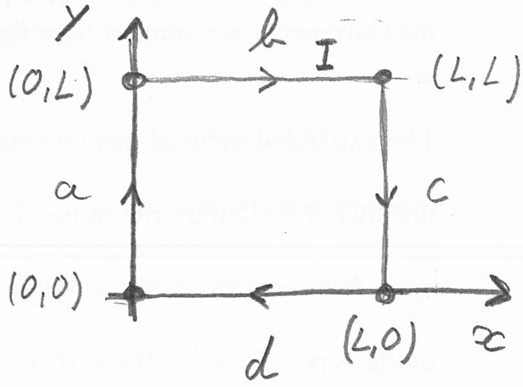
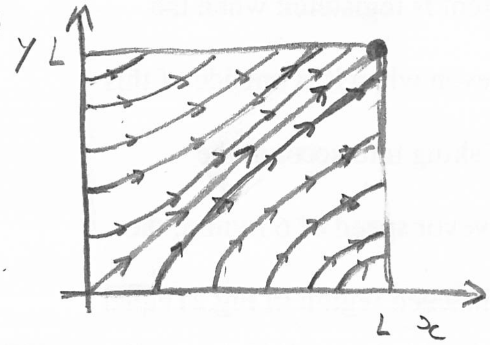
TORQUE SOBRE UMA ESPIRA DE CORRENTE EM UM CAMPO MAGNÉTICO NÃO UNIFORME

PROBLEMA 27.82



$$\underline{B} = B_0 \frac{y}{L}\,\hat{\underline{i}} + B_0 \frac{x}{L}\,\hat{\underline{j}}$$

(a)



(b) $\underline{dF} = I\,\underline{dl} \wedge \underline{B}$

LATO a

$\underline{dl}_a = dy\,\hat{\underline{j}}$ $\quad (x=0)$

$$\underline{dF}_a = I\,dy\,\hat{\underline{j}} \wedge \left(B_0 \frac{y}{L}\,\hat{\underline{i}} + B_0 \frac{x}{L}\,\hat{\underline{j}}\right) = I B_0 \frac{y}{L}\,dy\,\hat{\underline{j}} \wedge \hat{\underline{i}} =$$

$$= -I B_0 \frac{y}{L}\,dy\,\hat{\underline{k}}$$

$$\underline{F}_a = -\hat{\underline{k}} \int_0^L I B_0 \frac{y}{L}\,dy = -\hat{\underline{k}}\,\frac{1}{2} I B_0 \frac{y^2}{L}\Big|_0^L = -\frac{1}{2} I B_0 L\,\hat{\underline{k}}$$

LATO b

$\underline{dl}_b = dx\,\hat{\underline{i}}$ $\quad (y=L)$

$$\underline{dF}_b = I\,dx\,\hat{\underline{i}} \wedge \left(B_0 \frac{y}{L}\,\hat{\underline{i}} + B_0 \frac{x}{L}\,\hat{\underline{j}}\right) = I B_0 \frac{x}{L}\,dx\,\hat{\underline{i}} \wedge \hat{\underline{j}}$$

$$= I B_0 \frac{x}{L}\,dx\,\hat{\underline{k}}$$

$$\underline{F}_b = \hat{\underline{k}} \int_0^L I B_0 \frac{x}{L}\, dx = \hat{\underline{k}} \frac{1}{2} I B_0 \frac{x^2}{L}\Big|_0^L = \frac{1}{2} I B_0 L \hat{\underline{k}}$$

$\boxed{\text{LATO C}}$

$$\underline{dl}_c = -dy\, \hat{\underline{j}} \quad x = L$$

$$\underline{dF}_c = -I dy\, \hat{\underline{j}} \wedge \left(B_0 \frac{y}{L} \hat{\underline{i}} + B_0 \frac{x}{L} \hat{\underline{j}} \right) = -I B_0 \frac{y}{L} dy\, \hat{\underline{j}} \wedge \hat{\underline{i}} =$$

$$= I B_0 \frac{y}{L} dy\, \hat{\underline{k}}$$

$$\underline{F}_c = \hat{\underline{k}} \int_0^L I B_0 \frac{y}{L}\, dy = \hat{\underline{k}}\, \frac{1}{2} I B_0 \frac{y^2}{L}\Big|_0^L = \frac{1}{2} I B_0 L \hat{\underline{k}}$$

$\boxed{\text{LATO d}}$

$$\underline{dl}_d = -dx\, \hat{\underline{i}} \quad y = 0$$

$$\underline{dF}_d = -I dx\, \hat{\underline{i}} \wedge \left(B_0 \frac{y}{L} \hat{\underline{i}} + B_0 \frac{x}{L} \hat{\underline{j}} \right) = -I B_0 \frac{x}{L} dx\, \hat{\underline{i}} \wedge \hat{\underline{j}}$$

$$= -I B_0 \frac{x}{L} \cdot dx\, \hat{\underline{k}}$$

$$\underline{F}_d = -\hat{\underline{k}} \int_0^L I B_0 \frac{x}{L}\, dx = -\hat{\underline{k}}\, \frac{1}{2} I B_0 \frac{x^2}{L}\Big|_0^L = -\frac{1}{2} I B_0 L \hat{\underline{k}}$$

A FORÇA RESULTANTE É

$$\underline{F} = \underline{F}_a + \underline{F}_b + \underline{F}_c + \underline{F}_d = -\frac{1}{2} I B_0 L \hat{\underline{k}} + \frac{1}{2} I B_0 L \hat{\underline{k}}$$

$$+ \frac{1}{2} I B_0 L \hat{\underline{k}} - \frac{1}{2} I B_0 L \hat{\underline{k}} = 0$$

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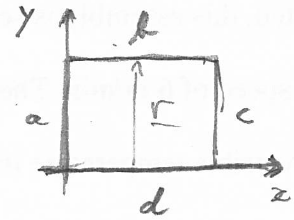
C

A espira é livre de girar livremente em torno do eixo Ox.

$$\underline{\tau} = \underline{r} \wedge \underline{F}$$



$\boxed{\text{LATO } a}$

$$d\underline{\tau}_a = \hat{\underline{j}}\, y \wedge d\underline{F}_a = \hat{\underline{j}}\, y \wedge \left(-I B_0 \frac{y}{L}\, dy\, \hat{\underline{k}}\right) =$$

$$= -I B_0 \frac{y^2}{L}\, dy\, \hat{\underline{j}} \wedge \hat{\underline{k}} = -I B_0 \frac{y^2}{L}\, dy\, \hat{\underline{i}}$$

$$\underline{\tau}_a = -I B_0\, \hat{\underline{i}} \int_0^L \frac{y^2}{L}\, dy = -\frac{1}{3} I B_0 L^2\, \hat{\underline{i}}$$

$\boxed{\text{LATO } b}$

$$d\underline{\tau}_b = \hat{\underline{j}}\, L \wedge d\underline{F}_b = \hat{\underline{j}}\, L \wedge \left(I B_0 \frac{x}{L}\, dx\, \hat{\underline{k}}\right) =$$

$$= I B_0\, x\, dx\, \hat{\underline{j}} \wedge \hat{\underline{k}} = I B_0\, x\, dx\, \hat{\underline{i}}$$

$$\underline{\tau}_b = I B_0\, \hat{\underline{i}} \int_0^L x\, dx = \frac{1}{2} I B_0 L^2\, \hat{\underline{i}}$$

$\boxed{\text{LATO } c}$

$$d\underline{\tau}_c = \hat{\underline{j}}\, y \wedge d\underline{F}_c = \hat{\underline{j}}\, y \wedge \left(I B_0 \frac{y}{L}\, dy\, \hat{\underline{k}}\right) =$$

$$= I B_0 \frac{y^2}{L}\, dy\, \hat{\underline{j}} \wedge \hat{\underline{k}} = I B_0 \frac{y^2}{L}\, dy\, \hat{\underline{i}}$$

$$\underline{\tau}_c = I B_0\, \hat{\underline{i}} \int_0^L \frac{y^2}{L}\, dy = \frac{1}{3} I B_0 L^2\, \hat{\underline{i}}$$

$\boxed{\text{LATO } d}$

$$\underline{\tau}_d = 0$$

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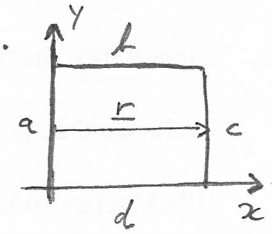
O TORQUE RESULTANTE É

$$\underline{\tau} = \underline{\tau}_a + \underline{\tau}_b + \underline{\tau}_c + \underline{\tau}_d = \frac{1}{2} I B_0 L^2 \hat{\underline{i}}$$

$$\boxed{\underline{\tau} = \frac{1}{2} I B_0 L^2 \hat{\underline{i}}}$$

d)

A espira é livre de girar livremente em torno do eixo Oy.



$$\underline{\tau} = \underline{r} \wedge \underline{F}$$

LATO a

$$\underline{\tau}_a = 0$$

LATO b

$$d\underline{\tau}_b = \hat{\underline{i}}\, x \wedge d\underline{F}_b = \hat{\underline{i}}\, x \wedge \left(I B_0 \frac{x}{L} dx\, \hat{\underline{k}} \right) =$$

$$= I B_0 \frac{x^2}{L} dx\, \hat{\underline{i}} \wedge \hat{\underline{k}} = - I B_0 \frac{x^2}{L} dx\, \hat{\underline{j}}$$

$$\underline{\tau}_b = - I B_0 \hat{\underline{j}} \int_0^L \frac{x^2}{L} dx = - \frac{1}{3} I B_0 L^2 \hat{\underline{j}}$$

LATO c

$$d\underline{\tau}_c = \hat{\underline{i}} L \wedge d\underline{F}_c = \underline{i} L \wedge \left(I B_0 \frac{x}{L} dx\, \hat{\underline{k}} \right) =$$

$$= I B_0\, x\, dx\, \hat{\underline{i}} \wedge \hat{\underline{k}} = - I B_0\, x\, dx\, \hat{\underline{j}}$$

$$\underline{\tau}_c = - I B_0 \hat{\underline{j}} \int_0^L x\, dx = - \frac{1}{2} I B_0 L^2 \hat{\underline{j}}$$

LATO d

$$d\underline{\tau}_d = \underline{i}\, x \wedge d\underline{F}_d = \hat{\underline{i}}\, x \wedge \left(- I B_0 \frac{x}{L} dx\, \hat{\underline{k}} \right) =$$

$$= - I B_0 \frac{x^2}{L} dx\, \hat{\underline{i}} \wedge \hat{\underline{k}} = I B_0 \frac{x^2}{L} dx\, \hat{\underline{j}}$$

$$\underline{\tau}_d = I B_0 \hat{\underline{j}} \int_0^L \frac{x^2}{L} dx = \frac{1}{3} I B_0 L^2 \hat{\underline{j}}$$

O TORQUE RESULTANTE É

$$\underline{\tau} = \underline{\tau}_a + \underline{\tau}_b + \underline{\tau}_c + \underline{\tau}_d = - \frac{1}{2} I B_0 L^2 \hat{\underline{j}}$$

$$\boxed{\underline{\tau} = - \frac{1}{2} I B_0 L^2 \hat{\underline{j}}}$$

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e

$$\underline{\tau} = \underline{\mu} \wedge \underline{B}$$

$$\boxed{\underline{\mu} = - I L^2 \hat{\underline{k}}}$$ MOMENTO DE DIPOLO MAGNÉTICO DA ESPIRA

O campo é não uniforme mas cada componente varia linearmente com a posição, portanto o valor medio é exatamente o valor do campo no centro da espira. Usando $\underline{B} = \frac{1}{2} B_0 \hat{\underline{i}} + \frac{1}{2} B_0 \hat{\underline{j}}$ a expressão $\underline{\tau} = \underline{\mu} \wedge \underline{B}$ fornece corretamente as componentes do torque

$$\underline{\tau} = - I L^2 \hat{\underline{k}} \wedge \left(\frac{1}{2} B_0 \hat{\underline{i}} + \frac{1}{2} B_0 \hat{\underline{j}} \right) = - \frac{1}{2} I B_0 L^2 \left(\hat{\underline{k}} \wedge \hat{\underline{i}} + \hat{\underline{k}} \wedge \hat{\underline{j}} \right)$$

$$= \underbrace{\frac{1}{2} I B_0 L^2 \hat{\underline{i}}}_{\text{ITEM C}} \underbrace{- \frac{1}{2} I B_0 L^2 \hat{\underline{j}}}_{\text{ITEM d}}$$

No caso geral de um campo não uniforme não é possível determinar a priori o ponto correto no interior da espira, mas em geral a formula $\underline{\tau} \approx \underline{\mu} \wedge \underline{B}$ fornece uma estimativa correta com um erro relativo que é da ordem da variação de B longo a superficie da espira.