Problema com Observações Parciais

Exemplo das Máquinas: Suponha agora que uma ação de controle U_n pode ser tomada no início do dia n.

C_t → não fazer nada

S_t → realiza inspeção detalhada e deixa máquina como nova se necessário

Vamos considerar agora:

$$P(C_t) = \begin{bmatrix} 2/3 & 1/3 \\ 0 & 1 \end{bmatrix}, \quad P(S_t) = \begin{bmatrix} 2/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}$$

$p_1 = 2/3$

$p_2 = 1/3$

Suponha também

$p(G|1) = 3/4$ (Π_{1G}) $p(G|2) = 1/4$ (Π_{2G})

$p(B|1) = 1/4$ (Π_{1B}) $p(B|2) = 3/4$ (Π_{2B})

Considere os seguintes custos $g(X_n, U_n)$:

$g(1, C_0) = 0$, $g(2, C_0) = 2$

$g(1, S_t) = 1$, $g(2, S_t) = 1$

Ou seja, paga 1 sempre para fazer inspeção, e paga 2 se não fizer e máquina estiver no estado 2, paga 0 caso contrário

Deseja-se U_1 e U_2 para minimizar

$E\left(g(X_1, U_1) + g(X_2, U_2)\right)$.

(B)

Informações: $I_1 = S_1$ → 2 possibilidades

$I_2 = (S_1, S_2, U_1)$ → $2 \times 2 \times 2 =$ 8 possibilidades

Objetivo: 2 decisões! $U_1(I_1)$, $U_2(I_2)$

$$\min E\left(g(X_1, U_1(I_1)) + g(X_2, U_2(I_2))\right)$$

Programação Dinâmica:

1) Último Estágio ($\pi = 2$) $\quad \downarrow U_2(I_2)$

$$J_2^*(I_2) = \min\left\{ \overset{\downarrow S}{1}, 2P(X_2 = 2 \mid I_2) \right\}$$

$$P(X_2 = y \mid S^2 = s^2, U_1 = u) =$$

$$\frac{P(S^2 = s^2, X_2 = y, U_1 = u)}{\left(\sum_{i=1}^{2} P(S^2 = s^2, X_2 = i, U_1 = u)\right)}$$

$$P(S^2=s^2, X_2=y, U_1=u) = P(S_2=s_2, S_1=s_1, X_2=y, U_1=u)$$

$$= P(S_2=s_2 \mid X_2=y, S_1=s_1, U_1=u)\left[P(X_2=y, X_1=1, S_1=s_1, U_1=u)\right.$$

$$\left. + P(X_2=y, X_1=2, S_1=s_1, U_1=u)\right]$$

Π_{ys_2}

$\cdot P(s_2|y)$

$P_{xy}(u)$

$$P(X_2=y, X_1=x, S_1=s_1, U_1=u) = P(X_2=y \mid X_1=x, S_1=s_1, U_1=u).$$

$$P(S_1=s_1 \mid X_1=x, U_1=u) \cdot \frac{P(X_1=x, U_1=u)}{P(X_1=x)} =$$

Π_{xs_1}

$$P_{xy}(u)\, P(s_1|x) \cdot P_x$$

Π_{ys_2} $\Pi_{s_1s_1}$ $\Pi_{s_2s_1}$

$$P(S^2=s^2, X_2=y, U_1=u) = P(s_2|y)\left[P_1 P_{1y}^{(u)} P(s_1|1) + P_2 P_{2y}^{(u)} P(s_1|2)\right]$$

$\rightarrow s_2=G,\ s_1=G,\ u=S_t,\ y=2$

$P(G|2)=1/4$, $P_{12}(S)=\frac{1}{3}$, $P_{22}(S)=\frac{1}{3}$; $P(G|1)=\frac{3}{4}$

$P(G|2)=1/4$

$P_1=2/3$, $P_2=1/3$

$$P(S_2=G, S_1=G, X_2=2, U_1=S_t) = \frac{1}{4}\left(\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{3}{4} + \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{4}\right)$$

$$= \frac{1}{4}\cdot\frac{1}{3}\left(\frac{2}{3}\cdot\frac{3}{4} + \frac{1}{3}\cdot\frac{1}{4}\right) = \frac{7}{(12)^2} = 0{,}0486$$

$$\begin{bmatrix} P(G|1) & 0 \\ P(B|1) & 0 \\ 0 & P(G|2) \\ 0 & P(B|2) \end{bmatrix} \begin{bmatrix} P_{11}(u) & P_{21}(u) \\ P_{12}(u) & P_{22}(u) \end{bmatrix} \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} \begin{bmatrix} P(G|1) & P(B|1) \\ P(G|2) & P(B|2) \end{bmatrix} =$$

$4 \times 2 \rightarrow 8$ elementos

$S_1 = \begin{cases} G \\ B \end{cases}$ $S_2 = \begin{cases} G \\ B \end{cases}$ $X_2 = \begin{cases} 1 \\ 2 \end{cases}$

$2 \times 2 \times 2 = 8$

u físico

$u = St$

$u = Co$

$$\begin{bmatrix} P(G|1) & 0 \\ P(B|1) & 0 \\ 0 & P(G|2) \\ 0 & P(B|2) \end{bmatrix} \begin{bmatrix} P_1 P_{11}(u) & P_2 P_{21}(u) \\ P_1 P_{12}(u) & P_2 P_{22}(u) \end{bmatrix} \begin{bmatrix} P(G|1) & P(B|1) \\ P(G|2) & P(B|2) \end{bmatrix} =$$

$X_2 = 1$ $X_2 = 2$

$$\begin{bmatrix} P(G|1) & 0 \\ P(B|1) & 0 \\ 0 & P(G|2) \\ 0 & P(B|2) \end{bmatrix} \begin{matrix} \leftarrow S_1 = G \\ \leftarrow S_1 = B \\ \leftarrow S_2 = G \\ \leftarrow S_2 = B \end{matrix}$$

$S_2 = G$ | $S_2 = B$

$S_1 = G$:

$P_1 P_{11}(u) P(G|1) + P_2 P_{21}(u) P(G|2)$ | $P_1 P_{11}(u) P(B|1) + P_2 P_{21}(u) P(B|2)$

$S_1 = B$:

$P_1 P_{12}(u) P(G|1) + P_2 P_{22}(u) P(G|2)$ | $P_1 P_{12}(u) P(B|1) + P_2 P_{22}(u) P(B|2)$

S_2, S_1, X_2

$S_1 = G$	$S_1 = B$
G, G, 1	G, B, 1
B, G, 1	B, B, 1
G, G, 2	G, B, 2
B, G, 2	B, B, 2

O mesmo para o outro "u".

$u = St$ e $u = Co$

$= U_0$

$$\begin{bmatrix} 3/4 & 0 \\ 1/4 & 0 \\ 0 & 1/4 \\ 0 & 3/4 \end{bmatrix} \begin{bmatrix} 2/3 & 0 \\ 1/3 & 1 \end{bmatrix} \begin{bmatrix} 2/3 & 0 \\ 0 & 1/3 \end{bmatrix} \begin{bmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{bmatrix} =$$

$$\begin{bmatrix} 3/4 & 0 \\ 1/4 & 0 \\ 0 & 1/4 \\ 0 & 3/4 \end{bmatrix} \begin{bmatrix} 2/3 & 0 \\ 1/3 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 1/6 \\ 1/12 & 1/4 \end{bmatrix} = \begin{bmatrix} 3/4 & 0 \\ 1/4 & 0 \\ 0 & 1/4 \\ 0 & 3/4 \end{bmatrix} \begin{bmatrix} 1/3 & 1/9 \\ 1/4 & 11/36 \end{bmatrix} =$$

$$\begin{bmatrix} 1/4 & 1/12 \\ 1/12 & 1/36 \\ 1/16 & 11/144 \\ 3/16 & 33/144 \end{bmatrix} = \begin{matrix} \lambda_2 \quad \lambda_1 \quad x_2 \\ \begin{bmatrix} G,G,1 & G,B,1 \\ B,G,1 & B,B,1 \\ G,G,2 & G,B,2 \\ B,G,2 & B,B,2 \end{bmatrix} \end{matrix}$$

$U_1 = U_0$

(7)

Co sempre aqui → $S_1 = \begin{cases} G \\ B \end{cases}$, $S_2 = \begin{cases} G \\ B \end{cases}$, $X_2 = \begin{cases} 1 \\ 2 \end{cases}$ → γ com $\gamma = 1, 2$

$$P(X_2 = \gamma \mid S_2 = s_2, S_1 = s_1) = \frac{P(S_2 = s_2, S_1 = s_1, X_2 = \gamma)}{P(S_2 = s_2, S_1 = s_1, X_2 = 1) + P(S_2 = s_2, S_1 = s_1, X_2 = 2)}$$

S_2	S_1	X_2	
G	G	1	$P(X_2 = 1 \mid S_2 = G, S_1 = G) = \frac{1/4}{1/4 + 1/16} = \frac{4}{5} = 0,8$
		2	$P(X_2 = 2 \mid S_2 = G, S_1 = G) = \frac{1/16}{1/4 + 1/16} = \frac{1}{5} = 0,2$
G	B	1	$P(X_2 = 1 \mid S_2 = G, S_1 = B) = \frac{1/12}{1/12 + 11/12^2} = \frac{12}{23} = 0,5217$
		2	$P(X_2 = 2 \mid S_2 = G, S_1 = B) = 11/23 = 0,4783$
B	G	1	$P(X_2 = 1 \mid S_2 = B, S_1 = G) = \frac{1/12}{1/12 + 3/16} = \frac{1}{1 + 9/4} = \frac{4}{13} = 0,307$
		2	$P(X_2 = 2 \mid S_2 = B, S_1 = G) = \frac{9}{13} = 0,693$
B	B	1	$P(X_2 = 1 \mid S_2 = B, S_1 = B) = \frac{1/36}{1/36 + 33/12^2} = \frac{1}{1 + 99/12} = \frac{12}{111} = 0,108$
		2	$P(X_2 = 2 \mid S_2 = B, S_1 = B) = \frac{99}{111} = 0,892$

$U_1 = C_0$ $\quad P(X_2 = 1 \mid S_2 = S_1, U_1 = u)$ (8)

S_2	S_1	X_2	Prob	$\min(1, 2\,\text{Prob})$	U_2	$J_2^{U_1}(I_2)$
G	G	1	0,8	1		1
G	G	(2)	0,2	0,4	C_0	0,4
G	B	1	0,5217	1		1
G	B	(2)	0,4783	0,9566	C_0	0,9566 (22/23)
B	G	1	0,3077	1		1
B	G	(2)	0,6923	1	S_t	1
B	B	1	0,1081	1		1
B	B	(2)	0,8919	1	S_t	1

$P(S_2 = G, S_1 = G, X_2 = 1, U_1 = C_0) = 7/24$,

$P(X_2 = 2 \mid S_2 = G, S_1 = G, U_1 = S_t) = \dfrac{7/12^2}{7/12^2 + 7/24} = \dfrac{1}{1 + \frac{12}{2}} = 1/7$

$= 0,142$

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$$\begin{bmatrix} 3/4 & 0 \\ 1/4 & 0 \\ 0 & 1/4 \\ 0 & 3/4 \end{bmatrix} \begin{bmatrix} 2/3 & 2/3 \\ 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1/2 & 1/6 \\ 1/12 & 1/4 \end{bmatrix} =$$

$$\begin{bmatrix} 3/4 & 0 \\ 1/4 & 0 \\ 0 & 1/4 \\ 0 & 3/4 \end{bmatrix} \begin{bmatrix} \frac{1}{3}+\frac{1}{18} & \frac{1}{9}+\frac{1}{6} \\ \frac{1}{6}+\frac{1}{36} & \frac{1}{18}+\frac{1}{12} \end{bmatrix} =$$

$\frac{2}{18}+\frac{3}{18}=\frac{5}{18}$

$\frac{1}{6}\left(\frac{1}{3}+\frac{1}{2}\right)=\frac{5}{36}$

$C_1 = 5t$

$$\begin{bmatrix} 3/4 & 0 \\ 1/4 & 0 \\ 0 & 1/4 \\ 0 & 3/4 \end{bmatrix} \begin{bmatrix} 7/18 & 5/18 \\ 7/36 & 5/36 \end{bmatrix} =$$

$1_2, 1_1, X_2$

$$\begin{bmatrix} 7/24 & 5/24 \\ 7/72 & 5/72 \\ 7/(12)^2 & 5/(12)^2 \\ 21/(12)^2 & 15/(12)^2 \end{bmatrix} =$$

G, G, 1 | G, B, 1
B, G, 1 | B, B, 1
G, G, 2 | G, B, 2
B, G, 2 | B, B, 2

ext

S_2	S_1	X_2	
G	G	1	$P(X_2=1 \mid S_2=G, S_1=G) = \frac{\frac{7}{24}}{\frac{7}{24}+\frac{7}{12^2}} = \frac{1}{1+1/6} = \frac{6}{7} = 0{,}857$
		2	$P(X_2=2 \mid S_2=G, S_1=G) = \frac{7/12^2}{7/24+7/12^2} = \frac{1}{7}$
G	B	1	$P(X_2=1 \mid S_2=G, S_1=B) = \frac{5/24}{5/24+5/12^2} = \frac{1}{1+1/6} = \frac{6}{7}$
		2	$P(X_2=2 \mid S_2=G, S_1=B) = 1/7$
B	G	1	$P(X_2=1 \mid S_2=B, S_1=G) = \frac{7/72}{\frac{7}{72}+\frac{21}{12^2}} = \frac{7}{7+\frac{21}{2}} = \frac{14}{35} = \frac{2}{5} = 0{,}4$
		2	$P(X_2=2 \mid S_2=B, S_1=G) = \frac{3}{5} = 0{,}6$
B	B	1	$P(X_2=1 \mid S_2=B, S_1=B) = \frac{5/72}{5/72+35/12^2} = \frac{1}{1+\frac{3}{2}} = \frac{2}{5}$
		2	$P(X_2=2 \mid S_2=B, S_1=B) = \frac{3}{5}$

$72 = 6 \cdot 12$

$U_1 = 21$

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S_2	S_1	X_2	Prob	Min(1, 2Prob)	U_2	$\Sigma_2^a(I_2)$
G	G	1	0,8571			
G	G	2	0,1429	0,2858	C	0,2858 (217)
G	B	1	0,8571			
G	B	2	0,1429	0,2858	C	0,2858 (217)
B	G	1	0,4			
B	G	2	0,6	1	S	1
B	B	1	0,4			
B	B	2	0,6	1	S	1

c) Primeira Estagio ($n=1$) $I_1 = S_1 = s_1$

$$J_1^*(s_1) = \min \Big\{ \underbrace{1 + E\Big(J_2^*\big(\overbrace{S_2, s_1, S_t}^{I_2}\big) \,\Big|\, S_1 = s_1, U_1 = S_t\Big)}_{S_t \ (I)},$$

$$\underbrace{2\,P(X_1 = 2 \mid S_1 = s_1) + E\Big(J_2^*\big(\overbrace{S_2, s_1, C_t}^{I_2}\big) \,\Big|\, S_1 = s_1, U_1 = C_t\Big)}_{C_t \ (J)} \Big\}$$

(II)

Note que

(I)

$$E\Big(J_2^*(S_2, s_1, \mu_1) \,\Big|\, S_1 = s_1, U_1 = \mu_1\Big) = P\Big(S_2 = G \,\Big|\, S_1 = s_1, U_1 = \mu_1\Big) \cdot$$

$$J_2^*(G, s_1, \mu_1) + P\Big(S_2 = B \,\Big|\, S_1 = s_1, U_1 = \mu\Big) \cdot J_2^*(B, s_2, \mu)$$

$$P\Big(S_2 = s_2 \,\Big|\, S_1 = s_1, U_1 = \mu_1\Big) = \frac{P(S_2 = s_2, S_1 = s_1, U_1 = \mu_1)}{P(S_1 = s_1, U_1 = \mu_1)} = \frac{a}{e}$$

$$a = P(S_2 = s_2, S_1 = s_1, U_1 = \mu_1) = \sum_{i=1}^{2} \sum_{j=1}^{2} P(S_2 = s_2, S_1 = s_1, X_2 = j, X_1 = i, U_1 = \mu_1)$$

$$= \sum_{j=1}^{2} p(s_2 \mid j)\Big(p_1\, p_{1j}(\mu_1)\, p(s_1 \mid 1) + p_2\, p_{2j}(\mu_1)\, p(s_1 \mid 2)\Big)$$

↑ página 4 e 5 para $j=1$ e $j=2$

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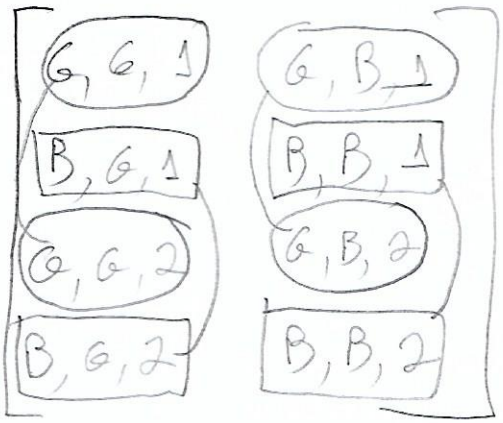

$$P(S_2 = s_2, S_3 = s_3, U_3 = u_3)$$

$U_1 = Co$

$$\begin{bmatrix} \frac{1}{4} \cdot \frac{1}{16} & \frac{1}{12} + \frac{11}{12^2} \\ \frac{1}{12} + \frac{3}{16} & \frac{1}{36} + \frac{33}{12^2} \end{bmatrix} = \begin{bmatrix} \frac{5}{16} & \frac{23}{12^2} \\ \frac{13}{48} & \frac{111}{3 \cdot 12^2} \end{bmatrix} = \begin{matrix} s_2 \, s_3 \\ \begin{bmatrix} G,G & G,B \\ B,G & B,B \end{bmatrix} \end{matrix}$$

$U_3 = St$

$$\begin{bmatrix} \frac{7}{24} + \frac{7}{12^2} & \frac{5}{24} + \frac{5}{12^2} \\ \frac{7}{72} + \frac{21}{12^2} & \frac{5}{72} + \frac{15}{12^2} \end{bmatrix} = \begin{bmatrix} \frac{49}{12^2} & \frac{35}{12^2} \\ \frac{35}{12^2} & \frac{25}{12^2} \end{bmatrix} = \begin{bmatrix} G,G & G,B \\ B,G & B,B \end{bmatrix}$$

$$= P(S_3=s_3, X_3=1, U_3=u_3) + P(S_3=s_3, X_3=2, U_3=u_3) =$$

$$P(S_3=s_3 \mid X_3=1, U_3=u_3) \cdot P(X_3=1, U_3=u_3) +$$

$$P(S_3=s_3 \mid X_3=2, U_3=u_3)\, P(X_3=2, U_3=u_3) =$$

$$P(s_3 \mid 1) \cdot P_1 + P(s_3 \mid 2) \cdot P_2$$

$s_3 = G$

$P = \frac{2}{3}\frac{2}{4} + \frac{1}{3}\frac{1}{4} = \frac{1}{2} + \frac{1}{12} = \frac{7}{12}$

$s_3 = B$

$P = \frac{1}{3}\frac{1}{4} + \frac{1}{3}\frac{3}{4} = \frac{1}{6} + \frac{1}{4} = \frac{5}{12}$

(I) ↑ Fmm

(II) ↓

$$P(X_3=2 \mid S_3=s_3) = \frac{P(X_3=2, S_3=s_3)}{P(S_3=s_3)} =$$

$$\frac{P(S_3=s_3 \mid X_3=2) \cdot P(X_3=2)}{P(S_3=s_3 \mid X_3=1) \cdot P(X_3=1) + P(S_3=s_3 \mid X_3=2) P(X_3=2)} =$$

$$\frac{P(s_3 \mid 2) \cdot P_2}{P(s_3 \mid 1) \cdot P_1 + P(s_3 \mid 2) \cdot P_2}$$

$s_3 = B$

$$P(X_3=2 \mid S_3=B) = \frac{(3/4)(1/3)}{(1/4)(2/3) + (3/4)(1/3)} =$$

$$\frac{1/4}{1/6 + 1/4} =$$

$$\frac{1}{2/3 + 1} = \frac{3}{5}$$

$s_3 = G \Longrightarrow P(G \mid 2) = 1/4, \; P(G \mid 1) = 3/4$

$$P(X_3=2 \mid S_3=G) = \frac{1/4 \cdot 1/3}{1/4 \cdot 1/3 + 3/4 \cdot 2/3} = \frac{1}{7}.$$

$y_1 = G$:

s_2	s_1	
G	G	$P(s_2=G \mid s_1=G) = \frac{\binom{15}{16}}{\binom{7}{12}} = \frac{15}{28}$
B	G	$P(s_2=B \mid s_1=G) = \frac{\binom{13}{48}}{\binom{7}{12}} = \frac{13}{28}$
G	B	$P(s_2=G \mid s_1=B) = \frac{23}{\binom{5}{12}} = \frac{23}{60}$ *
B	B	$P(s_2=B \mid s_1=B) = \frac{111}{\binom{5}{12}} = \frac{111}{180} = \frac{37}{60}$ **

$y_1 = st$

s_2	s_1	
G	G	$P(s_2=G \mid s_1=G) = \frac{49}{\binom{7}{12}} = \frac{7}{12}$
B	G	$P(s_2=B \mid s_1=G) = \frac{35}{\binom{7}{12}} = \frac{5}{12}$
G	B	$P(s_2=G \mid s_1=B) = \frac{35}{\binom{5}{12}} = \frac{7}{12}$
B	B	$P(s_2=B \mid s_1=G) = \frac{25}{\binom{5}{12}} = 5/12$

1) $S_1 = G$:

$$P(X_1 = 2 \mid S_1 = G) = 1/7$$

$$P(S_2 = G \mid S_1 = G, U_1 = C\&) = \frac{15}{28}, \quad \partial_2(G, G, C\&) = 2/5$$

$$P(S_2 = B \mid S_1 = G, U_1 = C\&) = \frac{13}{28}, \quad \partial_2(B, G, C\&) = 1$$

Continua: $\frac{2}{7} + \frac{15}{28} \cdot 2/5 + 1 \cdot \frac{13}{28} = \boxed{\frac{27}{28}}$

$$P(S_2 = G \mid S_1 = G, U_1 = S\&) = \frac{7}{12}, \quad \partial_2(G, G, S) = 2/7$$

$$P(S_2 = B \mid S_1 = G, U_1 = S\&) = \frac{5}{12}, \quad \partial_2(B, G, S) = 1$$

Para: $1 + \frac{7}{12} \cdot \frac{2}{7} + \frac{5}{12} \cdot 1 = \frac{19}{12}$

$$\partial_1^*(G) = \frac{27}{28}, \quad U_1^* = C\&.$$

$S_1 = B$

$P(X_1 = 2 \mid S_1 = C) = 3/5$

$P(S_2 = G \mid S_1 = B, U_1 = C\not{O}) = \frac{23}{60}^{*}$, $\sigma_2^{*}(G, B, C\not{O}) = 22/23$

$P(S_2 = \not{B} \mid S_1 = B, U_1 = C\not{O}) = \frac{37}{60}^{*}$, $\sigma_2^{*}(B, B, C\not{O}) = 1$

$P(S_2 = G \mid S_1 = B, U_1 = S_t) = \frac{7}{12}^{*}$, $\sigma_2^{*}(G, B, S_t) = 2/7$

$P(S_2 = B \mid S_1 = B, U_1 = S_t) = 5/12^{*}$, $\sigma_2^{*}(B, B, S_t) = 1$

Continuar: $\frac{6}{5}^{(2.315)} + \frac{22}{\not{23}} \cdot \frac{\not{23}}{60} + 1 \cdot \frac{37}{60} = \frac{131}{60}$

Parar: $1 + \frac{7}{12} \cdot \frac{2}{7} + \frac{5}{12} \cdot 1 = \boxed{\frac{19}{12}}$

$\sigma_1^{*}(B) = \frac{19}{12}$, $U_1^{*} = S_t$

$S_1 = G \Rightarrow U_1^* = C\theta \Rightarrow \begin{cases} S_2 = G \Rightarrow U_2^* = C\theta \\ S_2 = B \Rightarrow U_2^* = St \end{cases}$

$J_1^*(G) = \frac{27}{28}$

$S_1 = B \Rightarrow U_1^* = St \Rightarrow \begin{cases} S_2 = G \Rightarrow U_2^* = C\theta \\ S_2 = B \Rightarrow U_2^* = St \end{cases}$

$J_1^*(B) = \frac{19}{12}$