

Apêndice B

Derivada de $\log\left(\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}-a}\right)$

Vamos usar

$$\log(x/y) = \log(x) - \log(y) \quad (\text{B.1})$$

$$\frac{d}{dx} \log(x) = 1/x \quad (\text{B.2})$$

$$\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}} \quad (\text{B.3})$$

$$\frac{d}{dx} x^2 = 2x. \quad (\text{B.4})$$

Para

$$V = \log\left(\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}-a}\right) \quad (\text{B.5})$$

Temos

$$V = \log\left(\sqrt{x^2+a^2}+a\right) - \log\left(\sqrt{x^2+a^2}-a\right) \quad (\text{B.6})$$

e portanto

$$\begin{aligned} \frac{dV}{dx} &= \frac{x}{\sqrt{x^2+a^2}(\sqrt{x^2+a^2}+a)} - \frac{x}{\sqrt{x^2+a^2}(\sqrt{x^2+a^2}-a)} \\ &= \frac{x}{\sqrt{x^2+a^2}} \left(\frac{1}{\sqrt{x^2+a^2}+a} - \frac{1}{\sqrt{x^2+a^2}-a} \right) \\ &= \frac{x}{\sqrt{x^2+a^2}} \left(\frac{\sqrt{x^2+a^2}-a - \sqrt{x^2+a^2}-a}{(x^2+a^2)-a^2} \right) \\ &= \frac{x}{\sqrt{x^2+a^2}} \left(\frac{-2a}{x^2} \right) \\ &= -\frac{2a}{x\sqrt{x^2+a^2}} \end{aligned} \quad (\text{B.7})$$