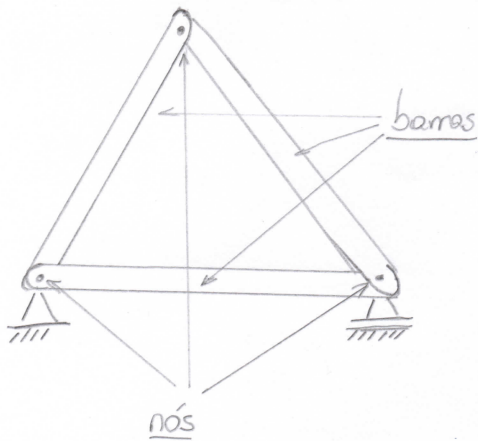
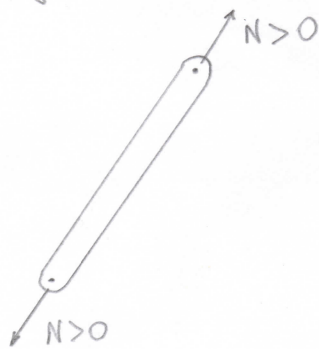


Trelças

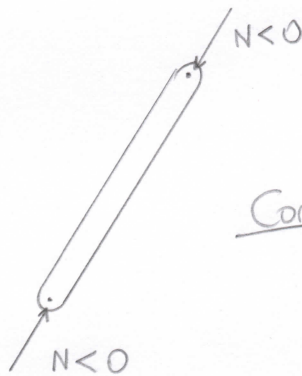
- Barras ligadas por conexões articuladas.
- Carregamentos exclusivamente nos nós.
- nós: pontos de conexão entre as barras.



Convenção de sinais:

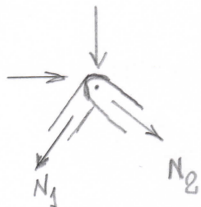


Tração



Compressão

Trelças Planas



- 2 equações de equilíbrio por nó

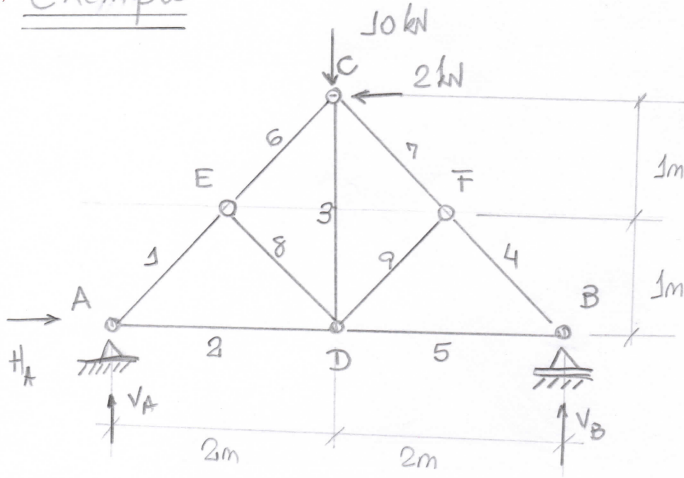
Logo para n nós, $2n$ equações de equilíbrio.

incógnitas:
} b : número de barras
} v : vínculos

Treligas isostáticas:

$$2n = b + r$$

Exemplo:



V_A	6
H_A	2
V_B	4
N_1	$-6\sqrt{2}$
N_2	4
N_3	0
N_4	$-4\sqrt{2}$
N_5	4
N_6	$-6\sqrt{2}$
N_7	$-4\sqrt{2}$
N_8	0
N_9	0

Reações de Apoio:

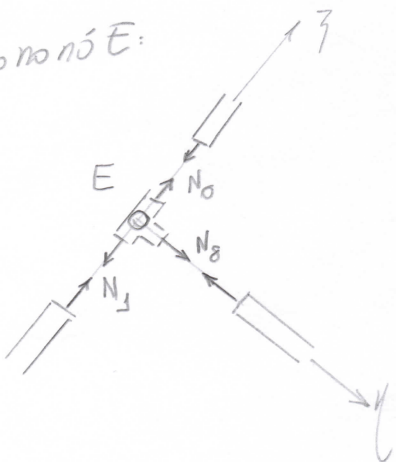
$$\sum F_x = 0 \Rightarrow H_A - 2 = 0 \Rightarrow \boxed{H_A = 2 \text{ kN}}$$

$$\sum F_y = 0 \Rightarrow V_A + V_B - 10 = 0$$

$$\sum M_A = 0 \Rightarrow -10 \cdot 2 + 2 \cdot 2 + V_B \cdot 4 = 0 \Rightarrow 4V_B - 20 + 4 = 0 \Rightarrow 4V_B = 16$$

$$\boxed{V_B = 4 \text{ kN}} \quad \boxed{V_A = 6 \text{ kN}}$$

Equilíbrio no nó E:



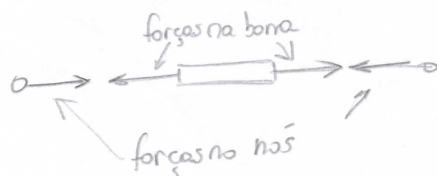
(Assumindo sempre $N > 0$)

$$\sum F_y = 0 \Rightarrow$$

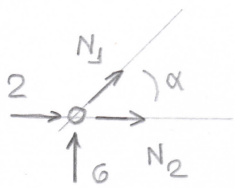
$$\sum F_y = 0 \Rightarrow \boxed{N_8 = 0}$$

$$\sum F_z = 0 \Rightarrow N_6 - N_1 = 0 \Rightarrow \boxed{N_6 = N_1}$$

força sendo no nó



Equilíbrio no nó A:



$$\cos \alpha = \sin \alpha = \frac{\sqrt{2}}{2}$$

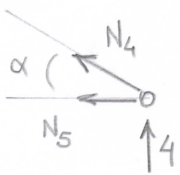
$$\sum F_x = 0 \Rightarrow 2 + N_2 + N_1 \cos \alpha = 0$$

$$\sum F_y = 0 \Rightarrow 6 + N_1 \sin \alpha = 0$$

$$\frac{\sqrt{2}}{2} N_1 = -6 \Rightarrow N_1 = \frac{-12}{\sqrt{2}} \Rightarrow \boxed{N_1 = -6\sqrt{2}}$$

$$N_2 = -2 - N_1 \frac{\sqrt{2}}{2} = -2 + 6 \Rightarrow \boxed{N_2 = 4}$$

Equilíbrio no nó B:



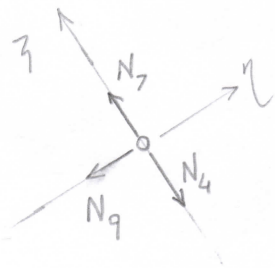
$$\sum F_x = 0 \Rightarrow -N_5 - N_4 \cos \alpha = 0$$

$$\sum F_y = 0 \Rightarrow 4 + N_4 \sin \alpha = 0 \Rightarrow N_4 \cdot \frac{\sqrt{2}}{2} = -4$$

$$N_4 = \frac{-8}{\sqrt{2}} \Rightarrow \boxed{N_4 = -4\sqrt{2}}$$

$$N_5 = -N_4 \cos \alpha = 4\sqrt{2} \cdot \frac{\sqrt{2}}{2} \Rightarrow \boxed{N_5 = 4}$$

Equilíbrio no nó E:



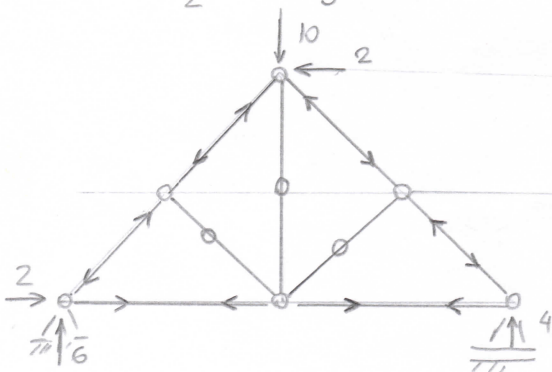
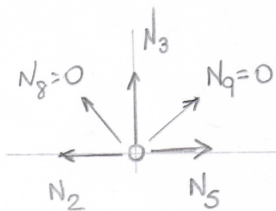
$$\sum F_y = 0 \Rightarrow \boxed{N_9 = 0}$$

$$\sum F_x = 0 \Rightarrow N_7 - N_4 = 0 \Rightarrow \boxed{N_7 = N_4}$$

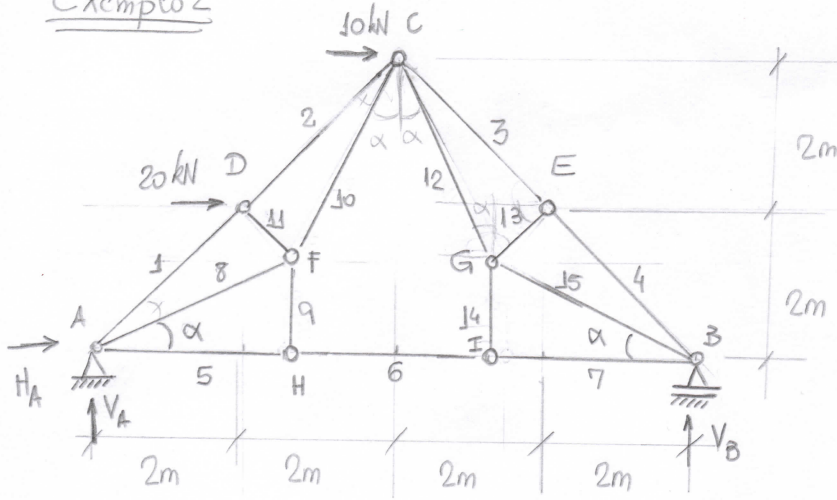
Equilíbrio no nó D:

$$\sum F_x = 0 \Rightarrow N_2 = N_5$$

$$\sum F_y = 0 \Rightarrow \boxed{N_3 = 0}$$



Exemplo 2



$\alpha = 30^\circ$

Reações de apoio:

$\sum F_x = 0 \Rightarrow H_A + 20 + 10 = 0 \Rightarrow H_A = -30 \text{ kN}$

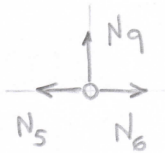
$\sum F_y = 0 \Rightarrow V_A + V_B = 0$

$\sum M_A = 0 \Rightarrow -20 \cdot 2 - 10 \cdot 4 + 8 \cdot V_B = 0$

$8V_B = 40 + 40 \Rightarrow V_B = 10 \text{ kN}$

$V_A = -10 \text{ kN}$

Nó H



$\sum F_x = 0 \Rightarrow N_5 = N_6$

$\sum F_y = 0 \Rightarrow N_9 = 0$

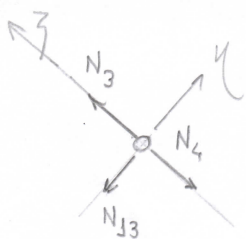
Nó I



$\sum F_x = 0 \Rightarrow N_6 = N_7$

$\sum F_y = 0 \Rightarrow N_{14} = 0$

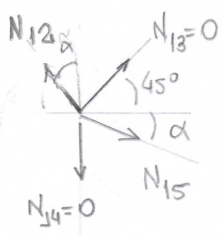
Nó E



$\sum F_y = 0 \Rightarrow N_{13} = 0$

$\sum F_z = 0 \Rightarrow N_3 = N_4$

Nó G



$\sum F_x = 0 \Rightarrow -N_{12} \cos \alpha + N_{15} \sin \alpha = 0 \Rightarrow N_{12} = \text{tg} \alpha \cdot N_{15}$

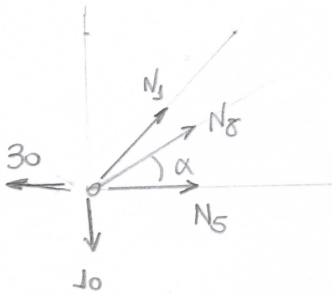
$\sum F_y = 0 \Rightarrow N_{12} \sin \alpha - N_{15} \cos \alpha = 0$

$\frac{\sin^2 \alpha}{\cos \alpha} \cdot N_{15} - N_{15} \cos \alpha = 0 \Rightarrow N_{15} (\sin^2 \alpha - \cos^2 \alpha) = 0$

Logo $N_{12} = N_{15} = 0$

N_1	-518
N_2	-19,32
N_3	$-10\sqrt{2}$
N_4	$-10\sqrt{2}$
N_5	10
N_6	10
N_7	10
N_8	27,32
N_9	0
N_{10}	27,32
N_{11}	$-10\sqrt{2}$
N_{12}	0
N_{13}	0
N_{14}	0
N_{15}	0

No' A



$$\sum F_x = 0: -30 + N_5 + N_1 \frac{\sqrt{2}}{2} + N_8 \frac{\sqrt{3}}{2} = 0$$

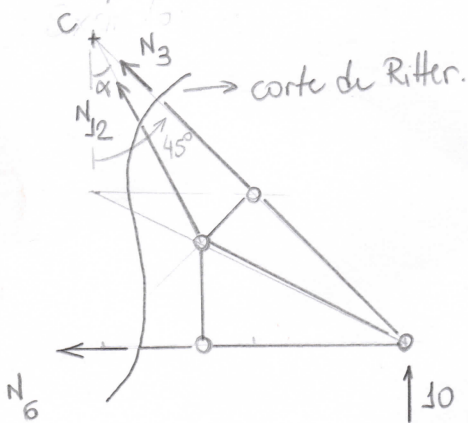
$$\sum F_y = 0: -10 + N_1 \frac{\sqrt{2}}{2} + N_8 \cdot \frac{1}{2} = 0$$

$$N_8 = 20 - N_1 \sqrt{2} \Rightarrow \boxed{N_8 = 27,3203}$$

Corte de Ritter (Método das seções)

- Corta-se a treliça em 2 partes.
- Verifica-se o equilíbrio em uma das partes.

Exemplo:

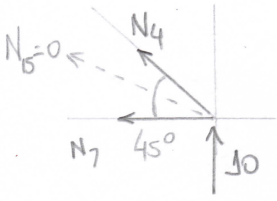


$$\sum F_x = 0 \Rightarrow -N_2 \cdot \text{sen } \alpha - N_3 \cdot \text{sen } 45^\circ - N_6 = 0$$

$$\sum F_y = 0 \Rightarrow +N_2 \cos \alpha + N_3 \cos 45^\circ + 10 = 0$$

$$\sum M_c = 0 \Rightarrow -N_6 \cdot 4 + 10 \cdot 4 = 0 \Rightarrow \boxed{N_6 = 10}$$

Nó B



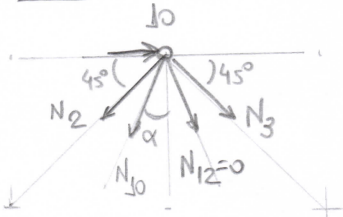
$$\sum F_x = 0 \Rightarrow -N_7 - N_4 \cos 45^\circ = 0$$

$$\sum F_y = 0 \Rightarrow 10 + N_4 \sin 45^\circ = 0$$

$$N_4 \cdot \frac{\sqrt{2}}{2} = -10 \Rightarrow \boxed{N_4 = -10\sqrt{2}}$$

$$N_7 = -N_4 \cdot \frac{\sqrt{2}}{2} \Rightarrow \boxed{N_7 = 10}$$

Nó C



$$\sum F_x = 0 \Rightarrow 10 - N_2 \cos 45^\circ + N_3 \cos 45^\circ - N_{10} \sin \alpha = 0$$

$$\sum F_y = 0 \Rightarrow -N_2 \sin 45^\circ - N_3 \sin 45^\circ - N_{10} \cos \alpha = 0$$

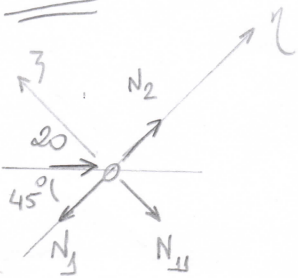
$$10 - \frac{\sqrt{2}}{2} N_2 + \frac{\sqrt{2}}{2} (-10\sqrt{2}) - N_{10} \cdot \frac{1}{2} = 0 \Rightarrow N_{10} = -\sqrt{2} N_2 \text{ or } N_2 = -\frac{\sqrt{2}}{2} N_{10}$$

$$-N_2 \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} (-10\sqrt{2}) - \frac{\sqrt{3}}{2} N_{10} = 0$$

$$-\left(-\frac{\sqrt{2}}{2} N_{10}\right) \frac{\sqrt{2}}{2} + 10 - \frac{\sqrt{3}}{2} N_{10} = 0 \Rightarrow N_{10} \frac{(1+\sqrt{3})}{2} = 10 \Rightarrow N_{10} = \frac{20}{\sqrt{3}-1}$$

$$\boxed{N_{10} = 27,3205} \Rightarrow \boxed{N_2 = -19,3184}$$

Nó D



$$\sum F_y = 0 \Rightarrow N_2 - N_{11} + 20 \cdot \frac{\sqrt{2}}{2} = 0$$

$$\sum F_x = 0 \Rightarrow -20 \cdot \frac{\sqrt{2}}{2} - N_{11} = 0 \Rightarrow N_{11} = -10\sqrt{2}$$

$$N_1 = 10\sqrt{2} + N_2 \Rightarrow \boxed{N_1 = -5,1762}$$