

① Polinômio de Taylor de $\ln x$ em $x=1$

$f(x) = \ln x$	$x_0 = 1$ $\ln(1) = 0$
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Em geral

$f'(x) = \frac{1}{x}$	$\frac{1}{1} = 1$
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$f''(x) = -\frac{1}{x^2}$	$-\frac{1}{1^2} = -1$
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$f^{(3)}(x) = \frac{+2}{x^3}$	$\frac{+2}{1^3} = +2$
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$$f^{(n)}(x) = (-1)^{n+1} \frac{1}{2 \times 3 \times 4 \times \dots \times n}$$

positivo no impar
negativo no par

$f^{(4)}(x) = -\frac{2 \times 3}{x^4}$	$-\frac{2 \times 3}{1^4} = -2 \times 3$
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Por, $f^{(2n)}(1) = -1 \times 2 \times 3 \times \dots \times 2n$

$f^{(5)}(x) = \frac{2 \times 3 \times 4}{x^5}$	$\frac{2 \times 3 \times 4}{1^5} = 2 \times 3 \times 4$
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$f^{(2n+1)}(1) = 2 \times 3 \times \dots \times (2n+1)$

$2 \times 3 \times 4$

$$② \quad f(1) = 0$$

$$f'(1) = 1 = 0!$$

$$f''(1) = (-1) \cdot 1 = (-1) \cdot 1!$$

$$f'''(1) = 2 = 2!$$

$$f^{(iv)}(1) = (-1) \cdot 2 \times 3 = 3!$$

$$f^{(v)}(1) = 2 \times 3 \times 4 = 4!$$

~~$$f^{(vi)}(1) = (-1) \cdot 2 \times 3 \times 4 \times 5 = 5!$$~~

$$f^{(vi)}(1) = (-1) \cdot 2 \times 3 \times 4 \times 5 = 5!$$

$$f^{(m)}(1) = \begin{cases} 0 & m=0 \\ (-1)^{m-1} (m-1)! & m \geq 1 \end{cases}$$

Polinômio de Taylor de grau 6

$$f(x) \approx f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots + \frac{f^{(iv)}(x_0)}{6!}(x-x_0)^6$$

No caso de $x_0 = 1$, $f(x) = \ln(x)$

③ Vamos calcular $\ln \frac{1}{2}$ • Order 6

$$\ln \frac{1}{2} \approx 0 + \frac{0!(\frac{1}{2}-1)}{1!} + \frac{(-1)1!(\frac{1}{2}-1)^2}{2!} + \frac{2!(\frac{1}{2}-1)^3}{3!} +$$

$$\frac{(-1)(3!)(\frac{1}{2}-1)^4}{4!} + \frac{(4!)(\frac{1}{2}-1)^5}{5!} + \frac{(-1)5!(\frac{1}{2}-1)^6}{6!}$$

$$\ln \frac{1}{2} \approx -\frac{1}{2} - \frac{1}{2}\left(\frac{1}{2}\right)^2 + \frac{(-\frac{1}{2})^3}{3} + \left(-\frac{1}{4}\right)\left(\frac{1}{2}\right)^4 +$$

$$\frac{1}{5}\left(-\frac{1}{2}\right)^5 + \left(-\frac{1}{6}\right)\left(\frac{1}{2}\right)^6$$

$$= -\left[\frac{1}{2} + \frac{1}{2} \frac{1}{2^2} + \frac{1}{3} \frac{1}{2^3} + \frac{1}{4} \frac{1}{2^4} + \frac{1}{5} \frac{1}{2^5} + \frac{1}{6} \frac{1}{2^6} \right]$$

4) $\ln \frac{1}{2} \approx - \left[\frac{1}{2} + \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{3} \frac{1}{8} + \frac{1}{4} \cdot \frac{1}{16} + \frac{1}{5} \cdot \frac{1}{32} + \frac{1}{6} \cdot \frac{1}{64} \right] = 0,6911458$

$$\text{Erro} = \frac{f^{(7)}(\bar{x}) (x-x_0)^7}{7!} = \frac{6! (\frac{1}{2}-1)^7}{\bar{x}^7 7!}$$

Erro em módulo

$$\frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{2^7}$$

No melhor caso $\frac{1}{7 \cdot 2^7}$

No pior caso $\frac{1}{7} \cdot \left(\frac{1}{2}\right)^7 \cdot \frac{1}{2^7} = \frac{1}{7} = 0,1428$

No calculadora $\ln \frac{1}{2} = 0,6931472$