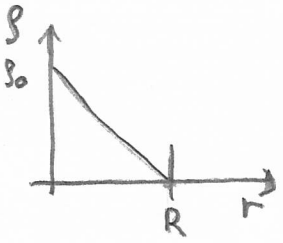
CAMPO DE UMA DISTRIBUÇÃO ESFÉRICA NÃO UNIFORME

PROBLEMA 22.57

$$\rho(r) = \begin{cases} \rho_0\left(1-\frac{r}{R}\right) & r \le R \\ 0 & r > R \end{cases} \qquad \rho_0 = \frac{3Q}{\pi R^3}$$



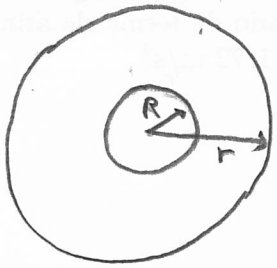
(a)

$$Q_{TOT} = \int_0^R \rho(r)\, 4\pi r^2 dr = \rho_0 \int_0^R \left(1-\frac{r}{R}\right) 4\pi r^2 dr =$$

$$= \rho_0 \left[\int_0^R 4\pi r^2 dr - \int_0^R 4\pi \frac{r^3}{R} dr \right] = \rho_0 \left(\frac{4}{3}\pi r^3 \Big|_0^R - \pi \frac{r^4}{R} \Big|_0^R \right) =$$

$$= \rho_0 \left(\frac{4}{3}\pi R^3 - \pi R^3 \right) = \frac{\pi}{3} \rho_0 R^3 = Q$$

(b)

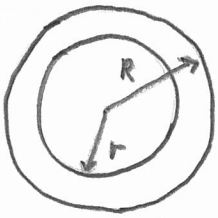


$$\Phi_E = \oiint E \cdot dA = 4\pi r^2 E$$

$$\Phi_E = \frac{Q_{LIQUIDA}}{\varepsilon_0} \qquad Q_{LIQUIDA} = Q$$

$$4\pi r^2 E = \frac{Q}{\varepsilon_0} \rightarrow E = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r^2}$$

(c)



$$\Phi_E = \oiint E \cdot dA = 4\pi r^2 E$$

$$\Phi_E = \frac{Q_{LIQUIDA}}{\varepsilon_0}$$

$$Q_{LIQUIDA} = \int_0^r \rho(r')\, 4\pi r'^2 dr' = \rho_0 \int_0^r \left(1-\frac{r'}{R}\right) 4\pi r'^2 dr' =$$

$$= \rho_0 \left[\int_0^r 4\pi r'^2 dr' - \int_0^r 4\pi \frac{r'^3}{R} dr' \right] =$$

PROBLEMA 22.57

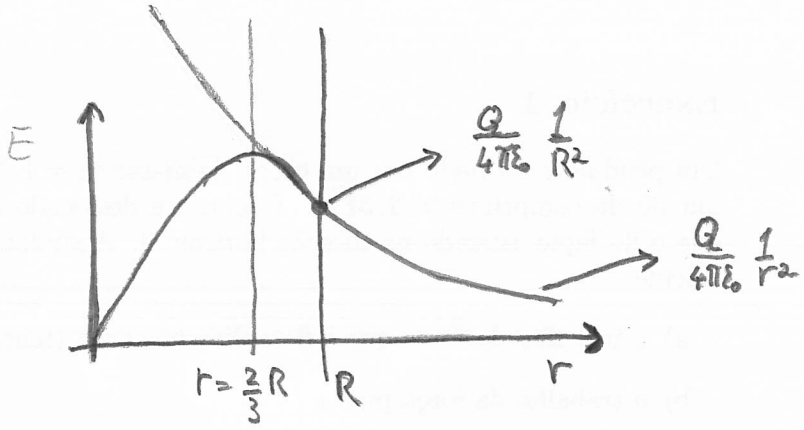
$$= \rho_0 \left[\frac{4}{3}\pi r'^3 \Big|_0^r - \frac{\pi r'^4}{R} \Big|_0^r \right] = \rho_0 \left(\frac{4}{3}\pi r^3 - \pi \frac{r^4}{R} \right) =$$

$$= \frac{3Q}{\pi R^3} \left(\frac{4}{3}\pi r^3 - \pi \frac{r^4}{R} \right) = Q \left(4\left(\frac{r}{R}\right)^3 - 3\left(\frac{r}{R}\right)^4 \right)$$

$$E = \frac{Q}{4\pi\varepsilon_0} \frac{1}{R^2} \left(4\frac{r}{R} - 3\left(\frac{r}{R}\right)^2 \right)$$

(d)

$r = 0 \quad E = 0$



(e)

$$\frac{dE}{dr} = \frac{Q}{4\pi\varepsilon_0} \frac{1}{R^2} \left(4\frac{1}{R} - 6\frac{r}{R^2} \right) = \frac{Q}{4\pi\varepsilon_0} \frac{1}{R^3} \left(4 - 6\frac{r}{R} \right) = 0$$

$$\frac{r}{R} = \frac{4}{6} = \frac{2}{3}$$

$$\boxed{\frac{r}{R} = \frac{2}{3}}$$

MÁXIMO DO CAMPO ELÉTRICO!