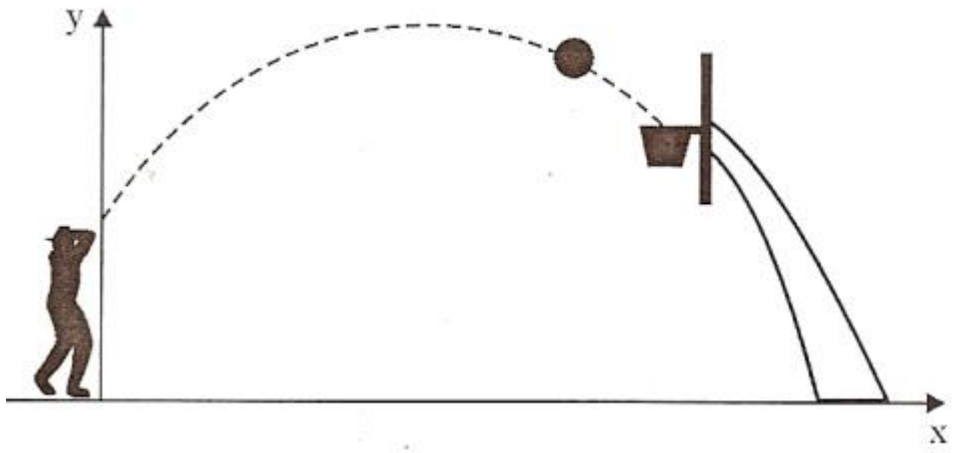
O mais importante:

Centro de Massa

Aula 23/04/2020

Como simplificar um sistema?

Jogar uma bolinha



Salto com vara

Como simplificar um sistema?

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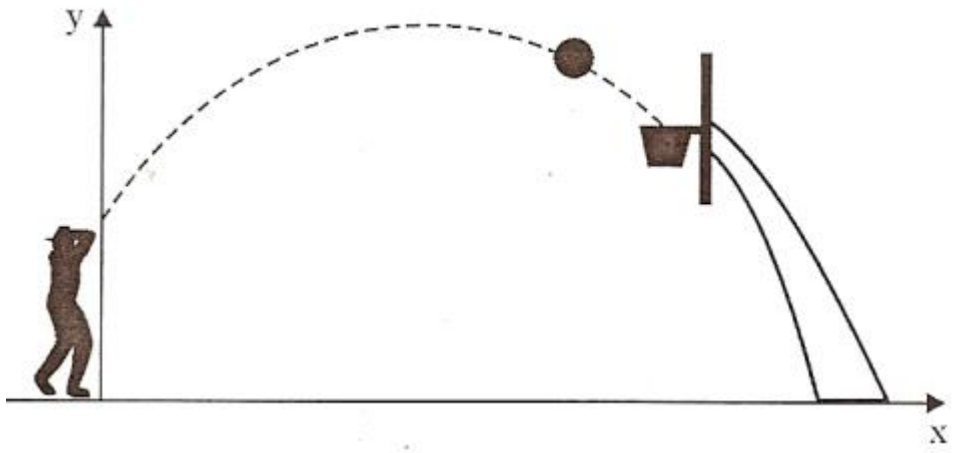

Salto com vara

Centro de massa!

Demonstração

!!

Demonstração

!!

Demonstração

!!

10,2 cm

8,5 cm

Centro de Massa

O centro de massa de um sistema de partículas é o ponto que se move como se:

→ *Toda a massa do sistema estivesse concentrada nesse ponto;*

→ *Todas as forças aplicadas estivessem nesse ponto.*

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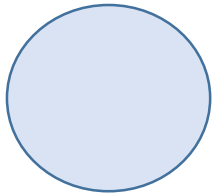
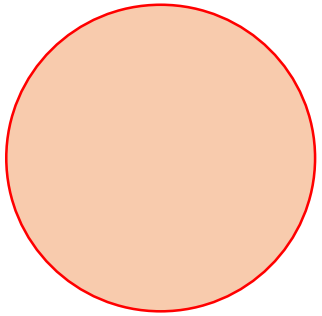
Sistemas de Partículas

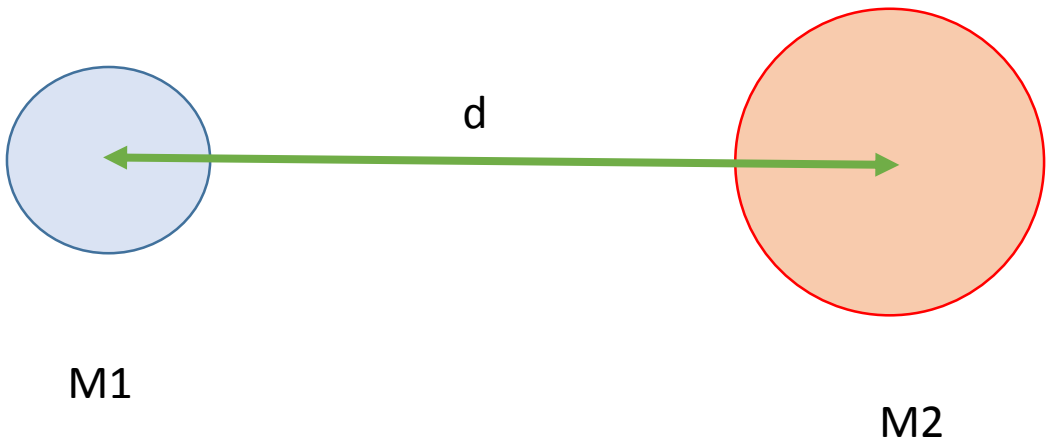


M1

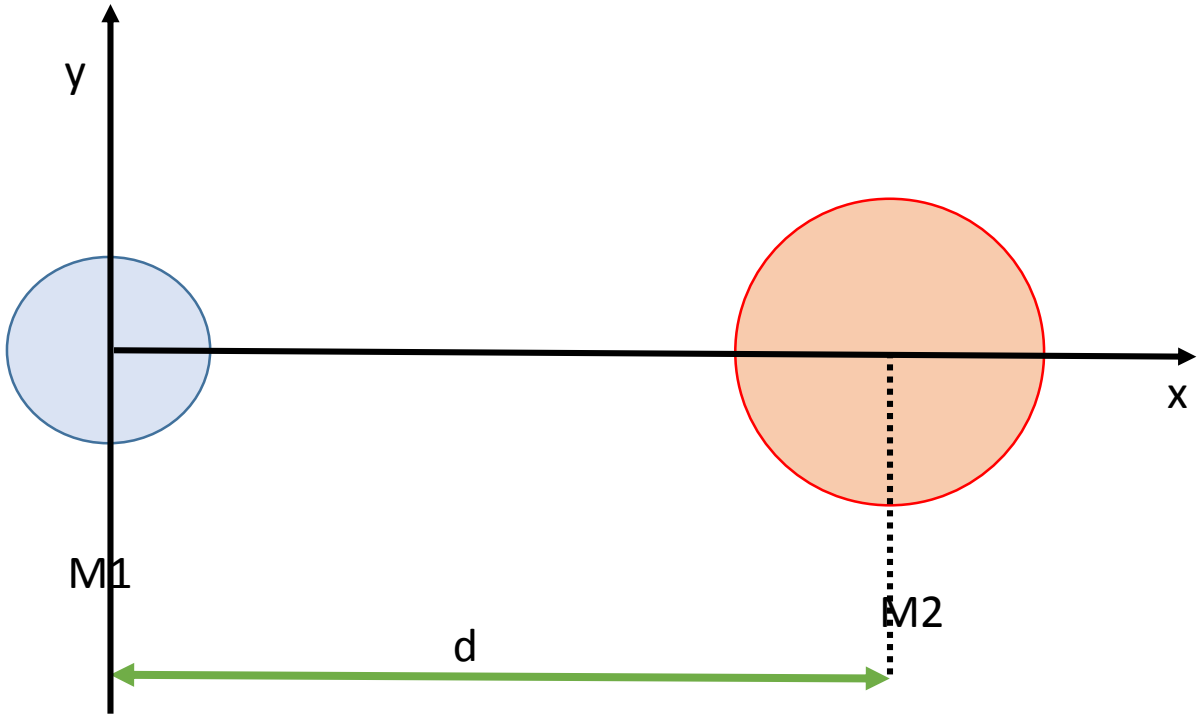


M2

Sistemas de Partículas



Sistemas de Partículas



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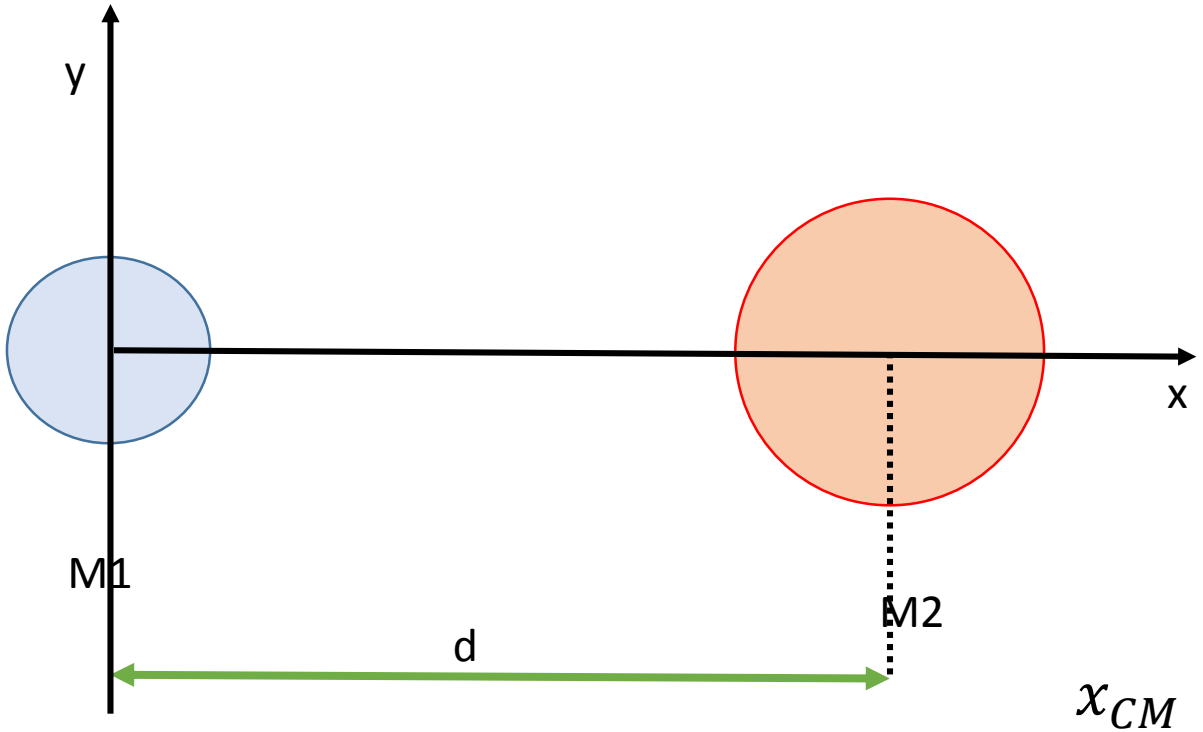
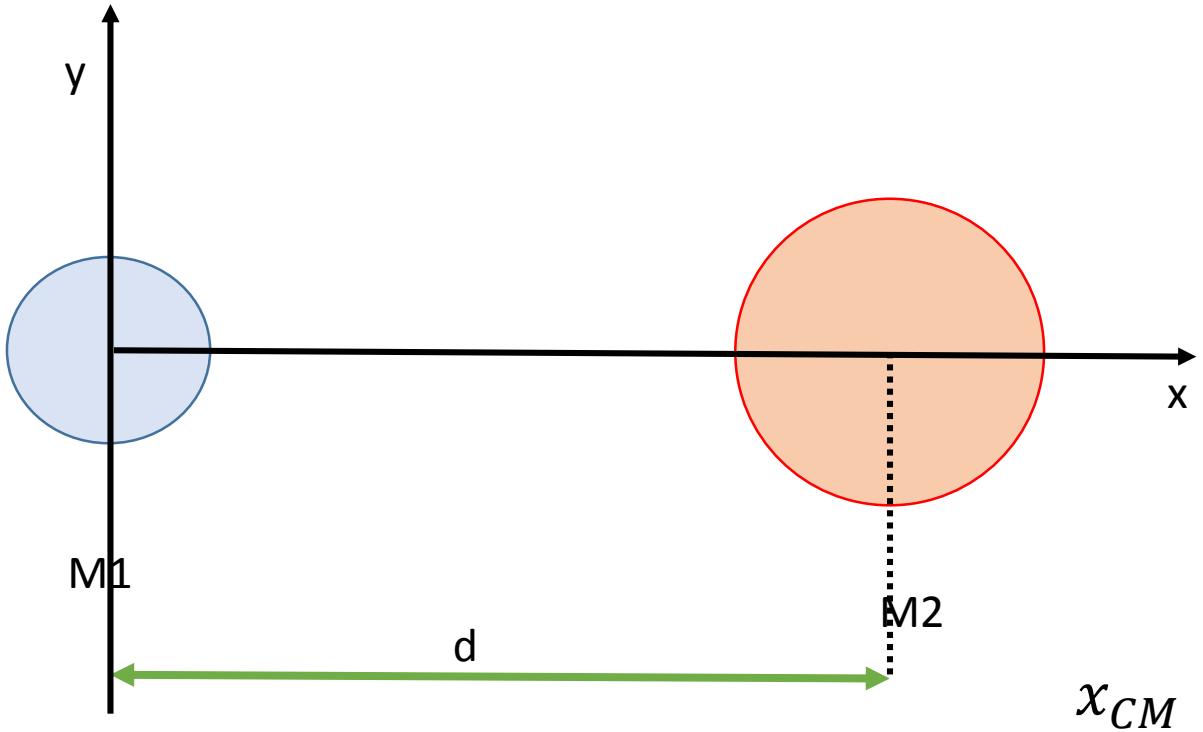


$$x_{CM} = \frac{m_2}{m_1 + m_2} d$$

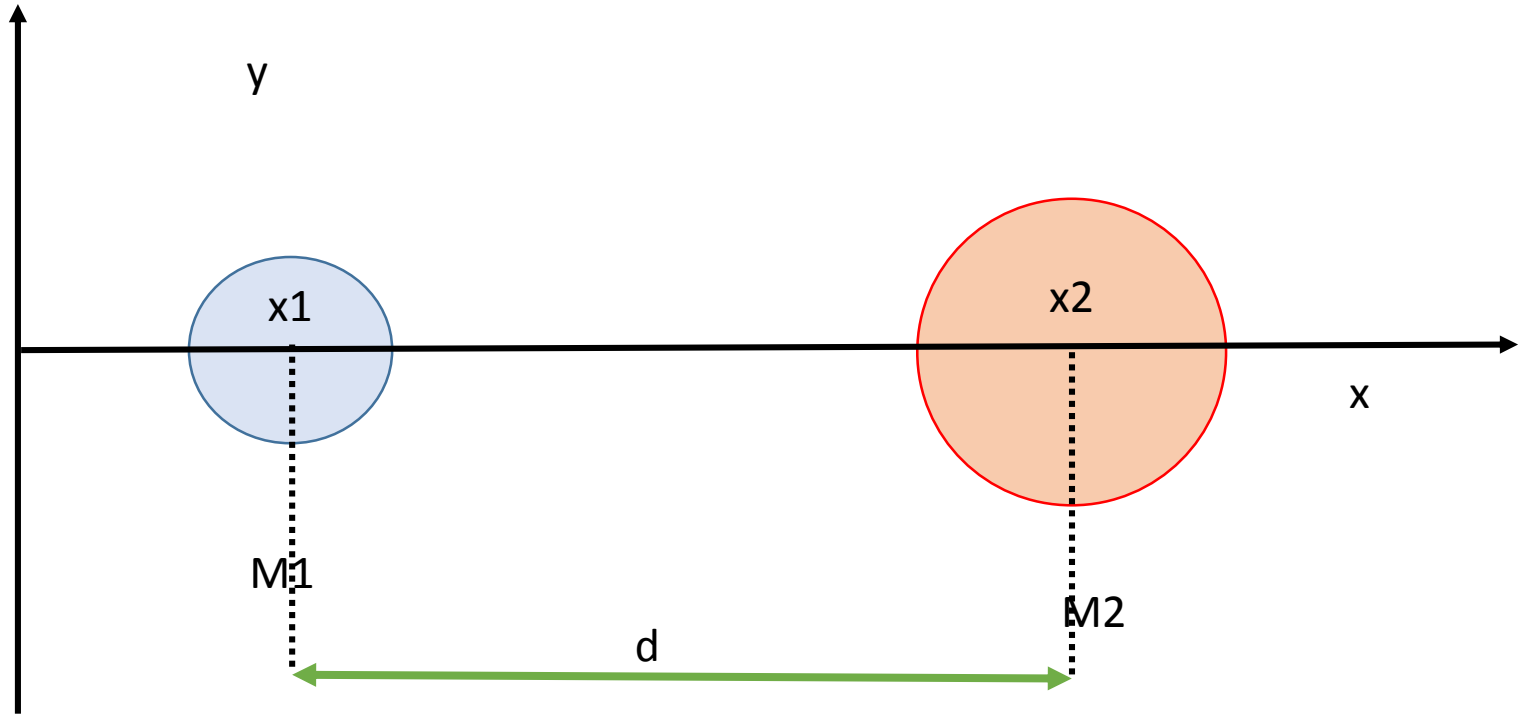
Sistemas de Partículas



$$x_{CM} = \frac{m_2}{m_1 + m_2} d$$

$se\ m_2 = 0?$ $se\ m_1 = 0?$ $se\ m_1 = m_2?$

Sistemas de Partículas



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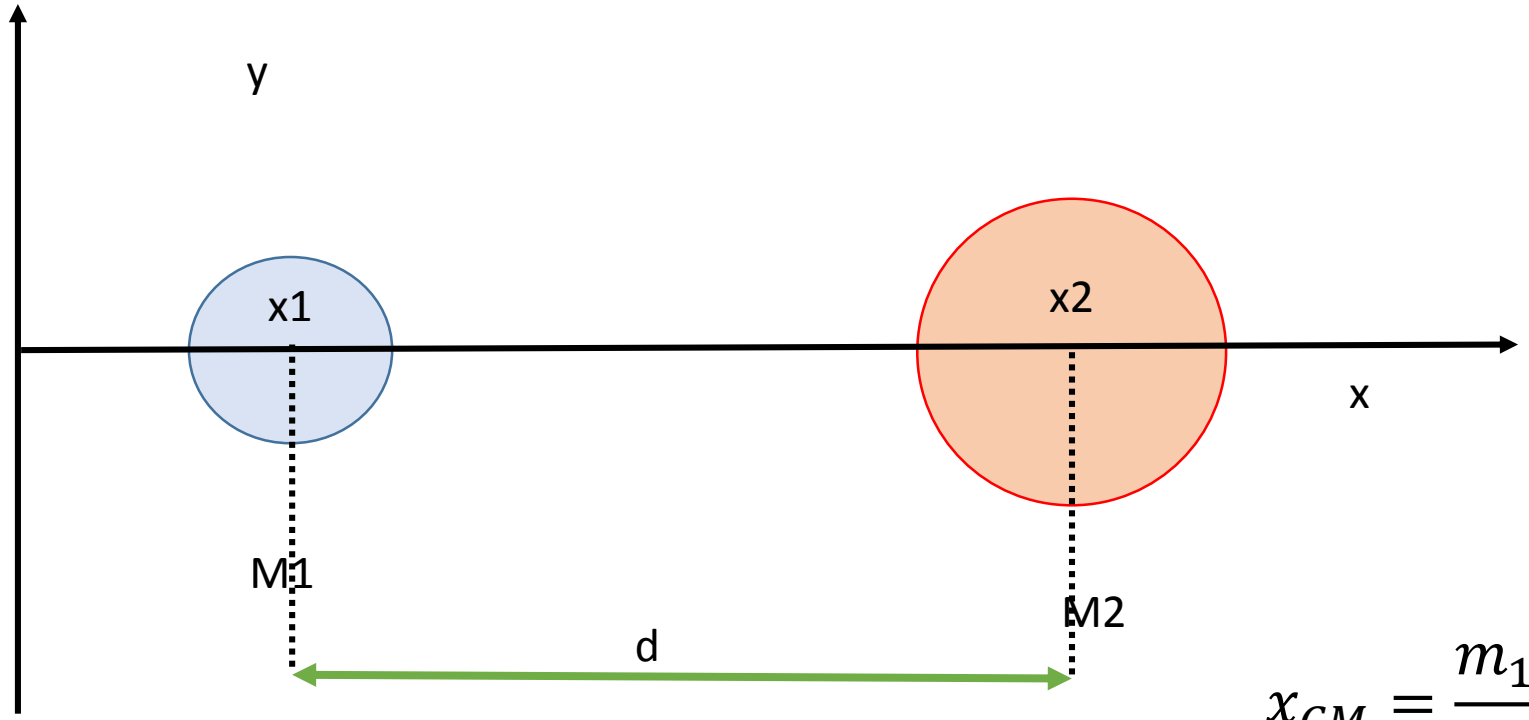
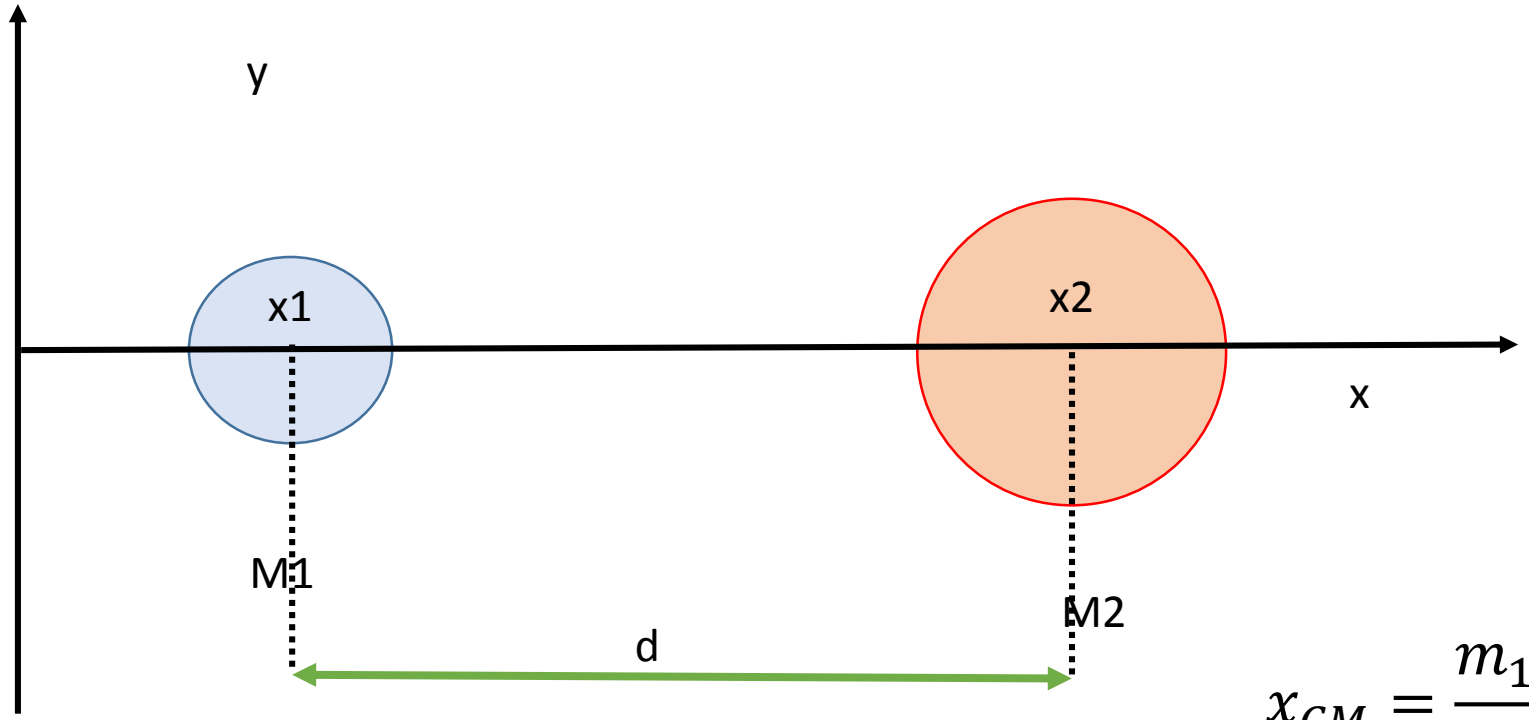


$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Sistemas de Partículas



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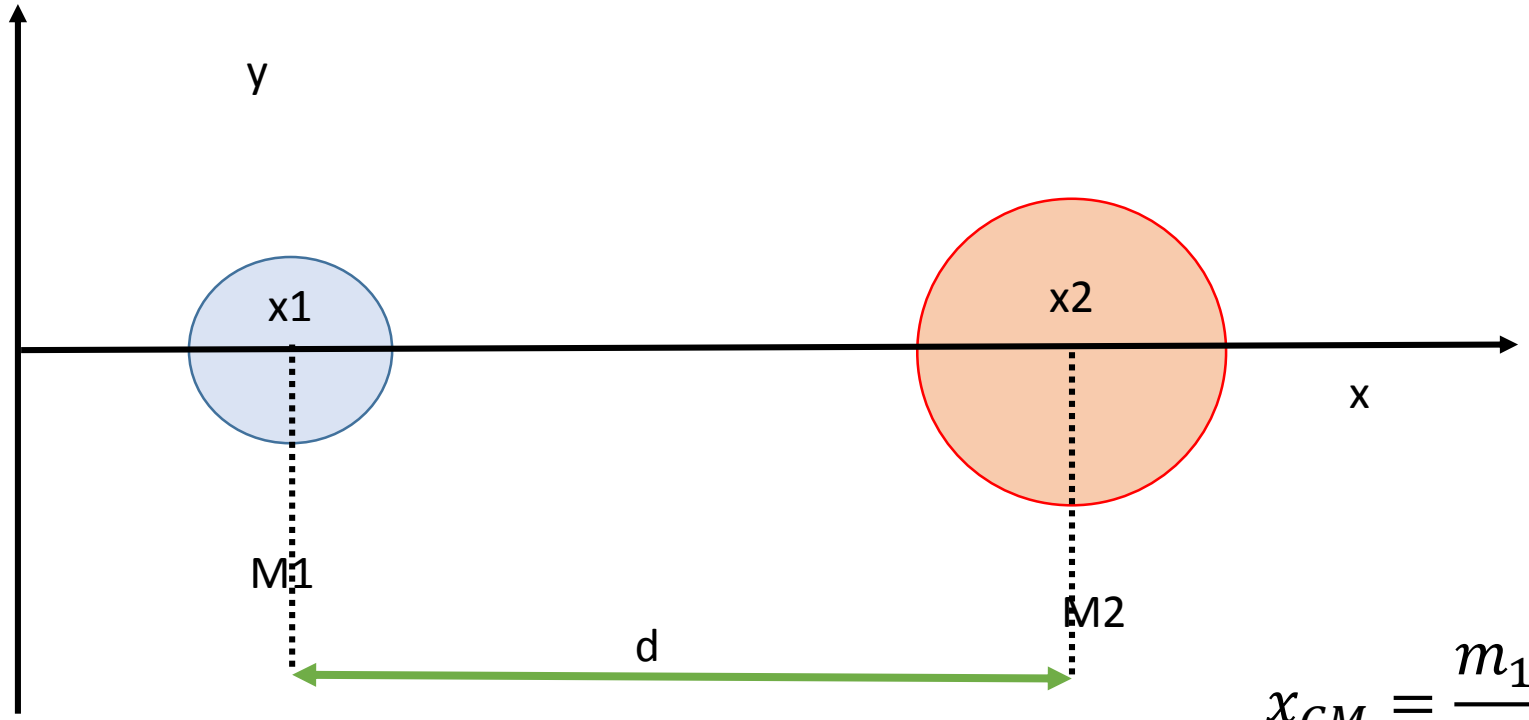

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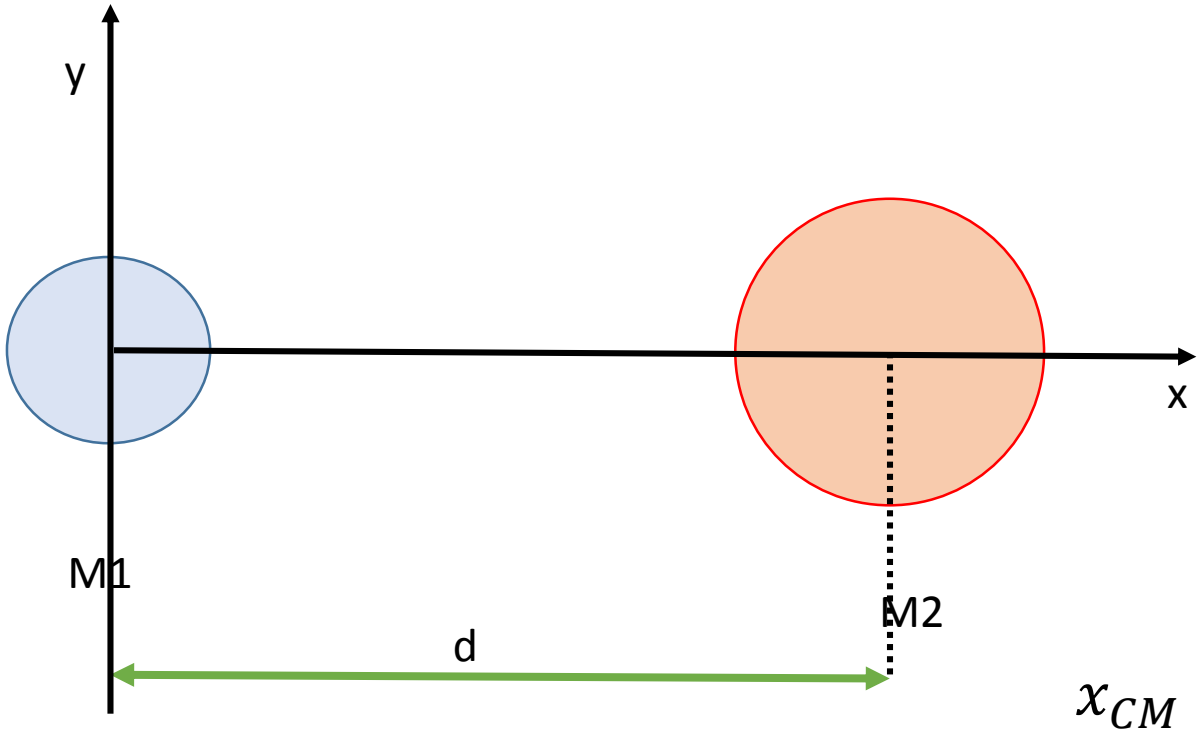

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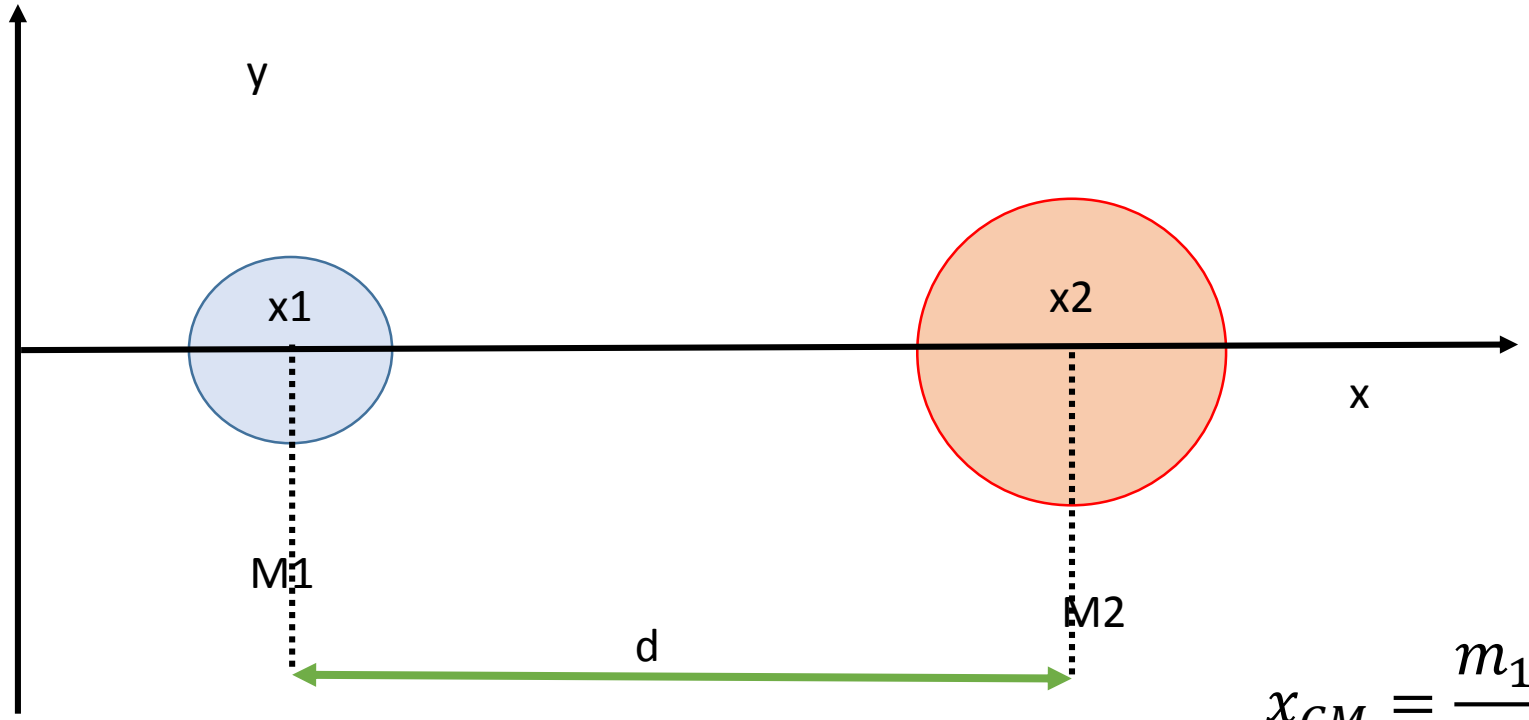

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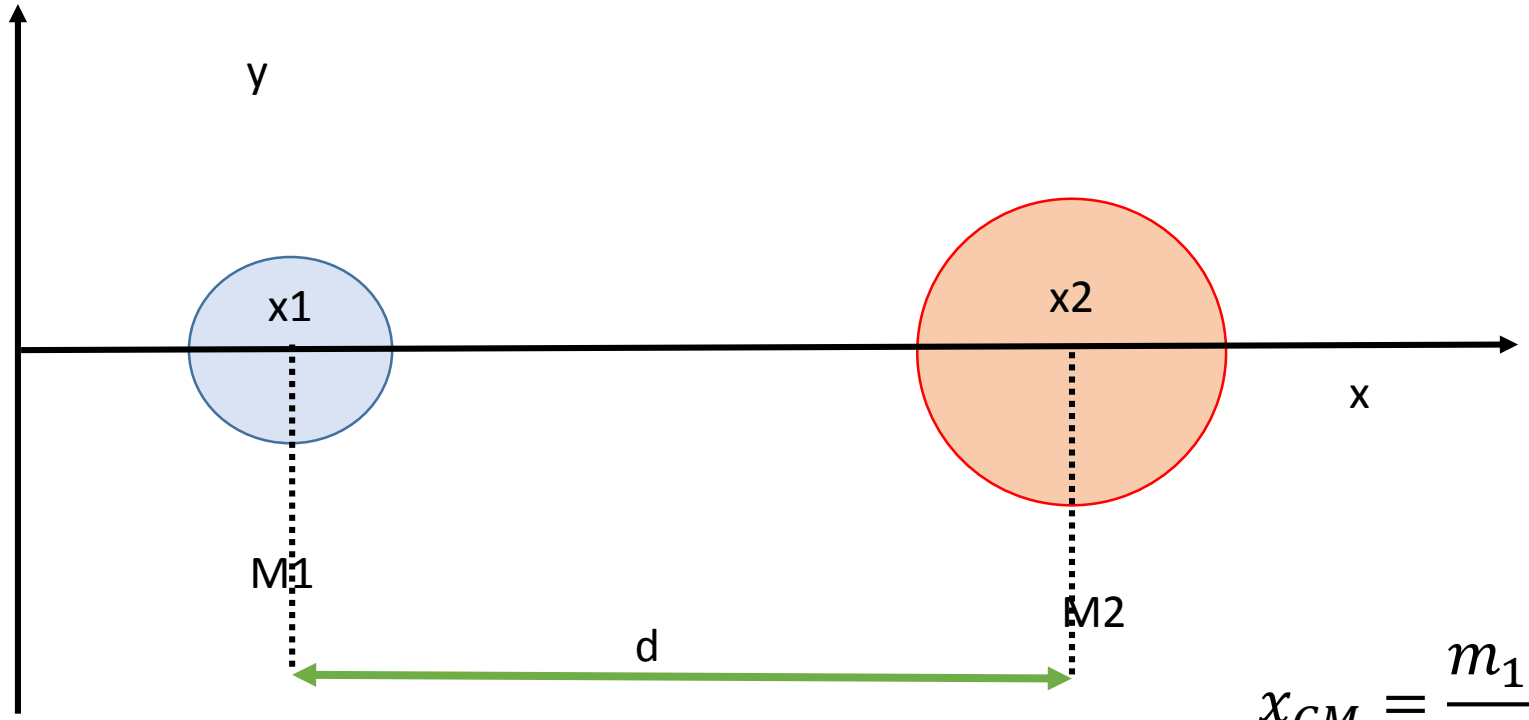
Considerando a massa total do sistema → M

Sistemas de Partículas



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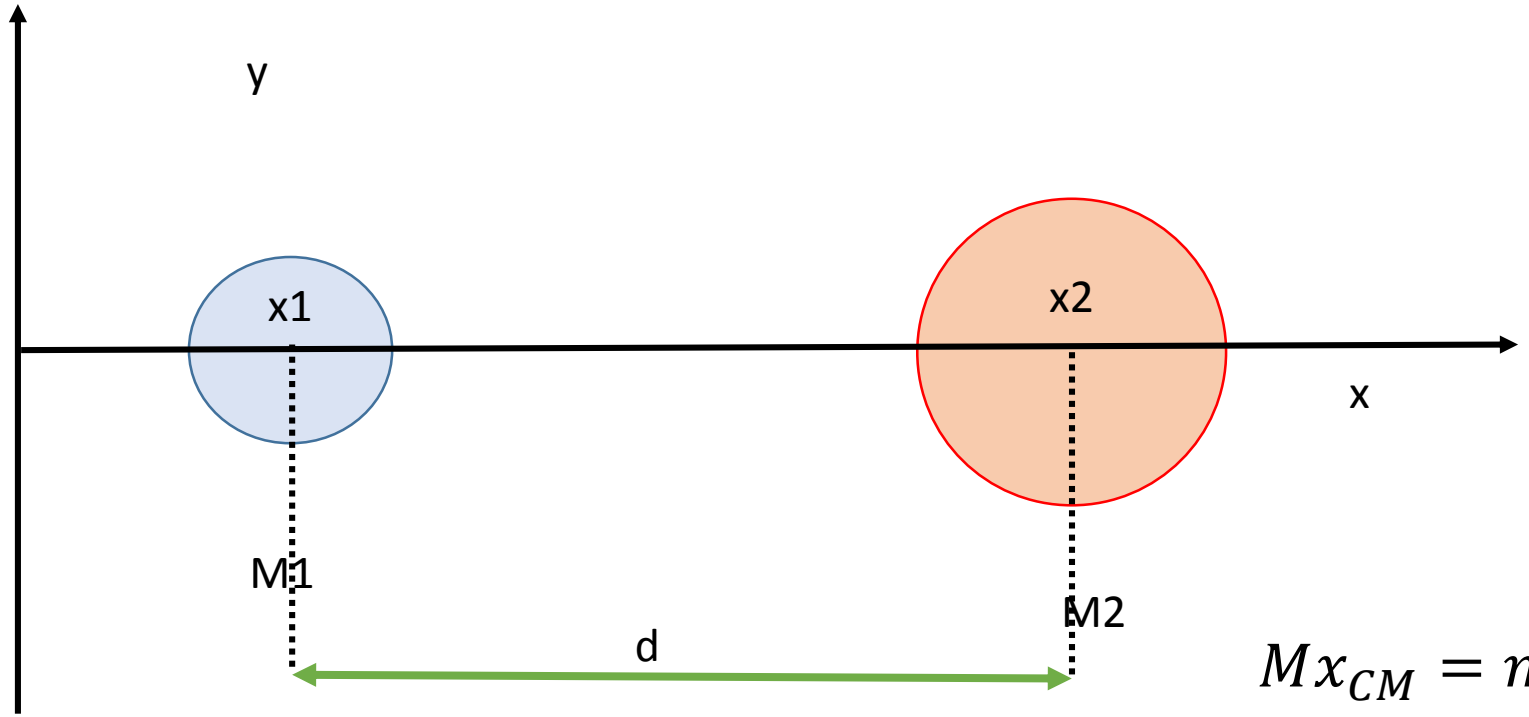

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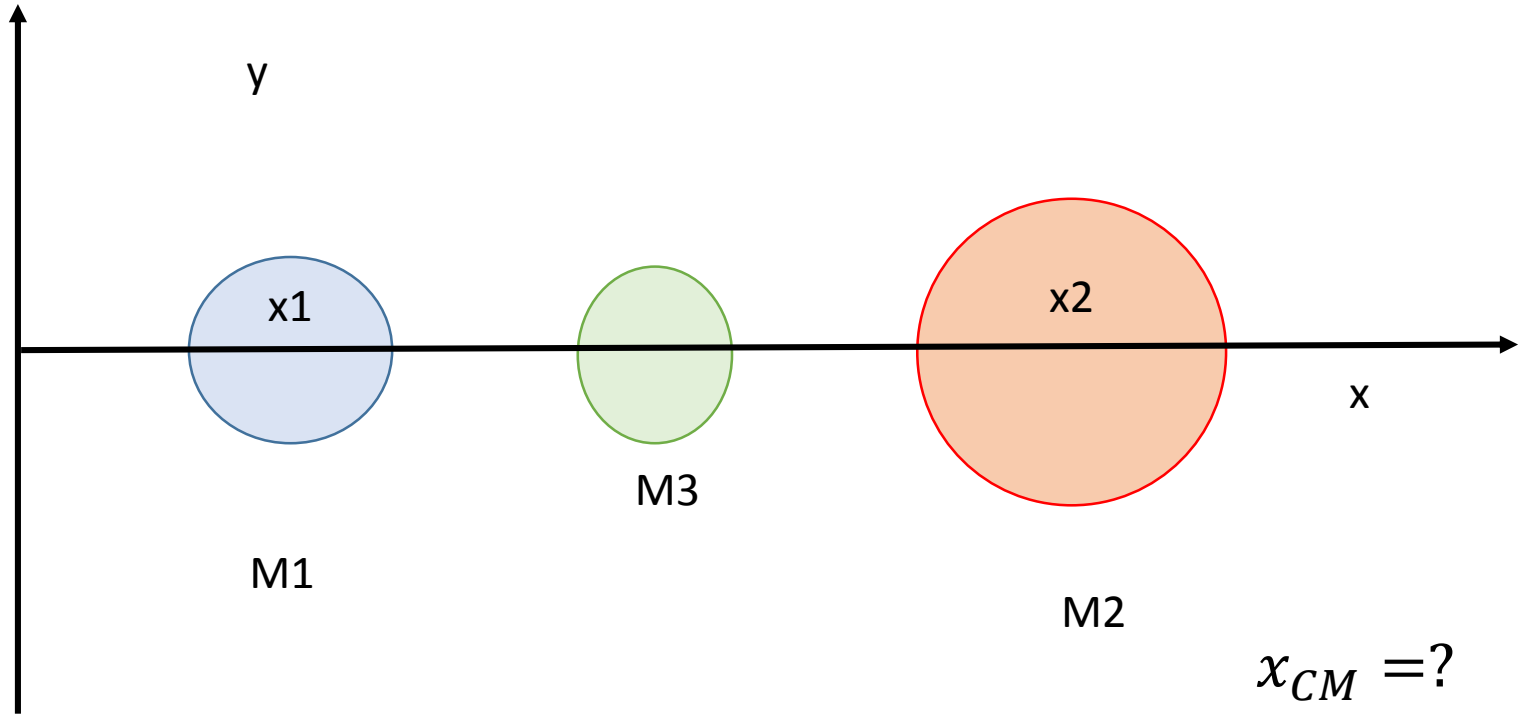

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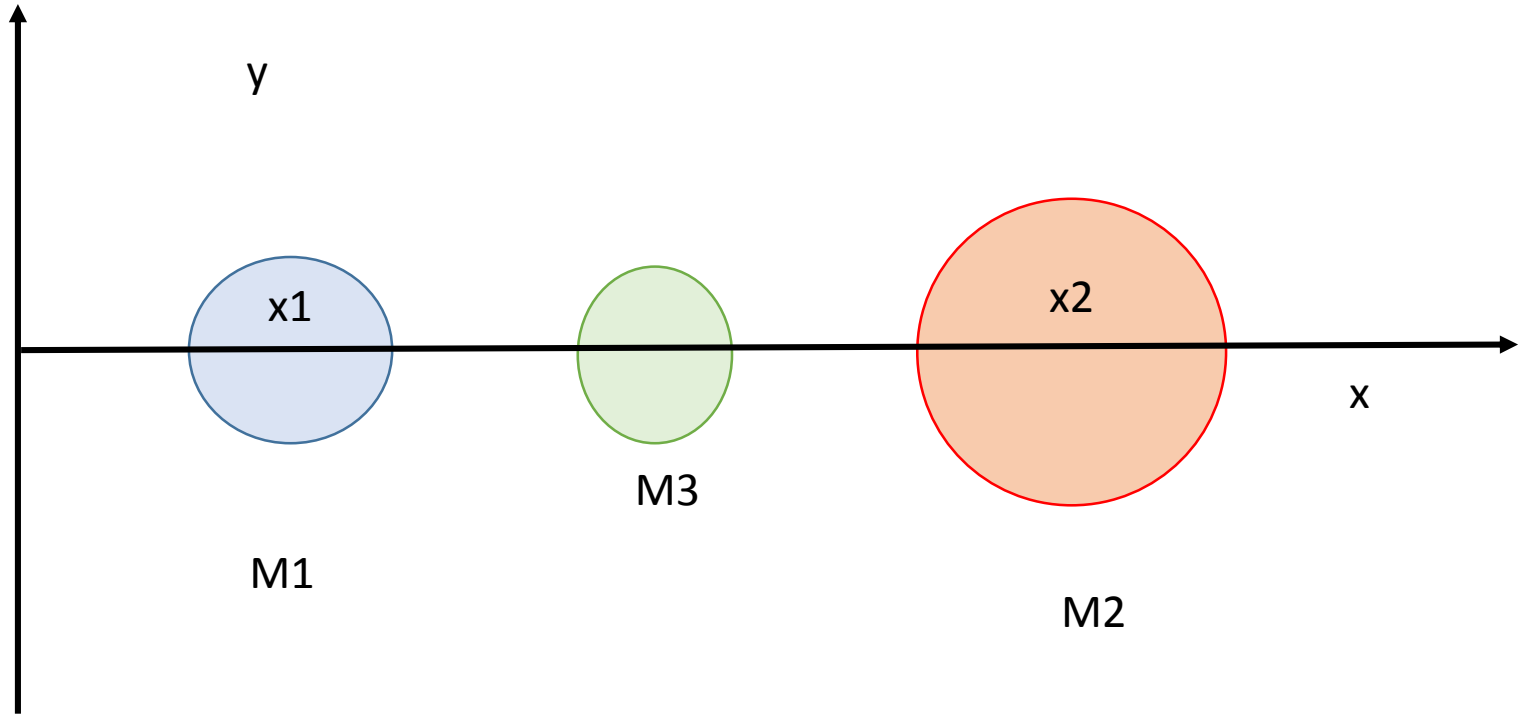
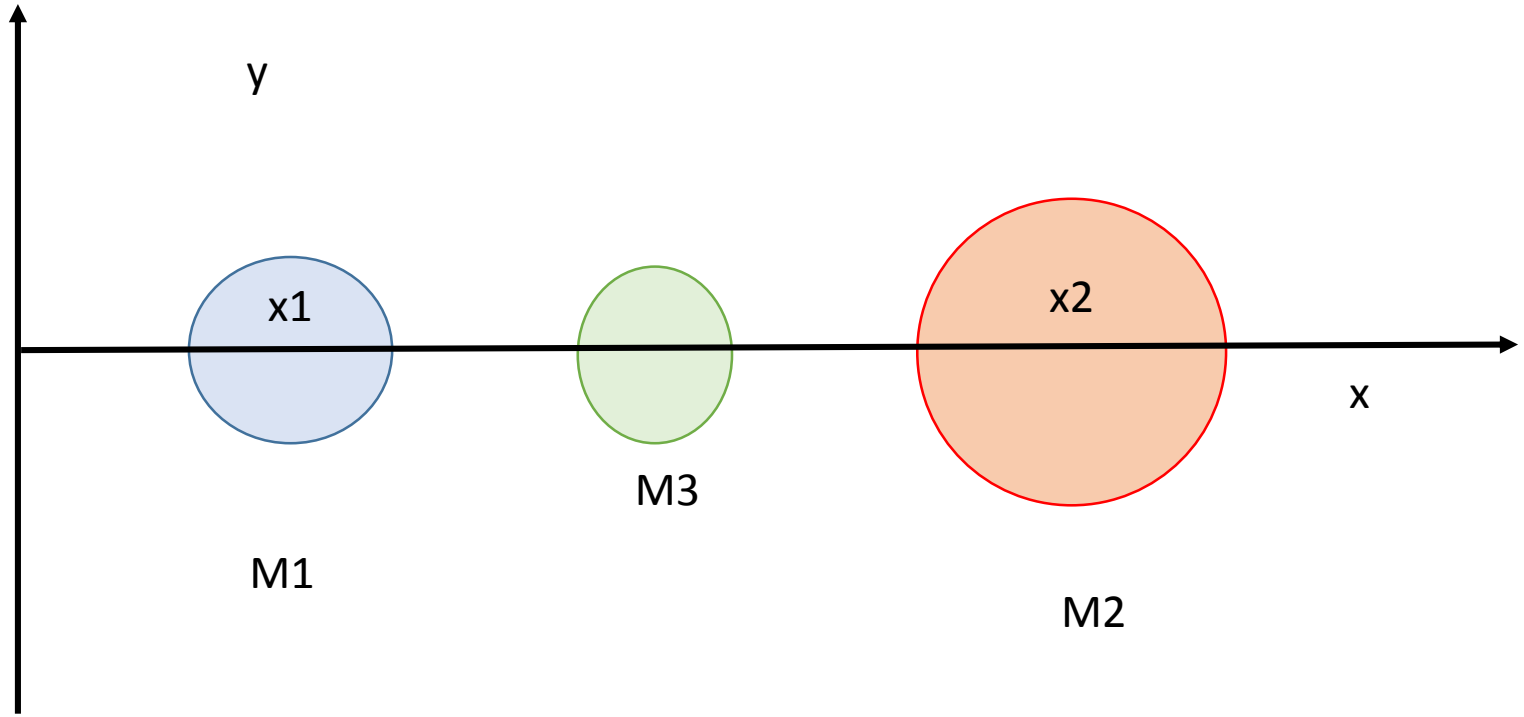


Considerando a massa total do sistema → M

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{M}$$

Sistemas de Partículas



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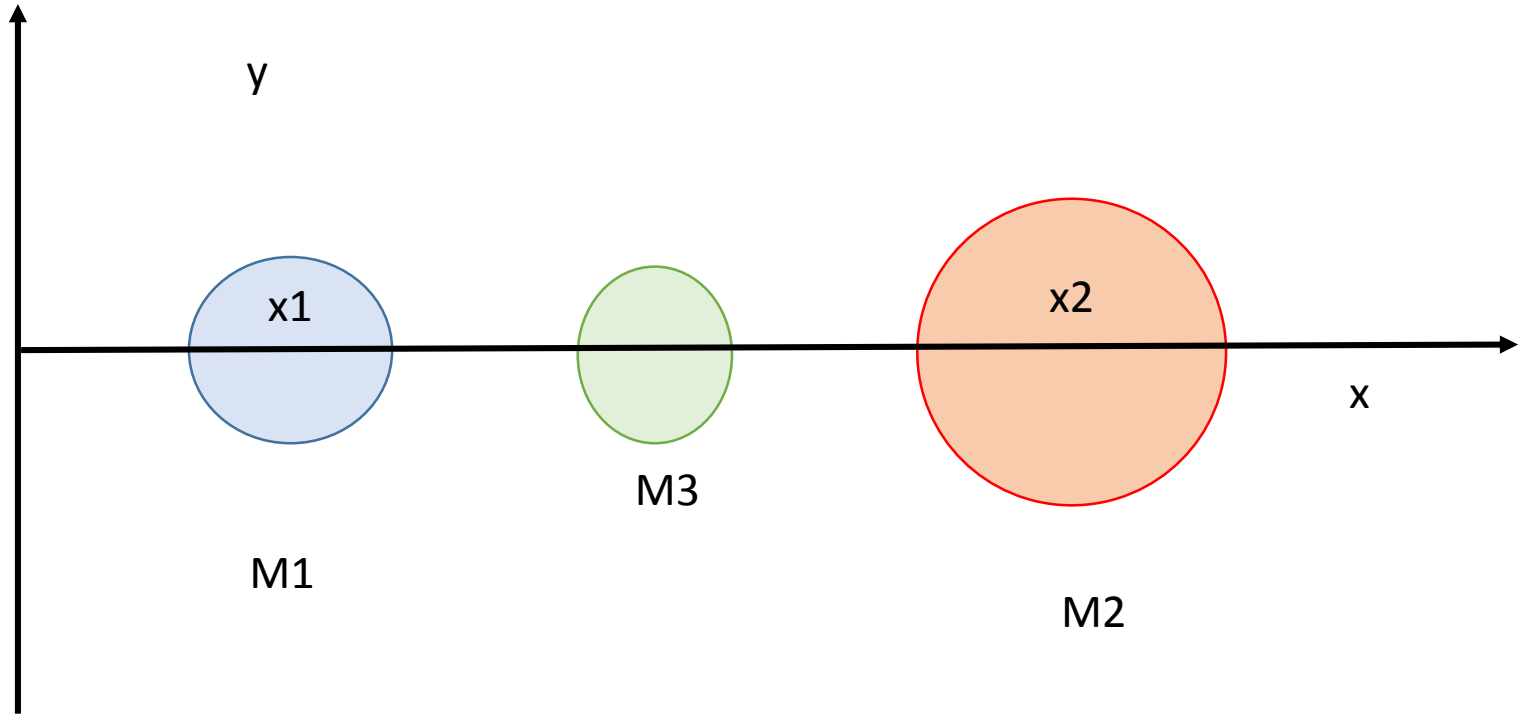
Sistemas de Partículas



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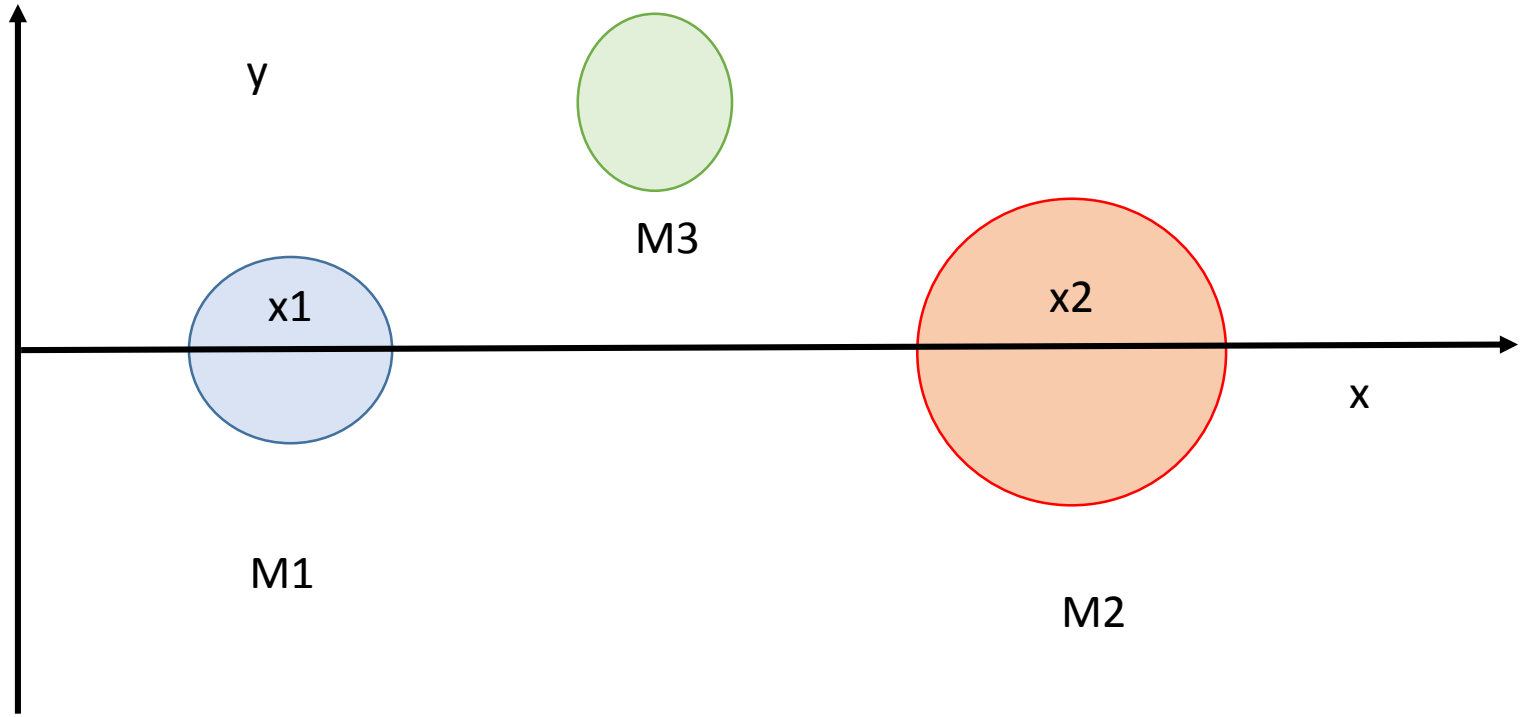
Sistemas de Partículas em 3 D



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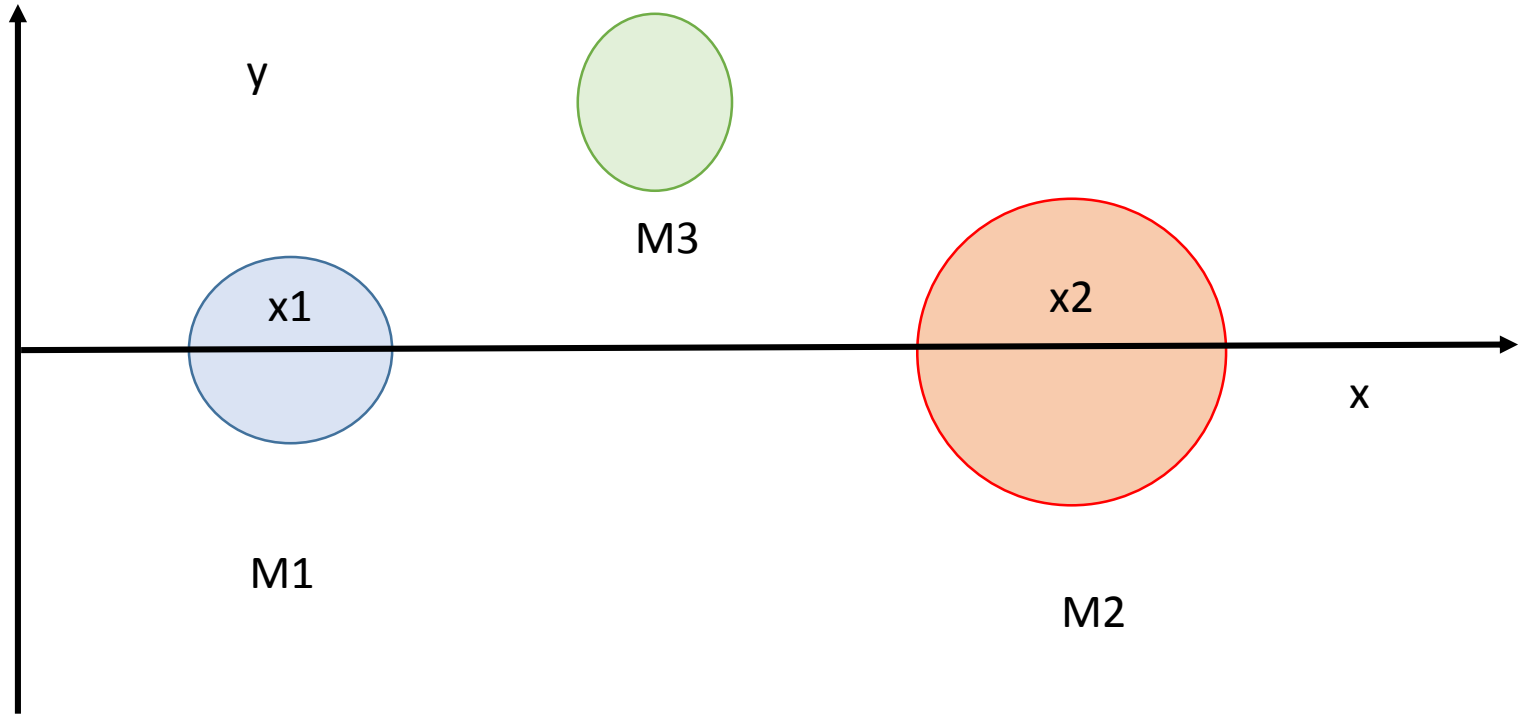

$$x_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \qquad y_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i \qquad z_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

Vetorialmente

$$\vec{r_i} = x_i\hat{\imath} + y_i\hat{\jmath} + z_i\hat{k}$$

$$\overrightarrow{r_{CM}} = x_{CM}\hat{\imath} + y_{CM}\hat{\jmath} + z_{CM}\hat{k}$$

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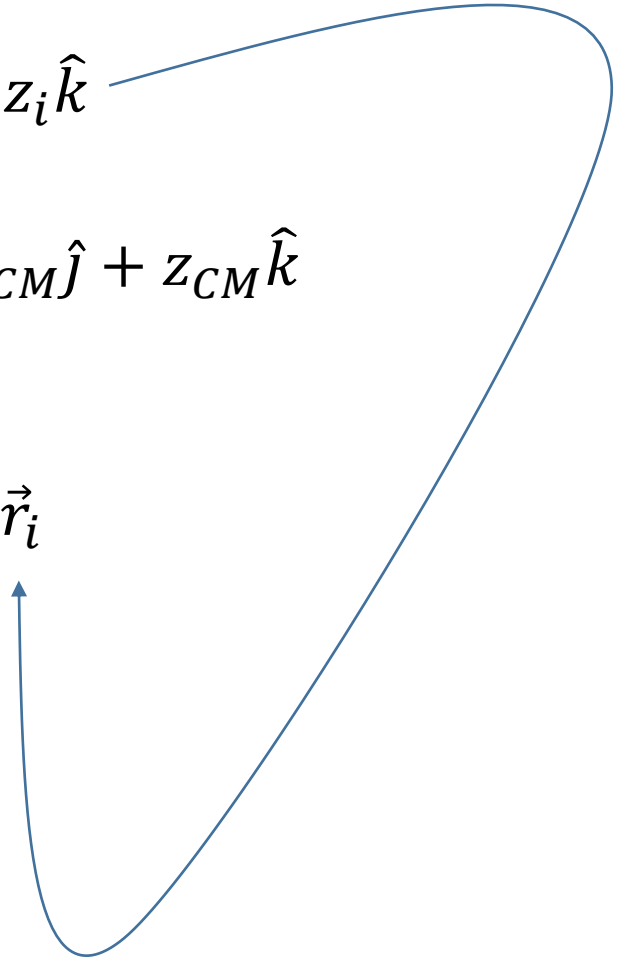

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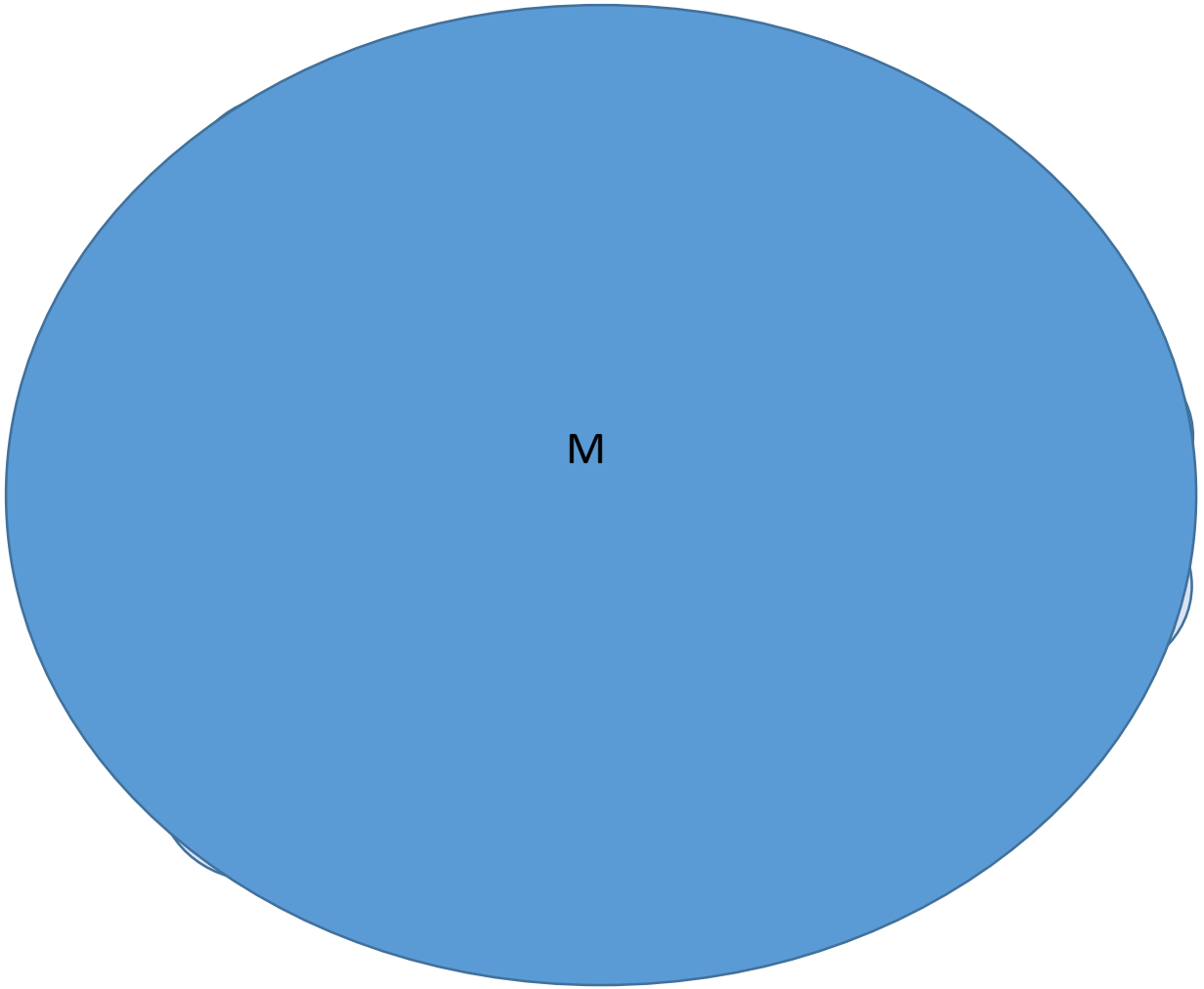

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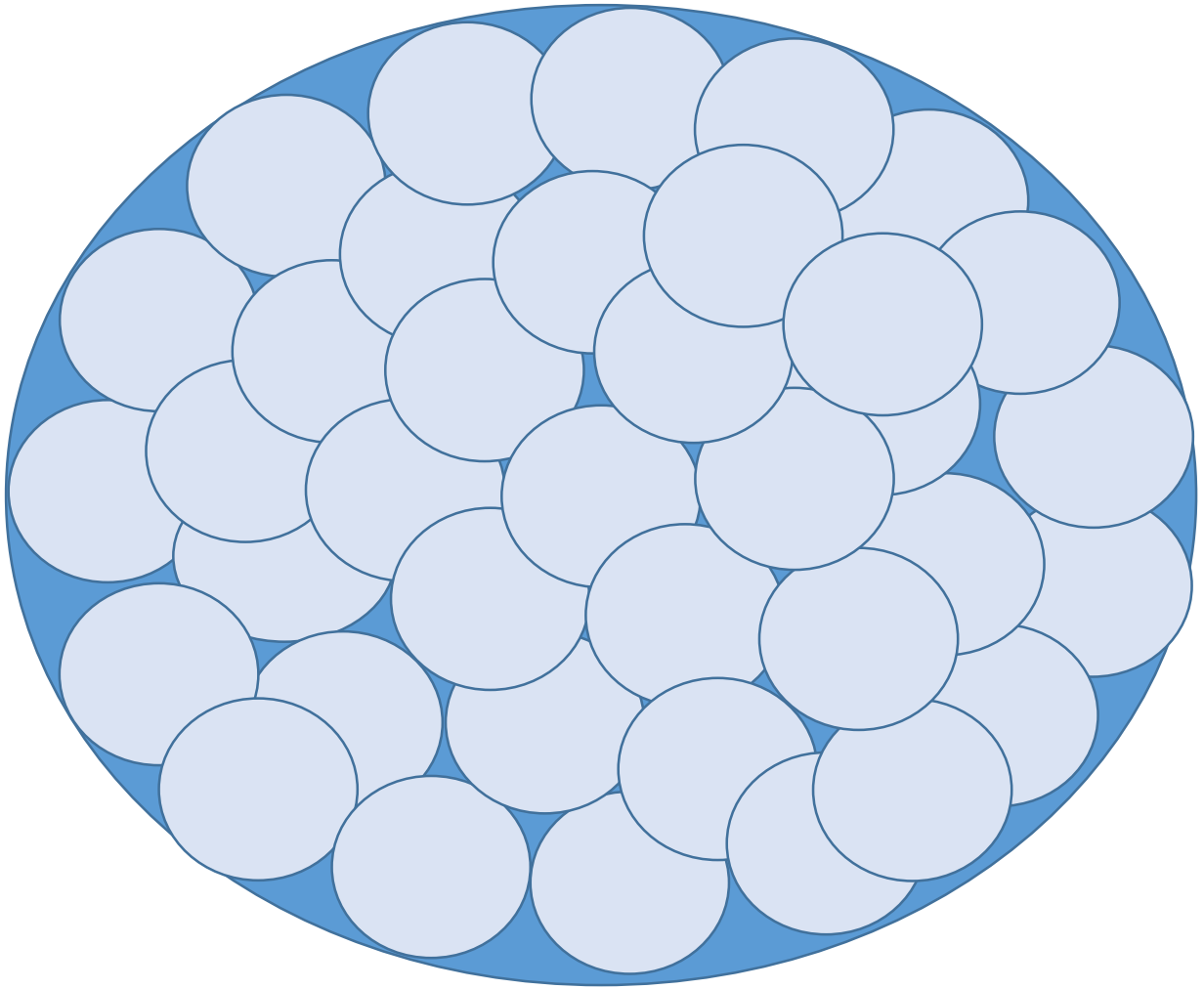
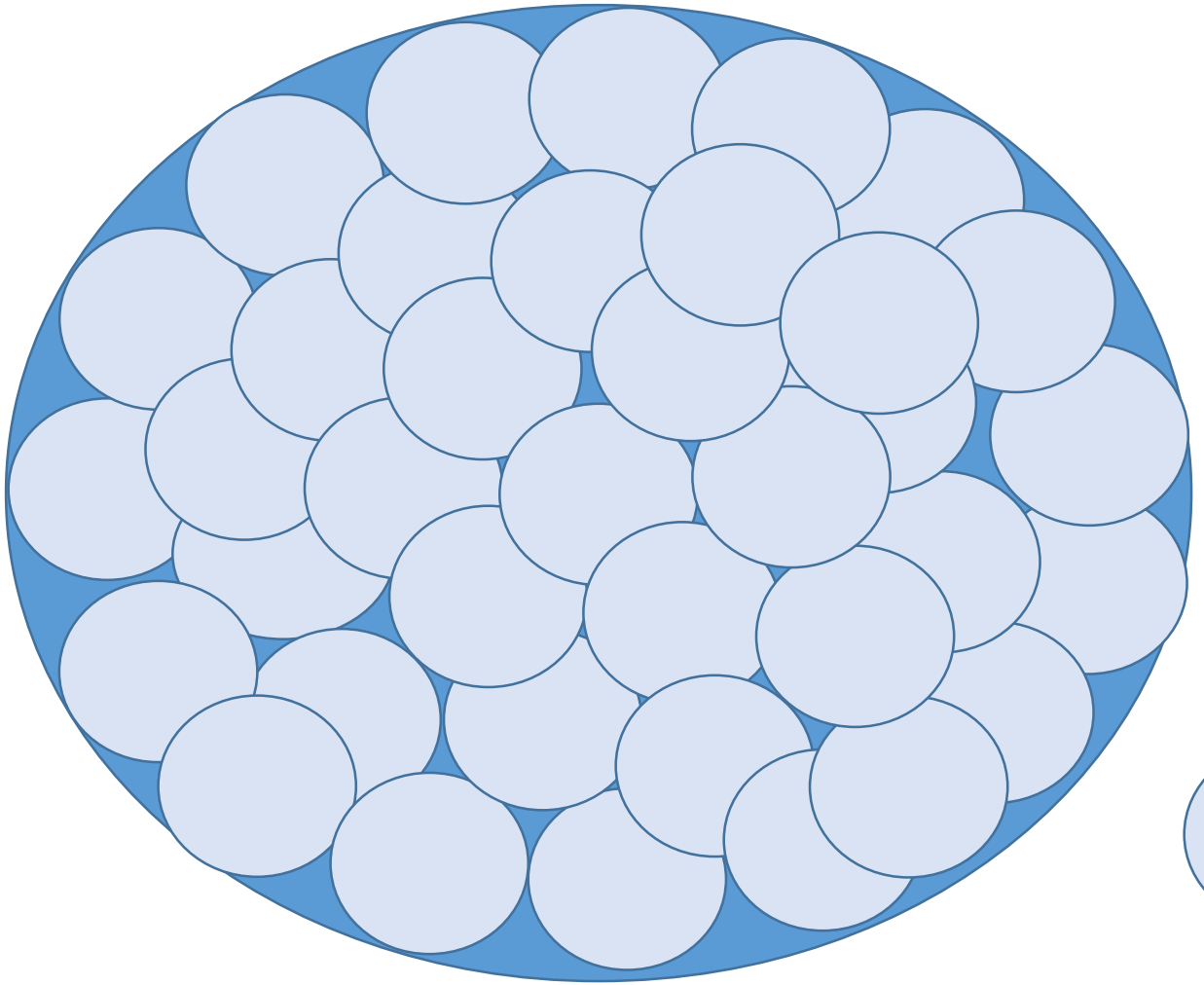


~distribuição contínua de átomos

Corpos Maciços



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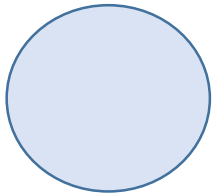
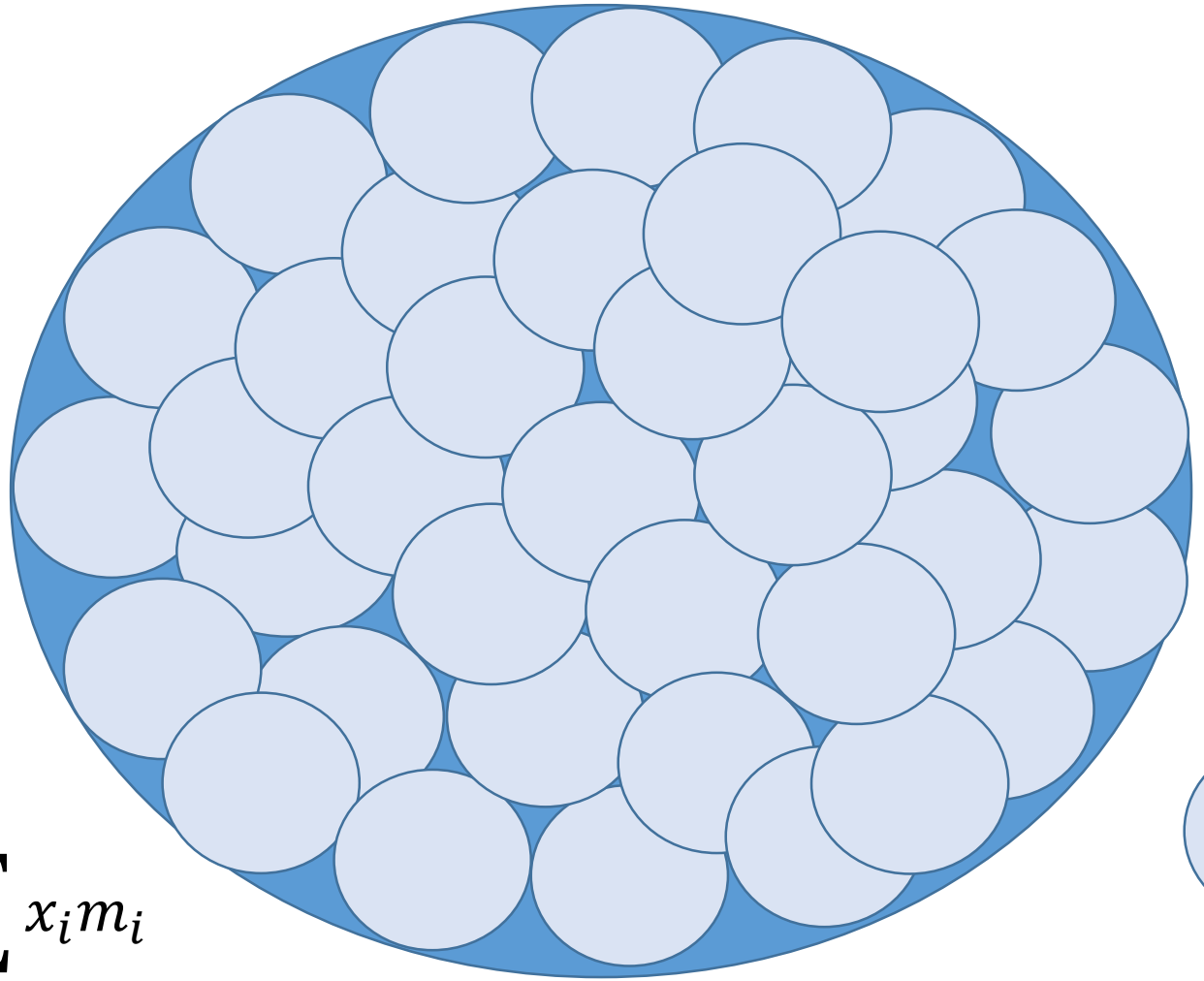


dm

Corpos Maciços



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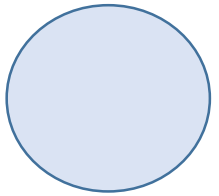
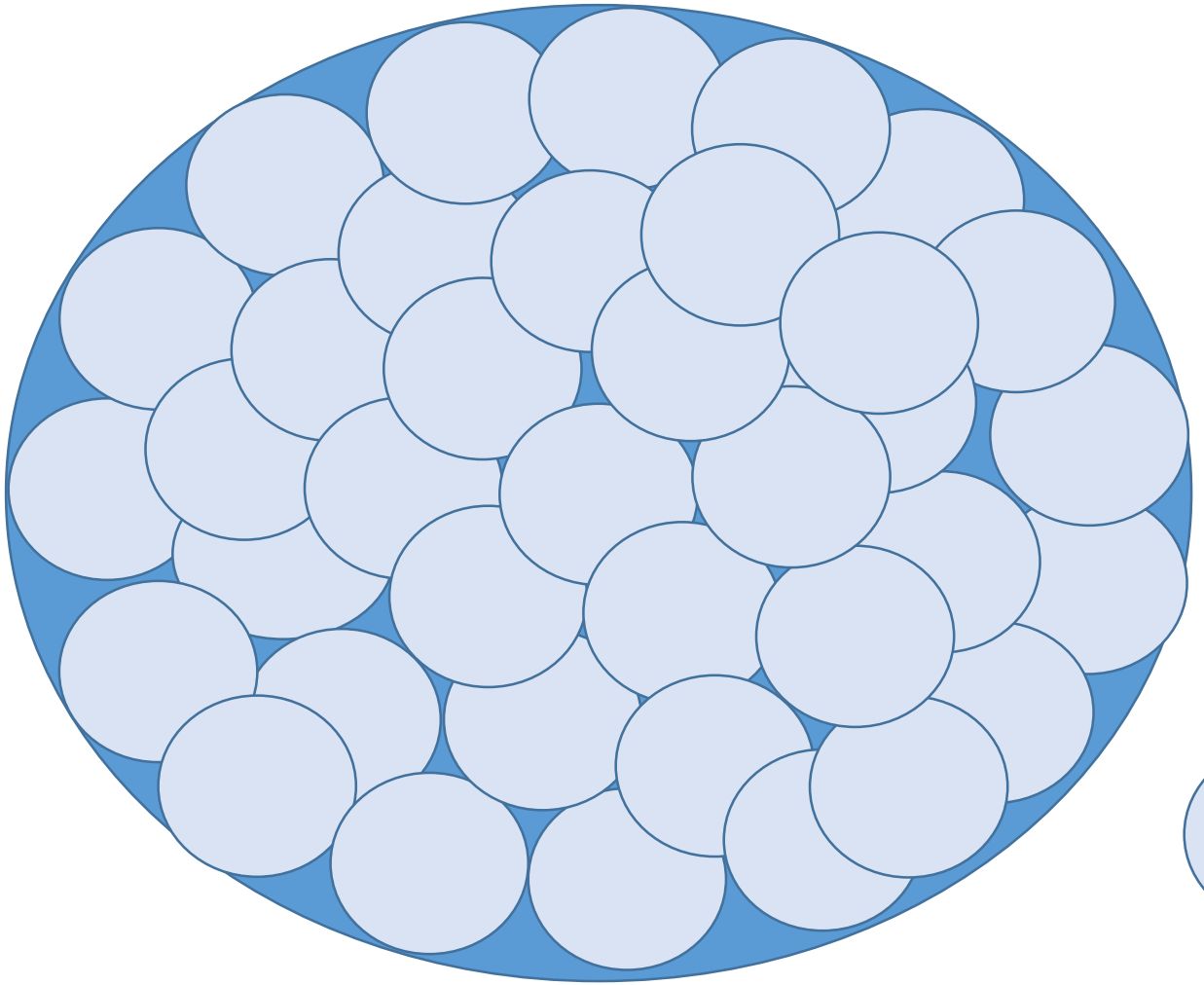


dm

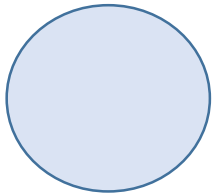
$$x_{CM} = \frac{1}{M}\sum_{i=1}^{n} x_i m_i$$

Corpos Maciços



~distribuição contínua de átomos

$$x_{CM} = \frac{1}{M}\int x dm$$



dm

Corpos Maciços

$$x_{CM} = \frac{1}{M}\int x dm$$

Corpos Maciços

$$x_{CM} = \frac{1}{M}\int x dm \qquad y_{CM} = \frac{1}{M}\int y dm \qquad z_{CM} = \frac{1}{M}\int z dm$$

Corpos Maciços

Qual o centro de massa?

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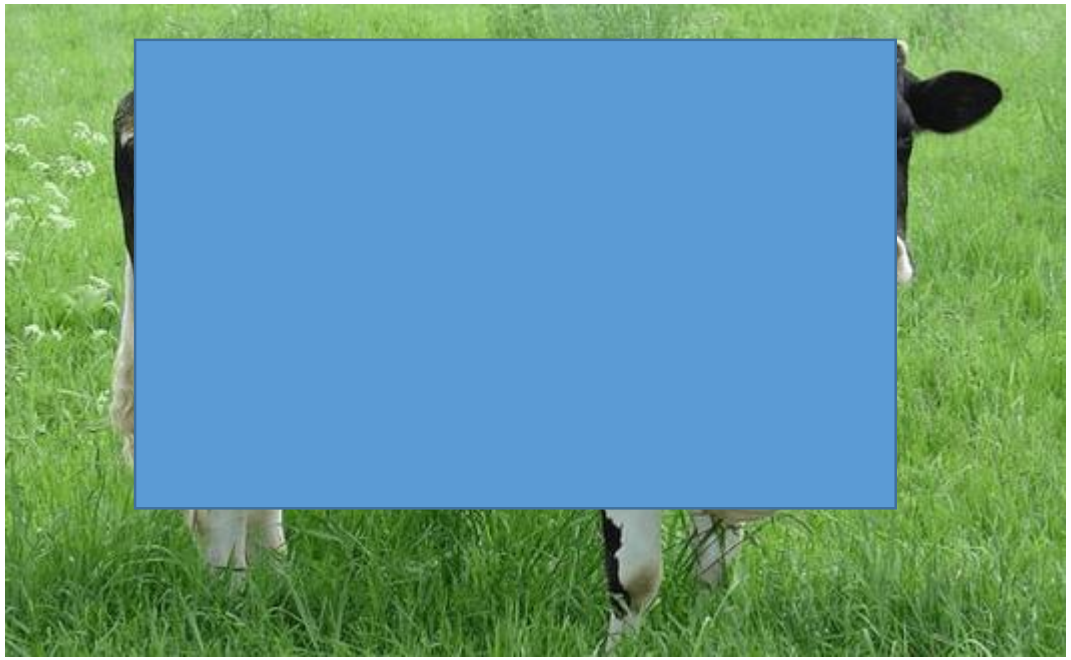

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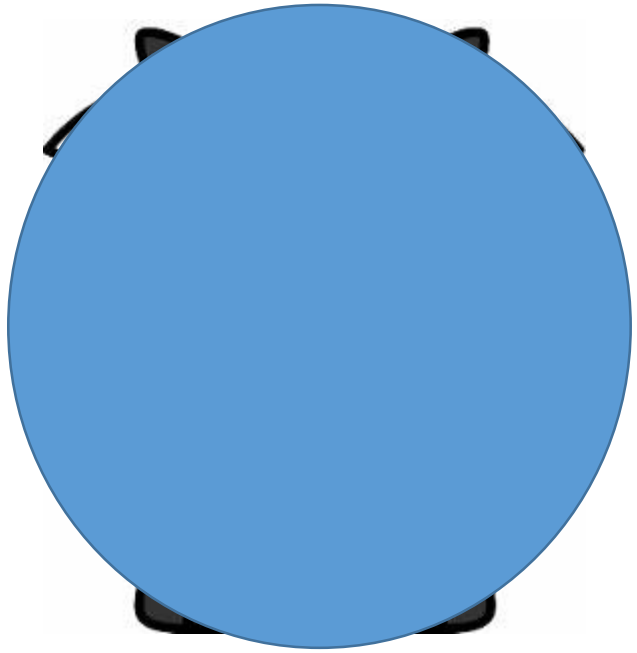

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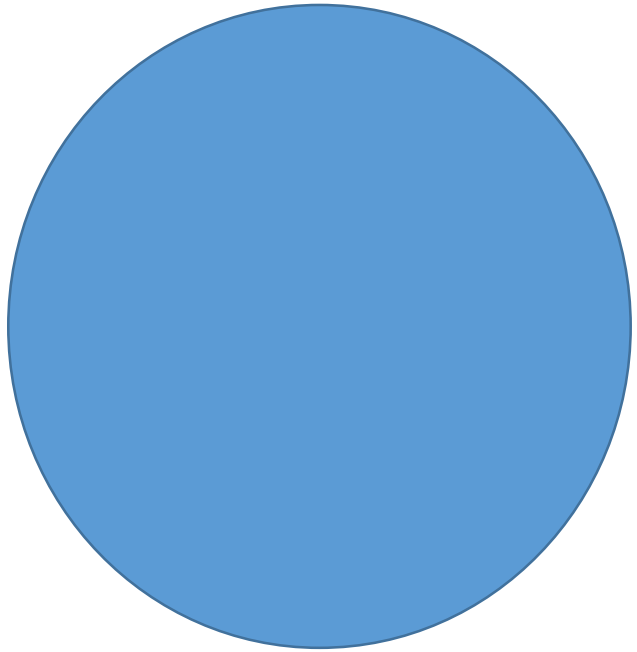
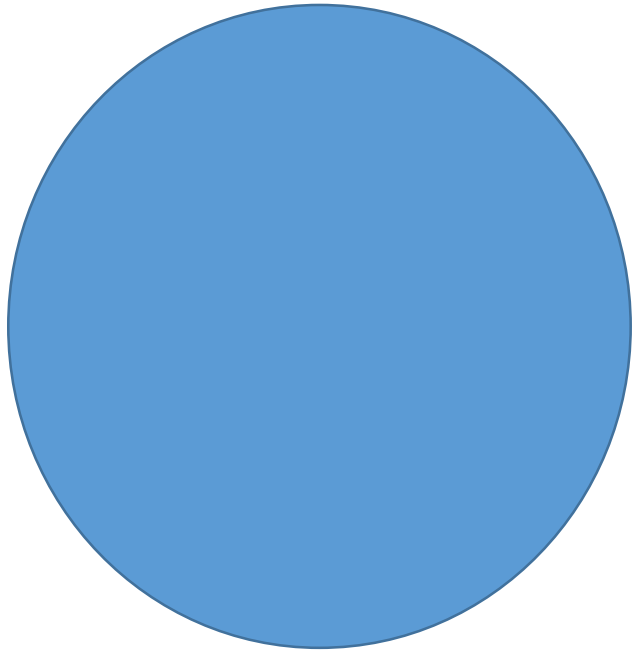


Qual o centro de massa?

Homogêneo → massa específica é a mesma para todo o objeto

Corpos Maciços



Qual o centro de massa?

Homogêneo → massa específica é a mesma para todo o objeto

Massa específica → $\rho = \frac{dm}{dV} = \frac{M}{V}$

Corpos Maciços

$$x_{CM} = \frac{1}{M}\int x dm \qquad y_{CM} = \frac{1}{M}\int y dm \qquad z_{CM} = \frac{1}{M}\int z dm$$

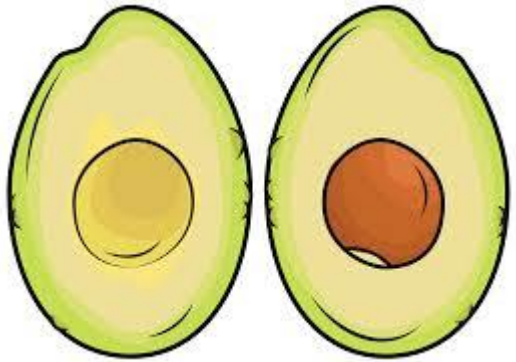
$$x_{CM} = \frac{1}{V}\int x dV \qquad y_{CM} = \frac{1}{V}\int y dV \qquad z_{CM} = \frac{1}{V}\int z dV$$

Simetria

O centro de massa está no ponto, linha ou plano de simetria

Esfera → ponto de simetria

Abacate → plano de simetria

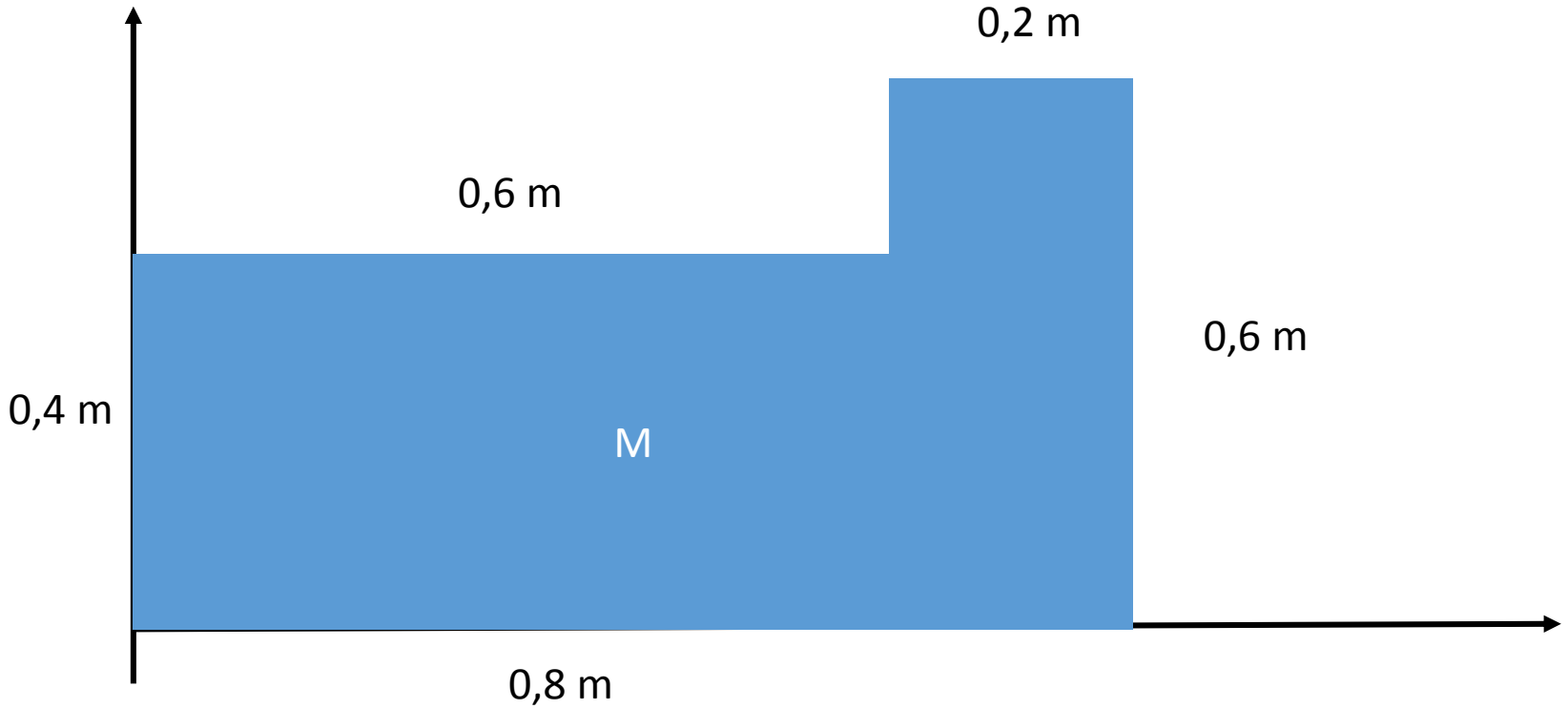


Simetria

Simetria

CM → Não tem massa!

Exemplo



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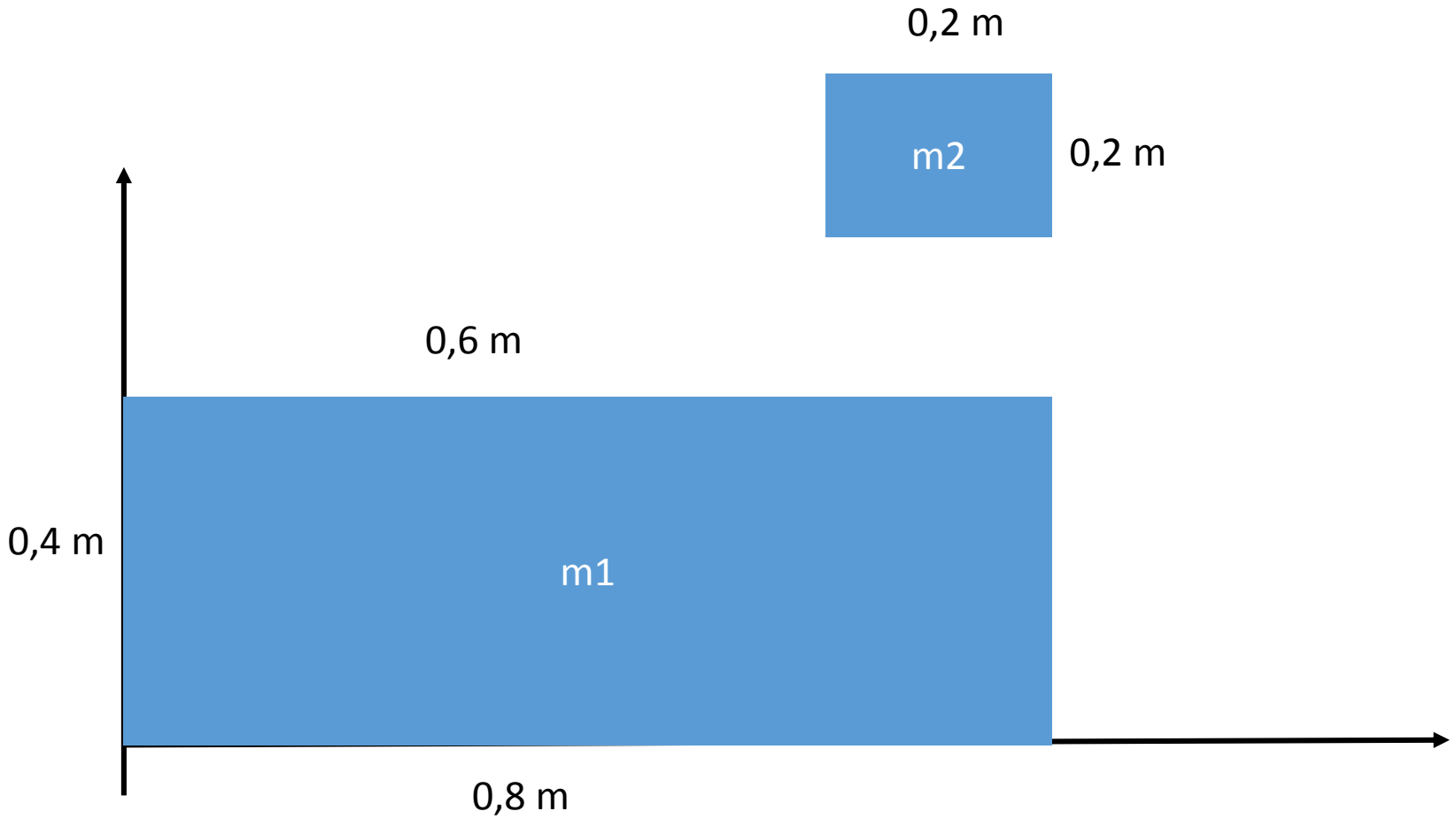

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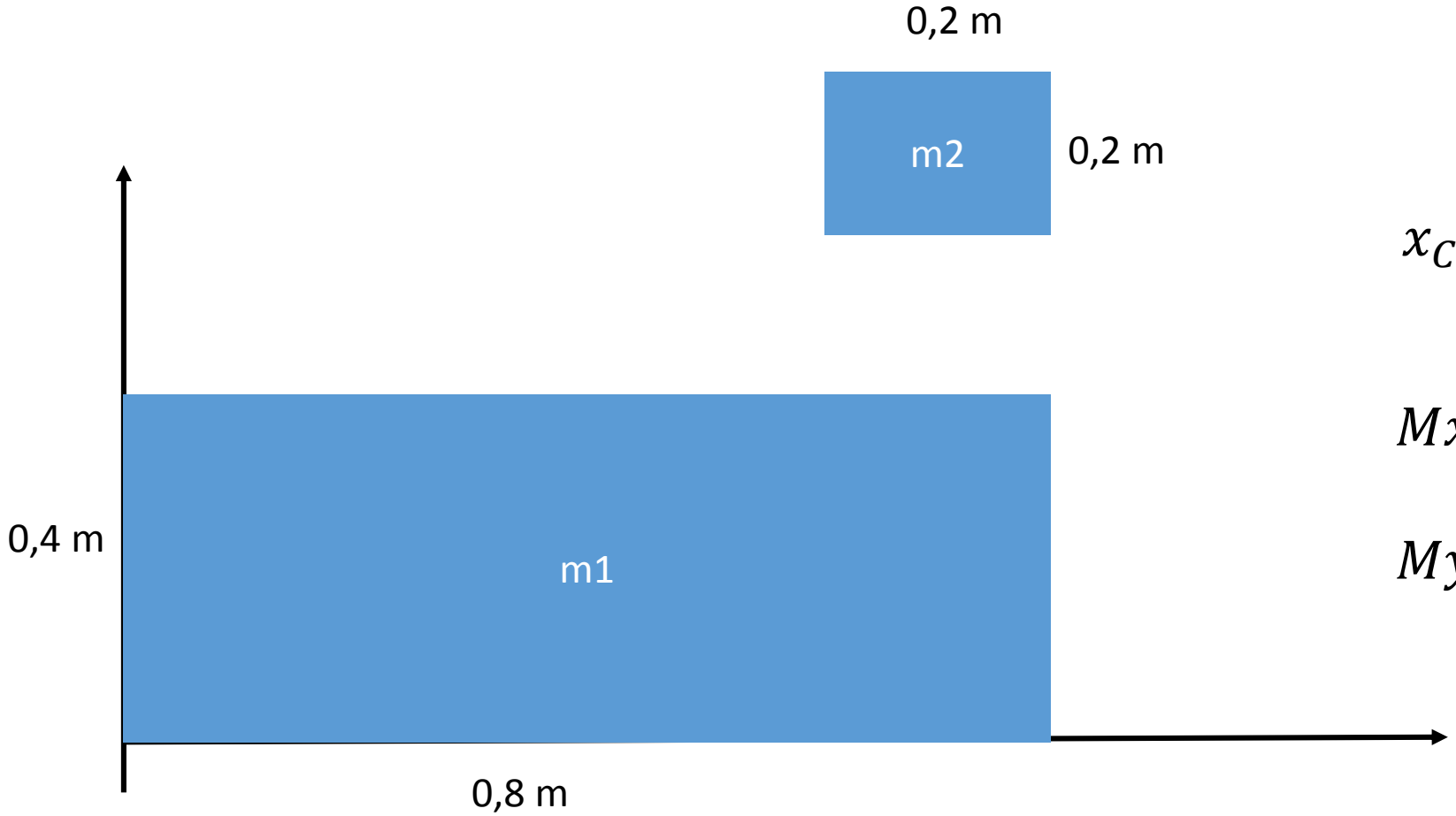
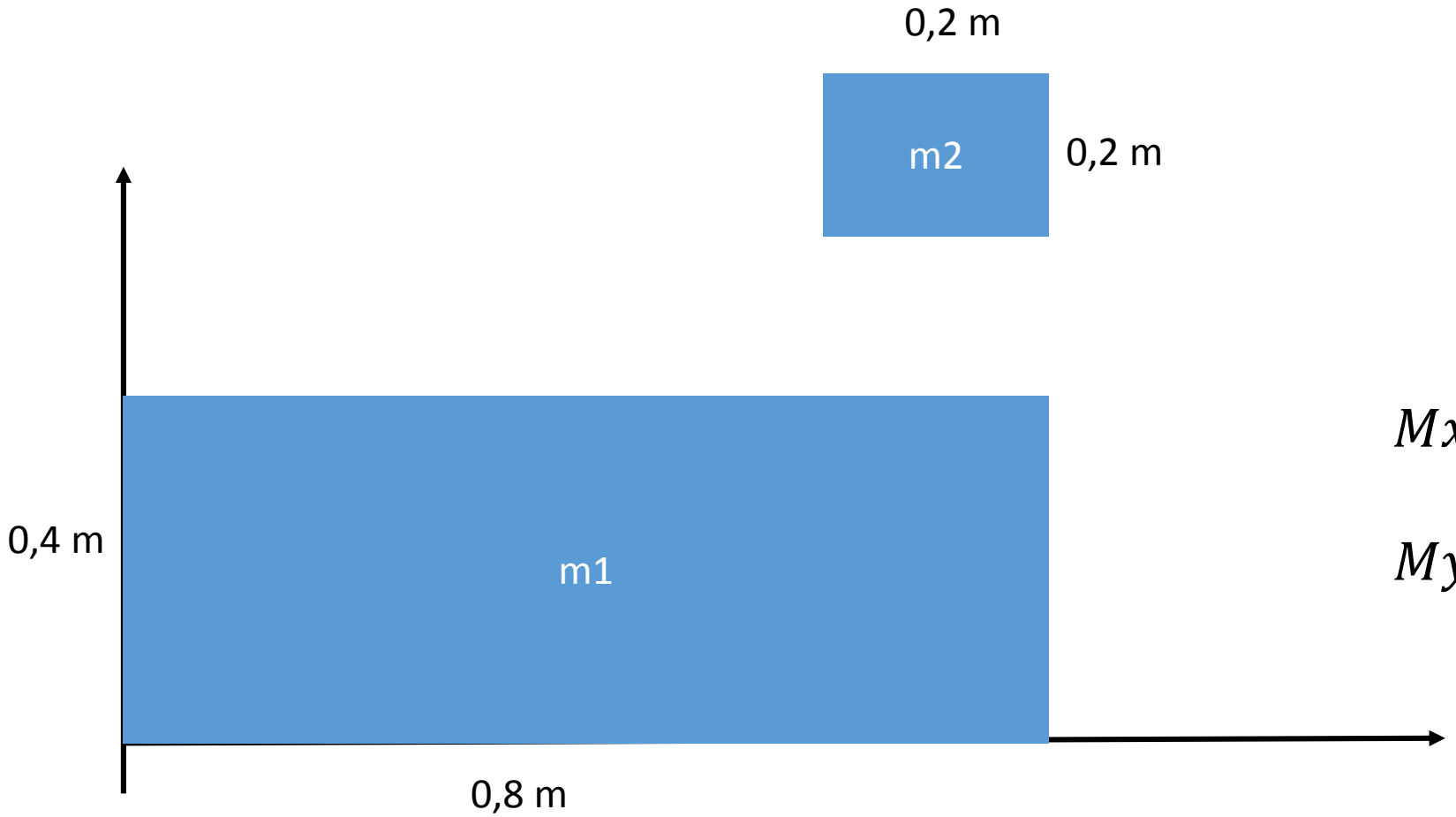


$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$M x_{CM} = m_1 x_1 + m_2 x_2$$

$$M y_{CM} = m_1 y_1 + m_2 y_2$$

Exemplo

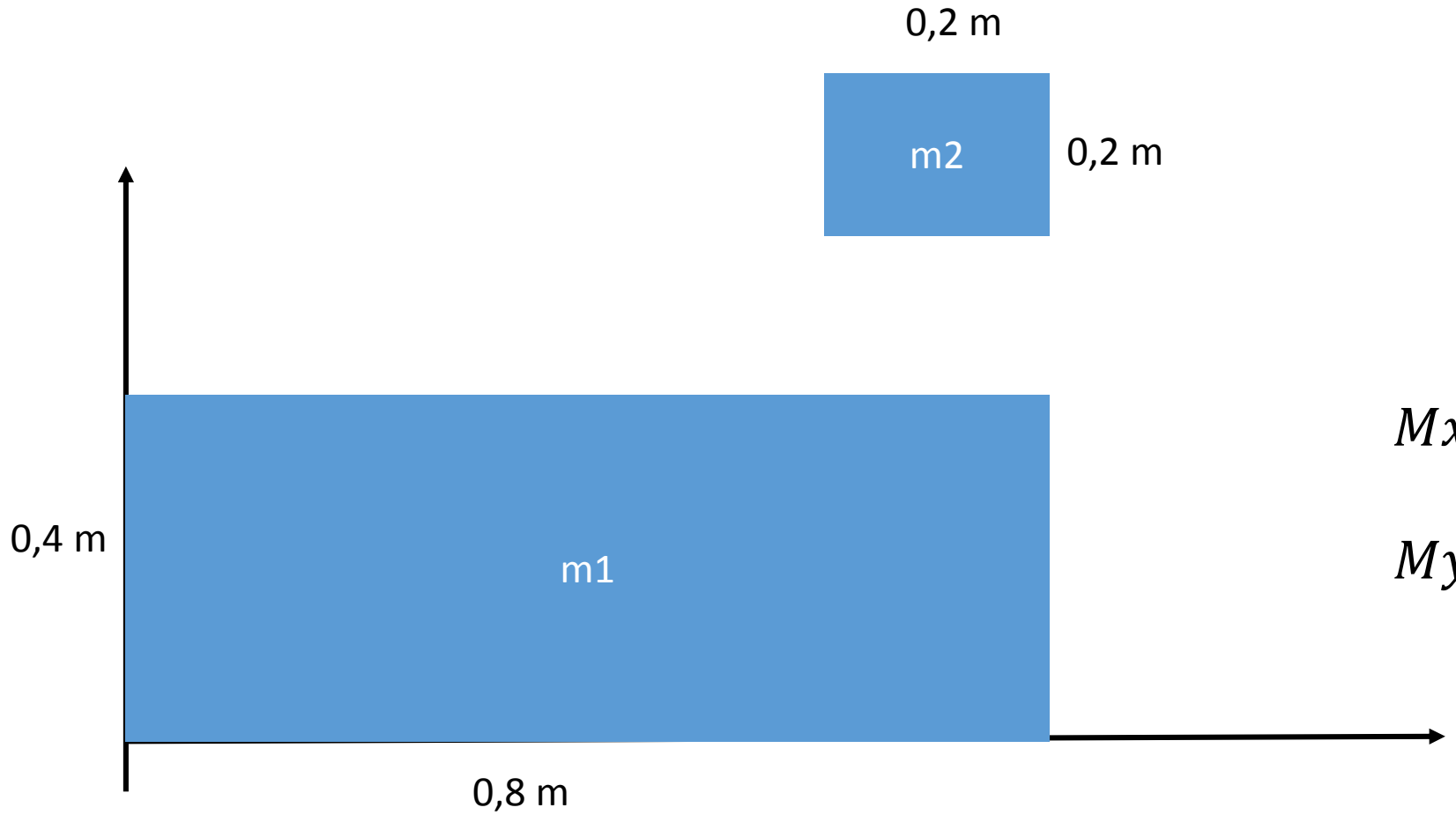


$$\frac{A_1}{A_2} = \frac{0{,}32}{0{,}04} = 8$$

$$Mx_{CM} = m_1x_1 + m_2x_2$$

$$My_{CM} = m_1y_1 + m_2y_2$$

Exemplo



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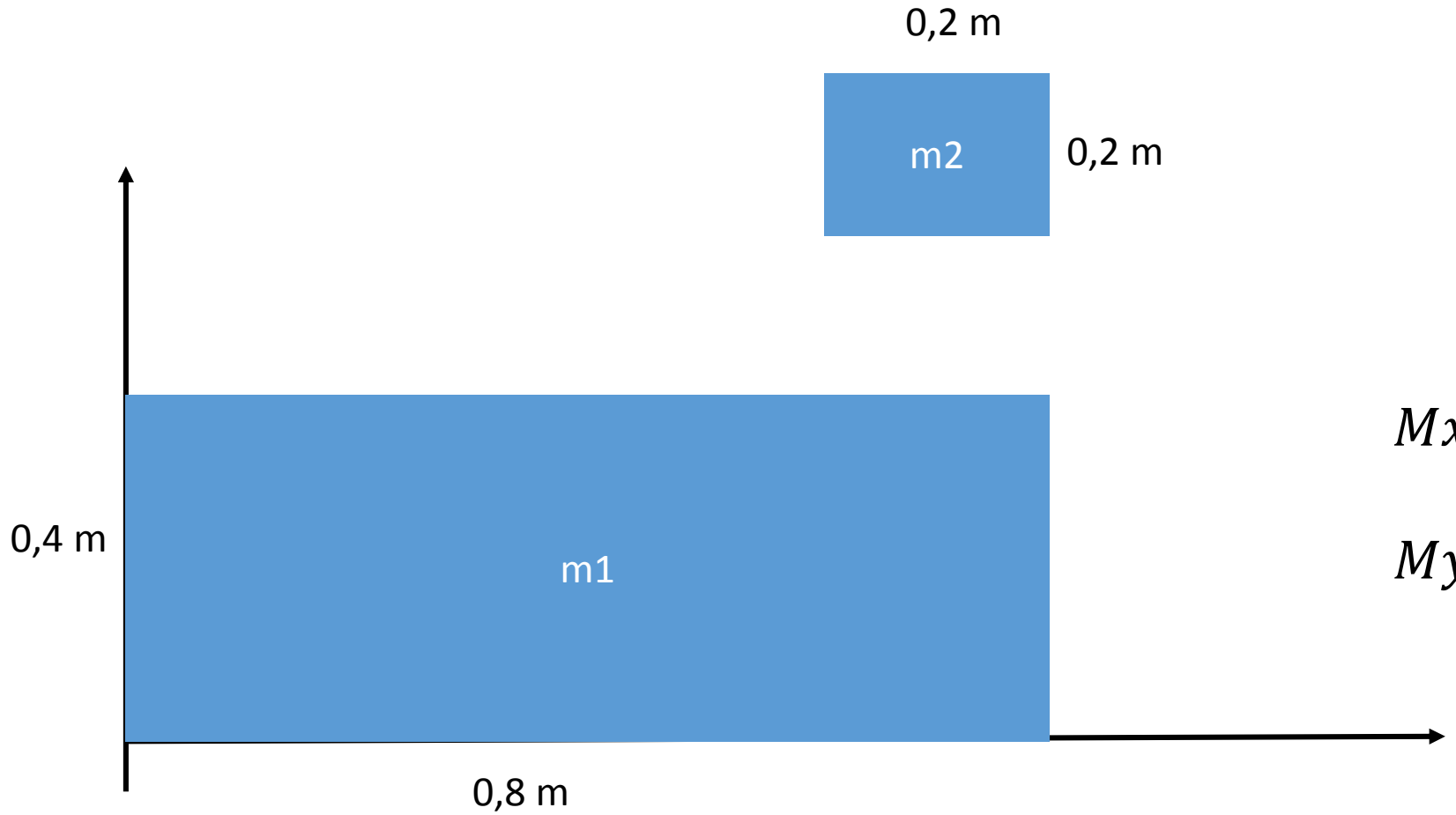
Objeto homogêneo: $\frac{A_1}{A_2} = \frac{m_1}{m_2} = 8 \rightarrow m_1 = 8m_2$

Exemplo



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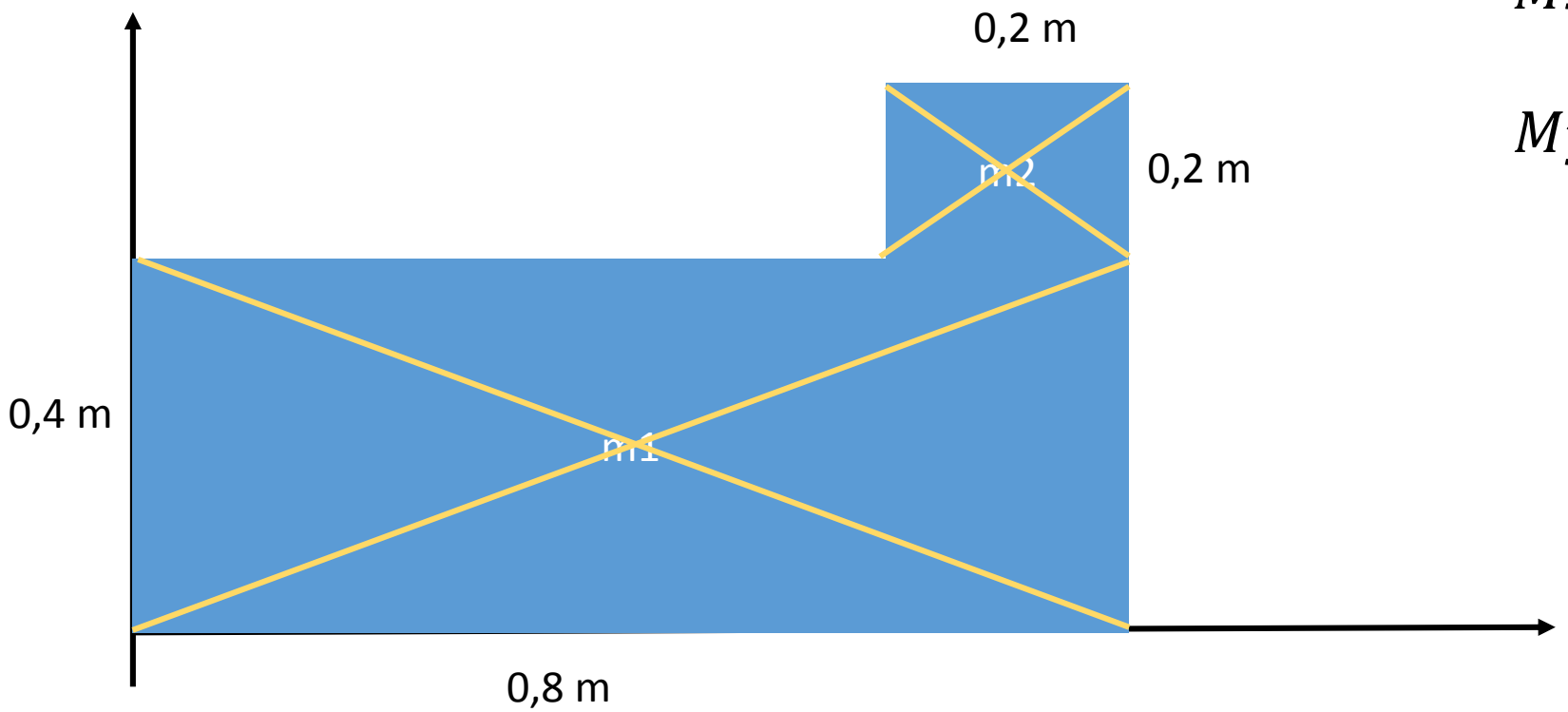
Objeto homogêneo: $\frac{A_1}{A_2} = \frac{m_1}{m_2} = 8 \rightarrow m_1 = 8m_2$

$$M = m_1 + m_2 = 9m_2$$

Exemplo

$$m_1 = 8m_2$$

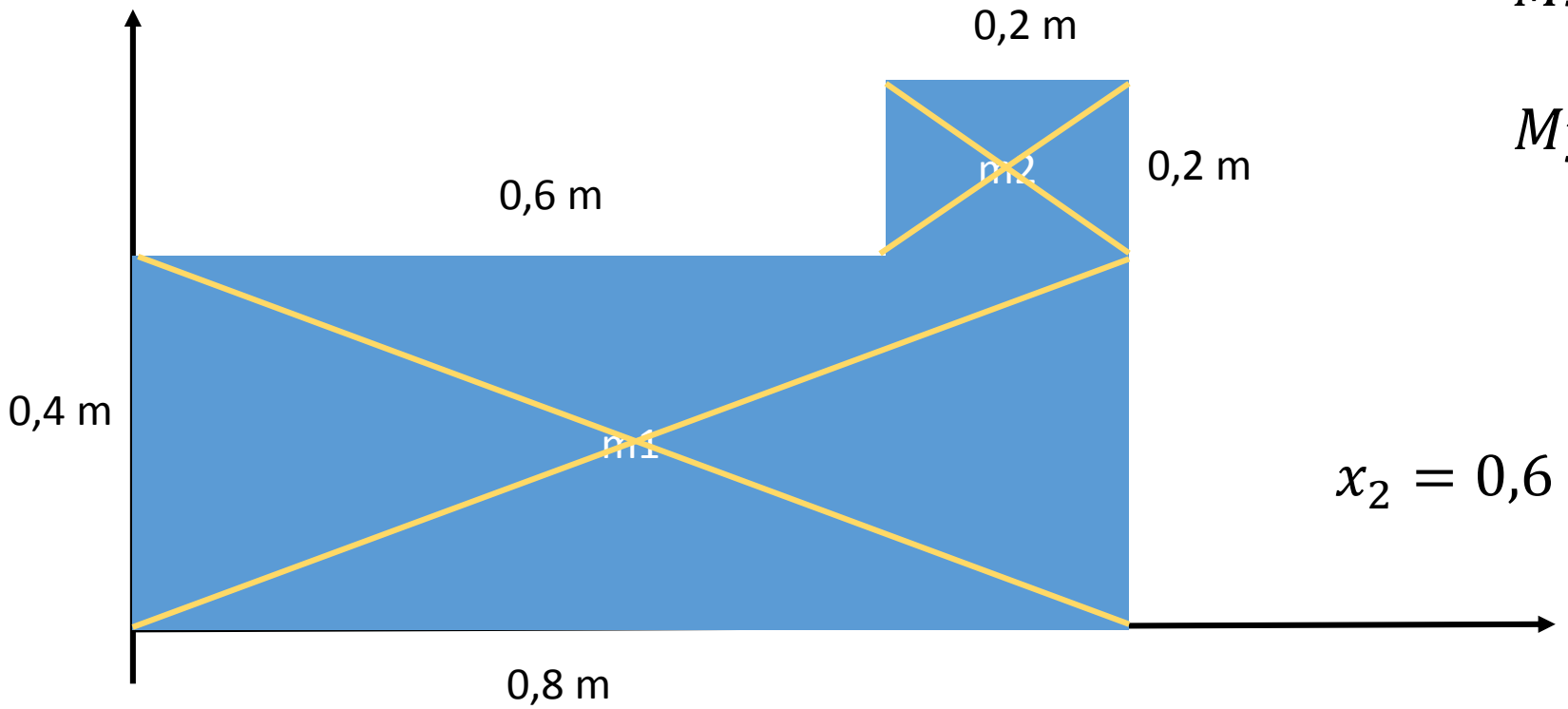
$$M = 9m_2$$



$$Mx_{CM} = m_1x_1 + m_2x_2$$

$$My_{CM} = m_1y_1 + m_2y_2$$

Exemplo



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$$x_1 = 0{,}4 \qquad y_1 = 0{,}2$$

$$x_2 = 0{,}6 + 0{,}1 = 0{,}7 \qquad y_2 = 0{,}5$$

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$$My_{CM} = m_1y_1 + m_2y_2$$

$$9m_2y_{CM} = 8m_20{,}2 + m_20{,}5$$

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$$My_{CM} = m_1y_1 + m_2y_2$$

$$9m_2y_{CM} = 8m_20{,}2 + m_20{,}5$$

$$y_{CM} = \frac{1{,}6 + 0{,}5}{9} = \frac{3{,}9}{9} = 0{,}233\ m$$

Exemplo

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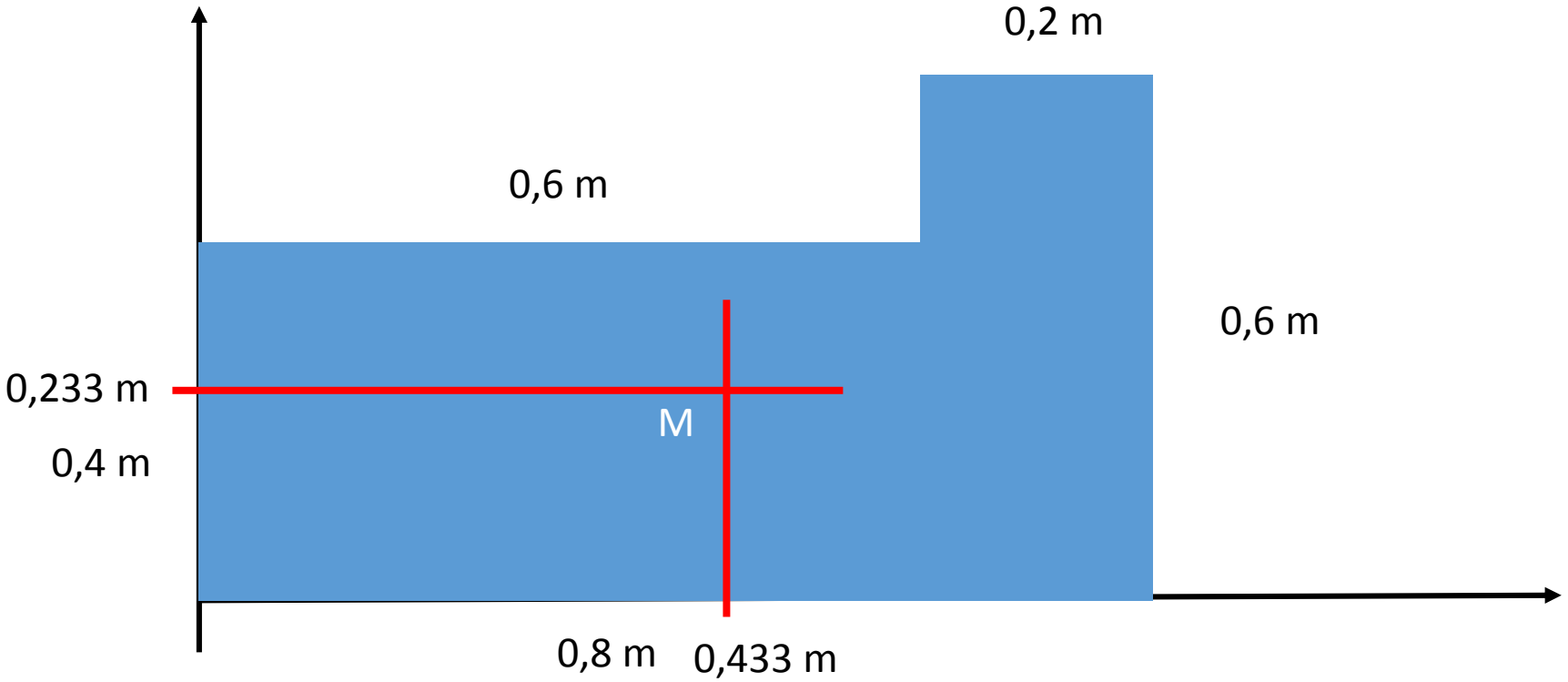
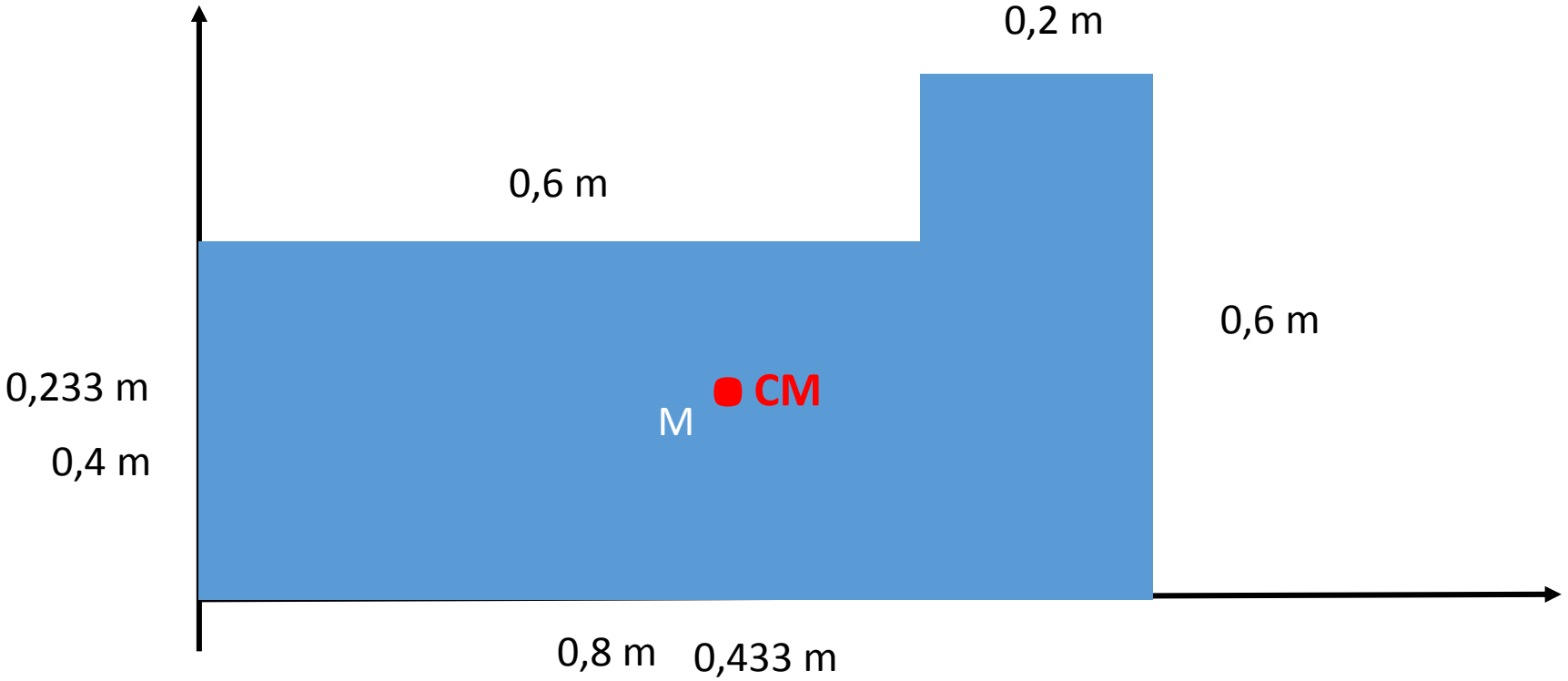
$$y_{CM} = 0{,}233\ m$$



Exemplo

$$x_{CM} = 0{,}433\ m$$

$$y_{CM} = 0{,}233\ m$$



Exemplo 2

Massa M