2ª maneira (geral):

$$\Rightarrow \quad Q(k) = E\left(x\,\tilde{y}'_{k|k-1}\right)\left(E\left(\tilde{y}_{k|k-1}\,\tilde{y}'_{k|k-1}\right)\right)^{-1}$$

$$\Rightarrow \quad \tilde{y}_{k|k-1} = y_k - \hat{y}_{k|k-1} = Hx + z_k - H\hat{x}_{k-1} =$$

$$H\left(x - \hat{x}_{k-1}\right) + z_k = H\tilde{x}_{k-1} + z_k$$

$\tilde{x}_{k-1} = x - \hat{x}_{k-1}$ é o erro no instante $k-1$

$$E\left(x\,\tilde{y}'_{k|k-1}\right) = E\left(x\,\tilde{x}'_{k-1}\right)H' + \cancel{E\left(x\,z'_k\right)}^{\,0} =$$

$$E\left(\tilde{x}_{k-1}\,\tilde{x}'_{k-1}\right)H'$$

já que $x = \hat{x}_{k-1} + \tilde{x}_{k-1}$ e $\hat{x}_{k-1} \perp \tilde{x}_{k-1}$. Seja

$$\Rightarrow \quad P(k-1) = \mathrm{cov}\left(\tilde{x}_{k-1}\right) = E\left(\tilde{x}_{k-1}\,\tilde{x}'_{k-1}\right).$$ Temos que

$$E\left(\tilde{y}_{k|k-1}\,\tilde{y}'_{k|k-1}\right) = E\left(\left(H\tilde{x}_{k-1} + z_k\right)\left(\tilde{x}'_{k-1}H' + z'_k\right)\right) =$$

$HP(k-1)H' + N$ $\qquad$ $\tilde{x}_{k-1} = x - \hat{x}_{k-1}$, x e $\hat{x}_{k-1} \perp z_k$

Como $N > 0$, $HP(k-1)H' + N > 0$ e portanto

$$\Rightarrow \quad Q(k) = P(k-1)H'\left(HP(k-1)H' + N\right)^{-1}$$

Da expressão acima, temos que

$$\tilde{x}_k = x - \hat{x}_k = \overbrace{x - \hat{x}_{k-1}}^{\tilde{x}_{k-1}} - Q(k)\overbrace{\left(H\tilde{x}_{k-1} + z_k\right)}^{= y_k - H\hat{x}_{k-1}} =$$

$$\left(I - Q(k)H\right)\tilde{x}_{k-1} - Q(k)z_k$$

$\tilde{x}_{k-1} \perp z_k$ já que $\tilde{x}_{k-1} \in \mathcal{L}(x, z_1, \dots, z_{k-1})$. Logo,

$$P(k) = E(\tilde{x}_k \tilde{x}_k') = E\Big(\big[(I - Q(k)H)\tilde{x}_{k-1} - Q(k) z_k\big] \cdot$$

$$\big[\tilde{x}_{k-1}'(I - H'Q'(k)) - z_k' Q'(k)\big]\Big) = (I - Q(k)H)P(k-1)(I - Q(k)H)'$$

$$+ Q(k) N Q'(k)$$

$$P(k) = (I - Q(k)H)P(k-1)(I - Q(k)H)' + Q(k)NQ'(k)$$

$$Q(k) = P(k-1)H'\big(HP(k-1)H' + N\big)^{-1}$$

$$P(k) = P(k-1) + Q(k)HP(k-1)H'Q'(k) - Q(k)HP(k-1) -$$

$$P(k-1)H'Q'(k) + Q(k)NQ'(k) =$$

$$P(k-1) + Q(k)\big(HP(k-1)H' + N\big)Q'(k) - \big(Q(k)HP(k-1) + P(k-1)H'Q'(k)\big)$$

$$P(k-1) + \cancel{P(k-1)H'Q'(k)} - \cancel{P(k-1)H'Q'(k)} - Q(k)HP(k-1) =$$

$$P(k-1) - P(k-1)H'\big(HP(k-1)H' + N\big)^{-1}HP(k-1)$$

Como $P(0) = P$, podemos calcular recursivamente $P(1), P(2), \ldots, P(k)$ e portanto $Q(k)$. Neste caso, pode-se mostrar que

$$P(k) = P - PH'\Big(HPH' + \frac{1}{k}N\Big)^{-1}HP$$

Para este problema em particular, as duas formulas

$$\hat{x}_k = \hat{x}_{k-1} + Q(k)(y_k - H\hat{x}_{k-1})$$

$$\hat{x}_k = \frac{1}{k}PH'\Big(HPH' + \frac{1}{k}N\Big)^{-1}\Big(\sum_{i=1}^{k} y_i\Big)$$

são equivalentes do ponto de vista computacional (deve-se

(110)

inverter uma matriz $r \times r$ em cada estágio). Note entretanto que os coeficientes $a(H)$ podem ser calculados antes de iniciar o processo.

$$(A+BRC)^{-1} = A^{-1} - A^{-1}B(R^{-1}+CA^{-1}B)^{-1}CA^{-1}$$

$$P(k) = P(k-1) - P(k-1)H'(HP(k-1)H'+N)^{-1}HP(k-1), \quad P(0)=P$$

$$= P(k-1)\left(\underset{A^{-1}}{I} - \underset{B}{H'}(\underset{C}{HP(k-1)}\underset{B}{H'}+\underset{R^{-1}}{N})^{-1}\underset{C}{HP(k-1)}\right)$$

$$= P(k-1)\left(I + H'N^{-1}HP(k-1)\right)^{-1}$$

<u>Indução</u>: $P(k) = P - PH'\left(HPH' + \frac{1}{k}N\right)^{-1}HP$

$$= P\left(I + kH'N^{-1}HP\right)^{-1}$$

$P(AB)^{-1} = B^{-1}A^{-1}$

$$P(k+1) = P\left(I+kH'N^{-1}HP\right)^{-1}\left(I + H'N^{-1}HP\left(I+kH'N^{-1}HP\right)^{-1}\right)^{-1}$$

$$= P\left(I + kH'N^{-1}HP + H'N^{-1}HP\right)^{-1}$$

$$= P\left(I + (k+1)H'N^{-1}HP\right)^{-1}$$ ✓ OK

$$P(1) = P\left(I + H'N^{-1}HP\right)^{-1}$$ ✓ OK