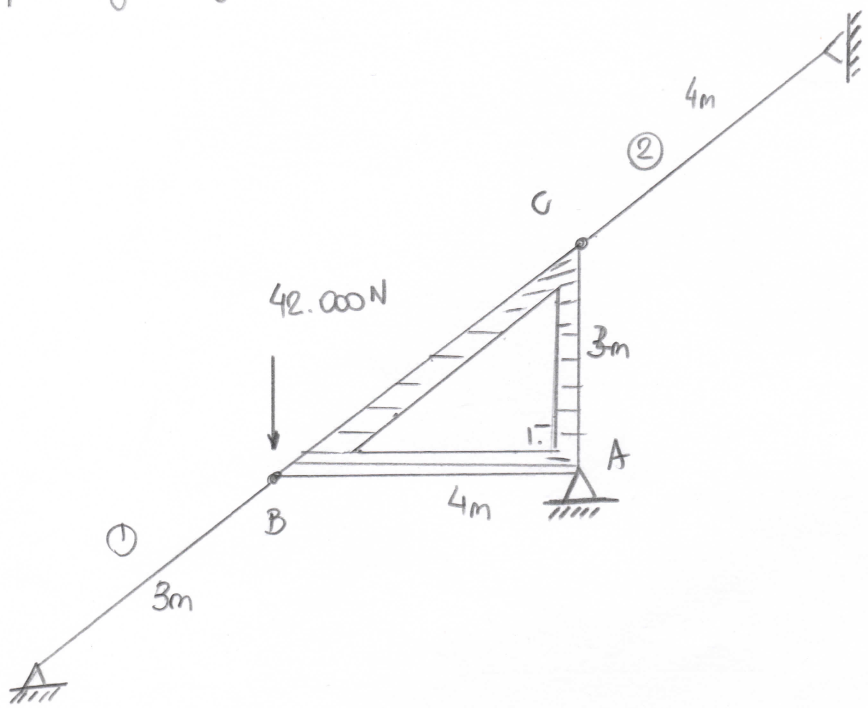
Exercício 19 (lista H. Britto)

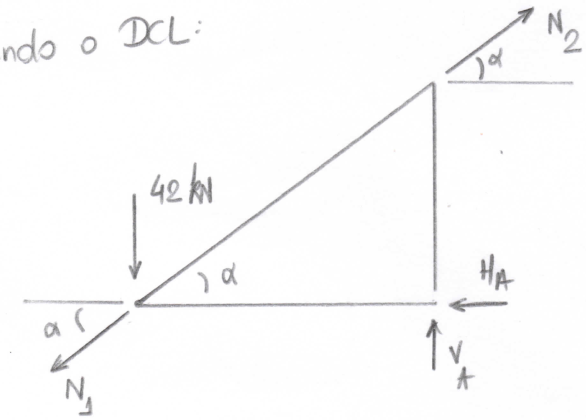
_ barras $E = 10^3 MPa$, $\bar{\sigma} = 40MPa$, mesma ST de área $A = ?$

- $v_B \leq 0,1m$

- chapa triangular rígida.



Montando o DCL:



$\Sigma F_H = 0$: $-N_1 \cos\alpha - H_A + N_2 \cos\alpha = 0$ (I)

$\Sigma F_V = 0$: $-N_1 \operatorname{sen}\alpha + V_A - 42 + N_2 \operatorname{sen}\alpha = 0$ (II)

$\Sigma M_B = 0$: $\boxed{V_A = 0}$

De (I): $N_2 - N_1 = \dfrac{H_A}{\cos\alpha}$

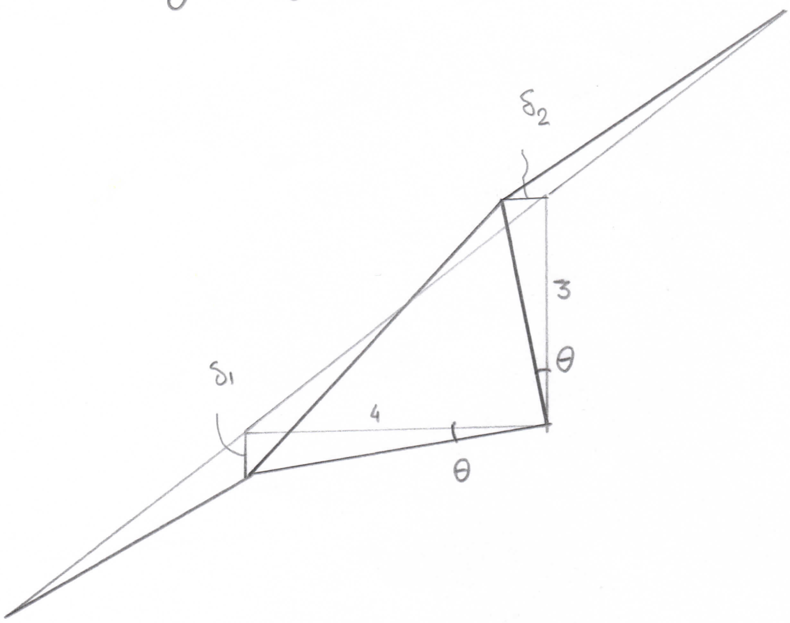
De (II): $N_2 - N_1 = \dfrac{42 - V_A}{\operatorname{sen}\alpha}$

$$\frac{H_A}{\cos\alpha} = \frac{42 - V_A}{\operatorname{sen}\alpha}$$

$H_A = \dfrac{42}{tg\alpha}$ como $\operatorname{sen}\alpha = 3/5$, $\cos\alpha = 4/5$ e $tg\alpha = 3/4$:

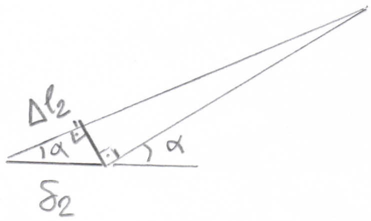
$H_A = 42 \cdot \dfrac{4}{3} \Rightarrow \boxed{H_A = 48 kN}$

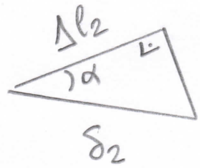
Precisamos estabelecer a compatibilidade entre N_1 e N_2. Considerando a estrutura deformada (ou seja, um giro θ em A):



Percebemos que $tg\,\theta = \frac{|\delta_1|}{4} = \frac{|\delta_2|}{3}$.

Observando o tendão ②:

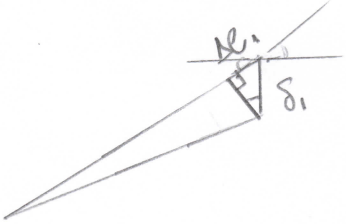


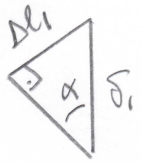


$$\cos\alpha = \frac{\Delta l_2}{\delta_2}$$

$$\therefore \delta_2 = \frac{\Delta l_2}{\cos\alpha}$$

Observando o tendão ①:





$$\operatorname{sen}\alpha = \frac{\Delta l_1}{\delta_1}$$

$$\delta_1 = \frac{\Delta l_1}{\operatorname{sen}\alpha}$$

$$\therefore \frac{|\Delta l_1|}{4\operatorname{sen}\alpha} = \frac{|\Delta l_2|}{3\cos\alpha} \Rightarrow |\Delta l_1| = |\Delta l_2|$$

Vemos que como a chapa gira no sentido anti-horário, o tendão ① encurta e o tendão ② estica.

Assim:

$$\Delta\ell_1 = -\Delta\ell_2$$

Como $\Delta\ell = \dfrac{N\ell}{EA}$, logo:

$$-\frac{N_1\ell_1}{EA} = \frac{N_2\ell_2}{EA} \quad e \quad -N_1 \cdot 3 = N_2 \cdot 4 \Rightarrow N_1 = -\frac{4}{3}N_2$$

Substituindo em II:

$$\frac{4N_2}{3}\operatorname{sen}\alpha + 0 - 42 + N_2\operatorname{sen}\alpha = 0$$

$$\frac{7}{3}N_2\operatorname{sen}\alpha = 42 \Rightarrow N_2 = \frac{18}{\operatorname{sen}\alpha} \Rightarrow N_2 = \frac{18}{3/5} \Rightarrow \boxed{N_2 = 30\ kN}$$

$$\boxed{N_1 = -40\ kN}$$

Considerando o caso de tensaS máxima:

$$\sigma = \frac{N}{A} \leq \bar{\sigma} \qquad \text{como } |N_2| < |N_1|:$$

$$\frac{|N_1|}{A} \leq \bar{\sigma} \Rightarrow \frac{A}{|N_1|} \geq \frac{1}{\bar{\sigma}} \Rightarrow A \geq \frac{|N_1|}{\bar{\sigma}}$$

$$A \geq \frac{40 \cdot 10^3}{40 \cdot 10^6} \Rightarrow \boxed{A \geq 10^{-3}\ m^2}$$

Considerando o deslocamento do ponto B:

$$V_B = \delta_1 = \frac{\Delta\ell_1}{\operatorname{sen}\alpha} \leq \bar{V}$$

$$\frac{N_1\ell_1}{EA} \cdot \frac{1}{\operatorname{sen}\alpha} \leq \bar{V} \qquad \frac{N_1\ell_1}{A} \leq E\bar{V}\operatorname{sen}\alpha$$

$$\frac{A}{|N_1|l_1} \geq \frac{1}{E\bar{v}\,\text{sen}\,\alpha} \Rightarrow A \geq \frac{|N_1|l_1}{E\bar{v}\,\text{sen}\,\alpha}$$, $E = 10^3 MPa = 10^9 MPa$

$$A \geq \frac{40.10^3 . 3}{10^9 . 0{,}1 . 3/5} \Rightarrow \boxed{A \geq 2.10^{-3} m^2}$$

Logo, a área deve ser $\boxed{A = 20.10^{-4} m^2}$

Problemas (pg. 42)

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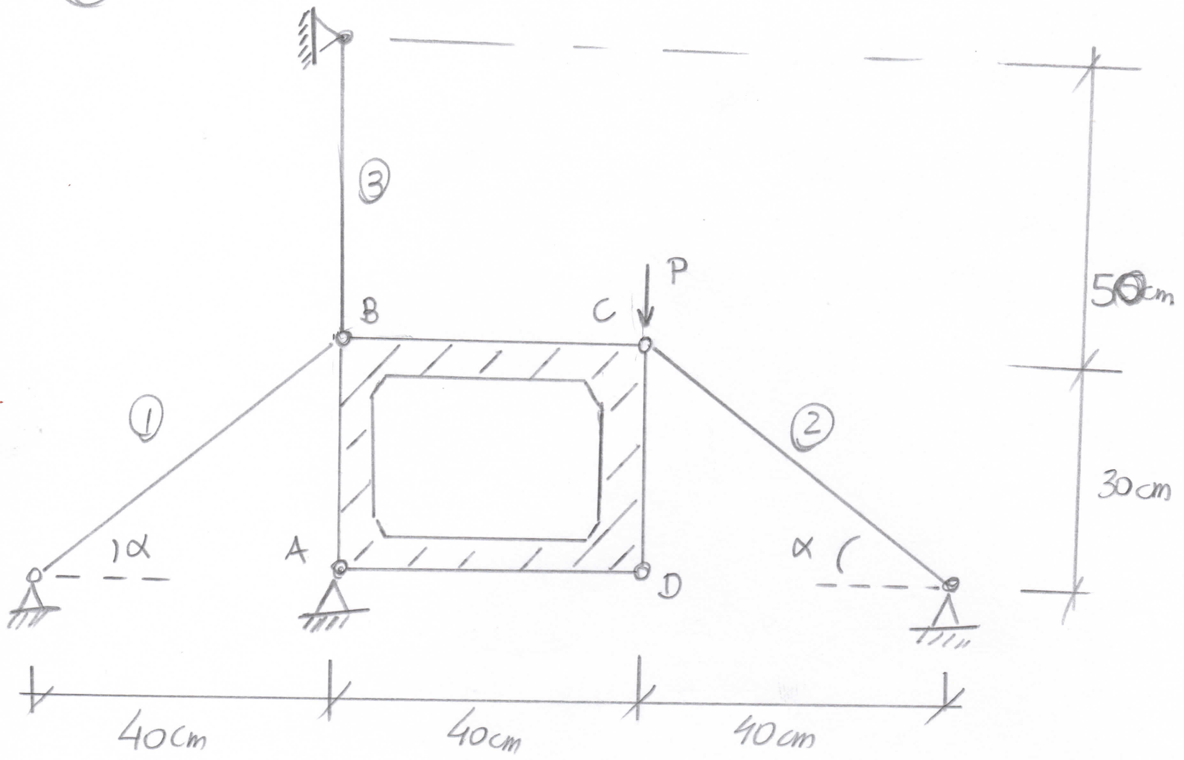
$P = 900\ kgf$

$\bar{\sigma} = 480\ kgf/cm^2$

$\bar{\varphi} = 0{,}0025\,rad$

$l = 50cm$

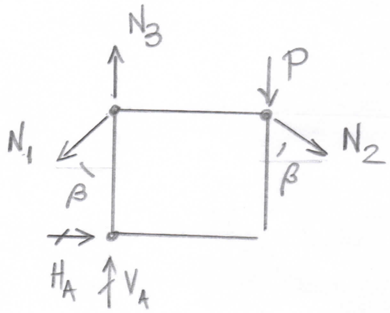
$E = 10^5\ kgf/cm^2$



$sen\alpha = cos\beta = 3/5$

$Cos\alpha = sen\beta = 4/5$

Equilíbrio:



$\sum F_H = 0: \ -N_1\, sen\beta + H_A + N_2\, sen\beta = 0$

$\sum F_V = 0: \ V_A + N_3 - N_1 \cos\beta - P - N_2 \cos\beta = 0$

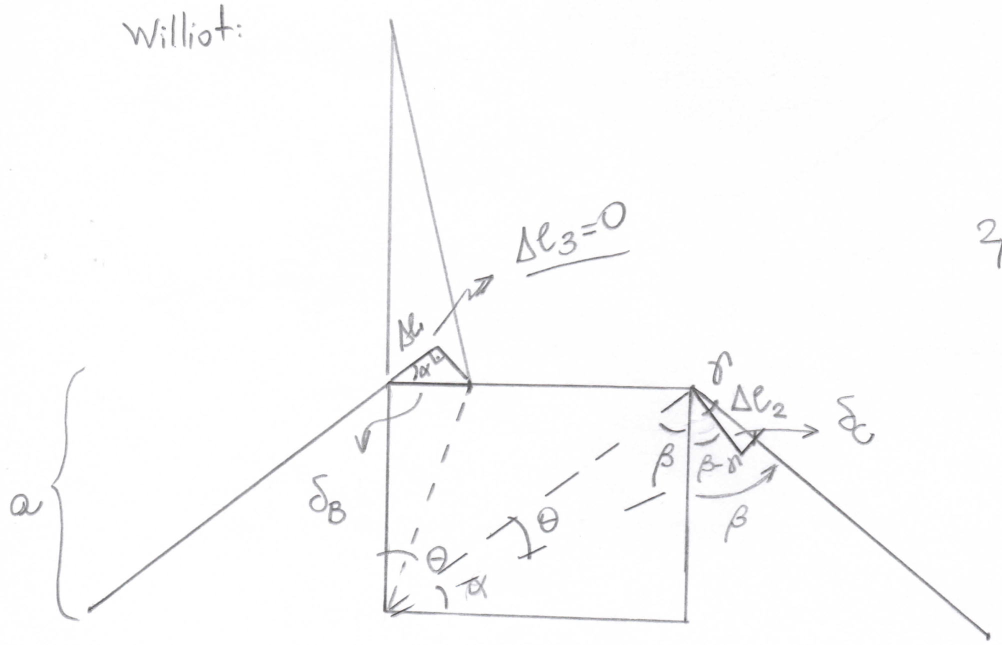
$\sum M_A = 0: \ N_1\, sen\beta . 30 - P.40 - N_2 \cos\beta . 40 - N_2\, sen\beta . 30 = 0$

$3N_1 \frac{4}{5} - N_2 \left[4.\frac{3}{5} + 3.\frac{4}{5} \right] = 4P$

$12N_1 - 24N_2 = 20P$

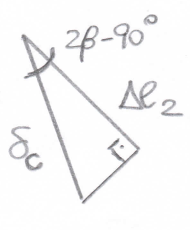
$3N_1 - 6N_2 = 5P$

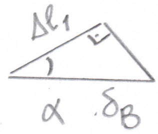
Williot:



$2\beta - \gamma = 90°$

$\gamma = 2\beta - 90°$



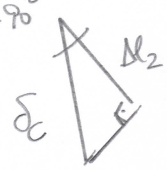


$$\cos\alpha = \frac{\Delta l_1}{\delta_B}$$

$$\therefore \delta_B = \frac{\Delta l_1}{\cos\alpha}$$

$\Delta l_2 = 0$

$\gamma = 2\beta - 90^\circ$



$$\cos\gamma = \frac{|\Delta l_2|}{\delta_C}$$

$$\delta_C = \frac{|\Delta l_2|}{\cos\gamma}$$

$$\cos(2\beta - 90^\circ) = \cos 2\beta \cos 90^\circ + \operatorname{sen} 2\beta \operatorname{sen} 90^\circ$$

$$\cos(2\beta - 90^\circ) = \operatorname{sen} 2\beta = 2 \operatorname{sen}\beta \cos\beta$$

$$\cos\gamma = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$$

Mas:

$$\operatorname{tg}\theta = \frac{\delta_B}{30} = \frac{\delta_C}{50} \Rightarrow \frac{\delta_B}{3} = \frac{\delta_C}{5}$$

$$\frac{1}{3} \cdot \frac{\Delta l_1}{\cos\alpha} = \frac{1}{5} \cdot \frac{|\Delta l_2|}{\cos\gamma}$$

$$\Delta l_1 = \frac{3}{5} \cdot \frac{\cos\alpha}{\cos\gamma} |\Delta l_2| = \frac{3}{5} \cdot \frac{4}{5} \cdot \frac{25}{24} |\Delta l_2|$$

$$\boxed{\Delta l_1 = \frac{|\Delta l_2|}{2}} \Rightarrow \Delta l_1 = -\frac{\Delta l_2}{2}$$

$$\frac{N_1 l_1}{E_1 A_1} = -\frac{1}{2} \frac{N_2 l_2}{E_2 A_2} \Rightarrow N_1 = -\frac{N_2}{2} \qquad N_3 = 0$$

Substituindo no equilíbrio:

$$3 \cdot N_1 - 6 \cdot (-2N_1) = 5P$$

$$15 N_1 = 5P$$

$$N_1 = \frac{P}{3} \Rightarrow N_1 = 300\ \text{kgf} \ ; \ N_2 = -600\ \text{kgf}$$

Dimensionamento:

P tensão

$$\sigma \leq \bar{\sigma} \Rightarrow \frac{|N_2|}{A} \leq \bar{\sigma} \Rightarrow A \geq \frac{|N_2|}{\bar{\sigma}} \Rightarrow A \geq \frac{600}{480}$$

$$A \geq 1{,}25 cm^2$$

2° rotação:

$$\theta \leq \bar{\varphi} \Rightarrow tg\theta \leq tg\bar{\varphi} \Rightarrow \frac{\delta_B}{a} \leq tg\bar{\varphi} \Rightarrow \frac{\Delta l_1}{a \cos\alpha} \leq tg\bar{\varphi}$$

$$\frac{N_1 . l}{EAa\cos\alpha} \leq tg\bar{\varphi} \Rightarrow A \geq \frac{N_1 l}{Ea\cos\alpha \, tg\bar{\varphi}} \qquad A \geq \frac{300.50}{10^5 .30. 4/5 . 0{,}0025}$$

$$A \geq 2{,}5 cm^2$$

Logo $A = 2{,}5 cm^2$.

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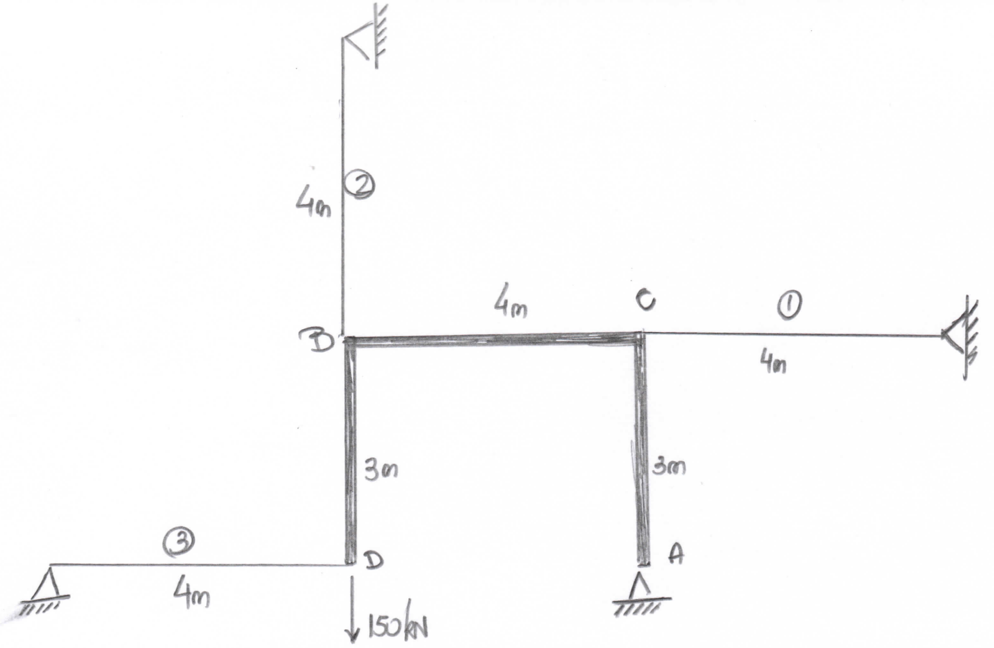


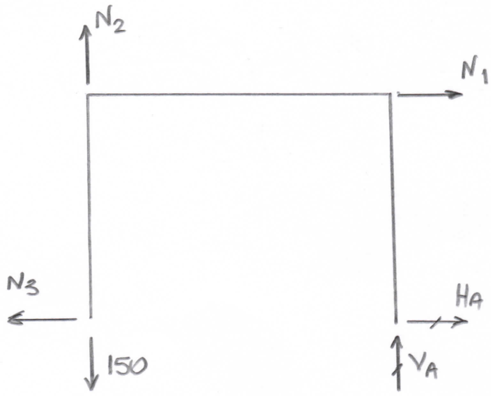
N_1, N_2, N_3 ?

$A = 8.10^{-4} m^2$

$E = 9,6. GPa$

$h_B = ?$

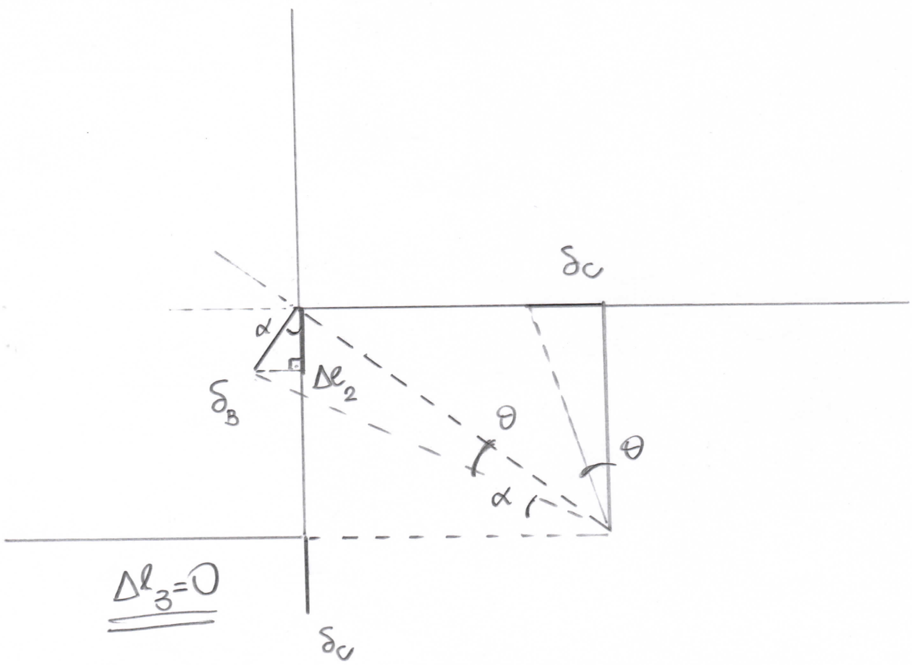
Equilíbrio.



$\curvearrowleft \sum M_A = 0 : -N_1 . 3 - N_2 . 4 + 150 . 4 = 0$

$$\boxed{3N_1 + 4N_2 = 600}$$

Williot



- $\Delta \ell_1 = \delta_c$

- $\cos \alpha = \frac{\Delta \ell_2}{\delta_B} \Rightarrow \delta_B = \frac{\Delta \ell_2}{\cos \alpha}$

- $tg\theta = \frac{\delta_c}{3} = \frac{\delta_B}{5}$

- $\Delta \ell_3 = 0$

$$\frac{\Delta l_1}{3} = \frac{\Delta l_2}{5\cos\alpha} \Rightarrow \frac{\Delta l_1}{3} = \frac{\Delta l_2}{4}$$

como $l_1 = l_2$, $E_1 = E_2$, $A_1 = A_2$:

$$\frac{N_1}{3} = \frac{N_2}{4} \Rightarrow N_1 = \frac{3}{4} N_2$$

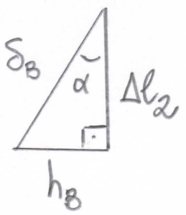
Assim:

$$3 \cdot \frac{3}{4} N_2 + 4N_2 = 600 \Rightarrow \frac{25}{4} N_2 = 600 \Rightarrow \boxed{N_2 = 96\ kN}$$

$$\boxed{N_1 = 72\ kN}$$

$$\boxed{N_3 = 0}$$

$$h_B = \delta_B \operatorname{sen}\alpha$$



$$h_B = \frac{\Delta l_2}{\cos\alpha} \operatorname{sen}\alpha = \Delta l_2 \operatorname{tg}\alpha$$

$$h_B = \frac{N_2 l_2}{EA} \cdot \operatorname{tg}\alpha$$

$$h_B = \frac{96 \cdot 10^3 \cdot 4}{9{,}6 \cdot 10^9 \cdot 8 \cdot 10^{-4}} \cdot \frac{3}{4}$$

$h_B = 0{,}0375 m$ (para a esquerda)

A barra ABC é rígida. As barras 1 e 2 são compostas de materiais G e F, conforme desenho.

Determinar P_{max} de modo que:

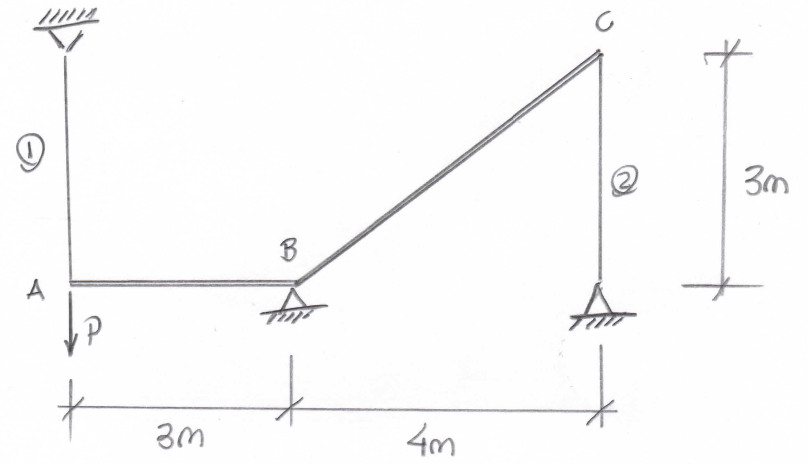
$\delta_C \leq \bar{\delta} = 1cm$

$\sigma_G \leq \bar{\sigma}_G = 15\ kN/cm^2$

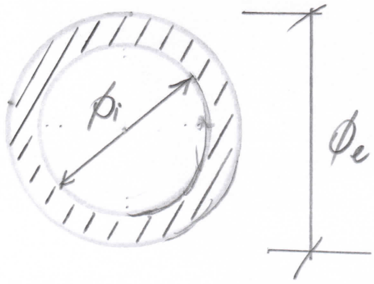
$\sigma_F \leq \bar{\sigma}_F = 5\ kN/cm^2$

$E_G = 21.000\ kN/cm^2$

$E_F = 2.100\ kN/cm^2$



Barras 1 e 2:



$\phi_i = 18cm$

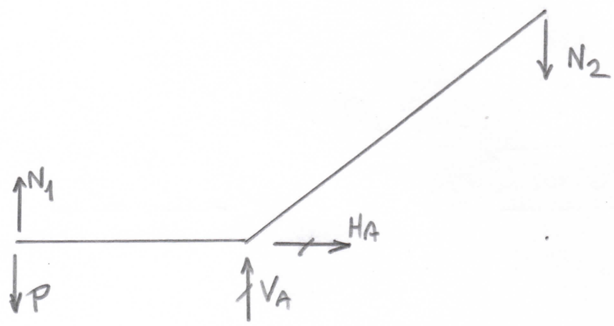
$\phi_e = 20cm$

$A_F = \frac{\pi \phi_i^2}{4} = 81\pi\ cm^2$

$A_G = \frac{\pi}{4}(\phi_e^2 - \phi_i^2) = 19\pi\ cm^2$

$(EA)_{eq} = E_G A_G + E_F A_F = 1.787.880\ kN\ \ (569.100\pi)$

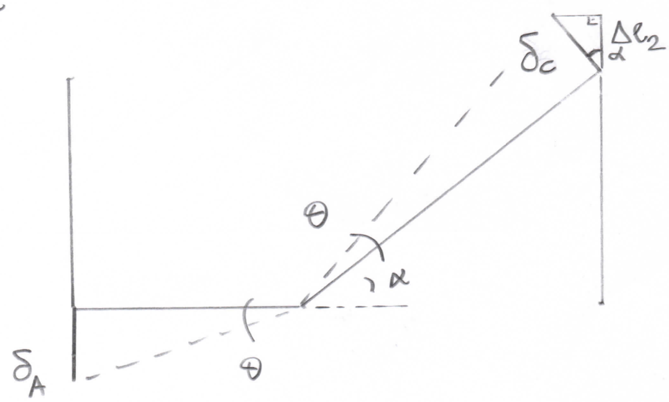
Equilíbrio:



$\sum M_A = 0:\ P.3 - N_1.3 - N_2.4 = 0$

$\boxed{3N_1 + 4N_2 = 3P}$

Williot



- $\Delta\ell_1 = \delta_A$
- $\cos\alpha = \frac{\Delta\ell_2}{\delta_C} \Rightarrow \delta_C = \frac{\Delta\ell_2}{\cos\alpha}$
- $tg\theta = \frac{\delta_A}{300} = \frac{\delta_C}{500}$ (δ em cm)

$$\frac{\Delta l_1}{300} = \frac{\Delta l_2}{500\cos\alpha} \Rightarrow \frac{\Delta l_1}{300} = \frac{\Delta l_2}{400}$$

Como $\Delta l_i = \frac{N_i l_i}{(EA)_{eq}}$:

$$\frac{N_1 \cancel{l_1}}{300\cancel{(EA)_{eq}}} = \frac{N_2 \cancel{l_2}}{400\cancel{(EA)_{eq}}} \Rightarrow \boxed{N_1 = \frac{3}{4}N_2}$$

No equilíbrio:

$$3\left(\frac{3}{4}N_2\right) + 4N_2 - 3P \Rightarrow \frac{25}{4}N_2 = 3P \Rightarrow \boxed{N_2 = \frac{12}{25}P} \Rightarrow \boxed{N_1 = \frac{9}{25}P}$$

Critérios

- maior tensão: fio 2 ($N_2 > N_1$)
- deslocamento C: $\delta_C = \frac{\Delta l_2}{\cos\alpha} \leq \overline{\delta_C}$

(1) como são 2 materiais, temos que verificar em ambos a deformação:

$$\varepsilon_F = \sigma_F / E_F = 2{,}38 \cdot 10^{-3}\ cm/cm$$

$$\varepsilon_G = \sigma_G / E_G = 7{,}14 \cdot 10^{-4}\ cm/cm$$

Como $\varepsilon \leq \varepsilon_F$ e $\varepsilon \leq \varepsilon_G$, ε_G limita.

$$\varepsilon = \frac{N}{(EA)_{eq}} \leq \varepsilon_G \Rightarrow \quad N_2 \leq (EA)_{eq}\,\varepsilon_G \Rightarrow N_2 \leq 1276{,}5\ kN$$

$$P \leq 2659{,}5\ kN$$

(2) Pelo deslocamento:

$$\frac{\Delta l_2}{\cos\alpha} \leq \overline{\delta_C} \qquad N_2 \leq 4767{,}7\ kN$$

$$\frac{N_2 l_2}{(EA)_{eq}\cos\alpha} \leq \overline{\delta_C} \qquad P \leq 9932{,}7\ kN$$

$$N_2 \leq \frac{(EA)_{eq}\cos\alpha\,\overline{\delta_C}}{l_2}$$