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$f: D \to \mathbb{R},\ D \subset \mathbb{R}$

Seja $\boxed{p \in D}$ tq

- $\lim_{x \to p} f(x) = L > 0$

$\underbrace{\forall a > 0}_{a=1} \underbrace{\exists \delta > 0}_{\delta_1} \forall y \in]p-\delta, p+\delta[\cap D$

$\underbrace{f(y)}_{} \in]L-a, L+a[$

$*\ a = L/2$

$a = L/2$ existe δ_a tq $\forall y \in]p-\delta_a, p+\delta_a[$

$f(y) \in]L/2, 3L/2[$

$\varepsilon = \delta_a$

$\forall y$ tq $|p-y| < \varepsilon$

$f(y) > 0 \quad (f \in]1/2, 3/2[)$

$\forall_{x,y \in \mathbb{R}} \; ||x|-|y|| \leq |x-y|$

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15)

$f(x) - L$

$$\underbrace{\lim_{x \to p} \left(f(x) - L \right)} = 0$$

$$\forall \varepsilon > 0 \ \exists \delta > 0 \ \forall y \in \left]p-\delta, p+\delta\right[\cap D$$

$$\left(f(y) - L\right) \in \left]-\varepsilon, \varepsilon\right[$$

$$\Updownarrow$$

$$f(y) \in \left]L-\varepsilon, L+\varepsilon\right[$$

$$\lim_{x \to p} \cos(x) = \cos(p)$$

$$\Updownarrow$$

$$\lim_{x \to p} \left(\cos(x) - \cos(p) \right) = 0$$

$$\parallel$$

$$\lim_{x \to p} -2 \operatorname{sen}\left(\frac{x+p}{2}\right) \cdot \operatorname{sen}\left(\frac{x-p}{2}\right)$$

$\parallel \to$ sen é cont

$$2 \cdot \operatorname{sen}\left(\frac{p+p}{2}\right) \cdot \operatorname{sen}\left(\frac{0}{2}\right)$$

$$\parallel$$

$$2 \cdot \underbrace{\operatorname{sen}(p)}_{\leq 1} \cdot \underbrace{\operatorname{sen}(0)}_{0}$$

$$\parallel$$

$$2 \cdot 1 \cdot 0 = 0$$