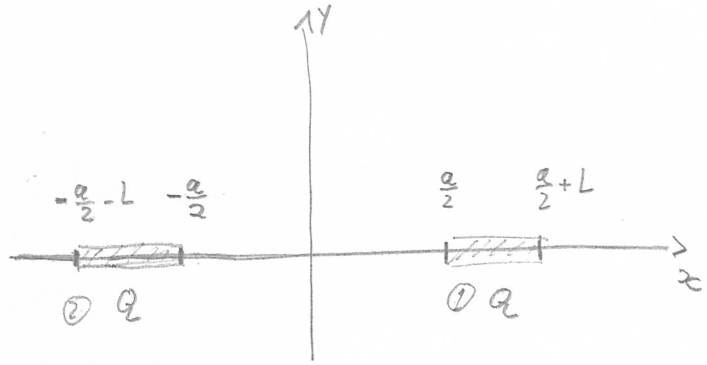
FORSA ENTRE DUAS BARRAS DELGADAS

PROBLEMA 21.107



A densidade linear de carga em cada barra é $\lambda = \frac{Q}{L}$.

(a)

O campo produzido pela barra ② na região $x>0$ é

$$dE_x = \frac{dQ}{4\pi\varepsilon_0}\frac{1}{(x+l)^2}$$

$$E_x = \frac{Q}{4\pi\varepsilon_0 L}\int_{-\frac{a}{2}-L}^{-\frac{a}{2}}\frac{dl}{(x-l)^2} = \frac{Q}{4\pi\varepsilon_0 L}\left(\frac{1}{(x-l)}\right)\Bigg|_{-\frac{a}{2}}^{-\frac{a}{2}-L}$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\left[\frac{1}{\left(x+\frac{a}{2}\right)} - \frac{1}{\left(x+\frac{a}{2}+L\right)}\right] =$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\frac{L}{\left(x+\frac{a}{2}\right)\left(x+\frac{a}{2}+L\right)} =$$

$$= \frac{Q}{4\pi\varepsilon_0}\frac{1}{\left(x+\frac{a}{2}\right)\left(x+\frac{a}{2}+L\right)}$$

$x\to\infty \quad E_x \to \frac{Q}{4\pi\varepsilon_0}\frac{1}{x^2}$

$L\to 0 \quad E_x \to \frac{Q}{4\pi\varepsilon_0}\frac{1}{\left(x+\frac{a}{2}\right)^2}$

(b)

O modulo da força que uma barra exerce sobre a outra é

$$F = \frac{Q}{L}\int_{\frac{a}{2}}^{\frac{a}{2}+L} E_x\,dx = \frac{Q^2}{4\pi\varepsilon_0 L^2}\int_{\frac{a}{2}}^{\frac{a}{2}+L}\left[\frac{1}{x+\frac{a}{2}} - \frac{1}{x+\frac{a}{2}+L}\right]dx$$

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$$F = \frac{Q^2}{4\pi\varepsilon_0 L^2}\left(\ln\left(x+\frac{a}{2}\right)\Big|_{\frac{a}{2}}^{\frac{a}{2}+L} - \ln\left(x+\frac{a}{2}+L\right)\Big|_{\frac{a}{2}}^{\frac{a}{2}+L}\right) =$$

$$= \frac{Q^2}{4\pi\varepsilon_0 L^2}\ \ln(a+L) - \ln(a) - \ln(a+2L) + \ln(a+L) =$$

$$= \frac{Q^2}{4\pi\varepsilon_0 L^2}\ \ln\frac{(a+L)^2}{a(a+2L)}$$

(c)

No limite $a >> L$

$$F = \frac{Q^2}{4\pi\varepsilon_0 L^2}\ \ln\frac{\left(1+\frac{L}{a}\right)^2}{1+2\frac{L}{a}} \approx \frac{Q^2}{4\pi\varepsilon_0 L^2}\ \ln\left(1+2\frac{L}{a}\right)^2\left(1-2\frac{L}{a}+\left(\frac{2L}{a}\right)^2+O\left(\frac{L}{a}\right)^3\right)$$

$$= \frac{Q^2}{4\pi\varepsilon_0 L^2}\ \ln\left(1+2\frac{L}{a}+\left(\frac{L}{a}\right)^2\right)\left(1-\frac{2L}{a}+\left(\frac{2L}{a}\right)^2+O\left(\frac{L}{a}\right)^3\right)$$

$$= \frac{Q^2}{4\pi\varepsilon_0 L^2}\ \ln\left(1+2\frac{L}{a}+\left(\frac{L}{a}\right)^2-2\frac{L}{a}-\left(2\frac{L}{a}\right)^2+\left(\frac{2L}{a}\right)^2+O\left(\frac{L}{a}\right)^3\right)$$

$$= \frac{Q^2}{4\pi\varepsilon_0 L^2}\ \ln\left(1+\left(\frac{L}{a}\right)^2+O\left(\frac{L}{a}\right)^3\right) \approx \frac{Q^2}{4\pi\varepsilon_0 L^2}\left(\frac{L}{a}\right)^2$$

$$= \frac{Q^2}{4\pi\varepsilon_0}\ \frac{1}{a^2}$$