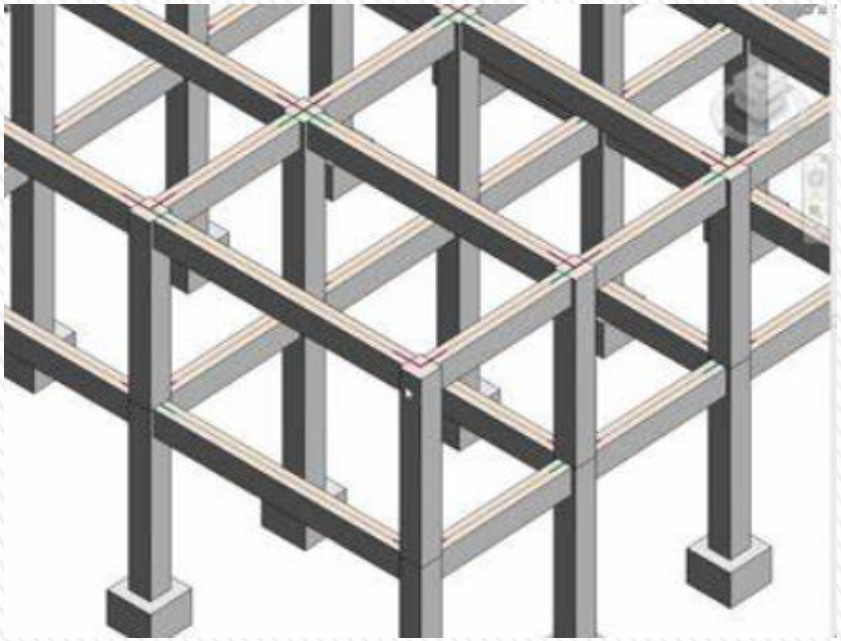
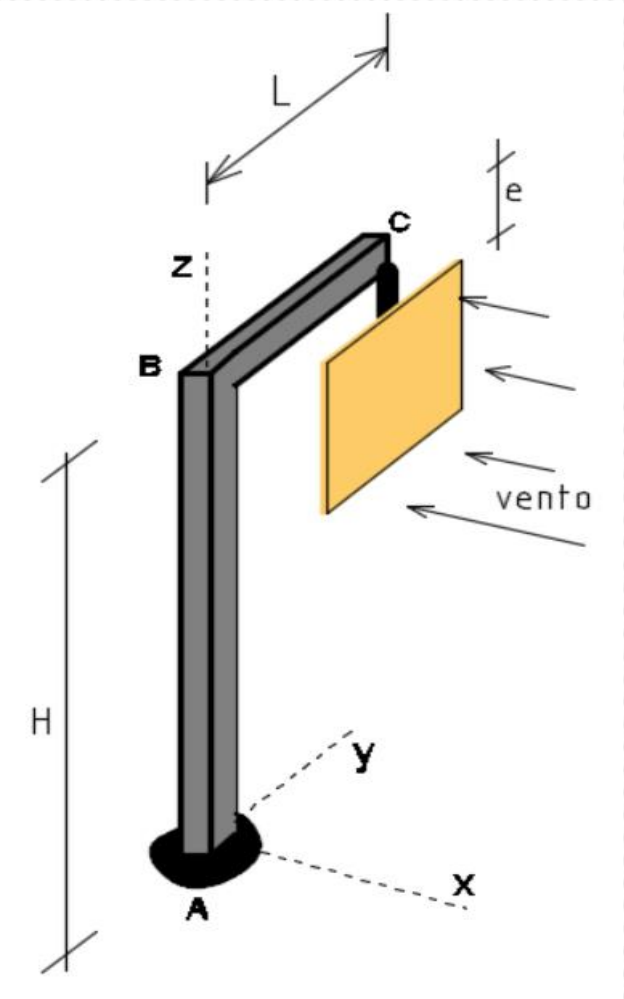
# *Quadros isostáticos espaciais*

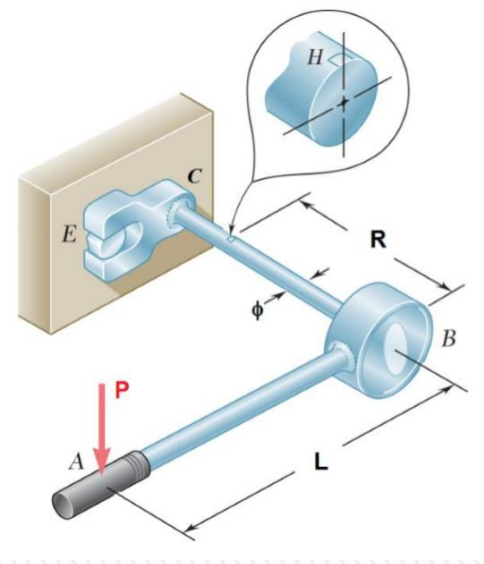
*Valério S. Almeida*

*Março/2020*

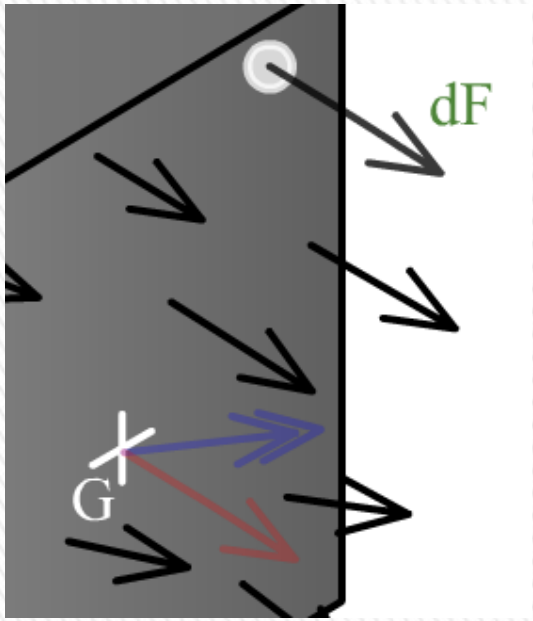
# *Quadros espaciais*





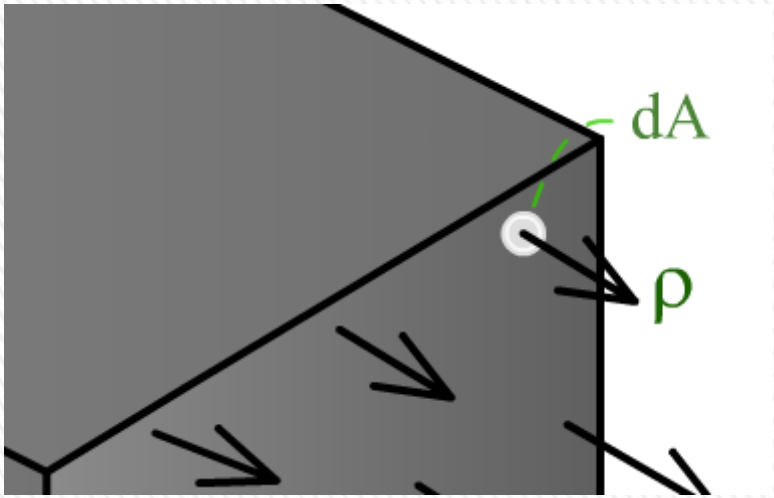



# ESFORÇOS SOLICITANTES 3D




**Tensão:** $\vec{\rho} = \vec{\sigma} + \vec{\tau}$

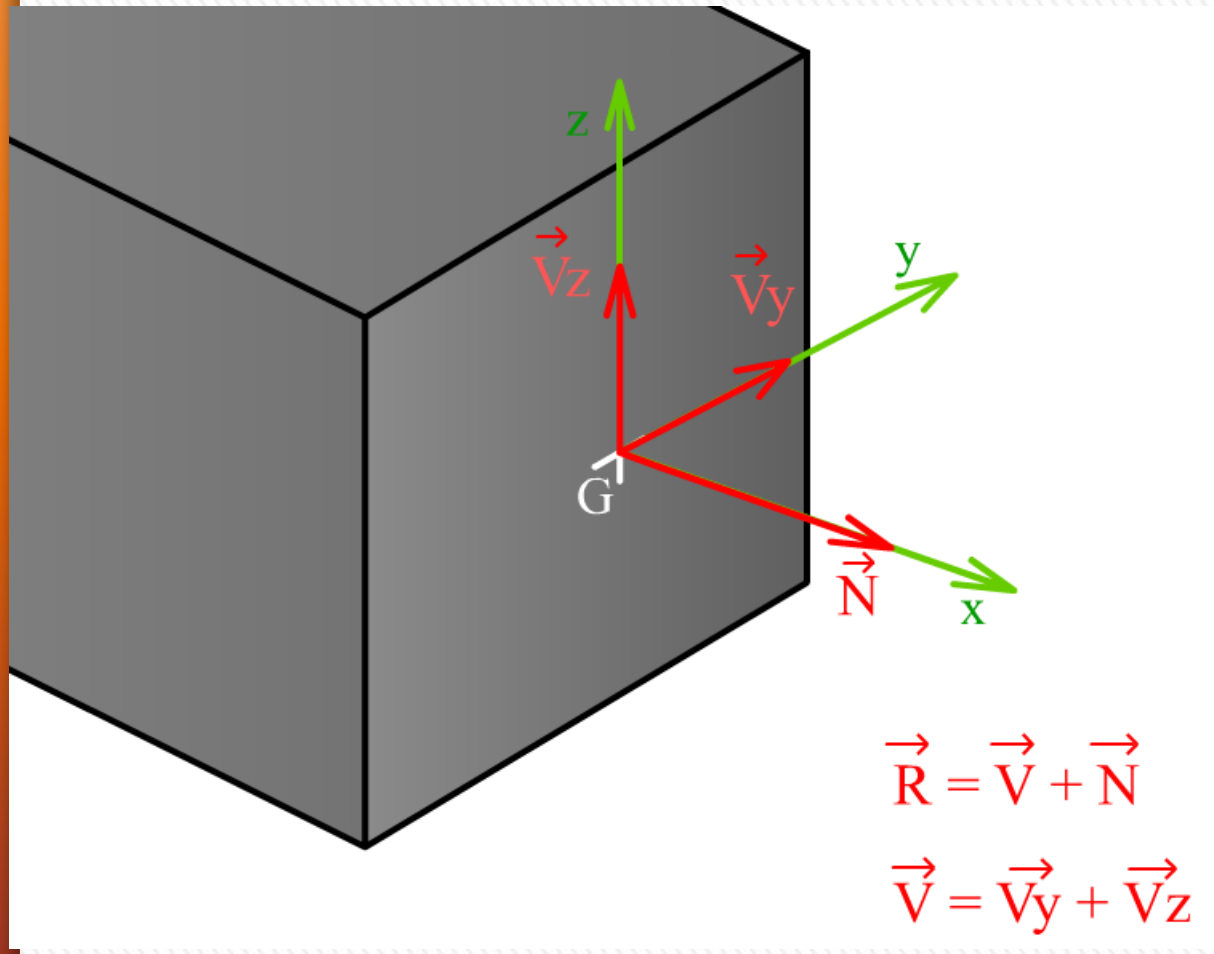
**Tensão normal a seção:** $\vec{\sigma}$




**Tensão paralela a seção:** $\vec{\tau}$

$$\vec{\rho}_{média} = \lim_{\Delta A \to 0} \frac{\overrightarrow{\Delta F}}{\Delta A}$$

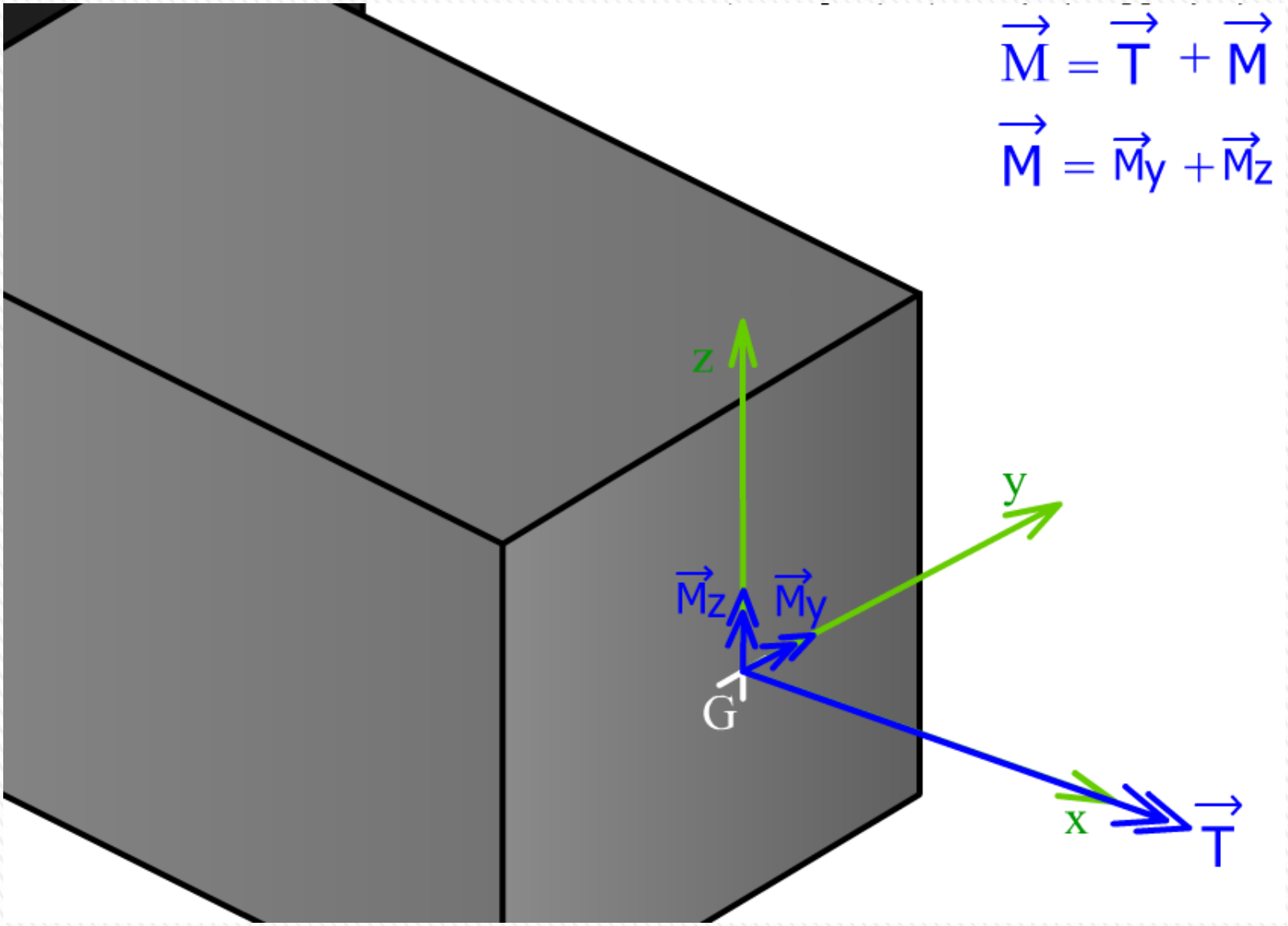
# ESFORÇOS SOLICITANTES 3D



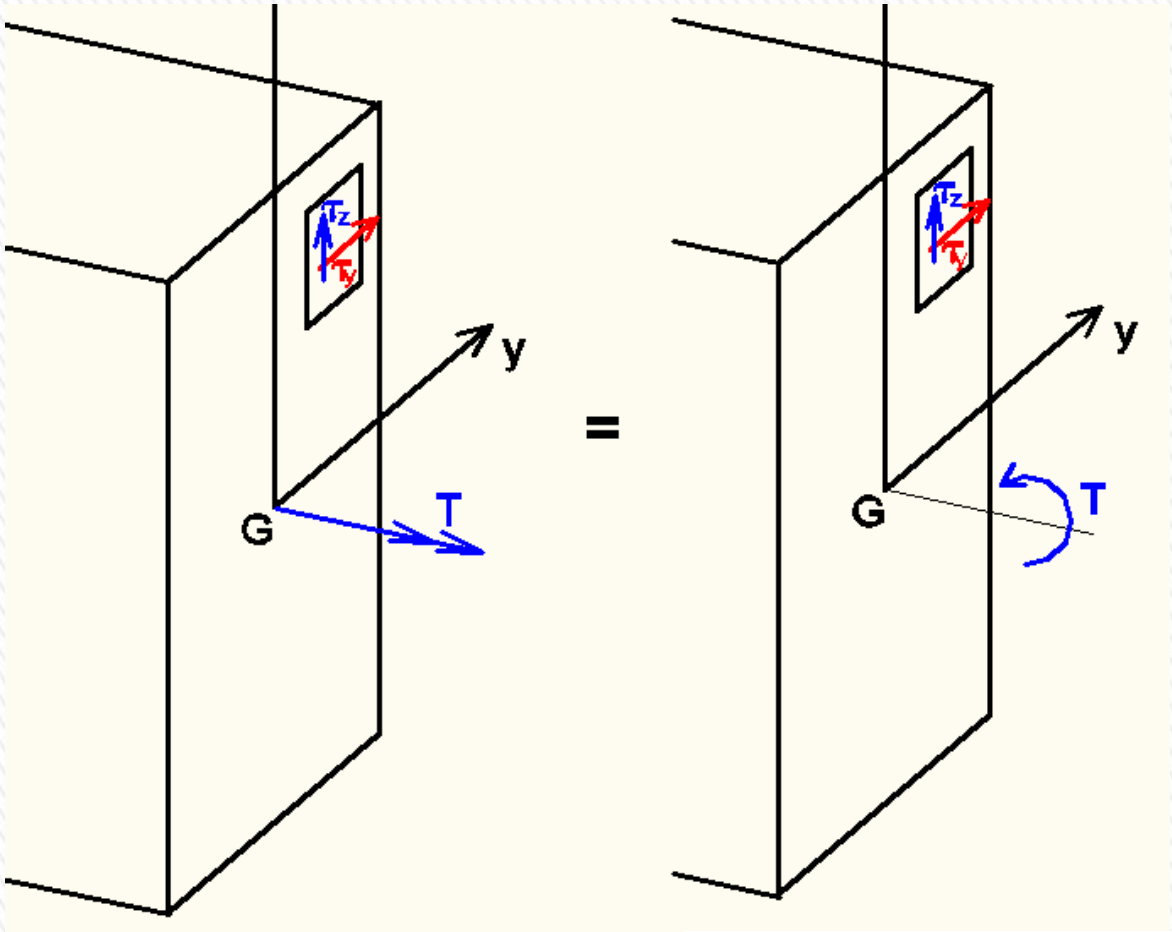

**N: Esforço Normal**

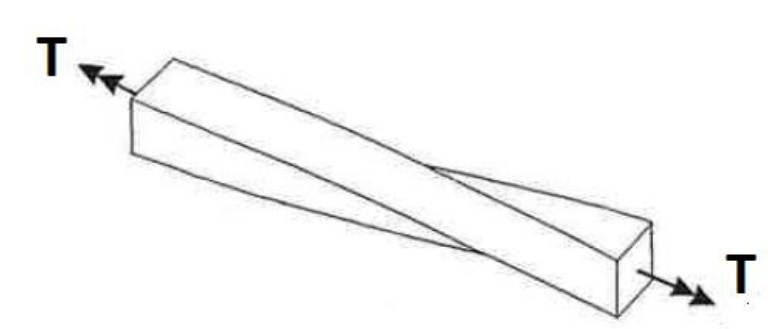
**V: Esforço Cisalhante ou Cortante**

# ESFORÇOS SOLICITANTES 3D

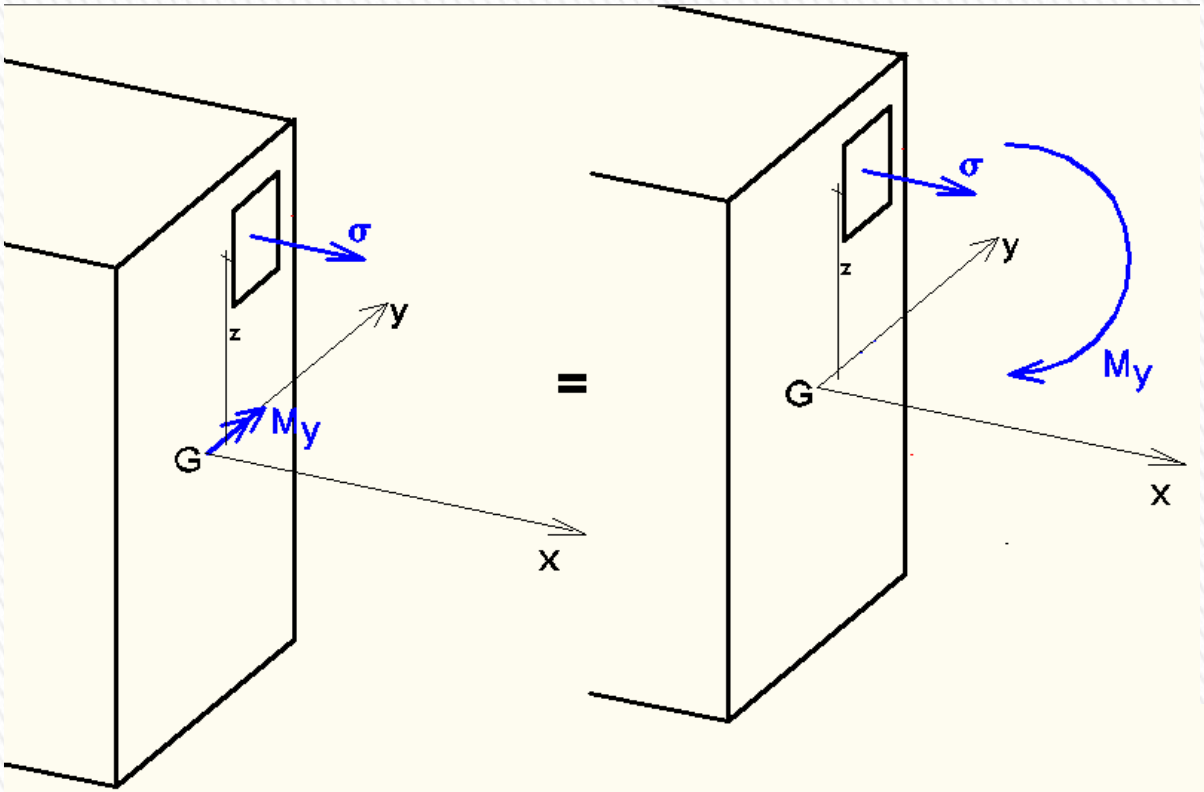


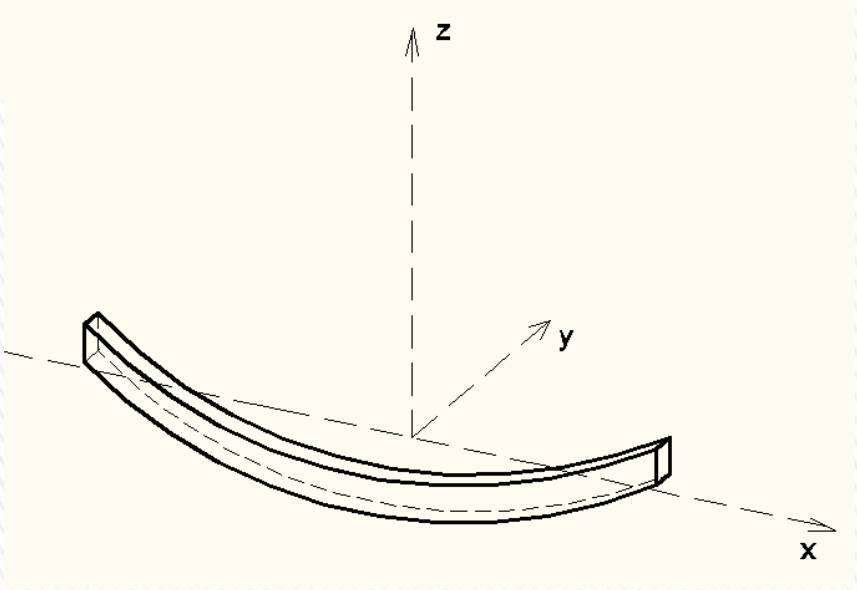
# MOMENTO TORÇOR (T)



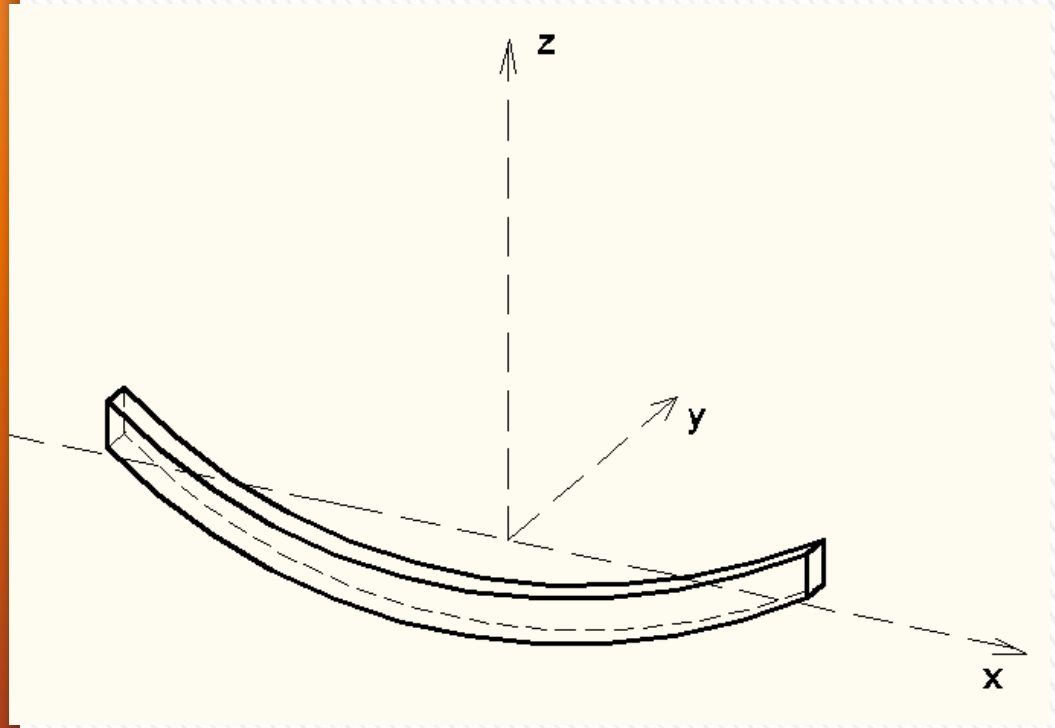
# MOMENTO FLETOR (My)

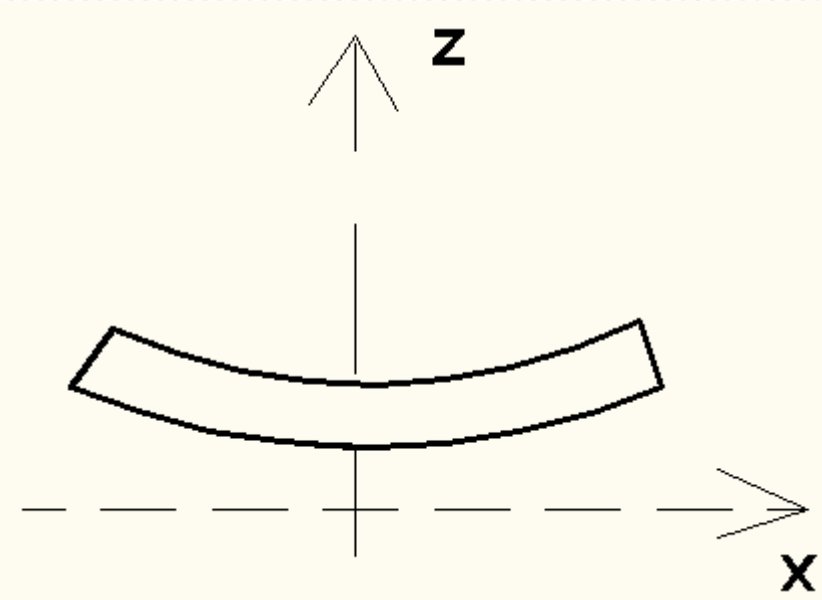


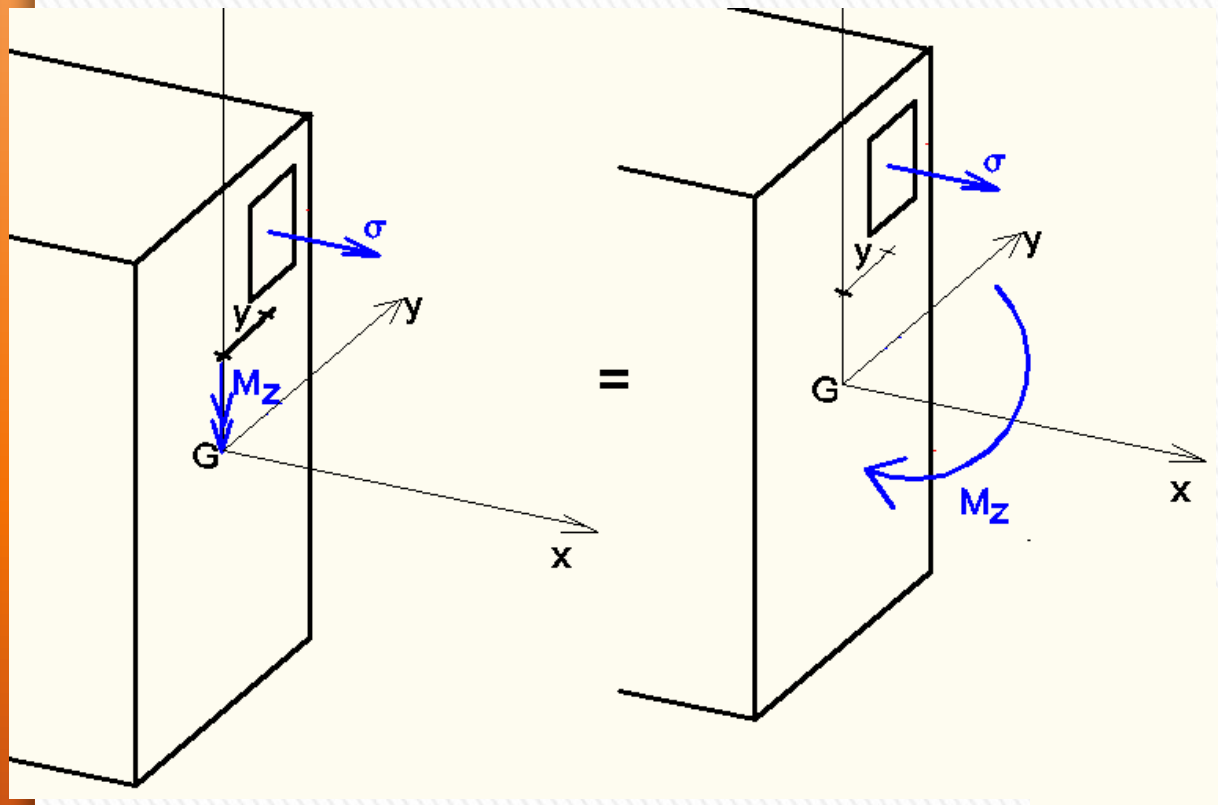
# MOMENTO FLETOR (My)

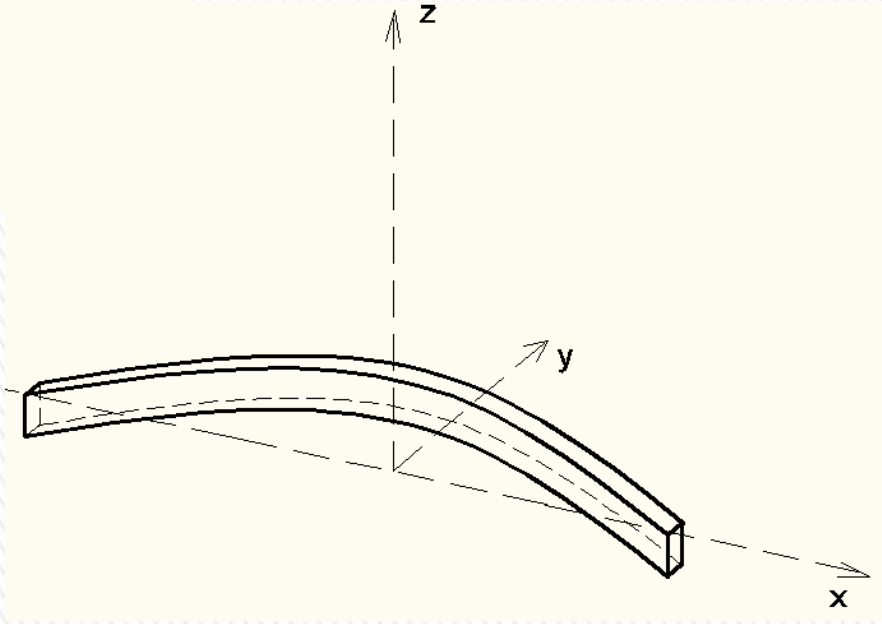
**Curvatura em torno de y**



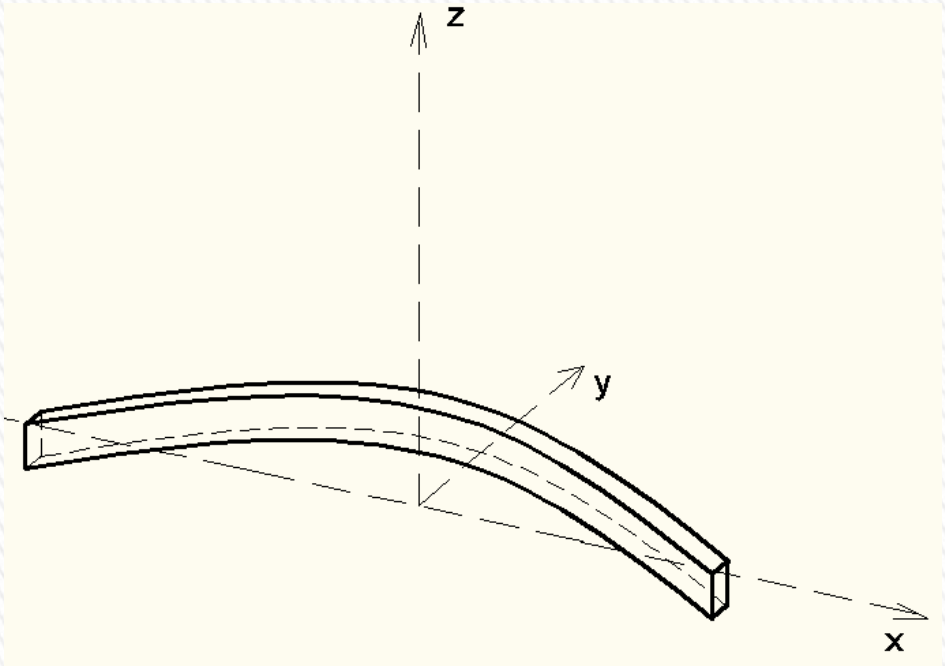



# MOMENTO FLETOR (Mz)

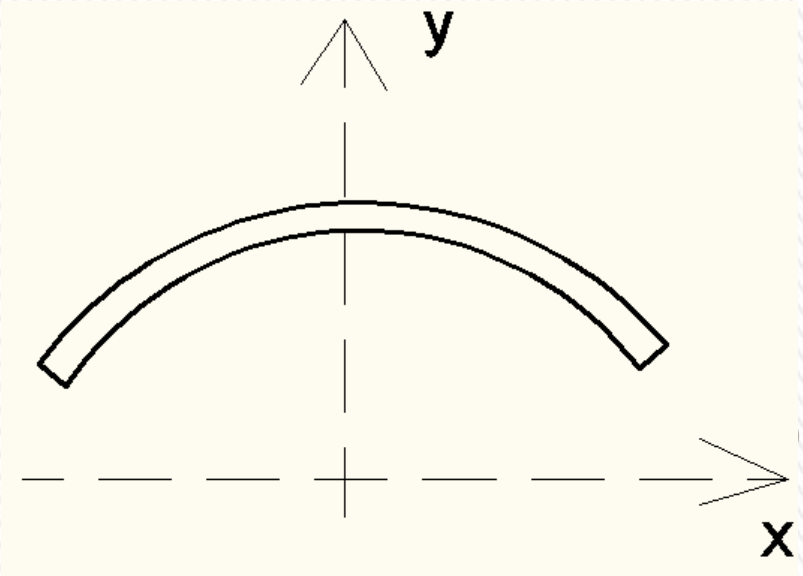


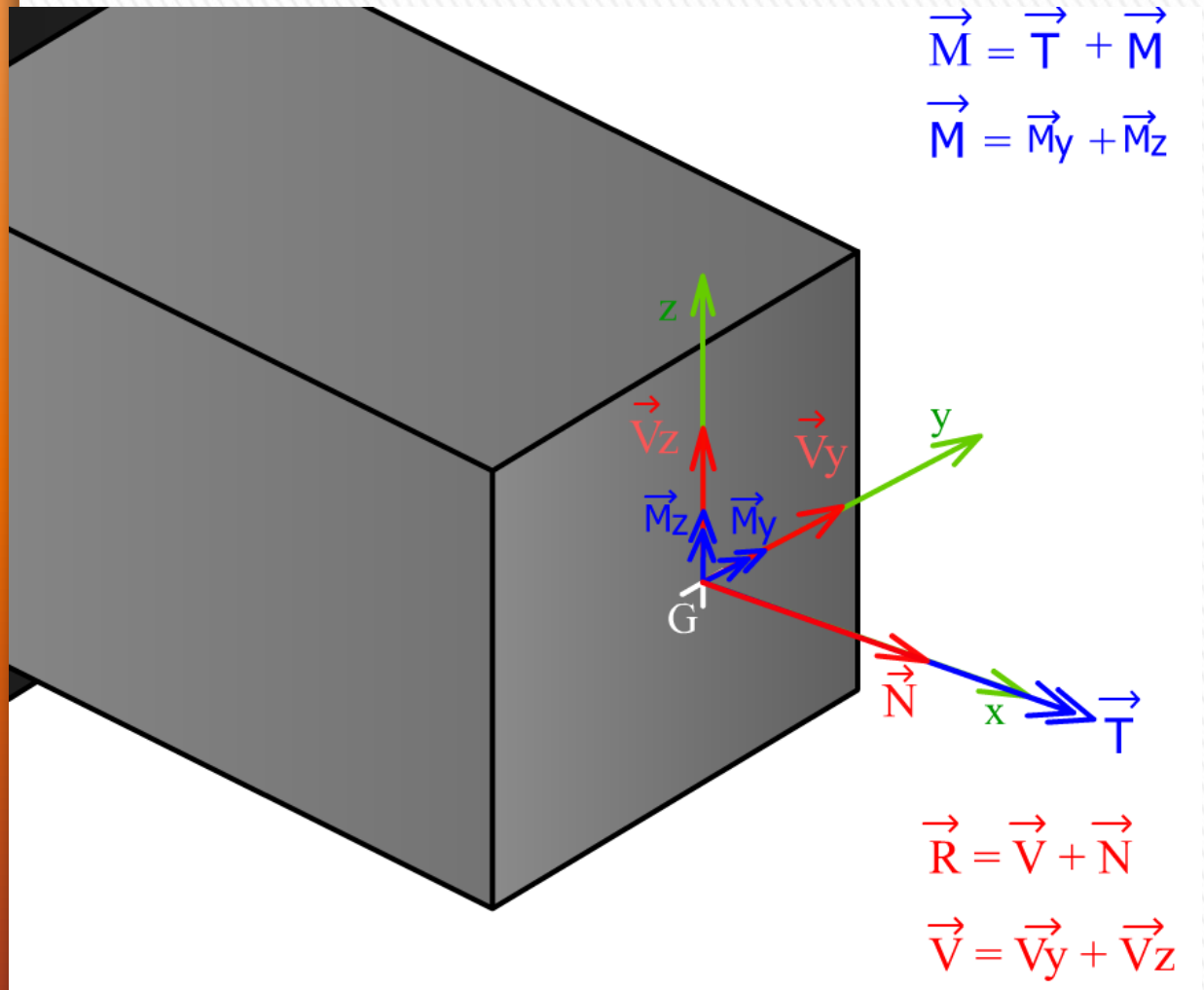
# MOMENTO FLETOR (Mz)




**Curvatura em torno de z**



# ESFORÇOS 3D




$$N = \int_A \sigma \, dA$$

$$V_y = \int_A \tau_y \, dA$$

$$V_z = \int_A \tau_z \, dA$$

$$M_z = \int_A \sigma \cdot y \, dA$$

$$M_y = \int_A \sigma \cdot z \, dA$$

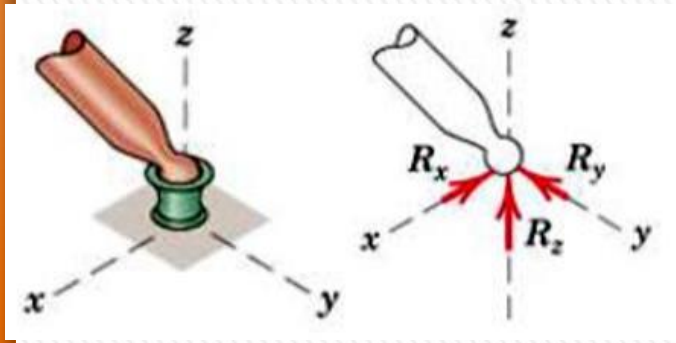
$$T = \int_A \left( \tau_z \, y - \tau_y \, z \right) dA$$

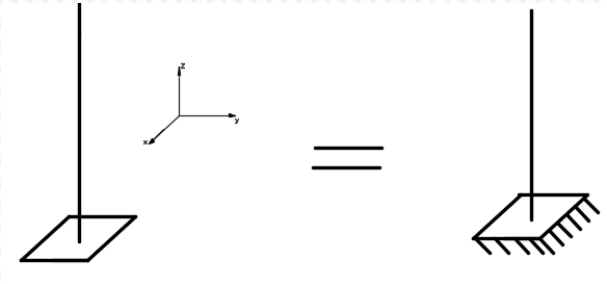
Laboratório de Mecânica Computacional da EPUSP. Todos os direitos reservados.

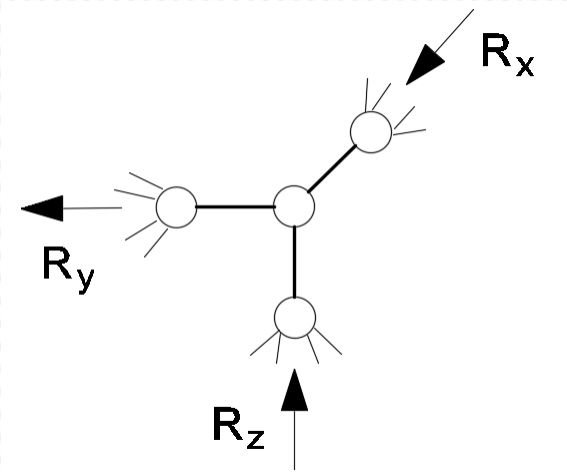
# VÍNCULOS

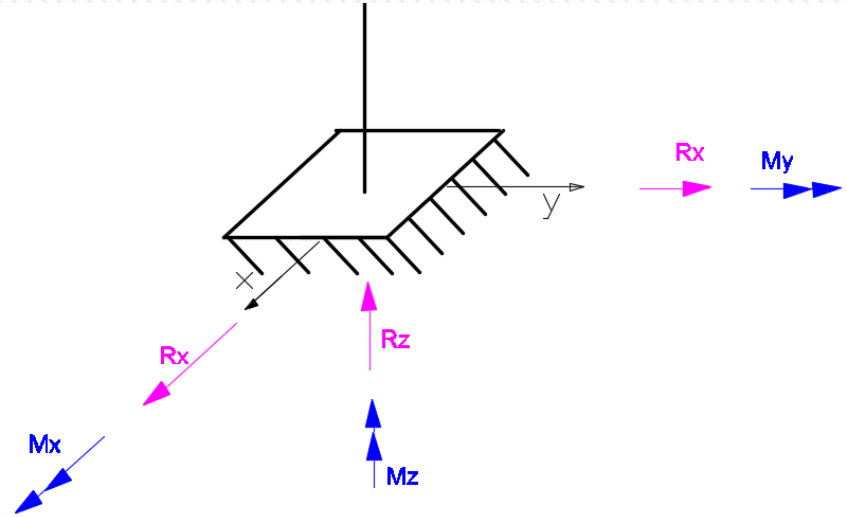
## Engaste

## Apoios fixos



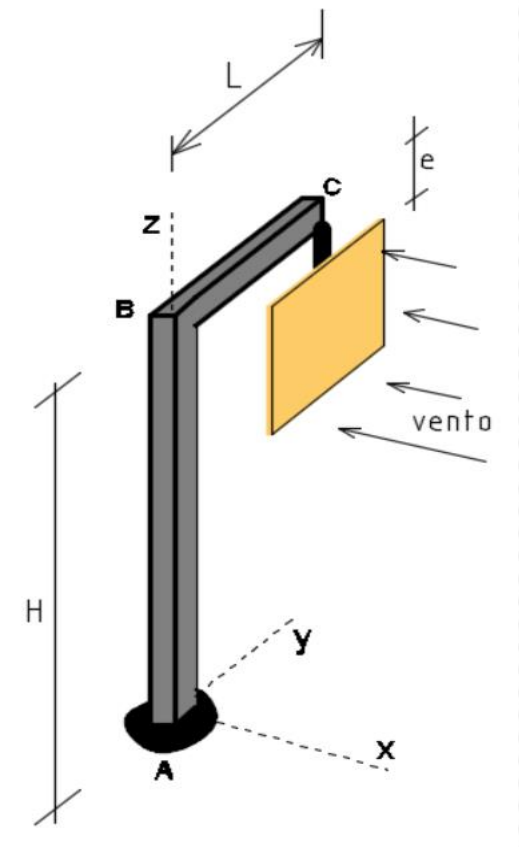


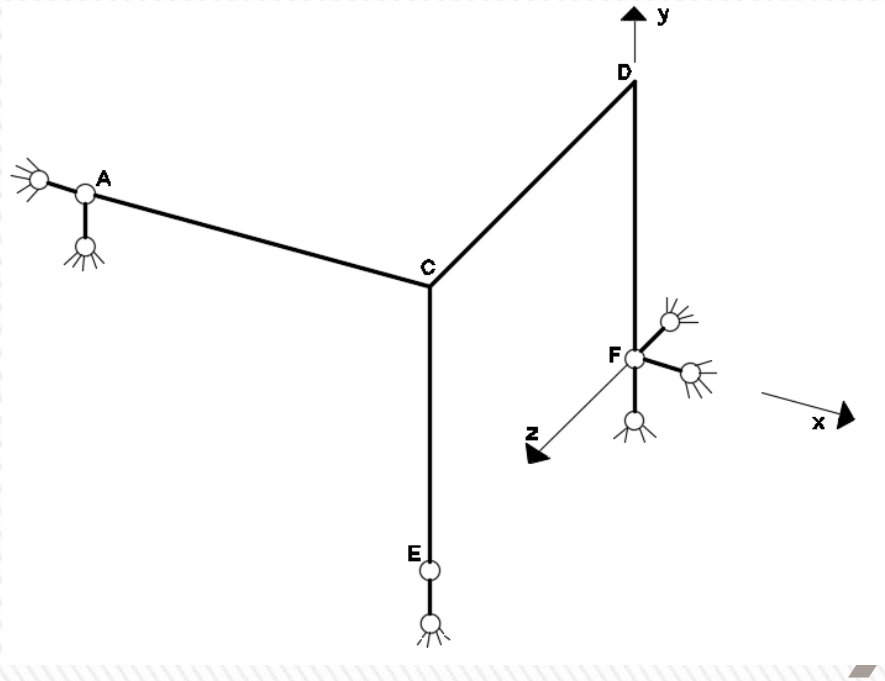



# Quadros isostáticos

Vínculos suficiente para manter estrutura em equilíbrio

Obtendo reações e esforços apenas com as equações da estática



# EQUAÇÕES DE EQUILÍBRIO DA ESTÁTICA

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0$$
$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0$$

# VETORES – PRODUTO VETORIAL

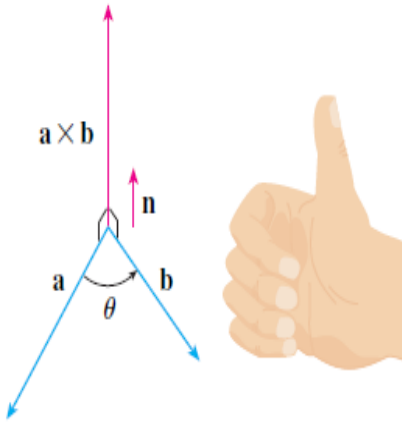
Definição: Se $a = \langle a_1, a_2, a_3 \rangle$ e $b = \langle b_1, b_2, b_3 \rangle$ então o produto vetorial de $a$ e $b$ é o vetor:

$$a \times b = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$

Para facilitar o cálculo e tornar mais fácil de lembrar esta definição o produto vetorial $(a \times b)$ é representado com a notação de determinante da seguinte forma:

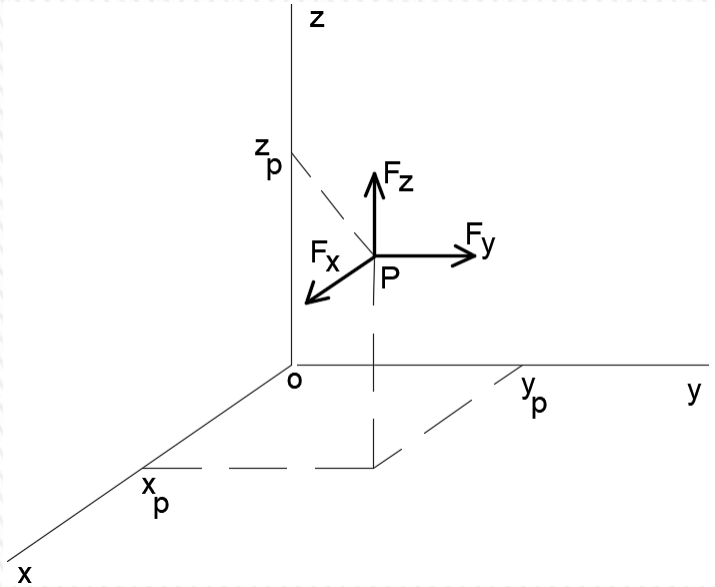
$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\mathbf{a} \times \mathbf{b} = \underbrace{(a_y b_z - a_z b_y)}_{\mathbf{c_x}}\mathbf{i} + \underbrace{(a_z b_x - a_x b_z)}_{\mathbf{c_y}}\mathbf{j} + \underbrace{(a_x b_y - a_y b_x)}_{\mathbf{c_z}}\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$



# Momento e Força – Produto vetorial

As forças em P geram que momento em “O”?




$$\boldsymbol{M_o = r \times F}$$

$$r = (x_p - x_o)i + (y_p - y_o)j + (z_p - z_o)k$$

$$F = F_x i + F_y j + F_z k$$

$$M_o = [(x_p - x_o)i + (y_p - y_o)j + (z_p - z_o)k] x \, [F_x i + F_y j + F_z k]$$

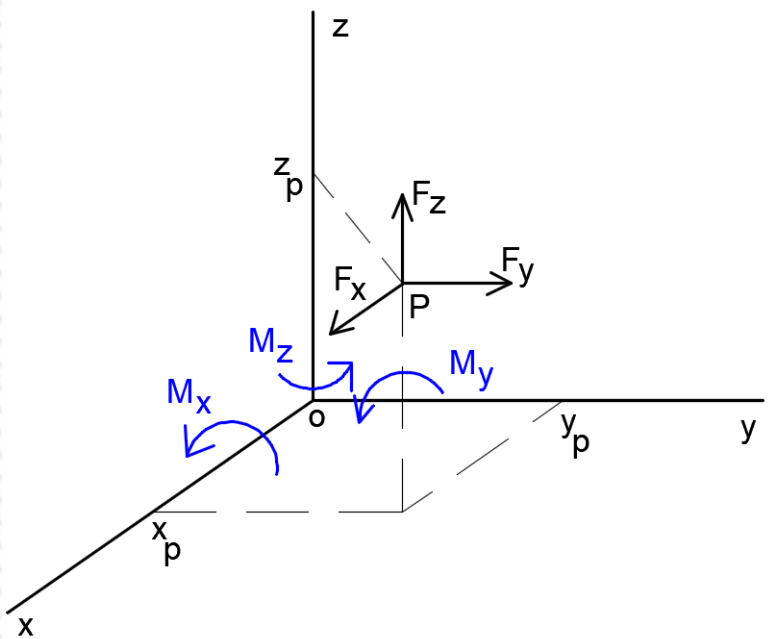
# Momento e Força – Produto vetorial

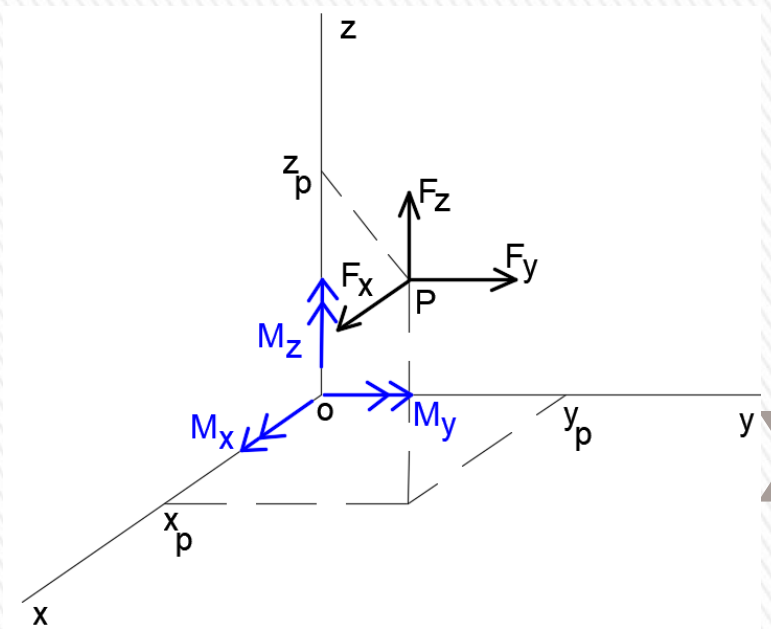
$$M_o = M_x i + M_y j + M_z k$$

$$M_x = (y_p - y_o)F_z - (z_p - z_o)F_y$$

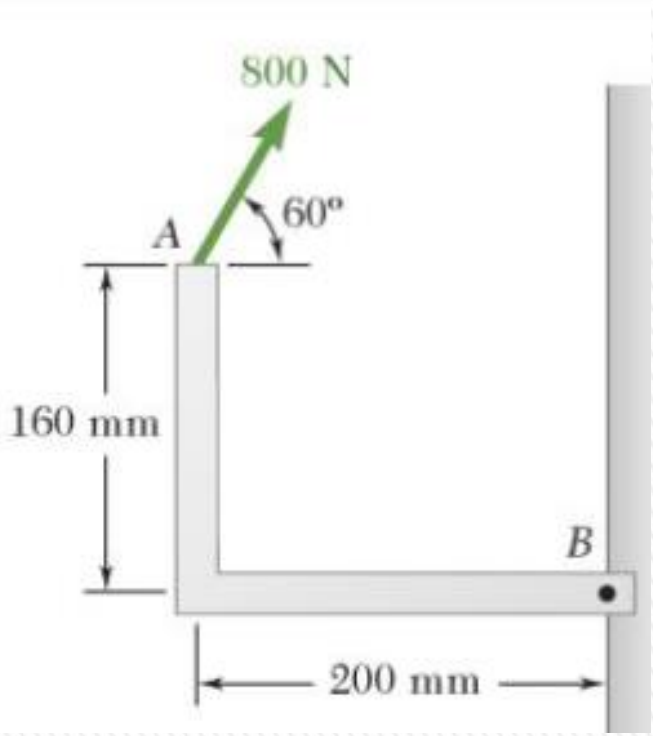
$$M_y = (z_p - z_o)F_x - (x_p - x_o)F_z$$

$$M_z = (x_p - x_o)F_y - (y_p - y_o)F_x$$



## EXEMPLO 1: Obter a resultante de momento em relação ao pólo B da estrutura plana

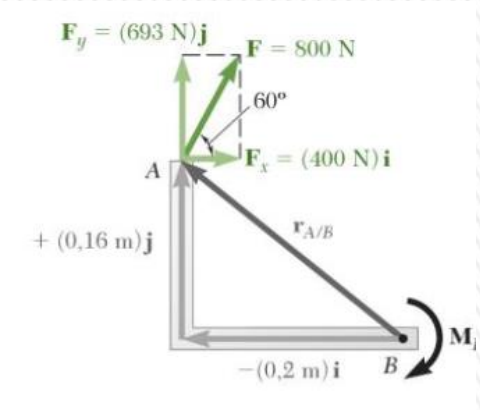



$$M_o = M_x i + M_y j + M_z k$$

$$M_x = (y_p - y_o)F_z - (z_p - z_o)F_y$$

$$M_y = (z_p - z_o)F_x - (x_p - x_o)F_z$$

$$M_z = (x_p - x_o)F_y - (y_p - y_o)F_x$$



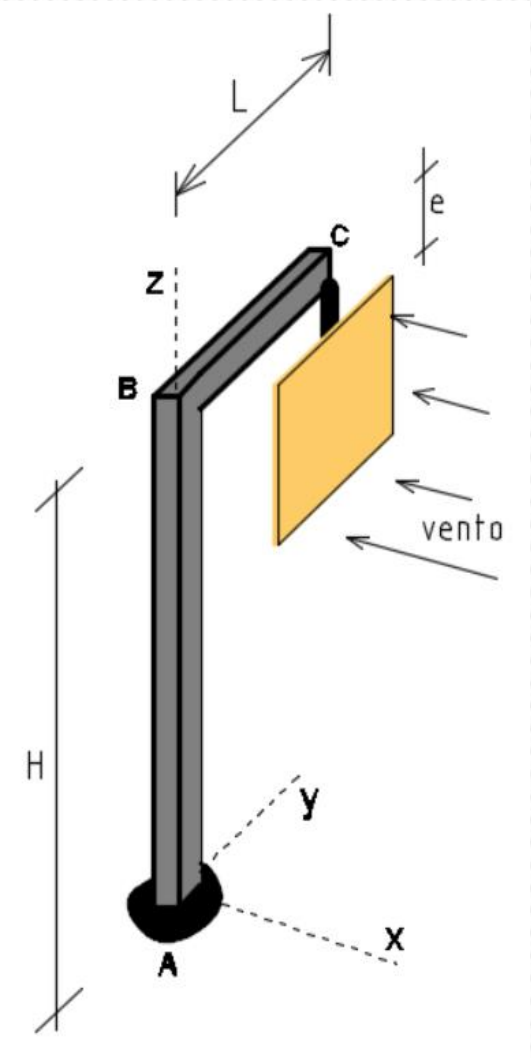

$$\mathbf{r}_{A/B} = -(0{,}2\ \text{m})\mathbf{i} + (0{,}16\ \text{m})\mathbf{j}$$

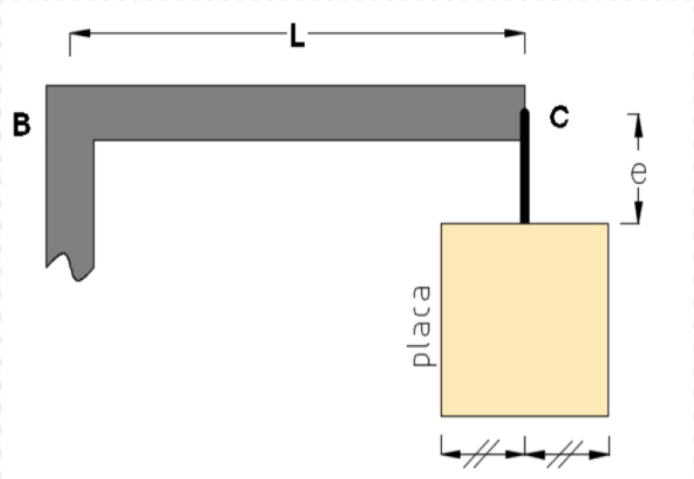
$$\mathbf{F} = (800\ \text{N})\cos 60°\mathbf{i} + (800\ \text{N})\,\text{sen}\,60°\mathbf{j}$$
$$= (400\ \text{N})\mathbf{i} + (693\ \text{N})\mathbf{j}$$

$$\mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{F} = [-(0{,}2\ \text{m})\mathbf{i} + (0{,}16\ \text{m})\mathbf{j}] \times [(400\ \text{N})\mathbf{i} + (693\ \text{N})\mathbf{j}]$$
$$= -(138{,}6\ \text{N}\cdot\text{m})\mathbf{k} - (64{,}0\ \text{N}\cdot\text{m})\mathbf{k}$$
$$= -(202{,}6\ \text{N}\cdot\text{m})\mathbf{k}$$

$$\mathbf{M}_B = 203\ \text{N}\cdot\text{m}\ \circlearrowright$$

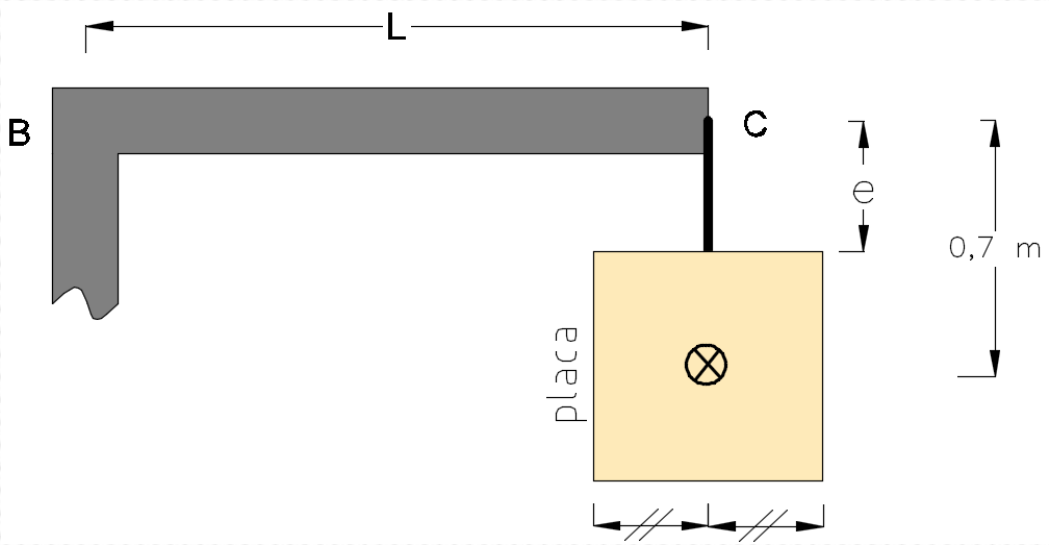
Exemplo 2: O poste está engastando em A. Peso da placa quadrada de lado 800 mm é de 1.000 N, a pressão na placa é 700 N/m$^2$ .
Dados: L = 2m, H = 5m, e = 30 cm. Obtenha as reações de momento no engaste.

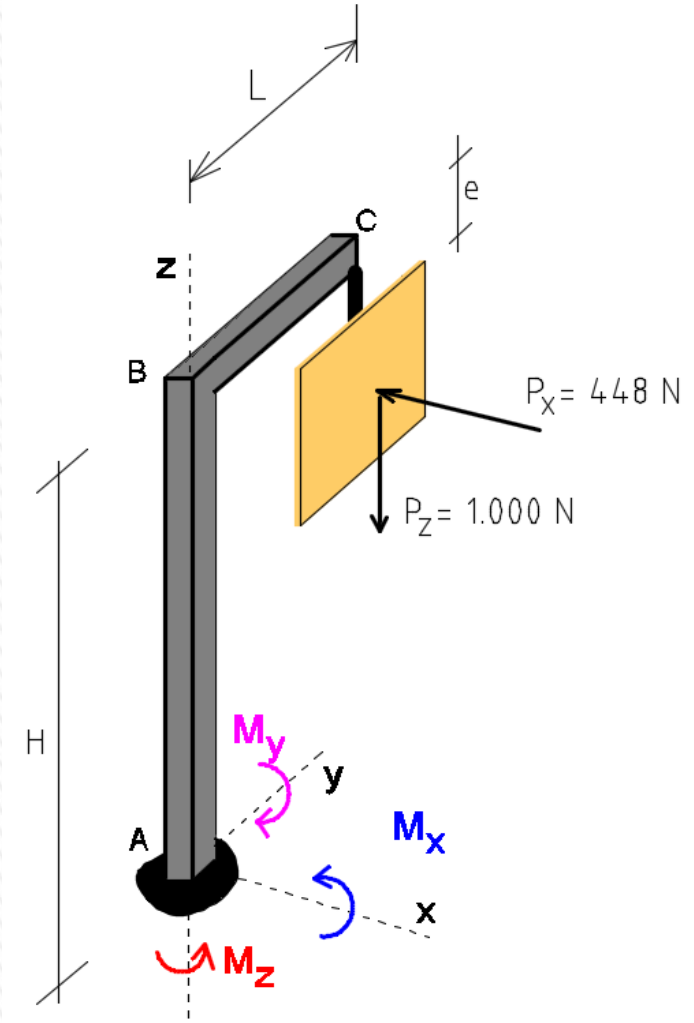


# Exemplo 2

**$P_x$ = 700 N/m². 0,8² (m²)= 448 N**



# Exemplo 2

$$M_o = M_x i + M_y j + M_z k$$

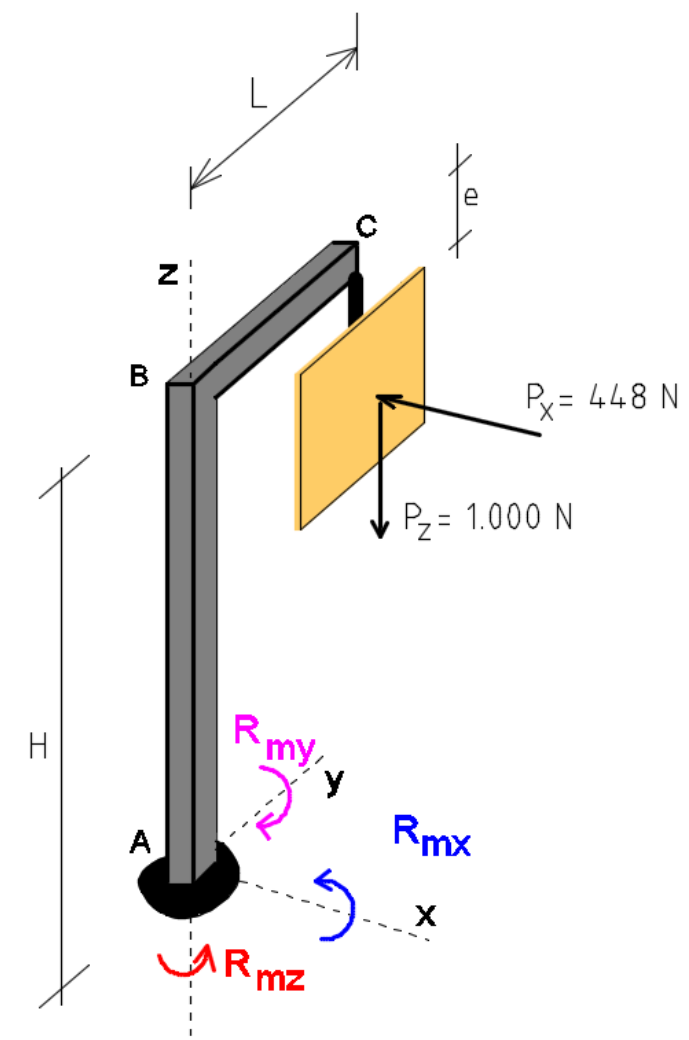
$$M_x = (y_p - y_o)F_z - (z_p - z_o)F_y$$

$$M_y = (z_p - z_o)F_x - (x_p - x_o)F_z$$

$$M_z = (x_p - x_o)F_y - (y_p - y_o)F_x$$

$x_p = 0$
$y_p = L = 2$ m
$z_p = H - 0{,}7 = 4{,}3$ m
$(x_0,y_0,z_0) = (0,0,0)$

$P_x = -448$ N
$P_y = 0$
$P_z = -1.000$ N




$M_{xA} = (2-0).(-1.000) - (4{,}3-0).0 = -2.000$ N.m
$M_{yA} = (4{,}3-0).(-448) - (0-0).(-1.000) = -1.926{,}4$ N.m
$M_{zA} = (0-0).(0) - (2-0).(-448) = 896$ N.m

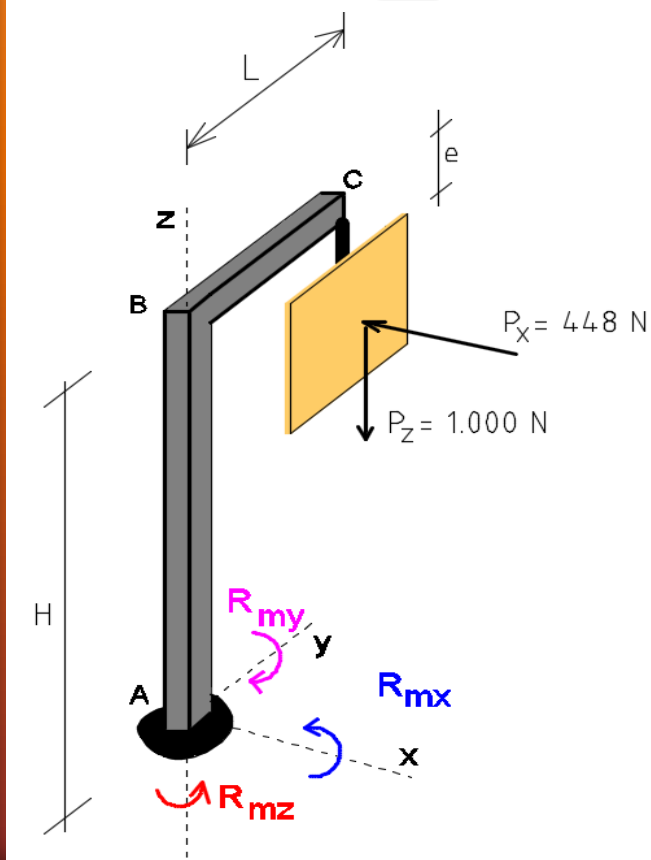
# Exemplo 2

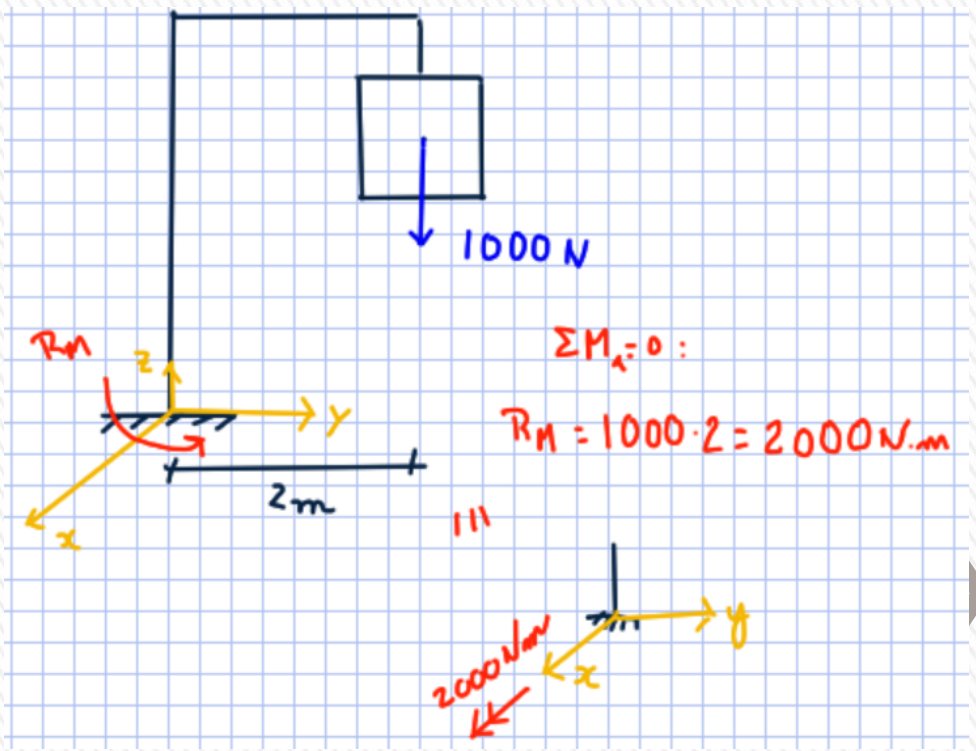
$M_{xA}$ = -2.000 N.m
$M_{yA}$ = -1.926,4 N.m
$M_{zA}$ = 896 N.m

$$\sum M_x = 0: \quad R_{mx} + M_x = 0 \rightarrow R_{mx} - 2000 = 0 \rightarrow \boldsymbol{R_{mx} = 2.000\ N.m}$$

$$\sum M_y = 0: \quad R_{my} + M_y = 0 \rightarrow R_{my} - 1926{,}4 = 0 \rightarrow \boldsymbol{R_{my} = 1.926{,}4\ N.m}$$

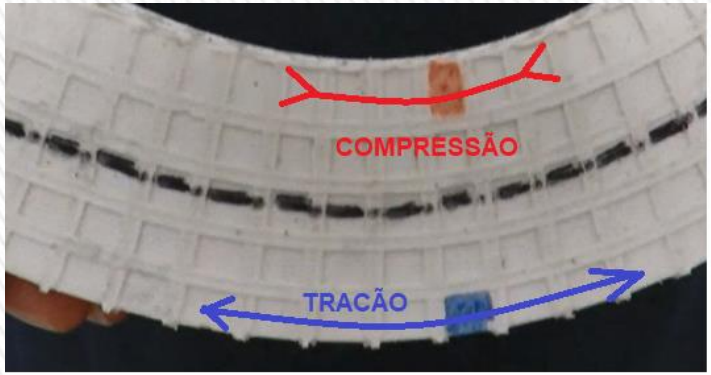
$$\sum M_z = 0: \quad R_{mz} + M_z = 0 \rightarrow R_{mz} + 896 = 0 \rightarrow \boldsymbol{R_{mz} = -896\ N.m}$$

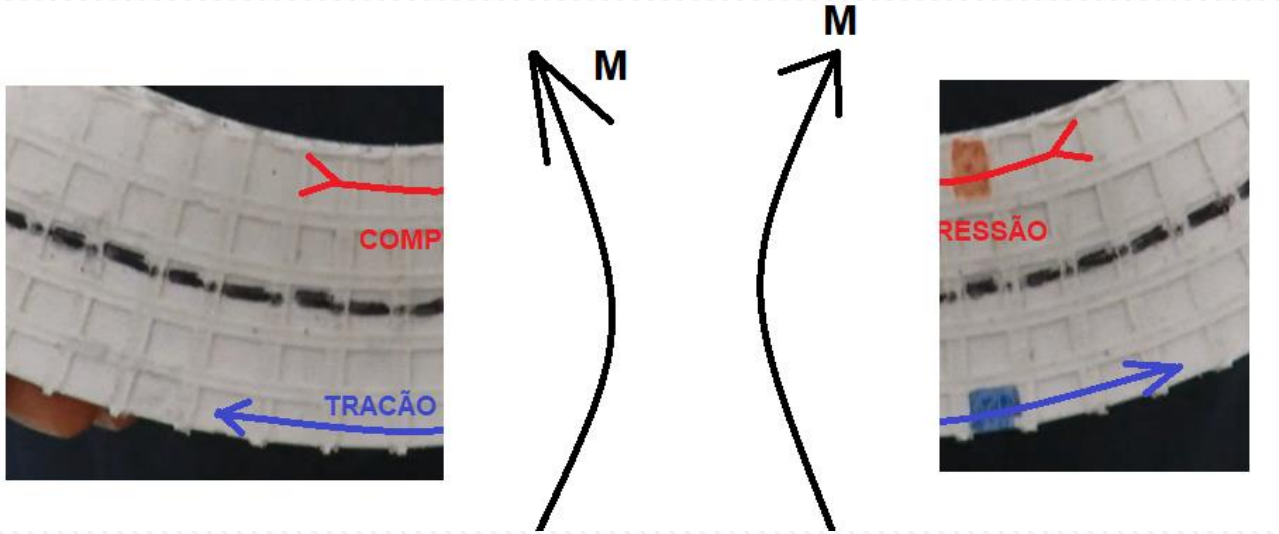


# Convenção de sinais para diagramas

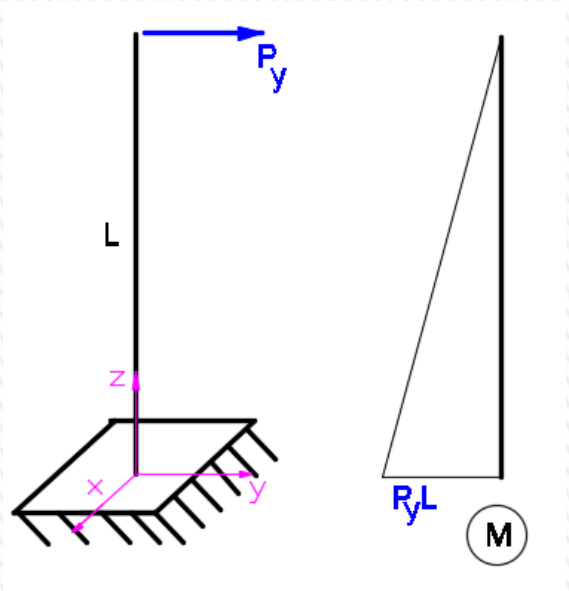
Momentos fletores: desenhar lado que traciona

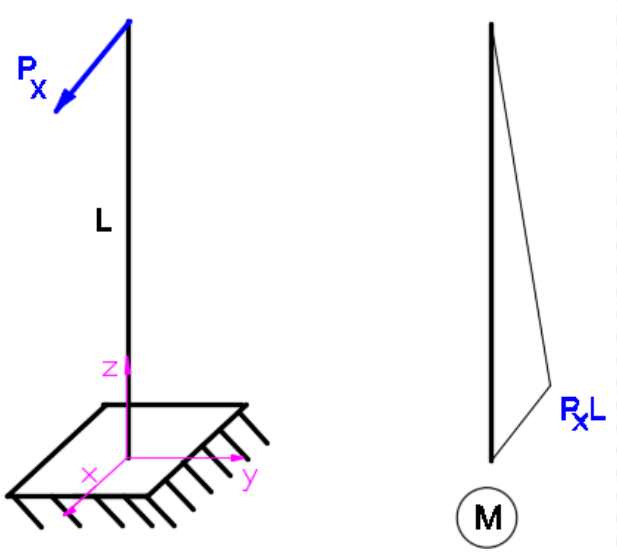


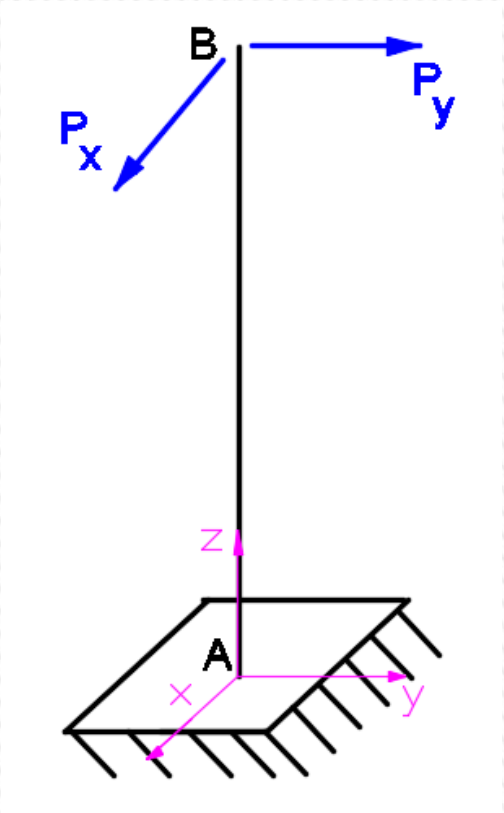



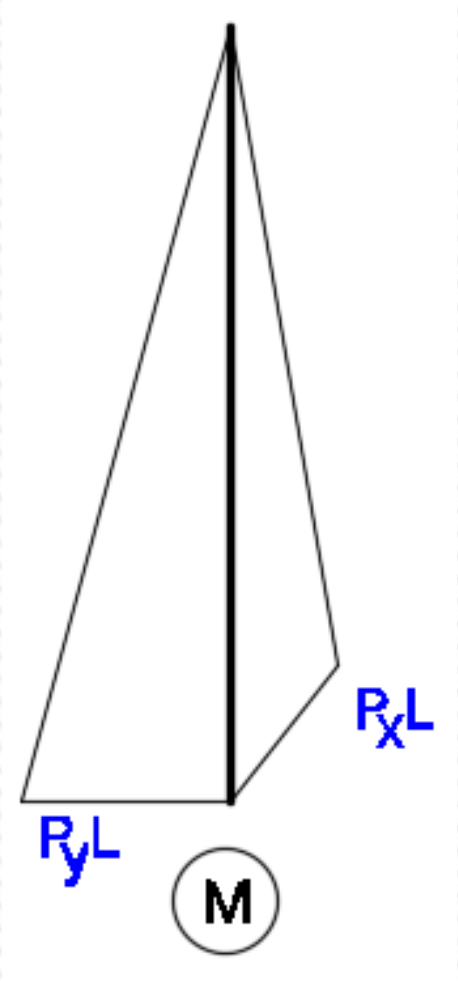
# Convenção de sinais para diagramas

Momentos fletores: desenhar lado que traciona



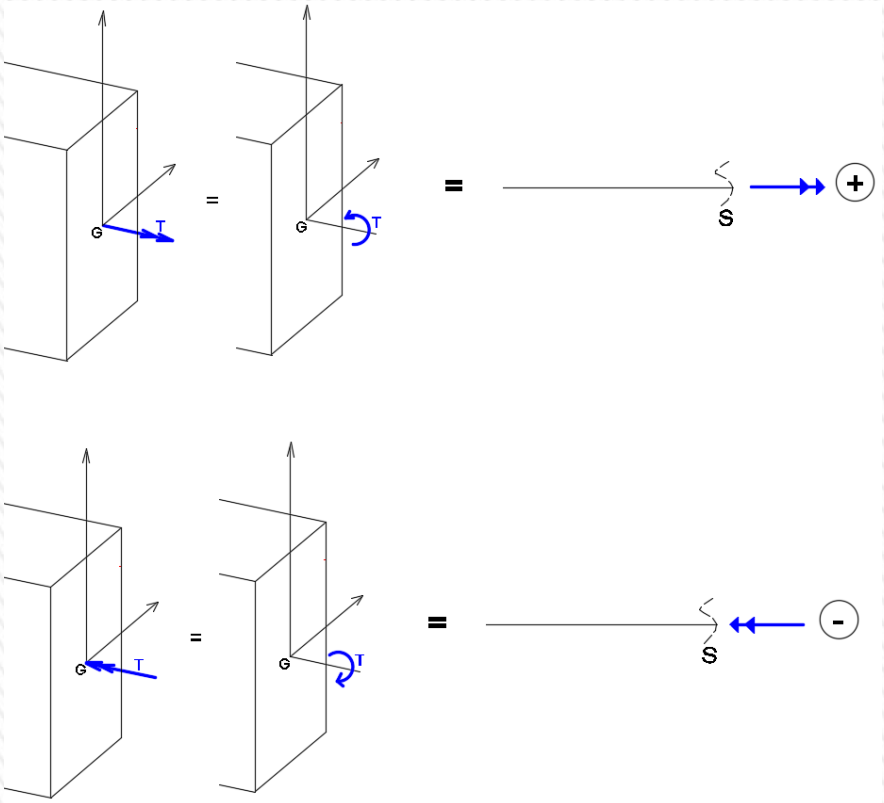



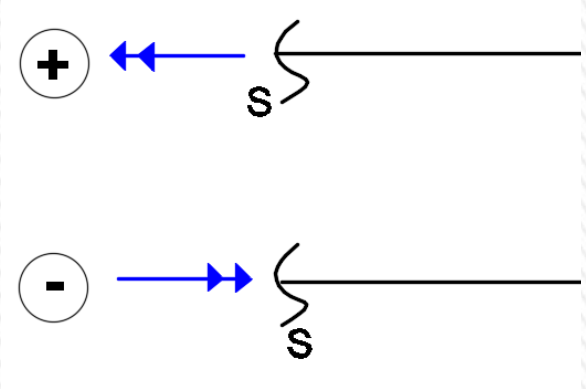



# Convenção de sinais para diagramas

Momento torçor (T) > 0 se vetor sai da seção



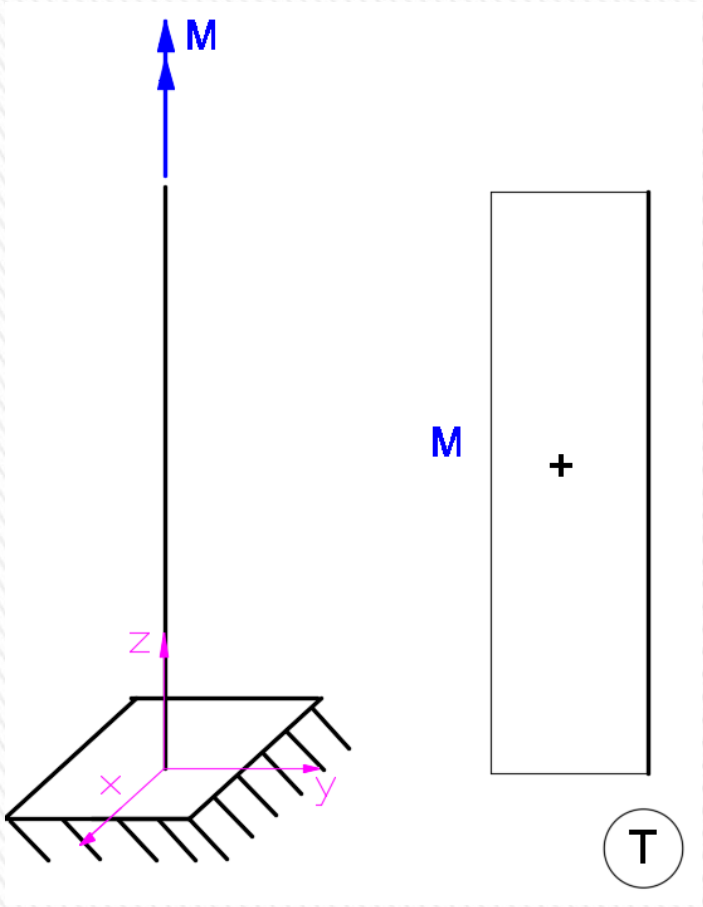

| | | |
|---|---|---|
| Momento de torção[2] | O vetor momento tem o sentido da normal externa à seção transversal em que atua | O vetor momento tem sentido contrário ao da normal externa à seção tranversal em que atua |

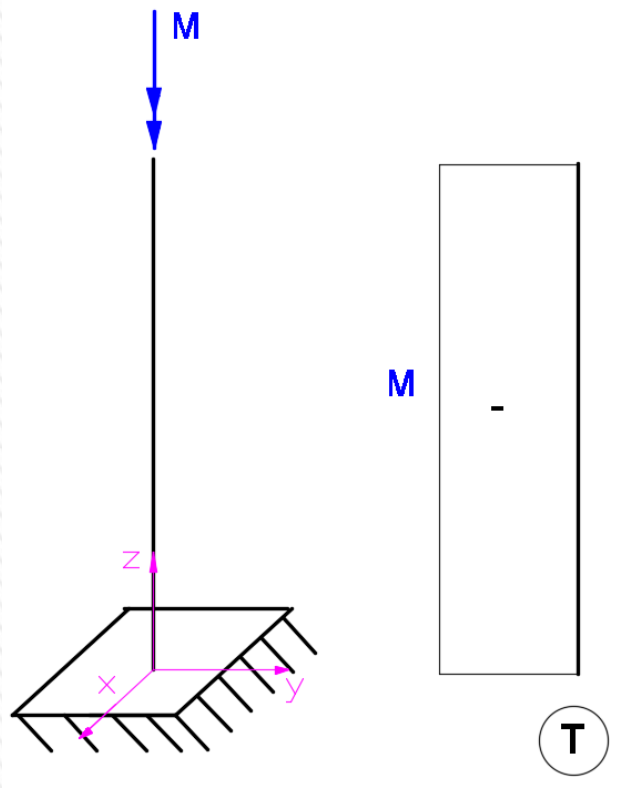


Rotacionar a seção no sentido anti-horário (T>0)

# Convenção de sinais para diagramas

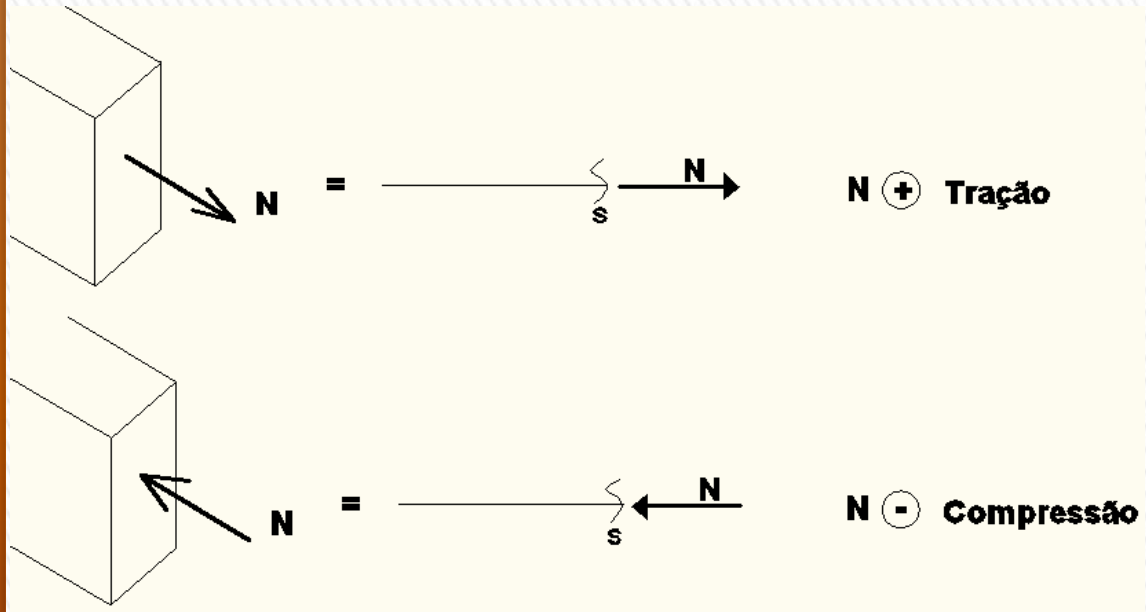
Momento torçor (T) > 0 se vetor sai da seção

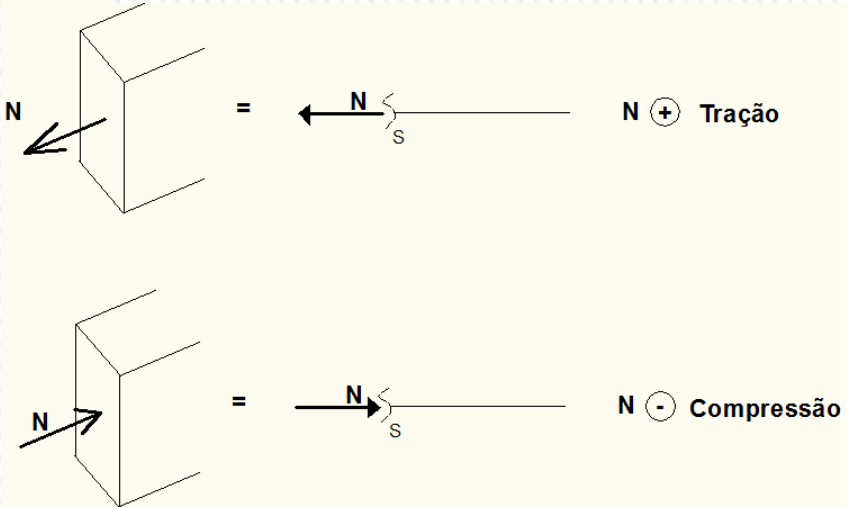


# Convenção de sinais para diagramas

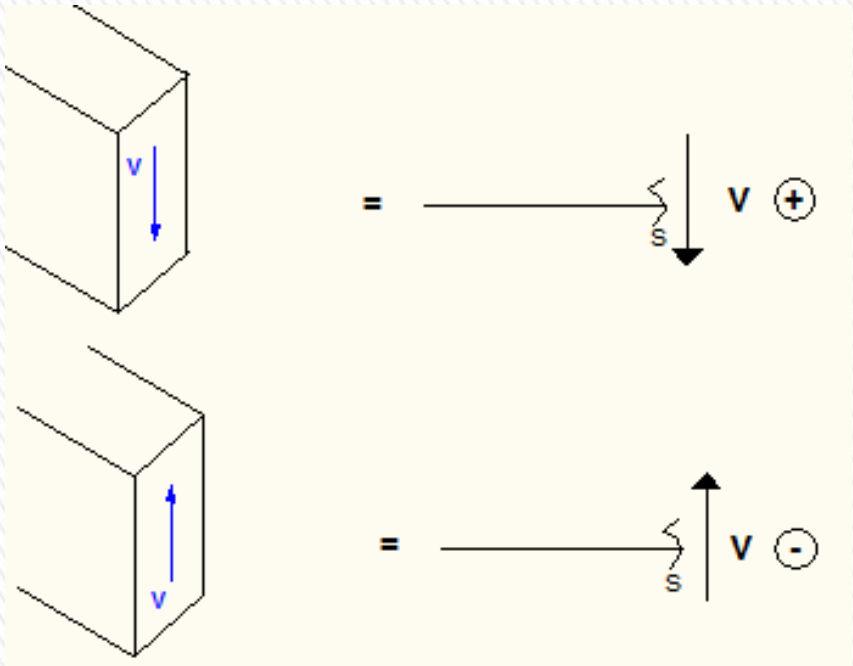
Normal (N) > 0 se vetor sai da seção

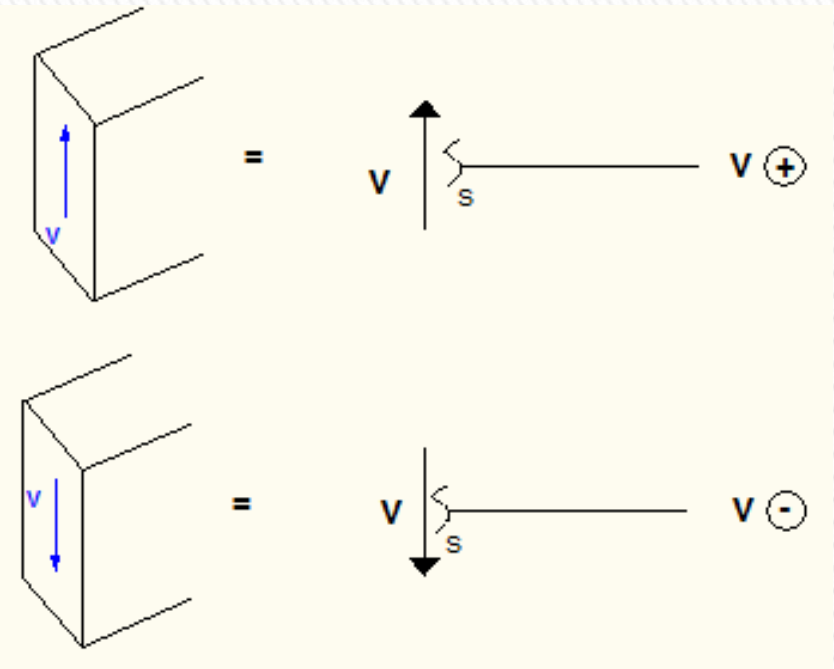


# Convenção de sinais para diagramas

Cortantes > 0 se gira a barra no sentido horário



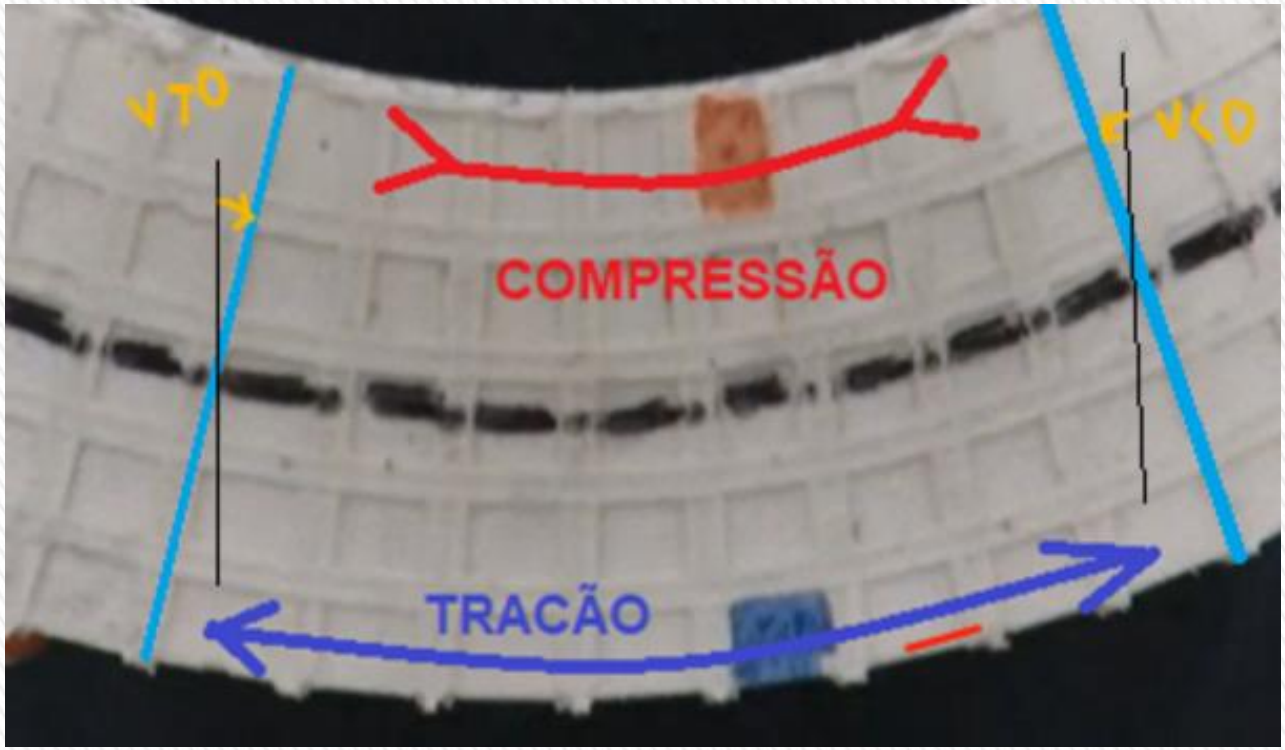




| | | |
|---|---|---|
| Força cortante | Gira o trecho de barra em que atua no sentido horário | Gira o trecho de barra em que atua no sentido anti-horário |

# Convenção de sinais para diagramas

Cortantes > 0 se gira a barra no sentido horário

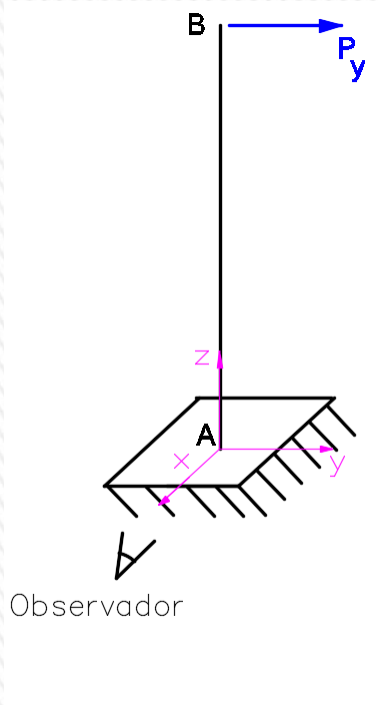
**V > 0** **V < 0**

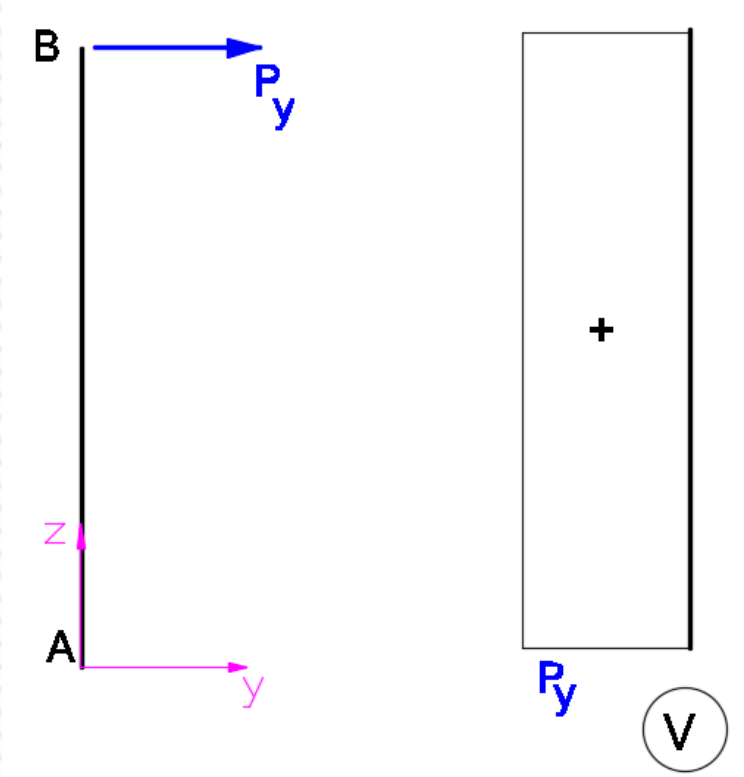


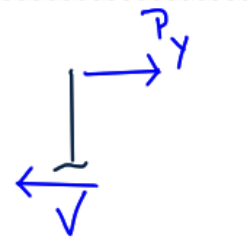
# Convenção de sinais para diagramas

Cortantes > 0 se gira a barra no sentido horário

**Visualizar a barra no plano pelo lado que está o eixo ortogonal positivo**



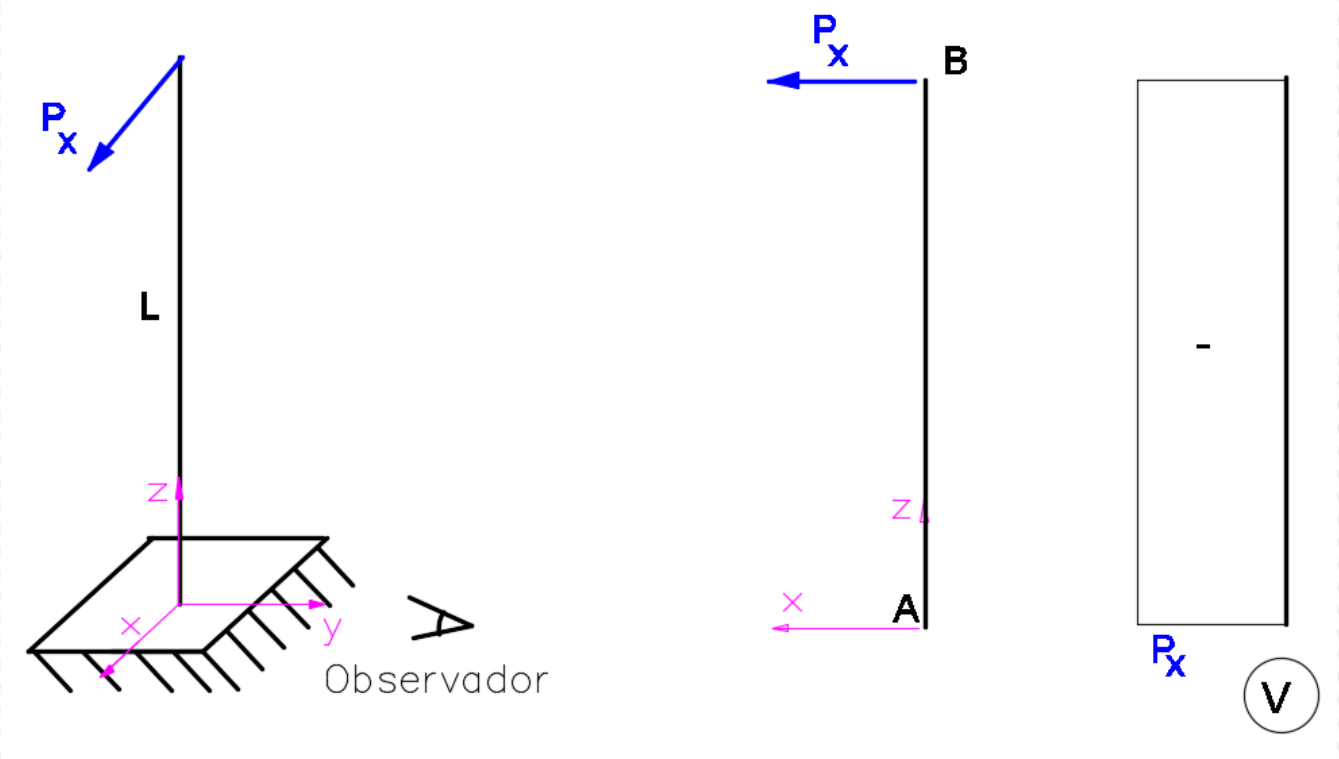



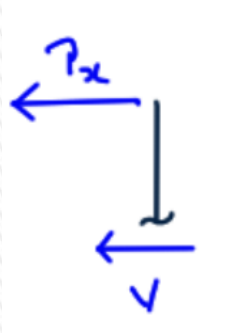



# Convenção de sinais para diagramas

Cortantes > 0 se gira a barra no sentido horário

**Visualizar a barra no plano pelo lado que está o eixo ortogonal positivo**

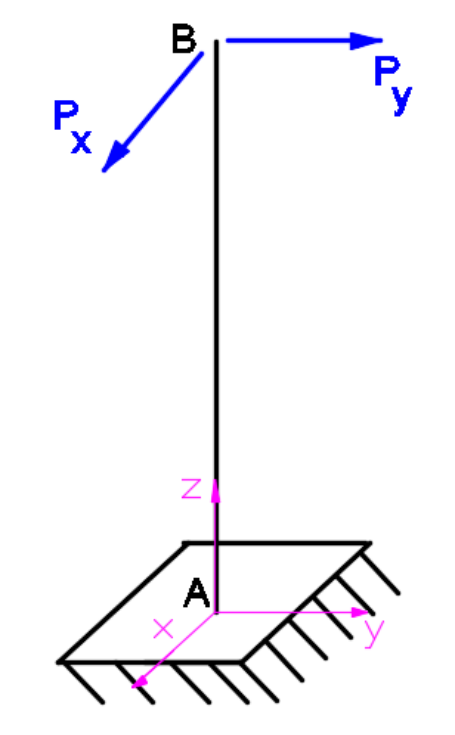


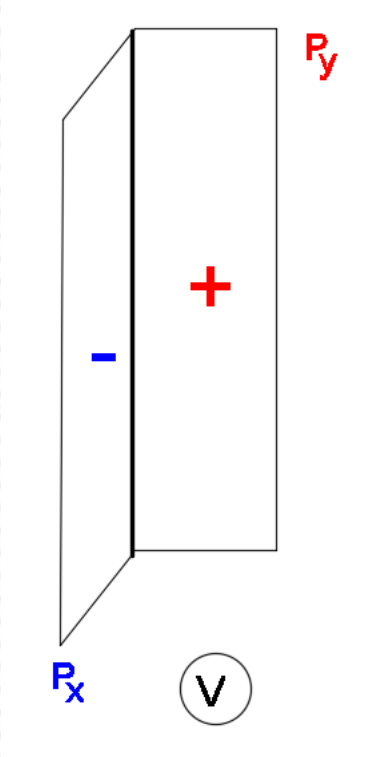



# Convenção de sinais para diagramas

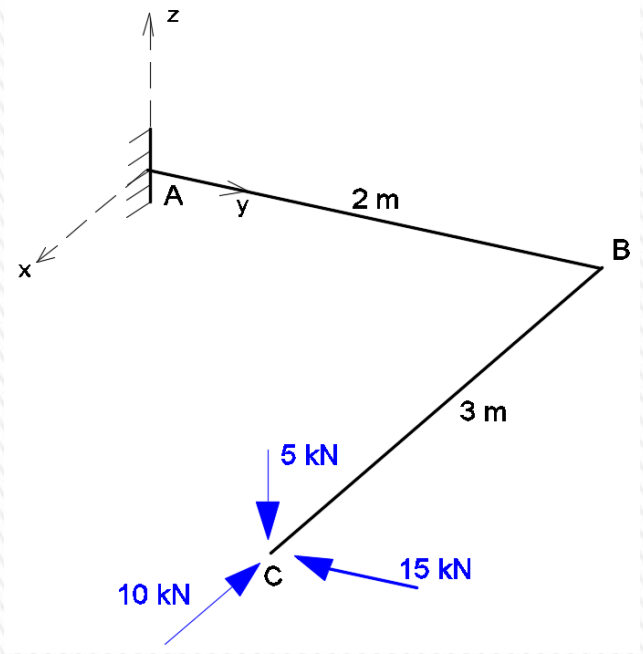
Cortantes > 0 se gira a barra no sentido horário

**Visualizar a barra no plano pelo lado que está o eixo ortogonal positivo**



# Exemplo 3




## 1) Reações

As forças causam os momentos:

$$M_o = M_x i + M_y j + M_z k$$

$$M_x = (y_p - y_o)F_z - (z_p - z_o)F_y$$

$$M_y = (z_p - z_o)F_x - (x_p - x_o)F_z$$

$$M_z = (x_p - x_o)F_y - (y_p - y_o)F_x$$

$$\mathbf{P_x = -10\ kN; P_y = -15\ kN; P_z = -5kN}$$

$$\mathbf{R_{Mx} + \sum_{i=1}^{nr.\,forças} (M_x)_i = 0}$$

E: $M_x = d_y P_z - d_z P_y$

NO QUAL: $P_x = -10\,kN$, $P_y = -15\,kN$, $P_z = -5\,kN$

ENTÃO: $M_x = (2)(-5) - \cancel{(0)(-15)}^{\,0} \Rightarrow$

$$\boxed{M_x = -10\,kNm}$$

ASSIM: $R_{M_x} - 10\,kNm = 0$ : $\boxed{R_{M_x} = 10\,kNm}$

# Exemplo 3

$$\mathbf{P_x = -10\ kN;\ P_y = -15\ kN;\ P_z = -5kN}$$

$$\mathbf{R_{My} + \sum_{i=1}^{nr.\ forças} (M_y)_i = 0}$$

ENTÃO: $M_y = \cancel{(0)}^{0}.(-10) - (3)(-5) \Rightarrow$

$$\boxed{M_y = 15\ kNm}$$

ASSIM: $R_{M_y} + 15\ kNm = 0$ : $\boxed{R_{M_y} = -15\ kNm}$

$$\mathbf{R_{Mz} + \sum_{i=1}^{nr.\ forças} (M_z)_i = 0}$$

E: $M_z = dx\,P_y - dy.P_x$

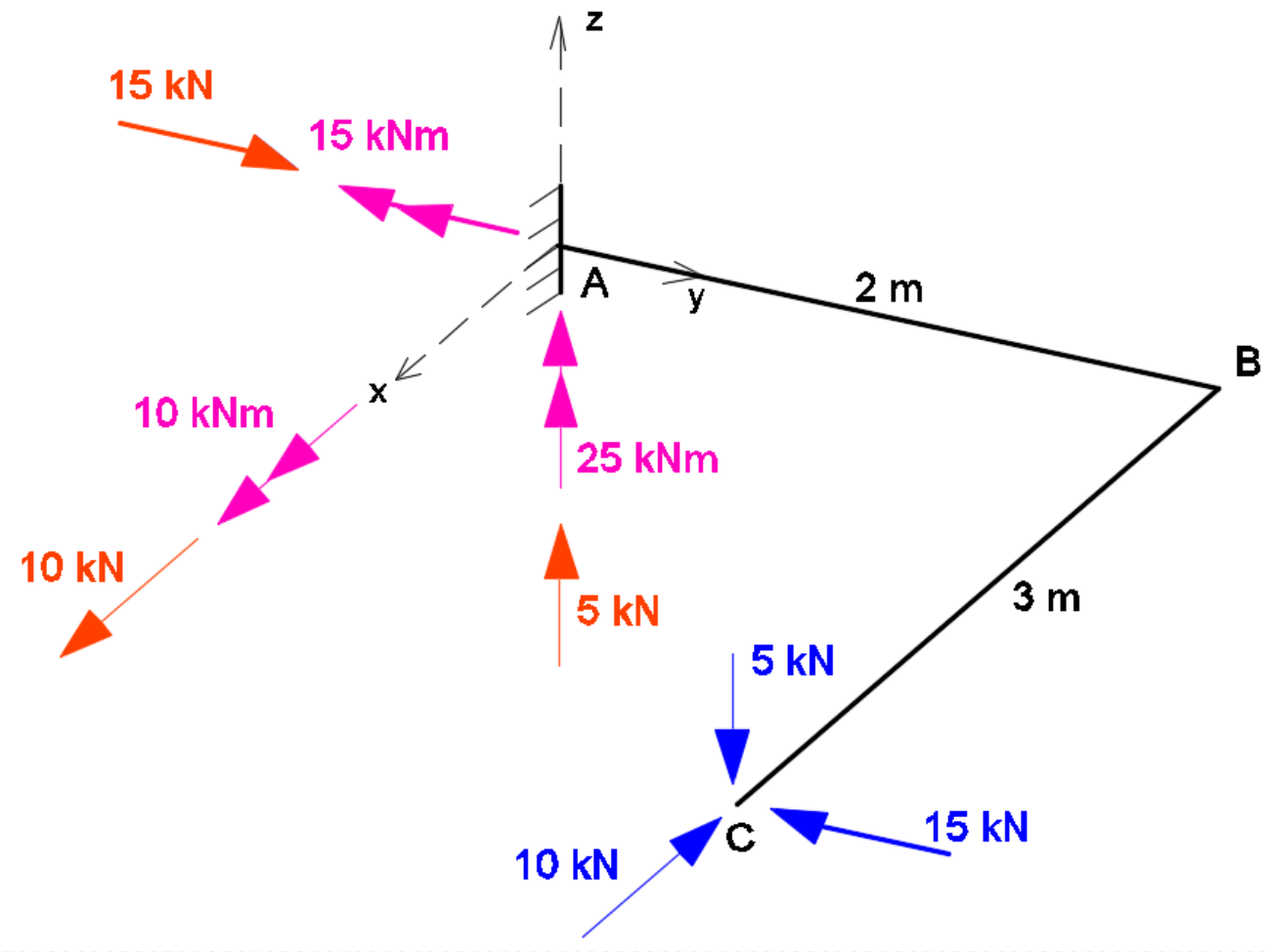
ENTÃO: $M_z = (3)(-15) - (2)(-10)$

$$\boxed{M_z = -25\ kNm}$$

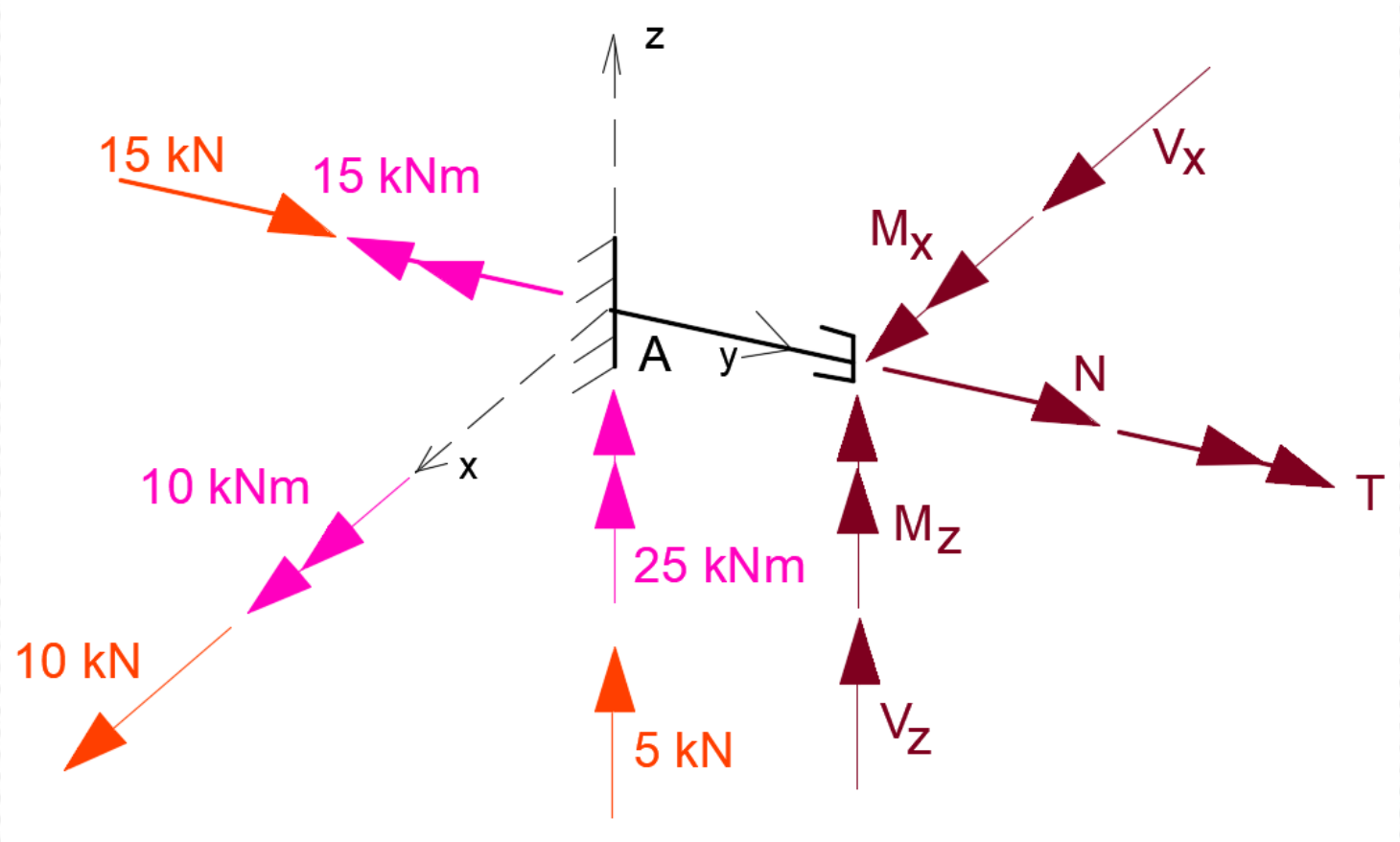
ASSIM: $R_{M_z} - 25 = 0$ : $\boxed{R_{M_z} = 25\ kNm}$

# Exemplo 3

## Reações



**Exemplo 3**

# 2) Esforços

a) Corte junto ao engaste em A