Cleaning Page

$$Q = UU' = \begin{pmatrix} 2 & -1/\sqrt{3} & -1/\sqrt{6} \\ \alpha & 5/2 & -1/\sqrt{12} \\ \alpha & \alpha & 3/2 \end{pmatrix} \qquad \begin{pmatrix} x \\ y_1 \\ y_2 \end{pmatrix} = Uz, \quad z = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}$$

$$U = \begin{pmatrix} u_1 & u_2 & u_3 \\ 0 & u_4 & u_5 \\ 0 & 0 & u_6 \end{pmatrix} \qquad \begin{cases} x = u_1 z_1 + u_2 z_2 + u_3 z_3 \\ y_1 = u_4 z_2 + u_5 z_3 \\ y_2 = u_6 z_3 \end{cases}$$

$$UU' = \begin{pmatrix} u_1 & u_2 & u_3 \\ 0 & u_4 & u_5 \\ 0 & 0 & u_6 \end{pmatrix} \begin{pmatrix} u_1 & 0 & 0 \\ u_2 & u_4 & 0 \\ u_3 & u_5 & u_6 \end{pmatrix} =$$

$$= \begin{pmatrix} u_1^2 + u_2^2 + u_3^2 & u_2 u_4 + u_3 u_5 & u_3 u_6 \\ \cancel{\alpha} & u_4^2 + u_5^2 & u_5 u_6 \\ \cancel{\alpha} & \cancel{\alpha} & u_6^2 \end{pmatrix}$$

$$u_6 = \sqrt{3/2}, \quad u_5 = \frac{-1/\sqrt{12}}{\sqrt{3/2}} = -\frac{1}{\sqrt{\frac{12 \cdot 3}{2}}} = -\frac{1}{\sqrt{18}} = -\frac{1}{3\sqrt{2}}$$

$$u_3 = \frac{-1/\sqrt{6}}{\sqrt{3/2}} = -\frac{1}{\sqrt{9}} = -\frac{1}{3}$$

$s = 5/2$

u_5^2

$$u_4^2 = \frac{5}{2} - \left(\frac{1}{3\sqrt{2}}\right)^2 = \frac{5}{2} - \frac{1}{18} = \frac{44}{18} = \frac{22}{9}$$

$$u_4 = \frac{\sqrt{22}}{3}$$

$$u_2 u_4 = -\frac{1}{\sqrt{2}} - u_3 u_5 \Rightarrow u_2 = \frac{-\frac{1}{\sqrt{2}} - \frac{1}{9\sqrt{2}}}{\sqrt{22}/3} =$$

$$\cdot -\frac{10}{\cancel{9}\sqrt{2}} \frac{\cancel{3}}{\sqrt{22}} = -\frac{10}{3\sqrt{44}} = -\frac{5}{3\sqrt{11}}$$

$$u_1^2 = 2 - \left(\frac{5}{3\sqrt{11}}\right)^2 - \left(\frac{1}{3}\right)^2 = 2 - \frac{25}{9 \cdot 11} - \frac{1}{9} =$$

$$= \frac{198 - 25 - 11}{9 \cdot 11} = \frac{162}{9 \cdot 11} = \frac{18}{11} \Rightarrow u_1 = \frac{3\sqrt{2}}{\sqrt{11}}$$

$$U = \begin{pmatrix} 3\sqrt{\frac{2}{11}} & -\frac{5}{3\sqrt{11}} & -\frac{1}{3} \\ 0 & \frac{\sqrt{22}}{3} & -\frac{1}{3\sqrt{2}} \\ 0 & 0 & \sqrt{3/2} \end{pmatrix}$$

$$X = u_1 Z_1 + u_2 \frac{\left(Y_1 - \frac{u_5}{u_6} Y_2\right)}{u_4} + \frac{u_3}{u_6} Y_2$$

$$= u_1 Z_1 + \frac{u_2}{u_4} Y_1 + \left(\frac{u_3}{u_6} - \frac{u_5 u_2}{u_4 u_6}\right) Y_2$$

$$= u_1 Z_1 + \frac{u_2}{u_4} Y_1 + \left(\frac{u_3 u_4 - u_5 u_2}{u_4 u_6}\right) Y_2$$

$$Z_1 \perp Y_1, Y_2 \Rightarrow \hat{X} = \left(\frac{u_2}{u_4}\right) Y_1 + \left(\frac{u_3 u_4 - u_5 u_2}{u_4 u_6}\right) Y_2$$

$$\frac{u_2}{u_4} = \frac{-\frac{5}{3\sqrt{11}}}{\sqrt{22}} = -\frac{5}{11\sqrt{2}} = -0,3214$$

$$\frac{u_3 u_4 - u_5 u_2}{u_4 u_6} = \frac{-\frac{1}{3} \cdot \frac{\sqrt{22}}{3} - \left(\frac{1}{3\sqrt{2}}\right)\left(\frac{5}{3\sqrt{11}}\right)}{\frac{\sqrt{22}}{3} \cdot \sqrt{2}} =$$

$$\frac{\left(-\frac{\sqrt{22}}{3} - \frac{5}{3\sqrt{22}}\right)\sqrt{2}}{\sqrt{66}} = \left(\frac{-22-5}{3 \cdot \sqrt{22} \cdot \sqrt{66}}\right)\sqrt{2} = \frac{\sqrt{33}}{}$$

$$= -\frac{27}{3\sqrt{22}\sqrt{33}} = -\frac{9}{\sqrt{2\cdot 11}\cdot\sqrt{3\cdot 11}} =$$

$$= \frac{-9}{\sqrt{6}\cdot 11} = -0,334$$

$$\hat{X} = -\frac{5}{11\sqrt{2}} Y_2 - \frac{9}{11\sqrt{6}} Y_2$$

Example: $\mathrm{cov}\begin{pmatrix} X \\ Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} 2 & -1/\sqrt{2} & -1/\sqrt{6} \\ + & 2{,}5 & -1/\sqrt{12} \\ \times & \times & 1{,}5 \end{pmatrix}$

$$E\left((XY_1, XY_2)\right) = -\left(\frac{1}{\sqrt{2}} \; \frac{4}{\sqrt{6}}\right)$$

$$\mathrm{cov}(Y) = \begin{pmatrix} 2{,}5 & -1/\sqrt{12} \\ \times & 1{,}5 \end{pmatrix}$$

$$\mathrm{cov}(Y)^{-1} = \begin{pmatrix} 0{,}4091 & 0{,}1707 \\ \times & 0{,}6818 \end{pmatrix} = \frac{3}{11}\begin{pmatrix} 3/2 & 1/\sqrt{12} \\ \times & 5/2 \end{pmatrix}$$

$$E\left((XY_1, XY_2)\right)\mathrm{cov}(Y)^{-1} = \left[-0{,}3214 \;\; -0{,}3340\right]$$
$$= -\frac{3}{11}\left[\frac{5}{3\sqrt{2}} \;\; \frac{3}{\sqrt{6}}\right]$$

$$\hat{X} = -0{,}3214\,Y_1 - 0{,}334\,Y_2 = -\frac{5}{11\sqrt{2}}Y_1 - \frac{9}{11\sqrt{6}}Y_2$$

$$E\left((X-\hat{X})Y_1\right) = E(XY_1) + 0{,}3214\,E(Y_1^2) + 0{,}334\,E(Y_1Y_2)$$
$$= -\frac{1}{\sqrt{2}} + 0{,}3214 \cdot 2{,}5 - 0{,}334\,\frac{1}{\sqrt{12}} \approx 0$$

$$E\left((X-\hat{X})Y_2\right) = E(XY_2) + 0{,}3214\,E(Y_1Y_2) + 0{,}334\,E(Y_2^2)$$
$$= -1/\sqrt{6} - 0{,}3214/\sqrt{12} + 0{,}334 \cdot 1{,}5 \approx 0$$