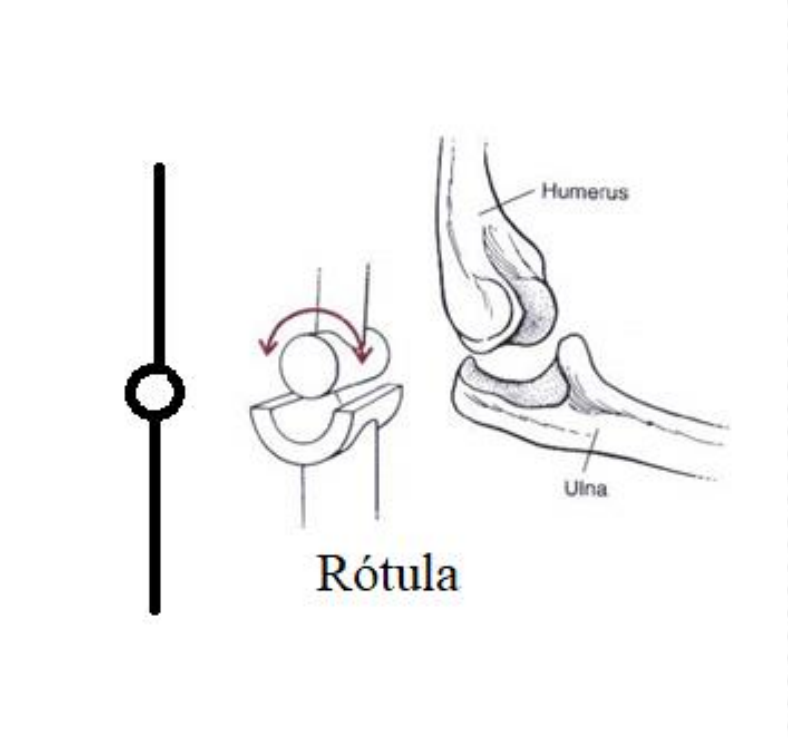
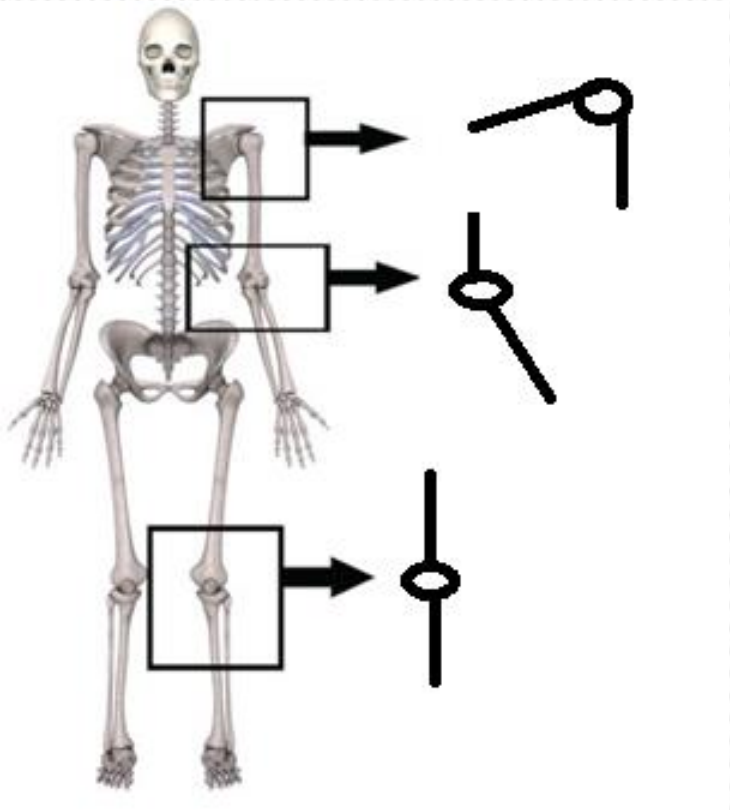
Quadros isostáticos: rótula e barra inclinada

Valério S. Almeida

Março/2020

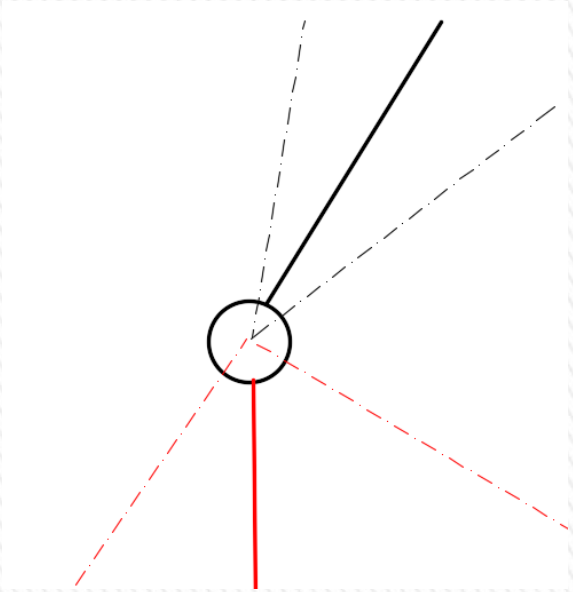
RÓTULAS

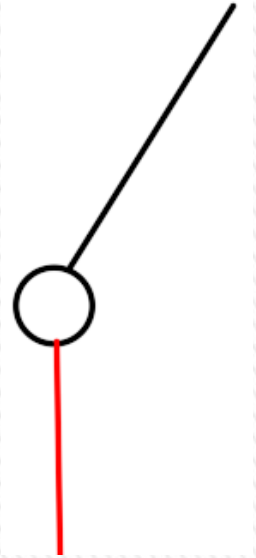




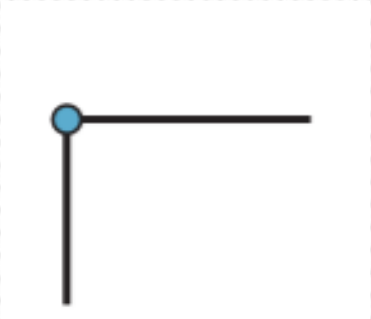
Conecta barras
Permite giro livre das barras

Rótulas

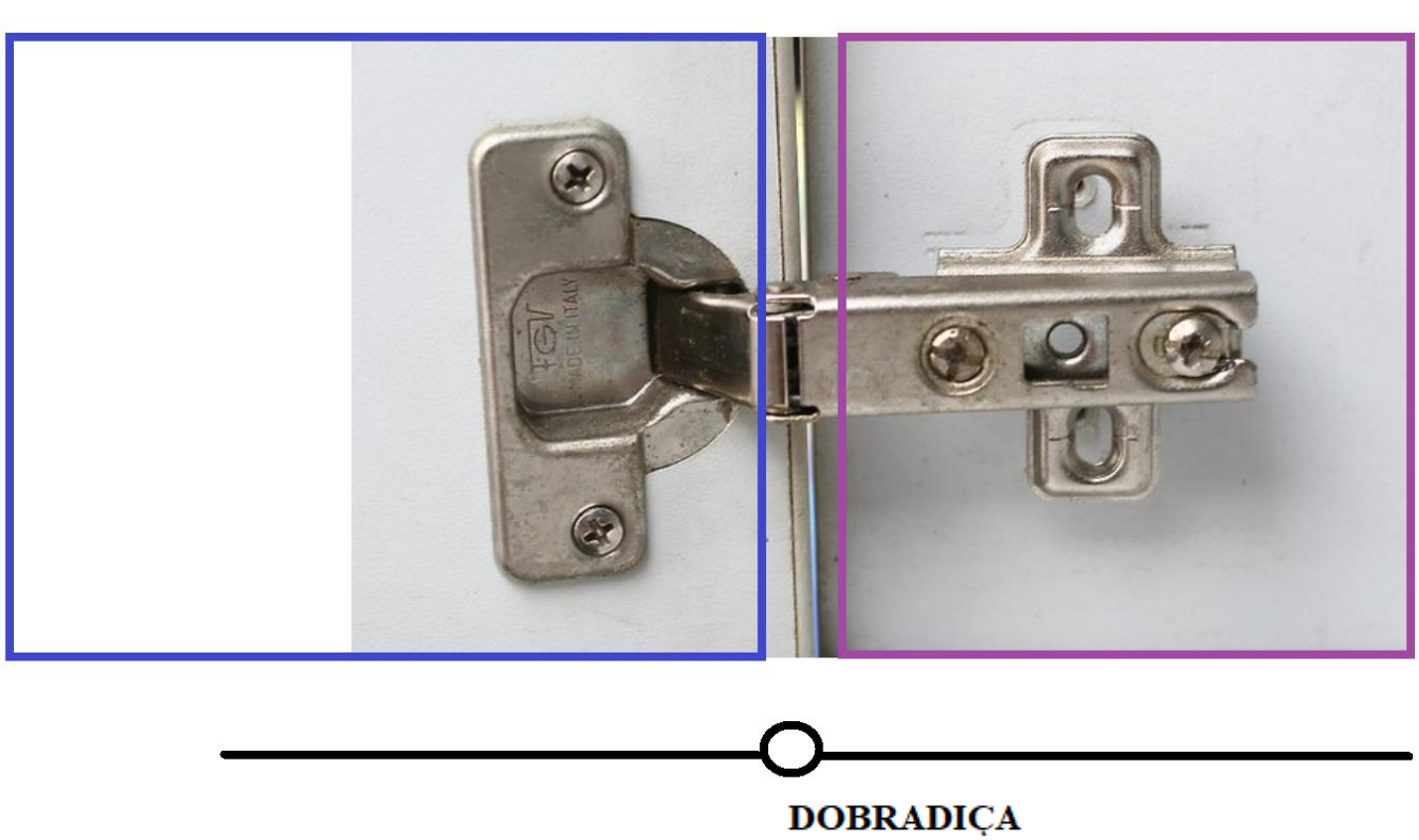




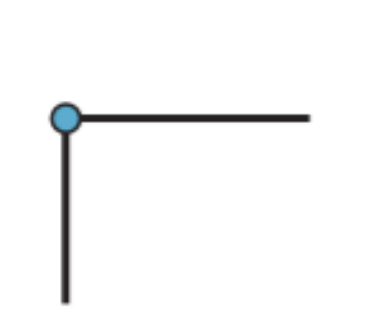
Conecta barras
Permite giro livre das barras



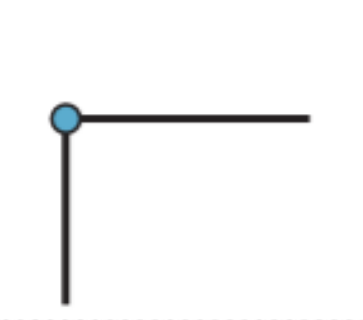
Rótulas



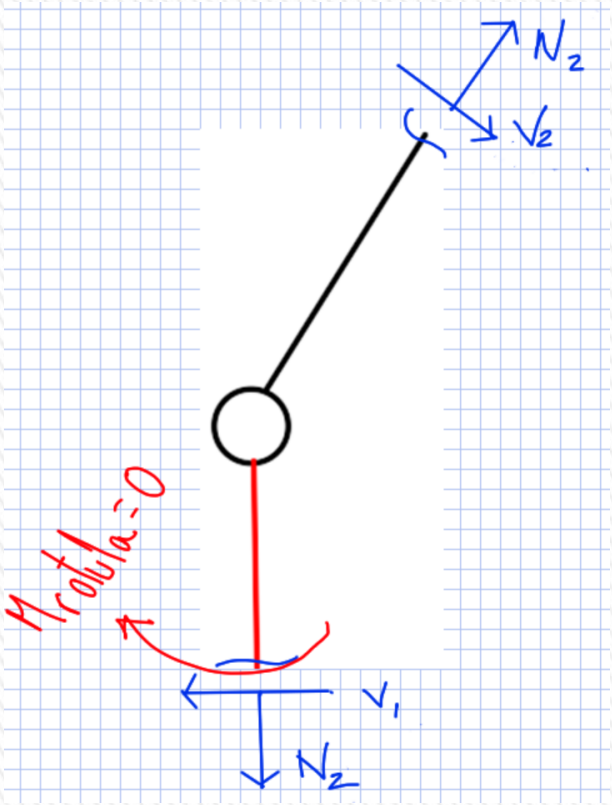
Conecta barras
Permite giro livre das barras



Rótulas



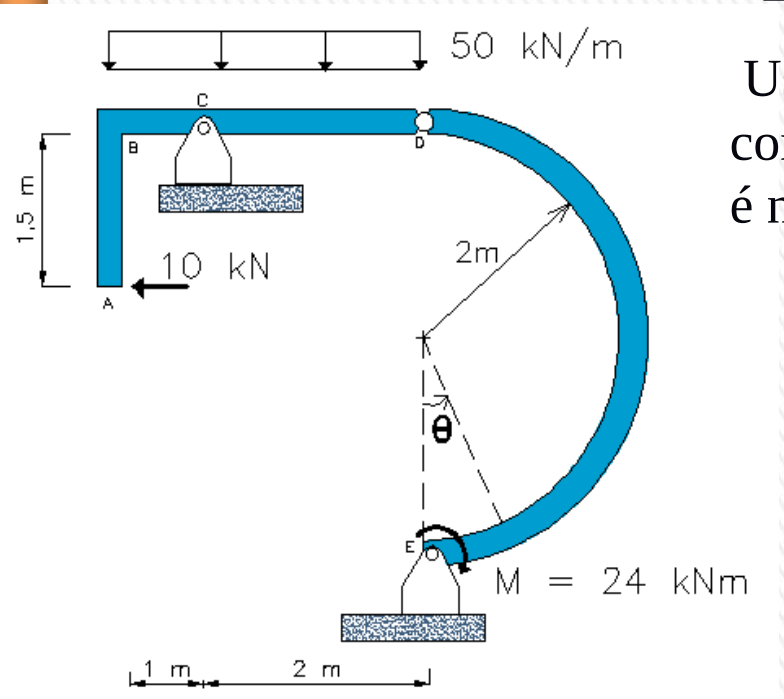
Conecta barras
Permite giro livre das barras
Não há momento fletor junto a seção que liga à rotula



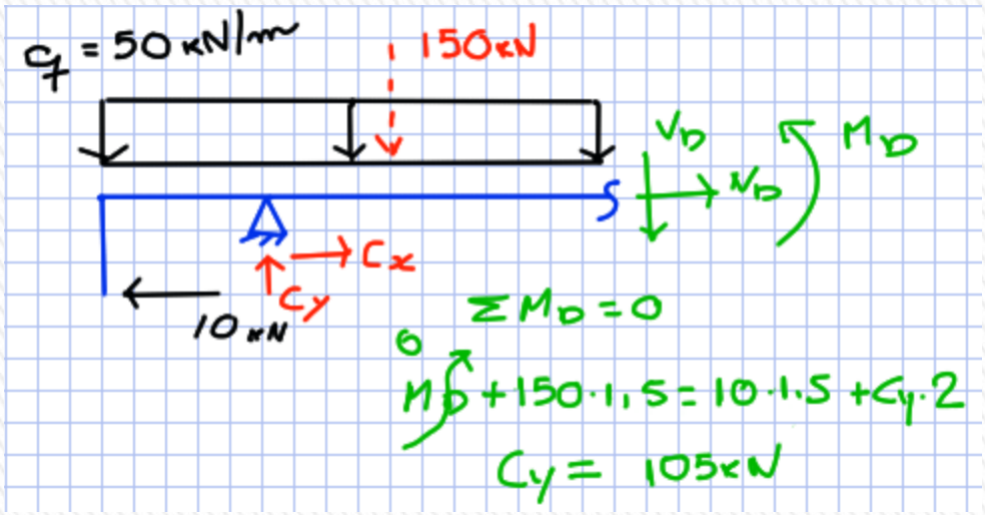
Corta-se a estrutura e usa-se a equação adicional:

$$M_{rótula} = 0$$

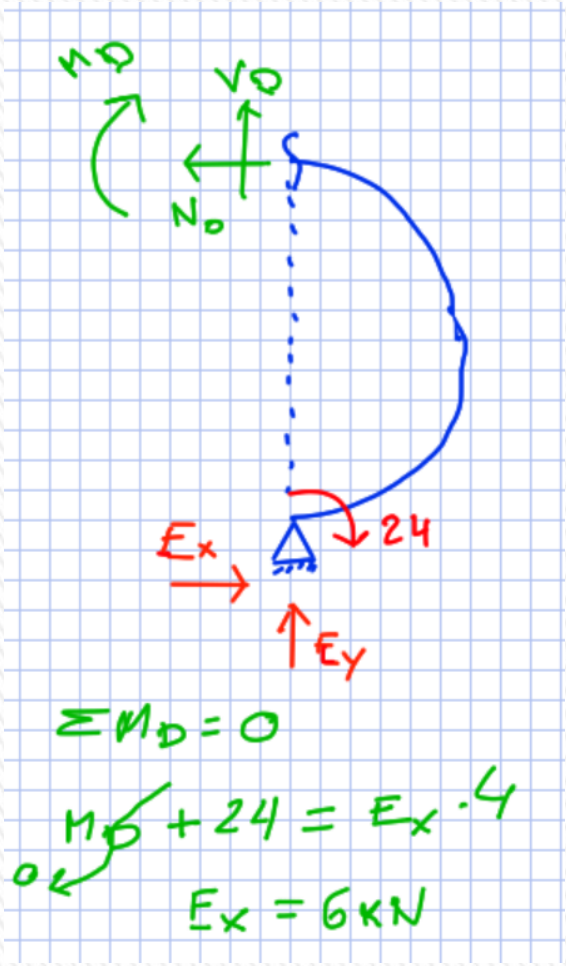
Exemplo 1



Usando as 3 equações de equilíbrio mais a equação complementar de que momento na seção em D (rótula) é nulo, determinam-se as reações:



Corte pela esquerda



Corte pela direita

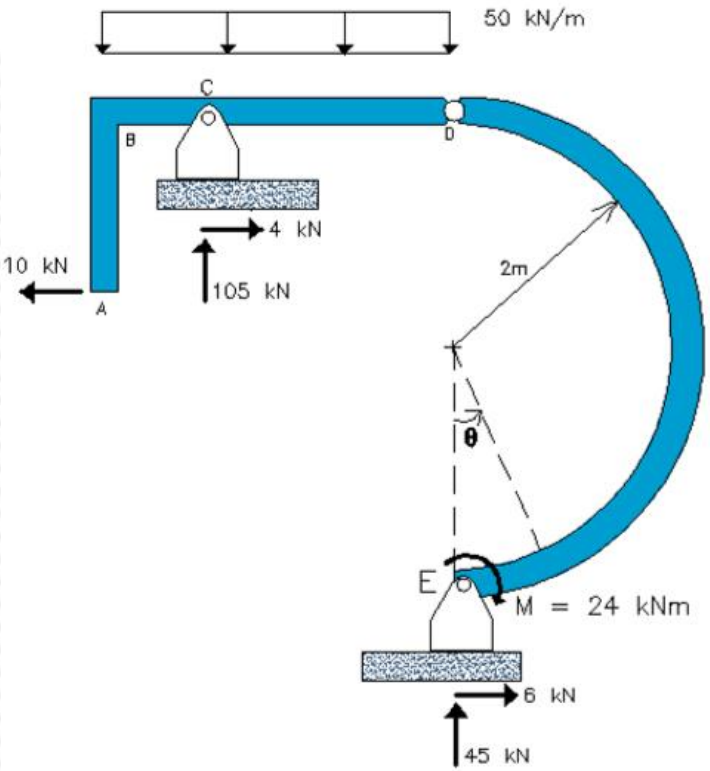
Exemplo 1

$$\sum M_D = 0: \rightarrow 4.E_x = 24 \rightarrow E_x = 6 \quad kN \quad (\rightarrow)$$

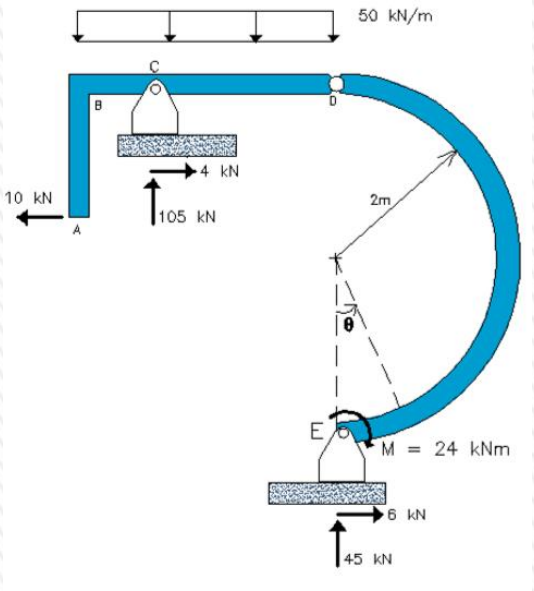
$$\sum F_X = 0: \rightarrow E_x + C_x = 10 \rightarrow C_x = 4 \quad kN \quad (\rightarrow)$$

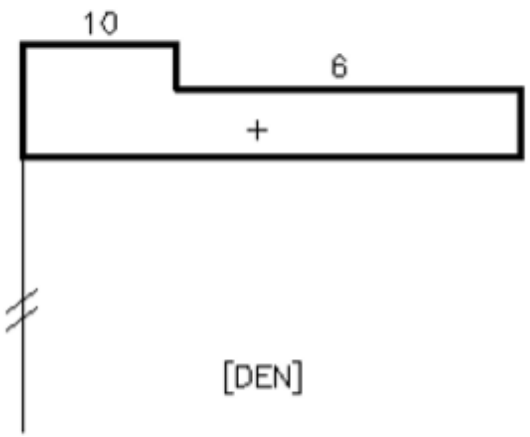
$$\sum M_E = 0: \rightarrow 2.C_y + 4.C_x + 24 = 50.3.1{,}5 + 10.2{,}5 \rightarrow C_y = 105 \quad kN \quad (\uparrow)$$

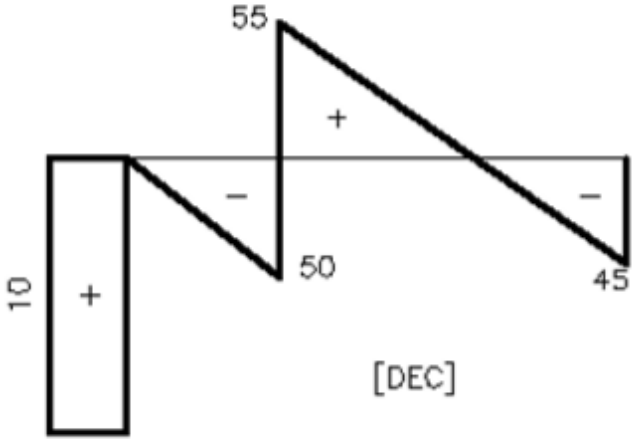
$$\sum F_y = 0: \rightarrow C_y + E_y = 50.3 \quad \rightarrow E_y = 45 \quad kN \quad (\uparrow)$$

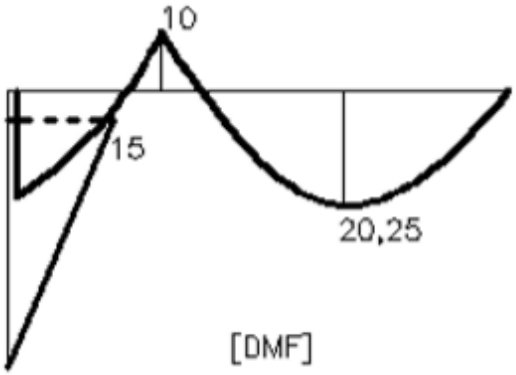


Exemplo 1



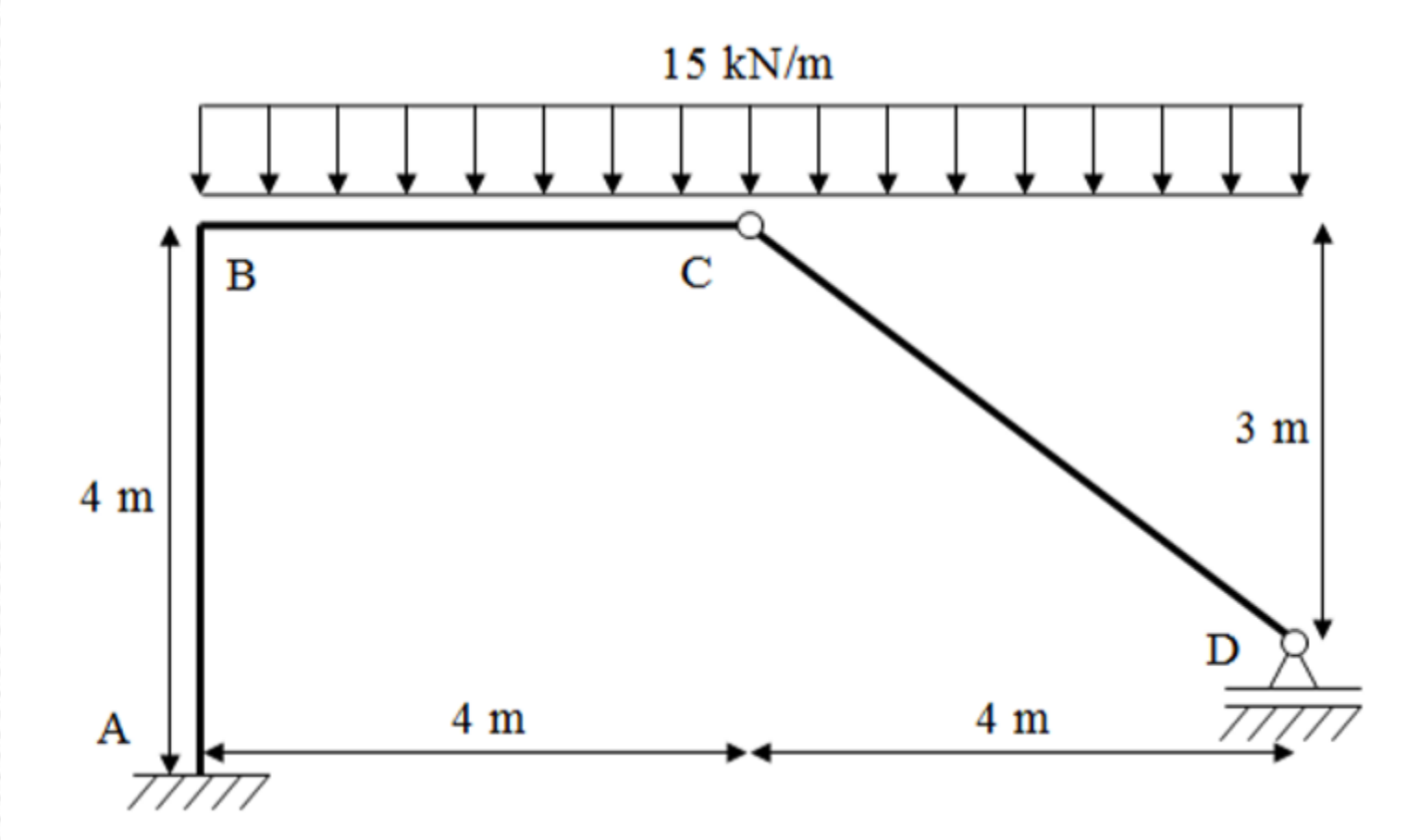




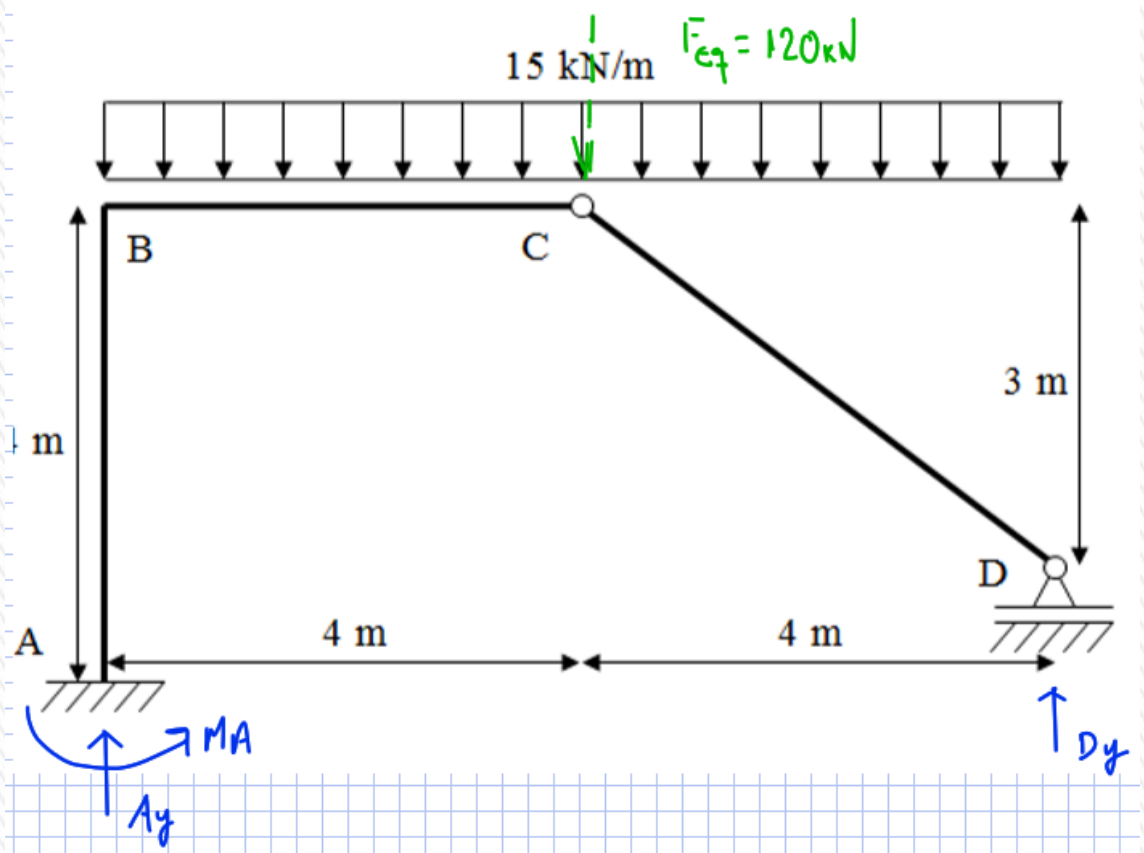


(kN,m)

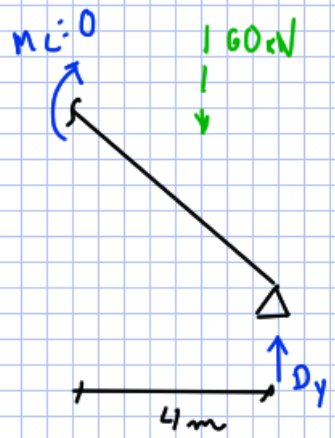
Exemplo 2



Exemplo 2



1) Cortar a estrutura em C:



$\Sigma M_C = 0 : \cancel{M_C} + 60 \cdot 2 = D_y \cdot 4$

$D_y = 30kN$

Exemplo 2

2) Equilibrio global:

$\Sigma M_A = 0$: $\circlearrowleft +$

$M_A + D_y \cdot 8 = 120 \cdot 4 \rightarrow M_A = 240$ kN·m

$\Sigma F_y = 0$: $A_y + D_y = 120 \rightarrow A_y = 90$ kN

3) 3 trechos: AB, BC, CD

AB:

$N_1 = -90$

$V_1 = 0$

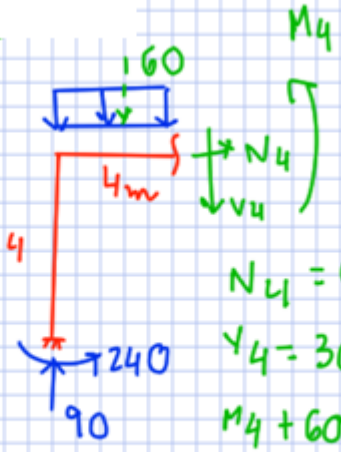
$M_1 = -240$

$N_2 = -90$

$V_2 = 0$

$M_2 = -240$

BC:



$N_3 = 0$

$V_3 = 90$

$M_3 = -240$

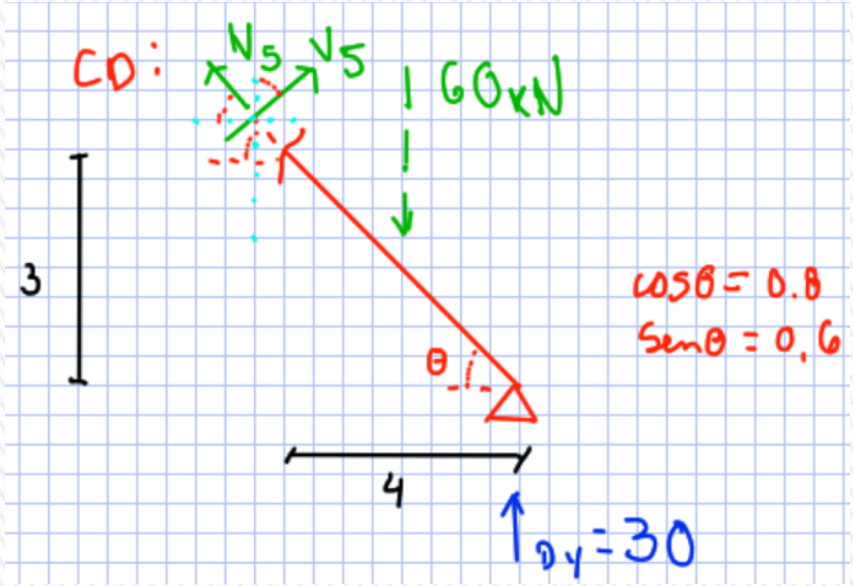
$N_4 = 0$

$V_4 = 30$

$M_4 + 60 \cdot 2 + 240 = 90 \cdot 4$

$M_4 = 0$ (Rótula!)

Exemplo 2



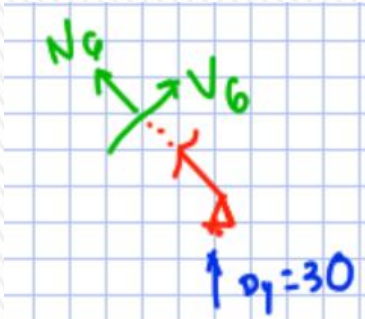
$\Sigma F_x = 0: \quad -N_5 \cos\theta + V_5 \operatorname{sen}\theta = 0 \rightarrow V_5 = 1.33 N_5$

$\Sigma F_y = 0 \quad N_5 \operatorname{sen}\theta + V_5 \cos\theta + 30 = 60$

Assim: $N_5 \cdot 0,6 + 1.33 N_5 \cdot 0,8 = 30 \rightarrow N_5 = 18\,kN$

$V_5 = 24\,kN$

$V_5 = 24k$

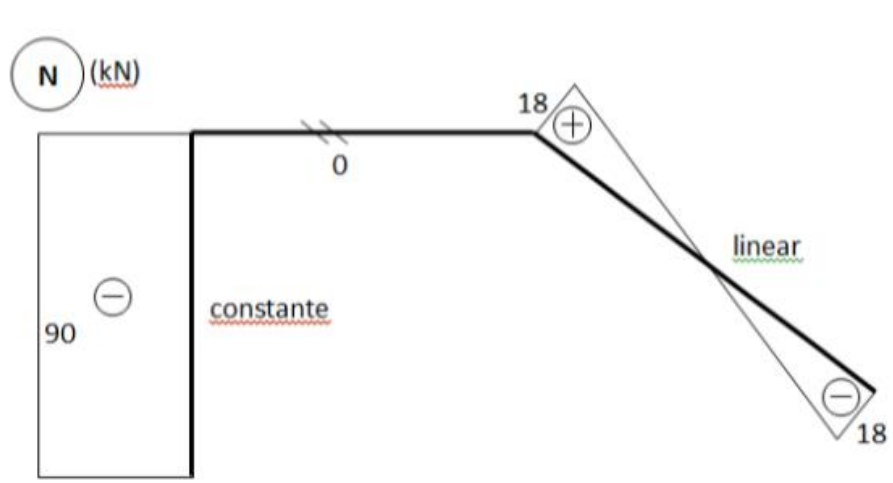


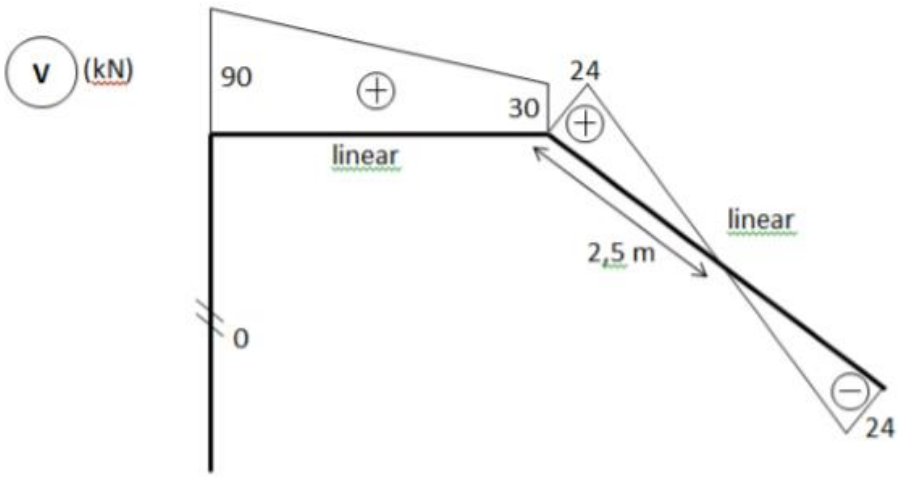
$\Sigma F_x = 0: \quad V_6 \operatorname{sen}\theta - N_6 \cos\theta = 0$

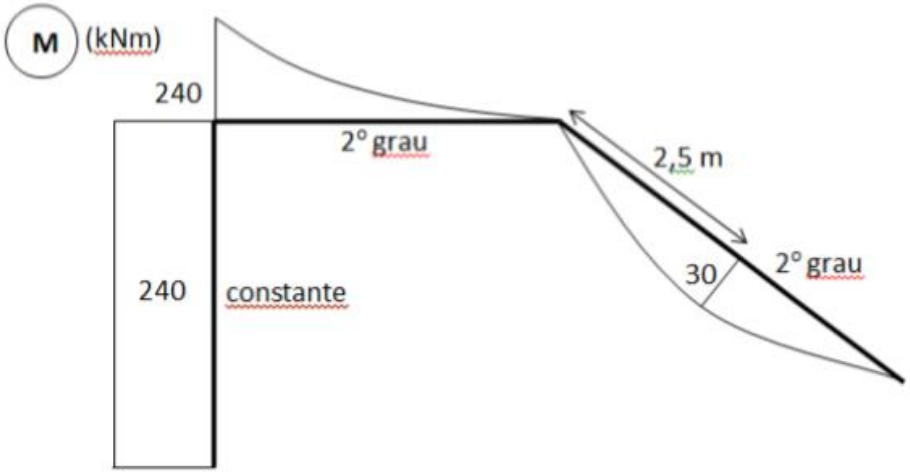
$\Sigma F_y = 0 \quad V_6 \cos\theta + N_6 \operatorname{sen}\theta + 30 = 0$

$\therefore N_6 = -18\,kN; \; V_6 = -24\,kN$

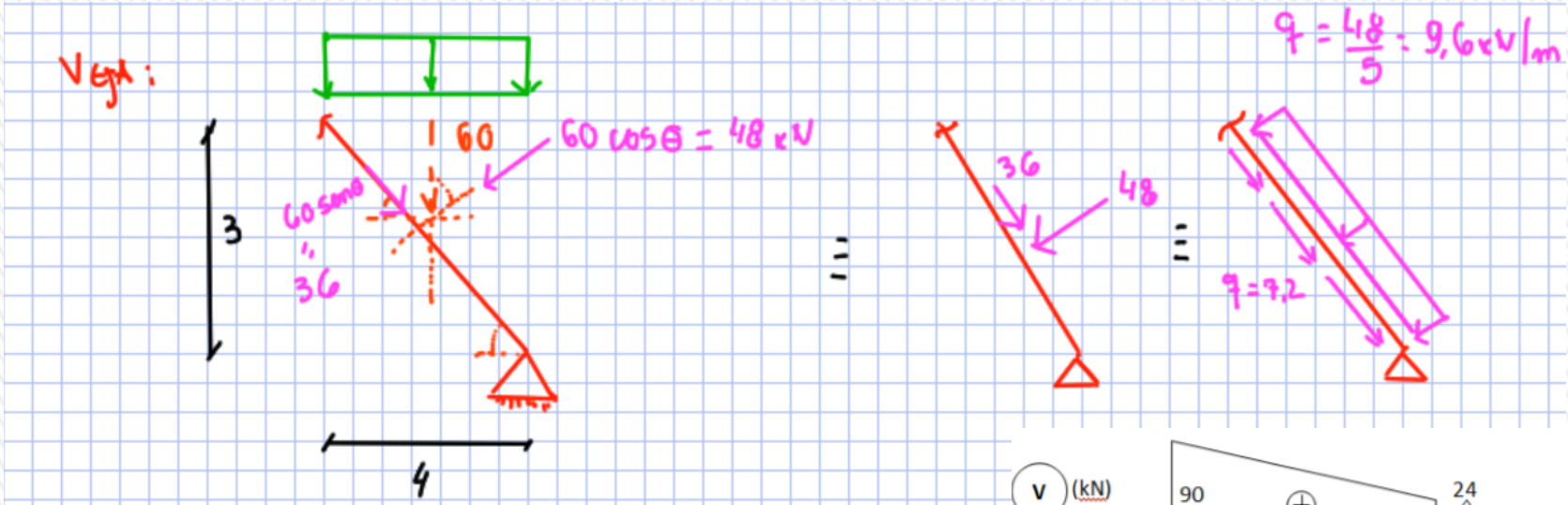
Exemplo 2


N (kN)
18
0
linear
90
constante
18


V (kN)
90
24
30
linear
linear
2,5 m
0
24


M (kNm)
240
2° grau
2,5 m
30
2° grau
240
constante

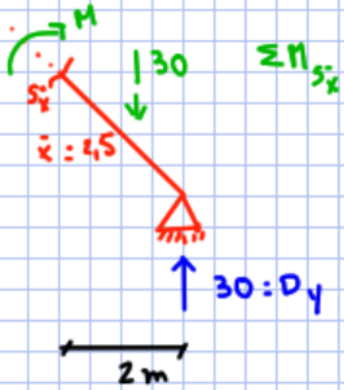
Exemplo 2



4) Pelo diagrama. trecho inclinado

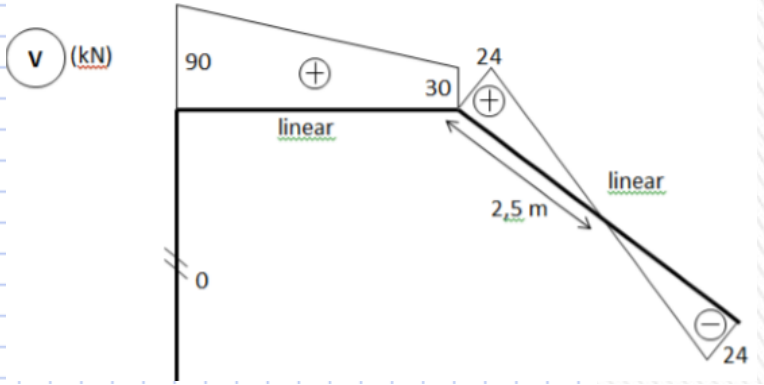
$V = 0 \rightarrow \bar{x} = 2,5\,m$ (de c)

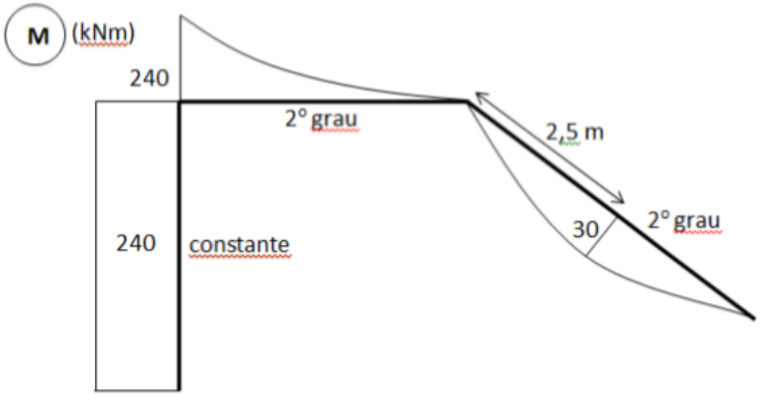
Façamos um corte em $\bar{x}$, p/ obter $M_{MÁX}$:



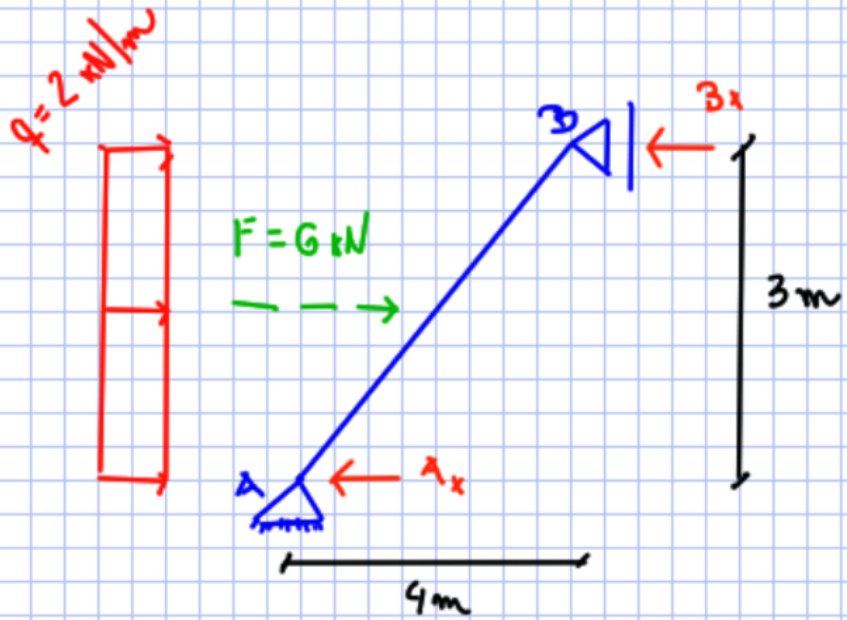
$\Sigma M_{S_{\bar{x}}} = 0: \; M + 30 \cdot 1 = 30 \cdot 2$

$M = 30\,kNm$





Exemplo 3

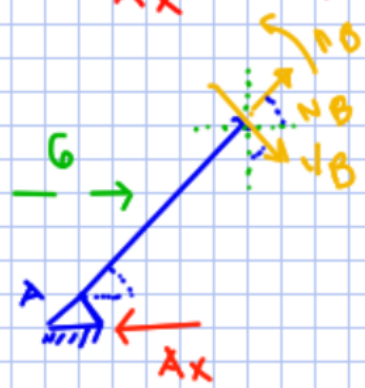


$\Sigma F_x = 0$:

$N_A \cos\alpha + V_A \operatorname{sen}\alpha = A_x$

$\Sigma F_y = 0$:

$N_A \operatorname{sen}\alpha - V_A \cos\alpha = 0$



$\Sigma F_x = 0$

$N_B \cos\alpha + V_B \operatorname{sen}\alpha + 6 = A_x$

$\Sigma F_y = 0$

$N_B \operatorname{sen}\alpha - V_B \cos\alpha = 0$