Uncertainties in Dynamical Systems

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Presentation Schedule

Uncertainties Representation

Motivation

In dynamical systems,a balance between capturing the true behavior of the physical system and generating mathematically tractable models requires a great effort in control designing due to *uncertainties*.[1]

For standard behavior the system is called *Nominal* and the composed of the nominal system and some kind of perturbation is generally referred to as the uncertain model or model set.

[1] A Garulli, A Tesi, and A Vicino. "UNCERTAINTY MODELS FOR ROBUSTNESS ANALYSIS". In: **CONTROL SYSTEMS, ROBOTICS AND AUTOMATION–Volume IX: Advanced Control Systems-III** (2009), p. 47

Structured and unstructured uncertainties

Usually in the literature it is possible to find two approaches in describing the uncertainty[1].

- Unstructured - Neglected or unmodelled dynamics, nonlinearities, high-frequency flexible models
- Structured - Uncertainty in the parameters, variations in the system operation (e.g. capacitors in electric circuits, damping coefficients).

Parametric uncertainties

Parametric uncertainties can be modeled by assuming some boundaries in each uncertain parameter of the system. This boundary lies in a region [α_{min}, α_{max}] and some example of normalization can be seen in 1:

$$\alpha_p = \bar{\alpha}(1 + r_\alpha \Delta) \tag{1}$$

where $\bar{\alpha}$ is the mean parameter value, $r_\alpha = \frac{(\alpha_{max} - \alpha_{min})}{(\alpha_{max} + \alpha_{min})}$ is the relative uncertainty in the parameter and Δ is any scalar so that $|\Delta| \leq 1$.

Polytopic and norm-bounded uncertainties

Polytopic and norm-bounded uncertainties in controller's synthesis are commonly seen in the literature. The former is useful to model different points of operation in a system, while the latter has its application in systems which there is an error in the actual and the nominal system. Polytopic uncertainties is more conservative than norm-bounded models [2], as can be seen in the figure 1.

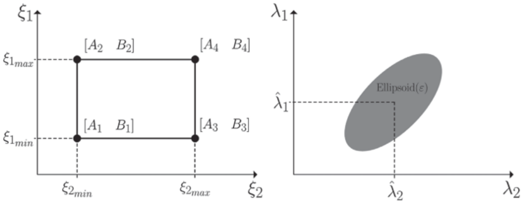

Figure: Uncertainty regions for polytopic and norm-bounded models [3]

[2] Rodrigo da Ponte Caun et al. "LQR-LMI control applied to convex-bounded domains". In: Cogent Engineering 5.1 (2018), p. 1457206

Polytopic uncertainty

For a system with the form 2 and a set with the shape $[\ A\ B\ ] \in \Omega$, the set Ω describes the uncertainty in the matrices A e B. A possible set Ω is 3:

$$\begin{aligned} \dot{x} &= Ax(t) + Bu(t) \\ y &= Cx(t) + Du(t) \end{aligned} \tag{2}$$

$$\Omega_p = Co\left\{\begin{bmatrix} A_1 & B_1 \end{bmatrix}, \begin{bmatrix} A_2 & B_2 \end{bmatrix}, \dots, \begin{bmatrix} A_N & B_N \end{bmatrix}\right\} \tag{3}$$

and N corresponds to a convex set of N vertices.

Polytopic uncertainty

If the system 2 is time-varying, another representation for it is 4:

$$\begin{aligned} \dot{x} &= A(t)x(t) + B(t)u(t) \\ y &= C(t)x(t) + D(t)u(t) \end{aligned} \tag{4}$$

Suppose a *system matrix*: $S(t) := \begin{pmatrix} A(t) & B(t) \\ C(t) & D(t) \end{pmatrix}$ $S(t)$ can be written as a *convex combination* of p system matrices[4] $S_1, S_2, \ldots, S_p$ such that 5 :

$$S(t) = \sum_{j=1}^{p} \alpha_j(t) S_j \tag{5}$$

where $\sum_{j=1}^{p} \alpha_j = 1$ and $S(t) := \begin{pmatrix} A_j & B_j \\ C_j & D_j \end{pmatrix}$

[4] Carsten Scherer and Siep Weiland. "Linear matrix inequalities in control". In: **Lecture Notes, Dutch Institute for Systems and Control, Delft, The Netherlands** 3.2 (2000)

Norm-bounded uncertainty

For coupled norm-bounded uncertainty, one representation can be described in 6:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + B_p p(t) \\ q(t) &= C_q x(t) + D_q u(t) \\ p(t) &= \Lambda q(t) \end{aligned} \tag{6}$$

where $\|\Lambda\| \leq 1$ is the pertubation. Due to the uncertainties in the system, the norm bounded time-varying uncertain matrix and regulated output can be described as $p(t)$, $q(t) \in \mathbb{R}^r$ respectively. The matrices B_p,C_q and D_q have appropriate dimensions for the problem and they are real constant matrices. $\Lambda = \begin{bmatrix} \Lambda_1 & \dots & 0 \\ \vdots & \Lambda_i & \vdots \\ 0 & \dots & \Lambda_r \end{bmatrix}$[3]. So, a set Ω_N to describe the norm-bounded uncertainties has the form:

$$\Omega_N = \left\{ \begin{bmatrix} A + B_p \Lambda C_q & B + B_p \Lambda D_q \end{bmatrix} : \|\Lambda_i\| \leq 1, \Lambda \quad \text{diagonal} \right\} \tag{7}$$

Norm-bounded uncertainty

An approach to describe norm-bounded uncertainty in a recursively way can be found in [5] [6] [7] In this case, the system 6 has the form 8:

$$x_{k+1} = (A + \delta A)x[k] + (B + \delta B)u[k]$$
$$y[k] = Cx[k] + Du[k] \tag{8}$$

And $\begin{bmatrix} \Delta A & \Delta B \end{bmatrix} = H\Delta \begin{bmatrix} E_a & E_b \end{bmatrix}$. Calling $H = B_p, \Delta = \Lambda$ and $\begin{bmatrix} C_q & D_q \end{bmatrix} = \begin{bmatrix} E_a & E_b \end{bmatrix}$, the systems 6 and 8 are very similar.

[5] Ali H Sayed and Vitor H Nascimento. "Design criteria for uncertain models with structured and unstructured uncertainties". In: **Robustness in identification and control**. Springer, 1999, pp. 159–173

[6] Marco H Terra, Joao P Cerri, and João Y Ishihara. "Optimal robust linear quadratic regulator for systems subject to uncertainties". In: **IEEE Transactions on Automatic Control** 59.9 (2014), pp. 2586–2591

[7] Filipe Marques Barbosa et al. "Robust path-following control for articulated heavy-duty vehicles". In: **Control Engineering Practice** 85 (2019), pp. 246–256

LMI - Overview

Definition

Linear Matrix Inequality

Suppose $A_0, A_1, \ldots, A_n$ symmetric $m \times m$ matrices. An LMI is expressed as:

$$A(x) < 0$$

with

$$A(x) = A_0 + \sum_{i=1}^{n} A_i x_i$$

and $x_i \in \mathbb{R}$ is the $i-th$ component vector x So $A : \mathbb{R}^n \rightarrow \mathbb{R}^{m \times m}$ is a linear function of x.

Schur complement

The Schur complement can be a useful LMI formulation when we have a nonlinear constraint, as in LQR-LMI problem (this example will be presented in the next section).

Schur complement formulation

Consider the symmetric block matrix $M(x)$ defined as follows:

$$M(x) = \begin{bmatrix} S(x) & V(x) \\ V(x)' & Q(x) \end{bmatrix} < 0$$

this is equivalent to the conditions (1) and (3) and $S(x) < 0$ if and only if $Q(x) < 0$.:

1. $Q(x) - V(x)'S(x)^{-1}V(x) < 0$
2. $S(x) - V(x)Q(x)^{-1}V(x)' < 0$

Schur complement

Schur complement of a Nine-Block Matrix

Consider the symmetric block matrix $M(x)$:

$$M(x) = \begin{bmatrix} S(x) & V(x) & W(x) \\ V(x)' & Q(x) & 0 \\ W(x)' & 0 & Z(x) \end{bmatrix} < 0$$

this is equivalent to the conditions (1) and $Q(x) < 0$ if and only if $Z(x) < 0$.:

1. $S(x) - V(x)Q(x)^{-1}V(x)' - W(x)Z(x)^{-1}W(x)' < 0$

Schur complement

Tips

Is it common to see the $*$ (star) symbol to represent an element in a symmetric $m \times m$ matrix. Consider $i = 0, 1, \ldots, m$ and $j = 0, 1, \ldots, m$ as the index of the a_{ij} element in $A_{m\times m}$ and $a_{ij} = *$ for some (i, j). The $*$ represents a'_{ji} element. For instance ($m = 2$):

$$M = \begin{bmatrix} A & * \\ B & C \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \begin{bmatrix} A & B' \\ B & C \end{bmatrix}$$

$$m_{12} = * = B' = m'_{21}$$

for $m = 3$:

$$M = \begin{bmatrix} A & * & * \\ B & C & * \\ D & E & F \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = \begin{bmatrix} A & B' & D' \\ B & C & E' \\ D & E & F \end{bmatrix}$$

$$m_{12} = B', m_{13} = D', m_{23} = E'$$

LMI in Matlab

The most common way to solve an LMI is the use of a LMI solver called SeDuMi and the LMI parser YALMIP. The installation is quite simple:

- Download SeDuMn package from http://sedumi.ie.lehigh.edu/;
- Download YALMIP from https://yalmip.github.io/;
- Extract both programs in a desired folder in your computer;
- In Matlab **Home** tab, go to **ENVIRONMENT** section and click on **Set Path**. In the following window, click on **Add with Subfolders...** and search for the folder you chose to extract SeDuMi and YALMIP;
- Click on **Save** button
- Now you have your LMI solver in Matlab!
- If you want to make sure you did the right procedure, tip **help sedumi** and **help yalmip** in the Matlab's command window. If you doní get any error, the installation was successful.

LMI code example

```
close all;
clear;
clc;

A = [-1 2;-3 -4]; % define matrix A
P = sdpvar(2,2); % create symmetric P with dimensions (2,2)

% Set LMI
T = A'*P+P*A;
F = [P > 0; T < 0; trace(P)==1]; % trace(P)=1 is an additional restriction
% to ensure that we have a matrix P with values not so close to zero.

about = optimize(F) % solve LMI
P feasible = double(P); % evaluate P
```

Linear Quadratic Regulator - LMI Approach

LQR - LMI Approach

Given the system 2, the optimal LQr controller can be obtained by using state-feedback gain **K** ($\tilde{u} = Kx$) that minimizes a performance index:

$$J = \int_0^{\infty} (x'Qx + u'Ru)dt, \quad Q = Q', Q \geq 0, R = R', R > 0. \tag{9}$$

So $J = \int_0^{\infty}(x'(Q+K'RK)x)dt$. Using trace operator $Tr(.)$, $J = \int_0^{\infty} Tr((Q+K'RK)xx')dt = Tr((Q+K'RK)P)$, where $P = \int_0^{\infty}(xx')dt$, $P = P'$, $P > 0$ and P must satisfy 10

$$(A+BK)P + P(A+BK)' + x_0x_0' = 0, x_0 \quad \text{state inital condition} \tag{10}$$

The optimal feedback gain K can be found by minimization of 11:

$$\begin{aligned} \min_{P,K} Tr(QP) + Tr(R^{\frac{1}{2}}KPK'R^{\frac{1}{2}}), \\ \text{subject to } (A+BK)P + P(A+BK)' + x_0x_0' < 0 \end{aligned} \tag{11}$$

Due to the bi-linearity of KP, it is required to make a change of variable so that the problem 11 will become a Linear Matrix Inequality (LMI). Introducing a new variable $Y = KP$, 11 can be rewritten as 12:

LQR - LMI

$$\min_{P,Y} Tr(QP) + Tr(R^{\frac{1}{2}} Y P^{-1} Y' R^{\frac{1}{2}}), \quad \text{subject to } AP + PA' + BY + Y'B' + x_0 x_0' < 0 \tag{12}$$

The term $R^{\frac{1}{2}} Y P^{-1} Y' R^{\frac{1}{2}}$ can be replaced for a second auxiliary variable X, and $X > R^{\frac{1}{2}} Y P^{-1} Y' R^{\frac{1}{2}}$ Using Schur's complement the complete LQR-LMI formulation is 13:

$$\begin{gathered} \min_{P,Y,X} Tr(QP) + Tr(X), \\ \text{subject to } AP + PA' + BY + Y'B' + I < 0 \\ \begin{bmatrix} X & R^{\frac{1}{2}} Y \\ Y' R^{\frac{1}{2}} & P \end{bmatrix} > 0 \end{gathered} \tag{13}$$

The whole process can be found in [8] and for discrete systems [9]

[8] Olalla, C et all, IEE transactions of industrial eletronics, *Vol*.56, 2009

[9] Dallali, H et al. Decentralized Feedback Design for a Compliant Robot ArmUKsim-Amss 8th European Modelling Symposium. 2014

LQR - LMI

The system response of continuous system is in the figure 2:

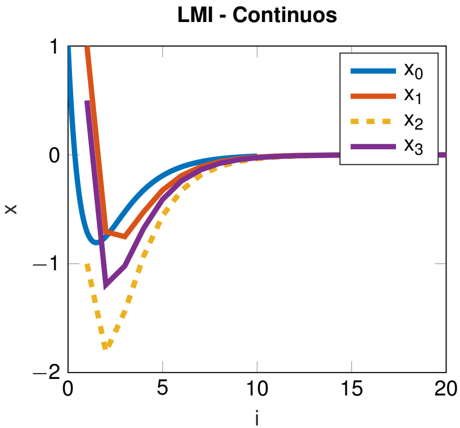

Figure: LQR-LMI Continuous response

LQR - LMI

The system response of discrete system is in the figure 3:

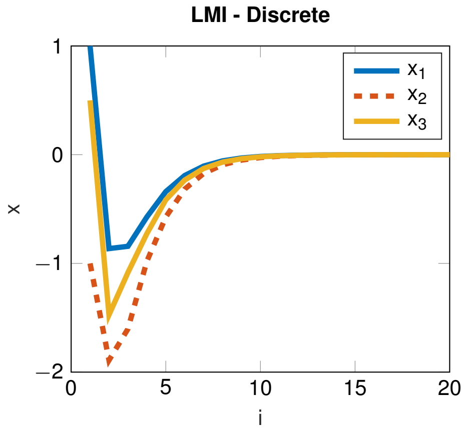

Figure: LQR-LMI Discrete response

Robust Linear Quadratic Regulator

Derivation of the upper bound using bounds on output energy

Consider the LQR problem given in 9. It is possible to define an energy function $V(x)$ such that 14:

$$V(x) + \int_0^{\infty} x(t)'(Q + K'RK)x(t)dt < 0, \tag{14}$$

And if $x(0)$ is known, 14 can be rewritten as 15:

$$V(x(\infty)) - V(x(0)) + \int_0^{\infty} x(t)'(Q + K'RK)x(t)dt < 0 \tag{15}$$

Considering that the systems has all real part negatives eigenvalues, $x(\infty) \to 0$. So, $\int_0^{\infty} x(t)'(Q + K'RK)x(t)dt < V(x(0)) = x(0)'Px(0)$. But $x(0)'Px(0) \leq \mu$. With Schur's complement, $W = P^{-1}$, so:

$$\begin{bmatrix} \mu & * \\ x(0) & W \end{bmatrix} \geq 0$$

For a polytope $\Omega_c = Co\{X_1(0), X_2(0), \ldots, X_p(0)\}$ the new upper bound is 16[10]:

$$\begin{bmatrix} \mu & * \\ x_i(0) & W \end{bmatrix} \geq 0, i = 1, \ldots, P \tag{16}$$

[10] Caun, R.P. Assunção, E.,Teixeira M.C.M. Caun, A. P. LQR-LMI control applied to convex-bounded domains. Cogent Engineering. 2018

Restriction of decay rate

In order to design a control law to attend performance requirements of a desired trajectory or behavior of output signals, it is important to give some new constraints in LMI formulation. Nonetheless, this method increases computational complexity, which is not desirable in any application. Hence. a different approach is necessary. For instance, it can be possible to incorporate the decay rate in the original LMI formulation, as can be seen in the following theorems, for Polytopic and norm-bounded uncertainty, respectively:[11]:

[11] Caun, R.P. Assunção, E.,Teixeira M.C.M. Caun, A. P. LQR-LMI control applied to convex-bounded domains. Cogent Engineering. 2018

Polytopic uncertainty

Theorem

[12]: *The system 2 is stable by* $u(t) = -Kx(t)$, *using decay rate greater than or equal to* α *and guaranteed cost J, less than* μ *if there exists matrices* $W = W' > 0 \in \mathbb{R}^{n\times n}$ *and* $Z \in \mathbb{R}^{m\times n}$ *such that*

$$\min_{\mu, W, Z} \mu$$

Subject to

$$\begin{bmatrix} G_i & * & * \\ W & -Q^{-1} & * \\ Z & 0 & -R^{-1} \end{bmatrix} \leq 0, \quad \begin{bmatrix} \mu & * \\ x(0) & W \end{bmatrix} \geq 0$$

$$G_i = A_i W + W A_i + B_i Z + Z' B_i' + 2\alpha W, K = -ZW^{-1}$$

[12] Caun, R.P. Assunção, E.,Teixeira M.C.M. Caun, A. P. LQR-LMI control applied to convex-bounded domains. Cogent Engineering. 2018

Polytopic uncertainty

The results of this theorem can be seen in the Figure 4. The system to be controlled was a 3*DOF* helicopter, the same as given in the paper. In the simulation, it was consider a polytope with two vertices such that the first vertex represents system's nominal behavior and the second describes 50% of failure in the actuators. As can be seen, $x(t) \to 0$ when $t \to 0$.

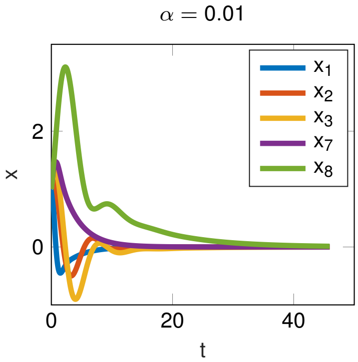

Figure: LMI-RLQR with restriction in decay rate response

Norm-bounded uncertain

Theorem

[13] *The same conditions of the previous thorem (Polytopic). However, the minimization problem is:*

$$\min_{\mu, W, Z, \Delta} \mu$$

Subject to

$$\begin{bmatrix} \Theta & * & * & * \\ W & -Q^{-}1 & * & * \\ Z & 0 & -R^{-}1 & * \\ C_q W + D_q Z & 0 & 0 & -\Delta \end{bmatrix} \leq 0, \quad \begin{bmatrix} \mu & * \\ x(0) & W \end{bmatrix} \geq 0$$

with $\delta_1, \ldots, \delta_r > 0$ *such that:*

[13] Caun, R.P. Assunção, E.,Teixeira M.C.M. Caun, A. P. LQR-LMI control applied to convex-bounded domains. Cogent Engineering. 2018

Norm-bounded uncertain

Theorem

$$\Delta = \begin{bmatrix} \delta_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \delta_r \end{bmatrix}$$

$$\Theta = AW + WA' + BZ + Z'B' + 2\alpha W + B_p \Delta B_p', K = -ZW^{-1}$$

Optimal robust linear quadratic regulator for systems subject to uncertainties

The problem is to obtain a robust linear quadratic regulator for discrete -time linear system subject to parametric uncertainties [14], in the form: $x_{k+1} = (A + \delta A)x[k] + (B + \delta B)u[k]$ and $\begin{bmatrix} \delta A & \delta B \end{bmatrix} = H\Delta \begin{bmatrix} E_a & E_b \end{bmatrix}$, in which $H_i \in R^{n \times p}$, E_f and E_g known matrices and Δ_i an arbitrary matrix, such that $\|\Delta_i\| \leq 1$.
The optimization problem to be solved is 17:

$$\min_{x_{i+1}, u_i} \max_{\delta F_i, \delta G_i} \left\{ J_i^{\mu}(x_{i+1}, u_i, \delta F_i, \delta G_i) \right\} \quad (17)$$

And 17 can be solved based on the solution of general robust regularized least-squares problem 18:

$$\min_{x} \max_{\delta A, \delta B} \left\{ \|x\|_Q^2 + \|(A + \delta A)x - (b + \delta B)\|_W^2 \right\} \quad (18)$$

where A is a nominal matrix, b is a measurement vector, $Q > 0$, $W > 0$ are weighting matrices, x is an unknown vector and $\begin{bmatrix} \delta A & \delta B \end{bmatrix} = H\Delta \begin{bmatrix} E_a & E_b \end{bmatrix}, \quad \|\Delta\| \leq 1.$

[14] Terra, M.H. Cerri, J.P. Ishihara, J.Y. **Optimal robust linear quadratic regulator for systems subject to uncertainties.** IEE transactions on Automatic Control

Optimal robust linear quadratic regulator for systems subject to uncertainties

Lemma 1: Consider the optimization problem (3), (4). The optimal solution for each $\mu > 0$ is given by

$$\begin{bmatrix} x_{i+1}^*(\mu) \\ u_i^*(\mu) \\ \tilde{J}_i^{\mu}\left(x_{i+1}^*(\mu), u_i^*(\mu)\right) \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & x_i^*(\mu)^T \end{bmatrix} \begin{bmatrix} L_{i,\mu} \\ K_{i,\mu} \\ P_{i,\mu} \end{bmatrix} x_i^*(\mu)$$

$$\begin{bmatrix} L_{i,\mu} \\ K_{i,\mu} \\ P_{i,\mu} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \\ 0 & 0 & -I & \mathcal{F}_i^T & 0 & 0 \end{bmatrix}$$

$$\times \begin{bmatrix} P_{i+1}^{-1} & 0 & 0 & 0 & I & 0 \\ 0 & R_i^{-1} & 0 & 0 & 0 & I \\ 0 & 0 & Q_i^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \Sigma_i(\mu, \lambda_i) & \mathcal{I} & -\mathcal{G}_i \\ I & 0 & 0 & \mathcal{I}^T & 0 & 0 \\ 0 & I & 0 & -\mathcal{G}_i^T & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ -I \\ \mathcal{F}_i \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

Figure: Lemma of robust LQR

Optimal robust linear quadratic regulator for systems subject to uncertainties

where $\quad \Sigma_{i,\mu} := \Sigma_i(\mu, \lambda_i) = \begin{bmatrix} \mu^{-1}I - \lambda_i^{-1}H_iH_i^T & 0 \\ 0 & \lambda_i^{-1}I \end{bmatrix}$,

$\mathcal{I} = \begin{bmatrix} I \\ 0 \end{bmatrix}, \quad \mathcal{G}_i = \begin{bmatrix} G_i \\ E_{G_i} \end{bmatrix}, \quad \mathcal{F}_i = \begin{bmatrix} F_i \\ E_{F_i} \end{bmatrix}$, and $\quad \lambda_i > \|\mu H_i^T H_i\|$.

Furthermore, alternatively one has

$$P_{i,\mu} = L_{i,\mu}^T P_{i+1} L_{i,\mu} + K_{i,\mu}^T R_i K_{i,\mu} + Q_i + (\mathcal{I}L_{i,\mu} - \mathcal{G}_i K_{i,\mu} - \mathcal{F}_i)^T \Sigma_{i,\mu}^{-1} (\mathcal{I}L_{i,\mu} - \mathcal{G}_i K_{i,\mu} - \mathcal{F}_i) \succ 0. \tag{12}$$

Figure: Lemma of robust LQR

Optimal robust linear quadratic regulator for systems subject to uncertainties

Uncertain Model: Consider (1), (2), and (4) with F_i, G_i, E_{F_i}, E_{G_i}, $Q_i \succ 0$, and $R_i \succ 0$ known for all i.

Initial Conditions: Set x_0 and $P_{N+1} \succ 0$.

Step 1: *(Backward)* Calculate for all $i = N, \dots 0$:

$$\begin{bmatrix} L_i \\ K_i \\ P_i \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -I \\ 0 & 0 & F_i \\ 0 & 0 & E_{F_i} \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}^T \begin{bmatrix} P_{i+1}^{-1} & 0 & 0 & 0 & 0 & I & 0 \\ 0 & R_i^{-1} & 0 & 0 & 0 & 0 & I \\ 0 & 0 & Q_i^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & -G_i \\ 0 & 0 & 0 & 0 & 0 & 0 & -E_{G_i} \\ I & 0 & 0 & I & 0 & 0 & 0 \\ 0 & I & 0 & -G_i^T & -E_{G_i}^T & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ -I \\ F_i \\ E_{F_i} \\ 0 \\ 0 \end{bmatrix}.$$

Step 2: *(Forward)* Obtain for each $i = 0, \dots N$:

$$\begin{bmatrix} x_{i+1}^* \\ u_i^* \end{bmatrix} = \begin{bmatrix} L_i \\ K_i \end{bmatrix} x_i^*,$$

with the total cost given by $J_r^* = x_0^T P_0 x_0$.

Figure: Algorithm of robust LQR

Optimal robust linear quadratic regulator for systems subject to uncertainties

$$F = \begin{bmatrix} 1.1 & 0 & 0 \\ 0 & 0 & 1.2 \\ -1.0 & 1.0 & 0 \end{bmatrix}, G = \begin{bmatrix} 0 & 1.0 \\ 1.0 & 1.0 \\ -1.0 & 0 \end{bmatrix}, H = \begin{bmatrix} 0.7 \\ 0.5 \\ -0.7 \end{bmatrix},$$
$$E_F = [0.4 \quad 0.5 \quad -0.6], \; E_G = [0.4 \quad -0.4], \; -1 \le \Delta_i \le 1$$

and $x_0 = [1 \; -1 \; 0.5]^T$. With respect to the quadratic cost function (4), the weighting matrices are given by $P_{N+1} = I_3$, $Q = I_3$, and $R = I_2$.

Figure: Example of robust LQR

Optimal robust linear quadratic regulator for systems subject to uncertainties

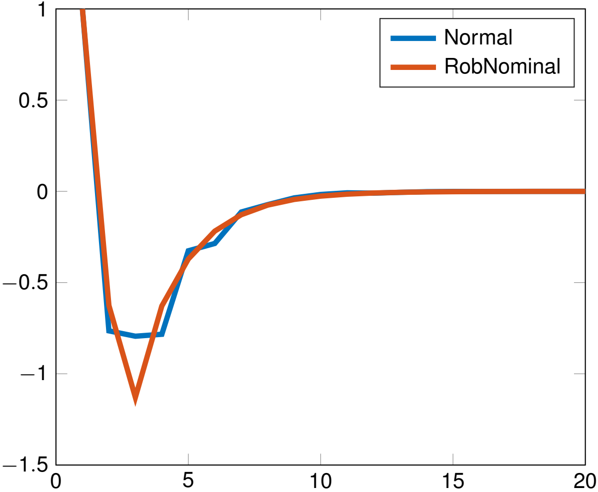

Figure: Robust LQR Response

Thanks

Thanks!