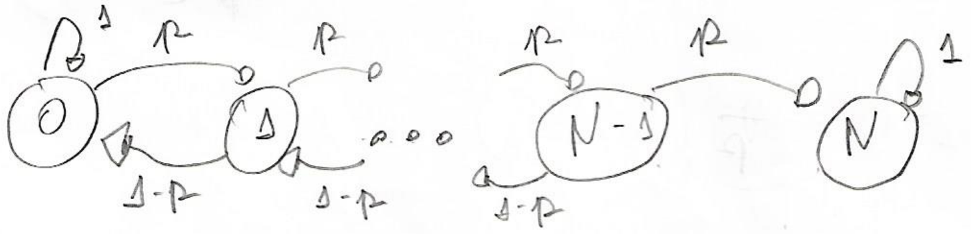
4.5. A ruína do jogador



$$P_{00} = P_{NN} = 1, \quad P_{i,i+1} = p = 1 - P_{i,i-1}, \quad i = 1, \dots, N-1$$

Três classes: $\{0\}, \{N\}$ → recorrentes

$\{1, \dots, N-1\}$ → transiente

X_n = fortuna do jogador na n-ésima jogada

$$Y = \begin{cases} 1 & \text{se jogador atinge } N \text{ em alguma jogada} \\ 0 & \text{se jogador nunca atinge } N \end{cases}$$

$P_i = P(Y = 1 / X_0 = i)$ → probabilidade da fortuna do jogador atingir eventualmente N começando de i

$$P_i = P(Y=1 / X_0 = i) = P(Y=1, X_1 = i+1 \,|\, X_0 = i) + P(Y=1, X_1 = i-1 / X_0 =$$

$$= \underbrace{P(Y=1 / X_1 = i+1, \cancel{X_0 = i})}_{P_{i+1}} \underbrace{P(X_1 = i+1 \,|\, X_0 = i)}_{p} + \underbrace{P(Y=1 \,|\, X_1 = i-1, \cancel{X_0 = i})}_{P_{i-1}} \underbrace{P(X_1 = i-1 |}_{1-p}$$

$$= P_{i+1}\, p + P_{i-1}(1-p), \quad i = 1, 2, \dots, N-1$$

$q = 1 - p$

$$\begin{cases} P_i = q P_{i-1} + p P_{i+1}, \quad i = 1, \dots, N-1 \end{cases}$$

$$P_i = pP_i + qP_i = pP_{i+1} + qP_{i-1} \therefore P_{i+1} - P_i = \frac{q}{p}\left(P_i - P_{i-1}\right)$$

$$P_0 = 0, \quad P_{i+1} - P_i = \frac{q}{p}\left(P_i - P_{i-1}\right), \quad i = 1, \ldots, N-1$$

$$P_2 - P_1 = \frac{q}{p}\left(P_1 - P_0\right) = \frac{q}{p} P_1$$

$$P_3 - P_2 = \frac{q}{p}\left(P_2 - P_1\right) = \left(\frac{q}{p}\right)^2 P_1$$

Lista Exercícios:

Cap. 4: 1, 2, 3, 5, 6, 8, 9, 10, 13, 14, 15, 17, 18

Cap. 5: 2, 5, 6, 7, 8, 9, 10, 12, 14, 18, 20, 21, 23, 24

$$P_i - P_{i-1} = \frac{q}{p}\left(P_{i-1} - P_{i-2}\right) = \left(\frac{q}{p}\right)^{i-1} P_1$$

$$P_N - P_{N-1} = \frac{q}{p}\left(P_{N-1} - P_{N-2}\right) = \left(\frac{q}{p}\right)^{N-1} P_1$$

$$\therefore P_i - P_1 = P_1\left(\frac{q}{p} + \left(\frac{q}{p}\right)^2 + \ldots + \left(\frac{q}{p}\right)^{i-1}\right)$$

$$\therefore P_i = \begin{cases} \dfrac{1-(q/p)^i}{1-q/p} P_1, & \dfrac{q}{p} \neq 1 \\ i P_1, & \dfrac{q}{p} = 1 \end{cases} ; \quad P_N = 1 \Rightarrow P_1 = \begin{cases} \dfrac{1-q/p}{1-(q/p)^N}, & \dfrac{q}{p} \neq 1 \\ \dfrac{1}{N}, & \dfrac{q}{p} = 1 \end{cases}$$

$$\therefore P_i = \begin{cases} \dfrac{1-(q/p)^i}{1-(q/p)^N}, & \dfrac{q}{p} \neq 1 \\ \dfrac{i}{N}, & \dfrac{q}{p} = 1 \end{cases}$$

Fazendo $N \to \infty$,

$$P_i \to \begin{cases} 1 - \left(\frac{q}{p}\right)^i, & p > \frac{1}{2} \\ 0, & p \leq \frac{1}{2} \end{cases}$$

$\to$ probabilidade positiva da fortuna crescer indefinidamente

$\to$ com prob. 1, o jogador vai quebrar