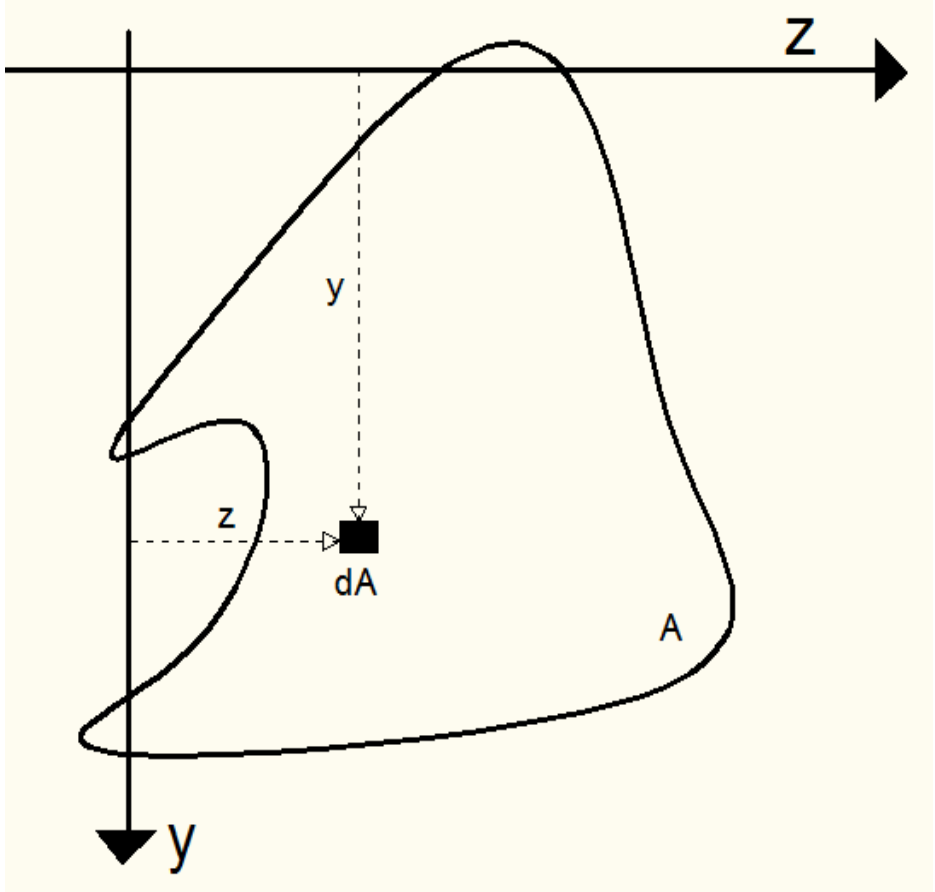
PEF3307

CARACTERÍSTICAS GEOMÉTRICAS DA SEÇÃO TRANSVERSAL

Momento estático e
Centro de Gravidade (baricentro)



- **Momento estático:**

$$M_{s_z} = \int_A y \cdot dA$$

Unidade do momento estático é L^3 : m^3, cm^3

$$M_{s_y} = \int_A z \cdot dA$$

- **Centro de Gravidade :**

$$z_{CG} = \frac{M_{s_y}}{A} = \frac{\int_A z \cdot dA}{\int_A dA} \qquad y_{CG} = \frac{M_{s_z}}{A} = \frac{\int_A y \cdot dA}{\int_A dA}$$

Centro de Gravidade (baricentro)

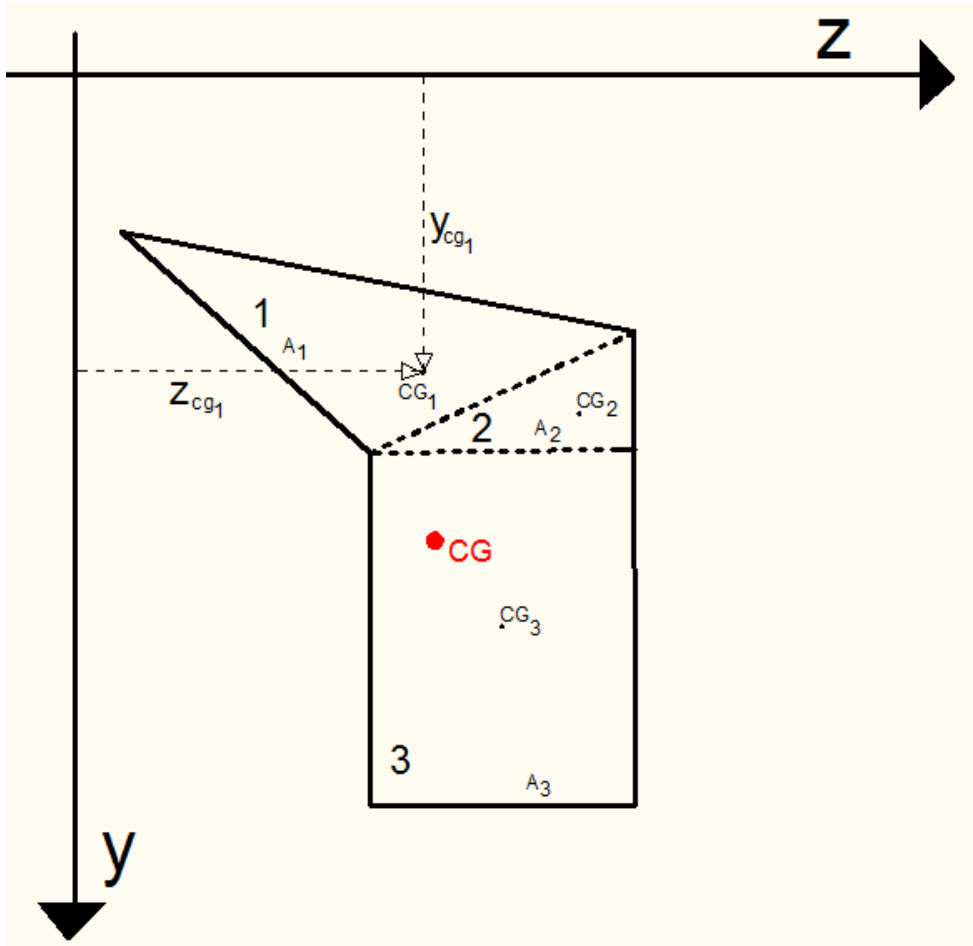
- **Cálculo do Centro de Gravidade:**
  - **seções formadas por *n* geometria básicas**

$$y_{CG} = \frac{M_{s_z}}{A} = \frac{\int_{A_1} y \cdot dA + \int_{A_2} y \cdot dA + \ldots + \int_{A_n} y \cdot dA}{\int_{A_1} dA + \int_{A_2} dA + \ldots + \int_{A_n} dA}$$

$$y_{CG} = \frac{\sum_i^n (y_{CG})_i \cdot A_i}{\sum_i^n A_i}$$

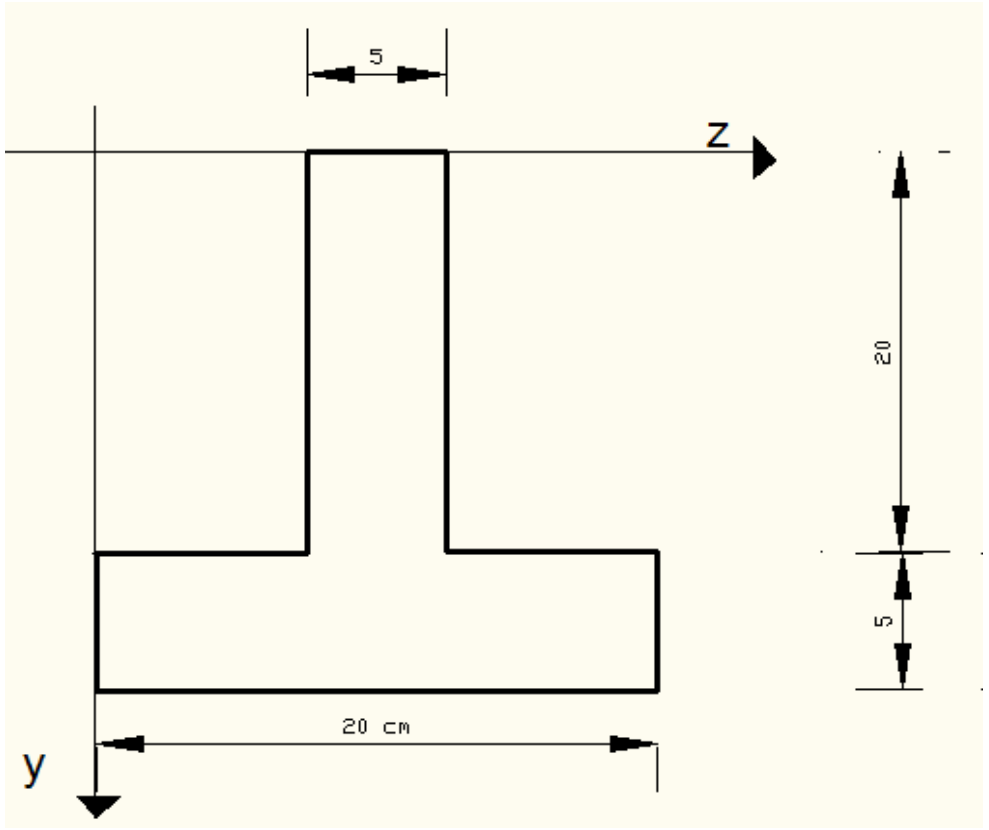
$$z_{CG} = \frac{M_{s_y}}{A} = \frac{\int_{A_1} z \cdot dA + \int_{A_2} z \cdot dA + \ldots + \int_{A_n} z \cdot dA}{\int_{A_1} dA + \int_{A_2} dA + \ldots + \int_{A_n} dA}$$

$$z_{CG} = \frac{\sum_i^n (z_{CG})_i \cdot A_i}{\sum_i^n A_i}$$



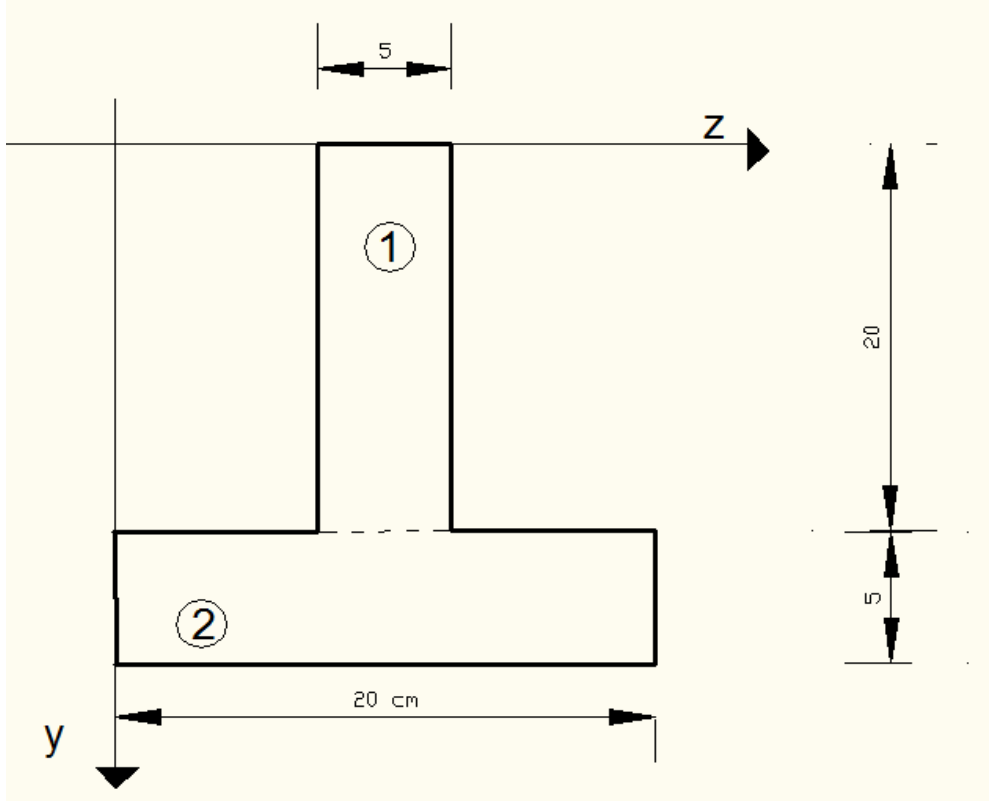
Exemplo:

Determinar o CG da seção transversal (ST) em perfil T invertido



a) **Dividir em geometrias básicas: n = 2**

Criar um sistema auxiliar yz



Exemplo:

Região 1:

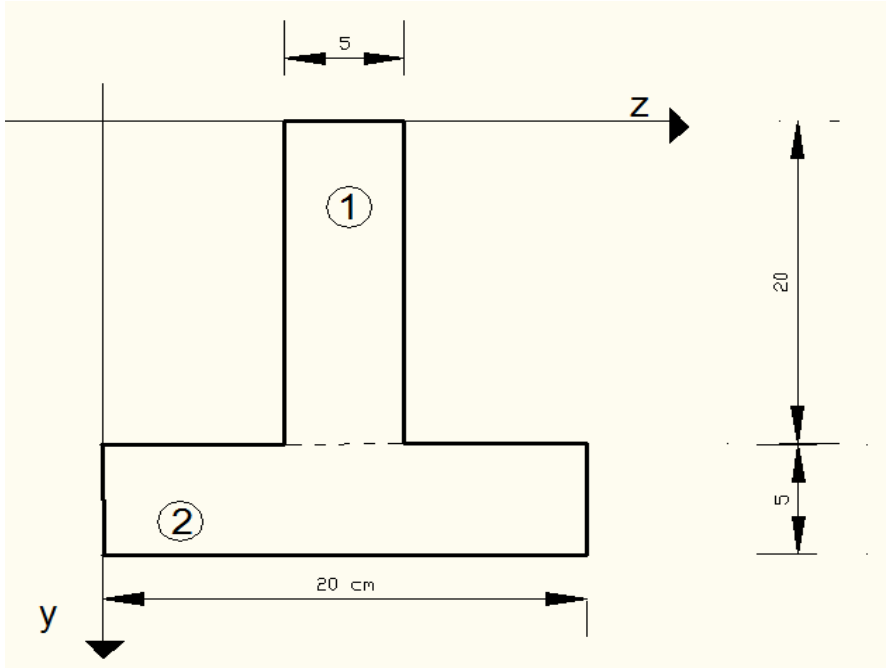
$(y_{CG})_1 = 10\ cm;\ (z_{CG})_1 = 10\ cm;\ (A)_1 = 100\ cm^2$

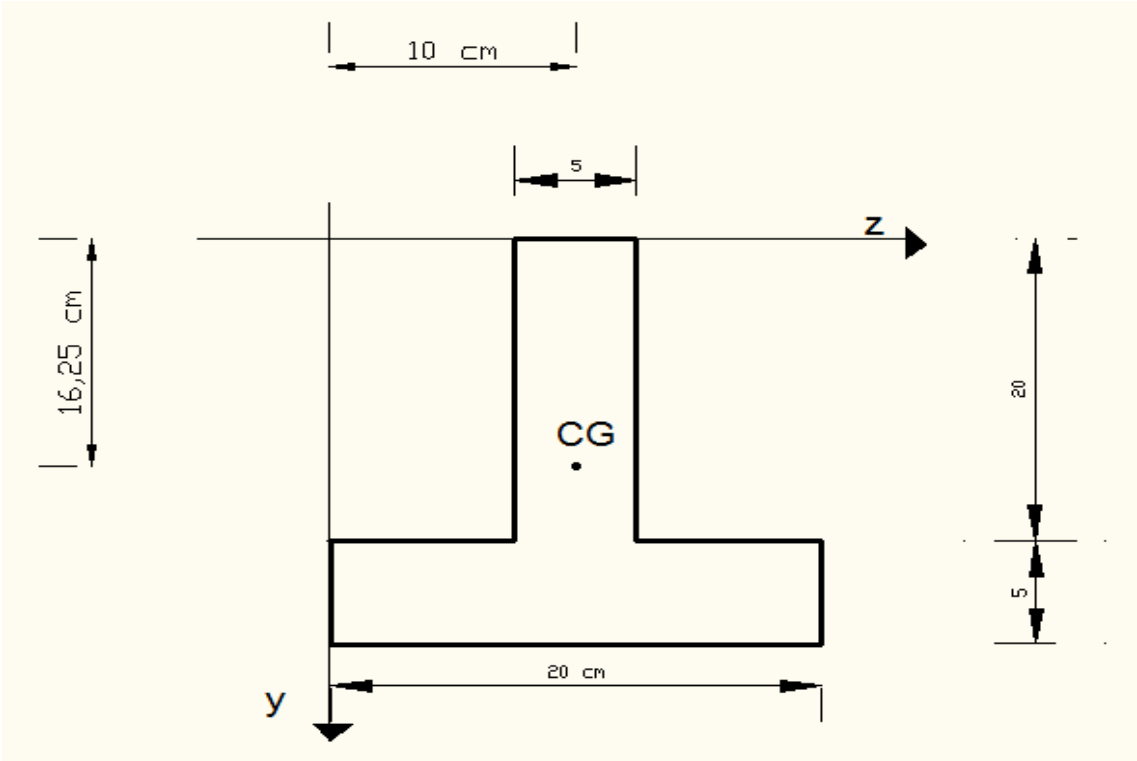
Região 2:

$(y_{CG})_2 = 22{,}5\ cm;\ (z_{CG})_2 = 10\ cm;\ (A)_2 = 100\ cm^2$

$$y_{CG} = \frac{(y_{CG})_1 \cdot A_1 + (y_{CG})_2 \cdot A_2}{A_1 + A_2} = \frac{10 \cdot 100 + 22{,}5 \cdot 100}{100 + 100} = 16{,}25\ cm$$

$$z_{CG} = \frac{(z_{CG})_1 \cdot A_1 + (z_{CG})_2 \cdot A_2}{A_1 + A_2} = \frac{10 \cdot 100 + 10 \cdot 100}{100 + 100} = 10\ cm$$





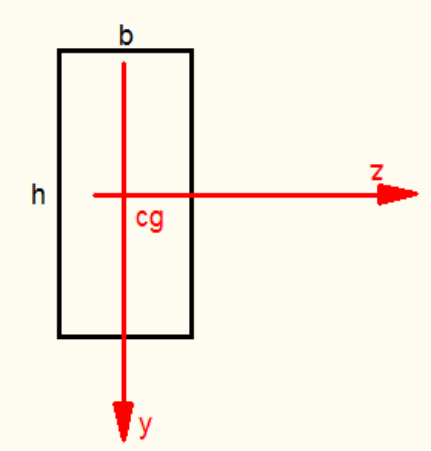
Momento de Inércia (2ª. ordem)

$$I_y = \int_A z^2 \cdot dA$$

$$I_z = \int_A y^2 \cdot dA$$

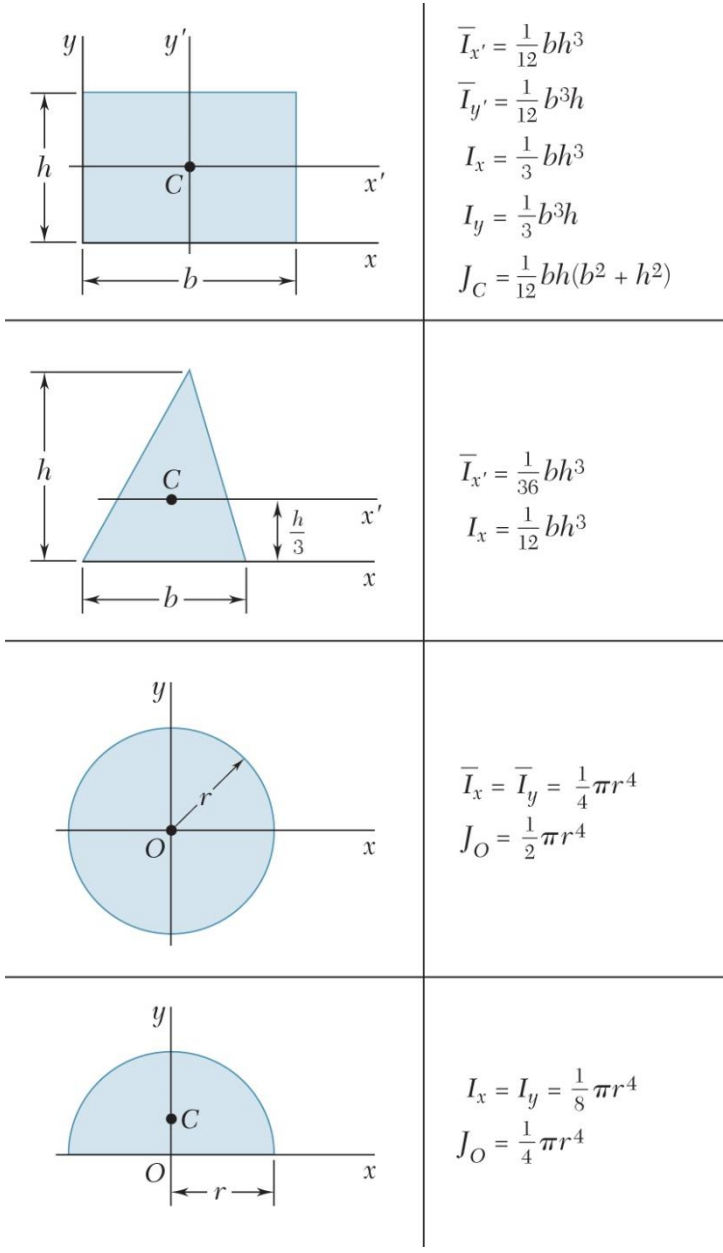
$$I_{yz} = \int_A z \cdot y \cdot dA$$

- **Geometrias básicas:**



$$I_z = I_{x'} = \frac{b.h^3}{12}$$

$$I_y = I_{y'} = \frac{h.b^3}{12}$$



I_y e I_z : momento de inércia em relação aos eixos y e z

I_{yz} : produto de inércia (m^4, cm^4)

Momento de Inércia (2ª. ordem)

- **Geometrias mais gerais:**
  - **Teorema dos eixos paralelos (Teorema de Steiner)**

$$I_y = \sum_{i=1}^{n} \left(I_{y_{cc}} + (\bar{z})^2 \cdot A \right)_i$$

$$I_z = \sum_{i=1}^{n} \left(I_{z_{cc}} + (\bar{y})^2 \cdot A \right)_i$$

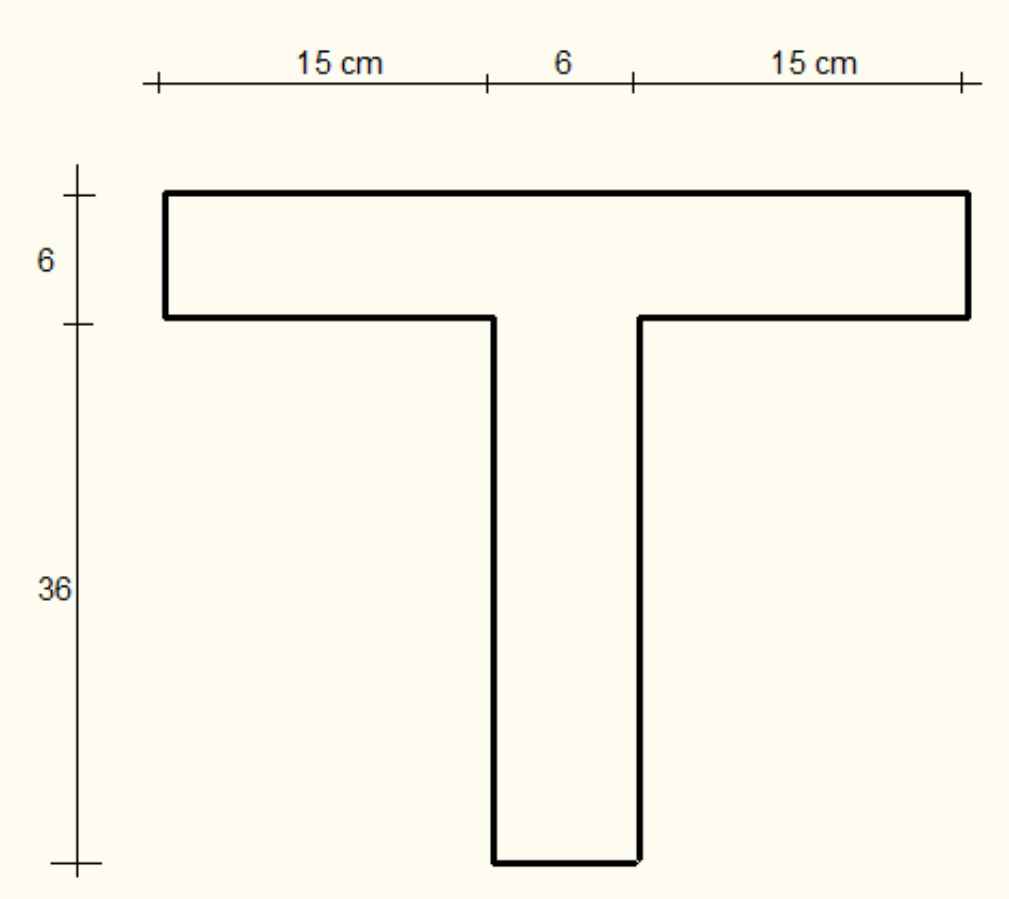
$$I_{yz} = \sum_{i=1}^{n} \left(I_{yz_{cc}} + (\bar{z}) \cdot (\bar{y}) \cdot A \right)_i$$

$(I_{y_{cc}})_i$, $(I_{z_{cc}})_i$ e $(I_{yz_{cc}})_i$ são os momentos de 2ª. ordem obtidos no CG da região *i*;

$(\bar{z})_i$ e $(\bar{y})_i$ são as coordenadas do CG da região *i*

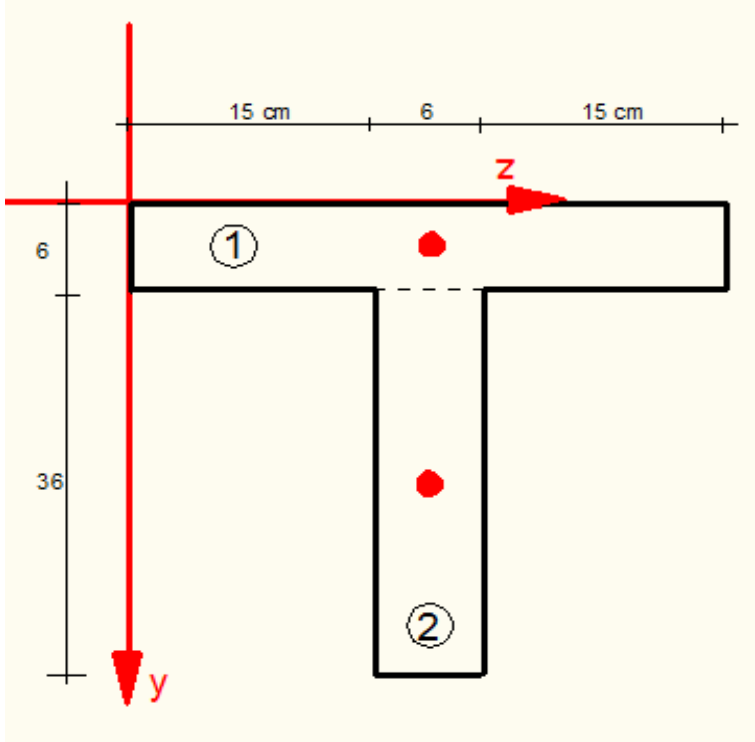
Exemplo 1:

Calcular momentos de inércia Iy e Iz da ST em relação ao CG:



a) Dividir em geometrias básicas: n = 2

Criar um sistema auxiliar yz



Exemplo 1:

b) Obter CG

Região 1:

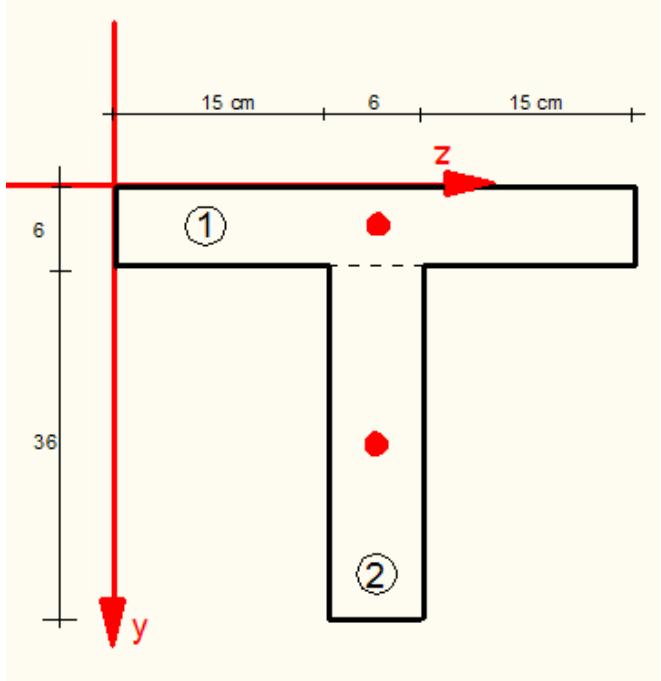
$(y_{CG})_1 = 3\ cm;\ (z_{CG})_1 = 18\ cm;\ (A)_1 = 216\ cm^2$

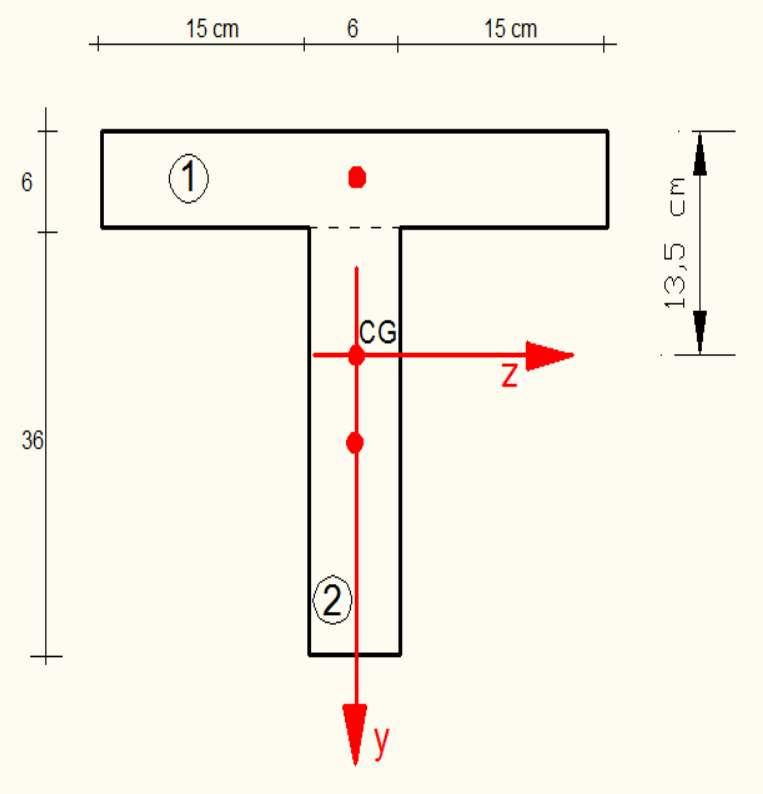
Região 2:

$(y_{CG})_2 = 24\ cm;\ (z_{CG})_2 = 18\ cm;\ (A)_2 = 216\ cm^2$

$$y_{CG} = \frac{(y_{CG})_1 \cdot A_1 + (y_{CG})_2 \cdot A_2}{A_1 + A_2} = \frac{3 \cdot 216 + 24 \cdot 216}{216 + 216} = 13{,}5\ cm$$

$$z_{CG} = \frac{(z_{CG})_1 \cdot A_1 + (z_{CG})_2 \cdot A_2}{A_1 + A_2} = \frac{18 \cdot 216 + 18 \cdot 216}{216 + 216} = 18\ cm$$





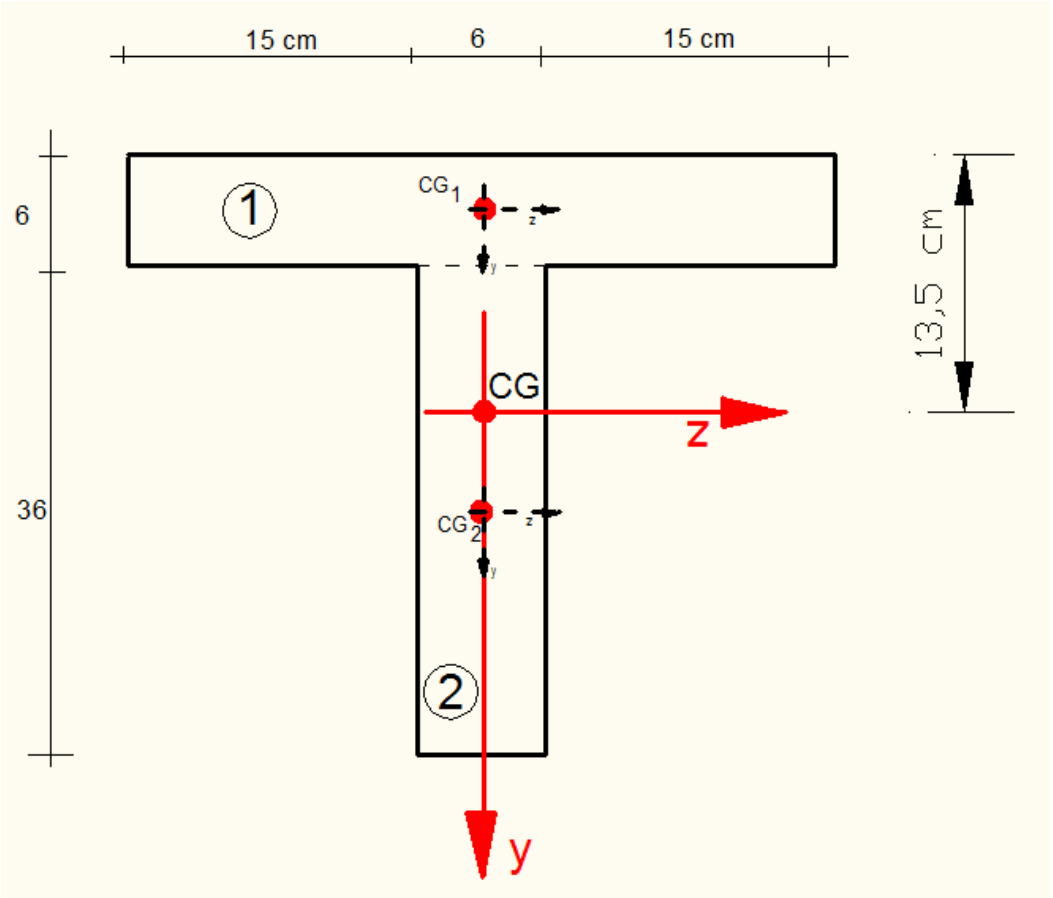
Exemplo 1:

c) Obter Momentos de Inércia:

$$I_y = \sum_{i=1}^{n} \left(I_{y_{CG}} + (\bar{z})^2 \cdot A \right)_i$$

$$I_{yz} = \sum_{i=1}^{n} \left(I_{yz_{CG}} + (\bar{z}) \cdot (\bar{y}) \cdot A \right)_i$$

$$I_z = \sum_{i=1}^{n} \left(I_{z_{CG}} + (\bar{y})^2 \cdot A \right)_i$$



Região 1:

$\bar{y}_1 = -10{,}5 \ \ cm$;

$\bar{z}_1 = 0$;

$$I_{z_{CG1}} = \frac{36.6^3}{12} = 648 \ \ cm^4; \ \ I_{y_{CG1}} = \frac{6.36^3}{12} = 23328 \ \ cm^4$$

Região 2:

$\bar{y}_2 = 10{,}5 \ \ cm$;

$\bar{z}_2 = 0$;

$$I_{z_{CG2}} = \frac{6.36^3}{12} = 23328 \ \ cm^4; \ \ I_{y_{CG2}} = \frac{36.6^3}{12} = 648 \ \ cm^4$$

$$(I_z)_{CG} = \left[648 + (-10{,}5)^2.216\right] + \left[23328 + (10{,}5)^2.216\right] = 71604 \ \ cm^4$$

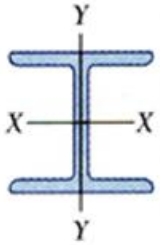
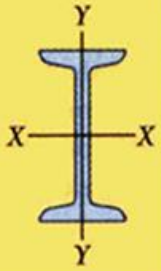
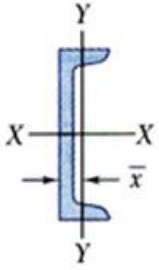
$$(I_y)_{CG} = \left[23328 + (0)^2.216\right] + \left[648 + (0)^2.216\right] = 23976 \ \ cm^4$$

$$(I_{yz})_{CG} = \left[0 + (-10{,}5.0).216\right] + \left[0 + (10{,}5.0).216\right] = 0$$

Tabelas de Perfis Padronizados

Beer | Johnston | DeWolf | Mazurek

Momentos de Inércia de Superfícies Compostas

		Designação	Área mm²	Altura mm	Largura mm	Eixo X-X $\bar{I}_x$ 10^6 mm⁴	Eixo X-X $\bar{k}_x$ mm	Eixo X-X $\bar{y}$ mm	Eixo Y-Y $\bar{I}_y$ 10^6 mm⁴	Eixo Y-Y $\bar{k}_y$ mm	Eixo Y-Y $\bar{x}$ mm
Perfil I		W460 × 74[1]	9.450	457	190	333	188		16,6	41,9	
		W410 × 60	7.580	407	178	216	169		12,1	40,0	
		W360 × 44	5.730	352	171	122	146		8,18	37,8	
		W200 × 35,9	4.580	201	165	34,4	86,7		7,64	40,8	
Perfil Duplo T		S460 × 104[1]	13.300	457	159	385	170		10,4	27,5	
		S310 × 74	9.480	305	139	126	115		6,69	26,1	
		S250 × 52	6.670	254	126	61,2	95,8		3,59	22,9	
		S150 × 25,7	3.270	152	91	10,8	57,5		1,00	17,2	
Perfil C		C310 × 37[1]	5.690	305	77	59,7	112		1,83	19,7	17,0
		C250 × 30	3.780	254	69	32,6	92,9		1,14	17,4	15,3
		C200 × 27,9	3.560	203	64	18,2	71,5		0,817	15,1	14,3
		C150 × 15,6	1.980	152	51	6,21	56,0		0,347	13,2	12,5

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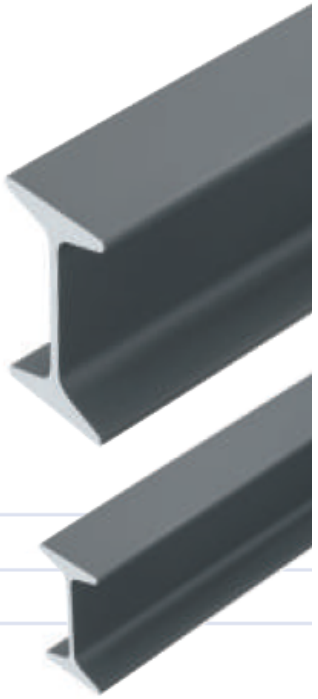
grupo a
Conhecimento que transforma.

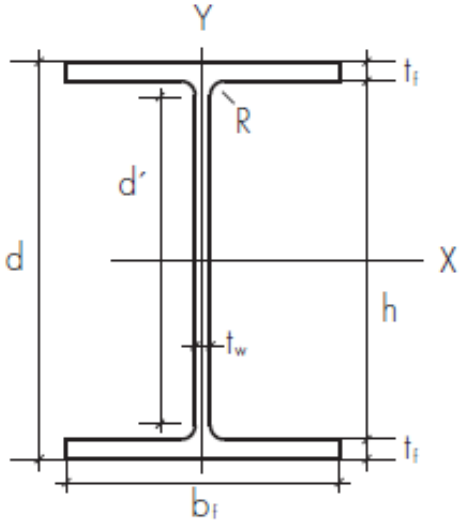
Tabelas de Perfis Padronizados

Perfil I

Dimensão		Aba		Alma	Área	Peso
		Largura	Espessura	Espessura		
pol.	mm	mm	mm	mm	mm²	kg/m
3"	76,00	59,00	6,60	4,30	1080	8,48
				6,38		9,67
4"	102,00	68,00	7,40	4,90	1450	11,46
				6,43		12,65
	102,00	71,00	7,40	8,30	1800	14,14
5"	127,00	76,00	8,30	5,40	1880	14,88
				8,81		18,24
6"	152,00	85,00	9,10	5,90	2360	18,60
				8,71		21,96
	152,00	91,00	9,10	11,80	3270	25,67

Dimensão		Eixo X-X			Eixo Y-Y				
		J	W	r	J	W	r	rt	d Af
pol.	mm	cm⁴	cm³	cm	cm⁴	cm³	cm	cm	cm-1
3"	76,00	105,10	27,60	3,12	18,90	6,40	1,33	1,45	1,95
4"	102,00	252,00	49,70	4,17	31,70	9,40	1,48	1,68	2,02
		266,00	52,40	4,06	34,30	9,90	1,46	1,83	1,98
	102,00	283,00	55,60	3,96	37,60	10,60	1,45	1,74	1,92
5"	127,00	511,00	80,40	5,21	50,20	13,20	1,63	1,88	2,01
		570,00	89,80	4,95	58,60	14,70	1,59	1,92	1,92

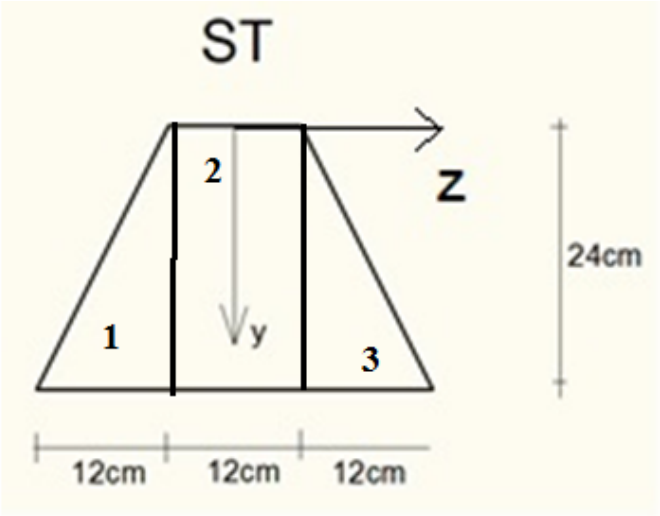




d = altura do perfil
d' = altura livre da alma
h = altura interna
b_f = largura da aba do perfil
t_f = espessura da aba
t_w = espessura da alma
R = raio de concordância

- Comprimentos de 6m e 12m.
- Atende à Norma ASTM A 572. Grau 50.
- Outros tamanhos poderão ser fornecidos mediante consulta prévia.

Exemplo 2:



Região 1 = Região 3:

$(y_{CG})_1 = 16\ cm\ ;\ A_1 = 144\ cm^2$

Região 2:

$(y_{CG})_2 = 12\ cm\ ;\ A_2 = 288\ cm^2$

$$y_{CG} = \frac{2\cdot(y_{CG})_1\cdot A_1 + (y_{CG})_2\cdot A_2}{2A_1 + A_2} = \frac{2.16.144 \cdot +12.288}{2.144 + 288} = 14\ cm$$

$$I_z = \sum_{i=1}^{n}\left(I_{z_{CG}} + (\bar{y})^2 \cdot A\right)_i$$

Região 1 = Região 3:

$(y_{CG})_1 = 16\ cm\ ;\ A_1 = 144\ cm^2$

$\bar{y}_1 = 2\ cm\ ;$

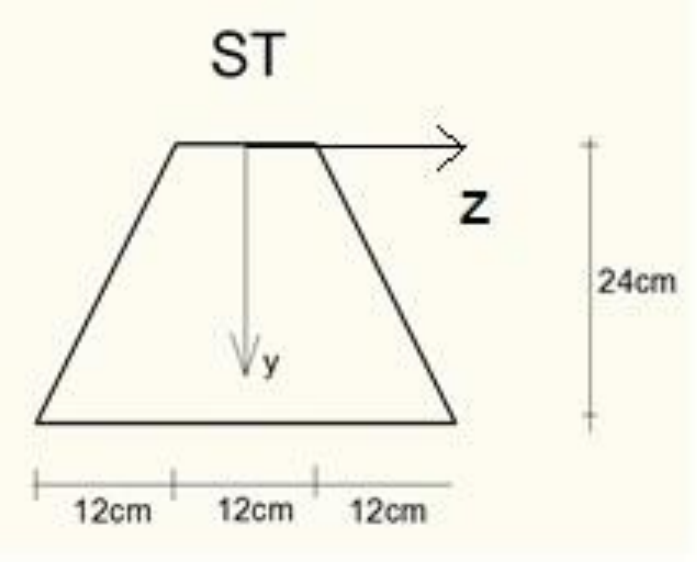
$$I_{z_{CG1}} = \frac{12.24^3}{36} = 4.608\ cm^4;$$

Região 2:

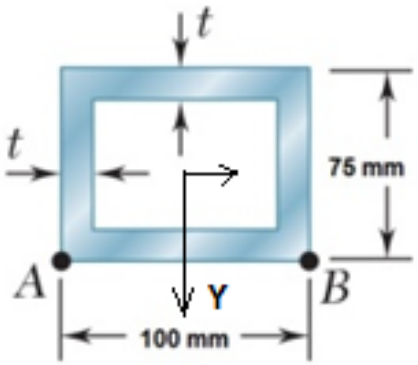
$\bar{y}_2 = -2\ cm\ ;\ A_2 = 288\ cm^2$

$$I_{z_{CG2}} = \frac{12.24^3}{12} = 13.824\ cm^4;$$

$$(I_z)_{CG} = 2\left[4.608 + (2)^2.144\right] + \left[13.824 + (-2)^2.288\right] = 25.344\ cm^4$$



Seções vazadas

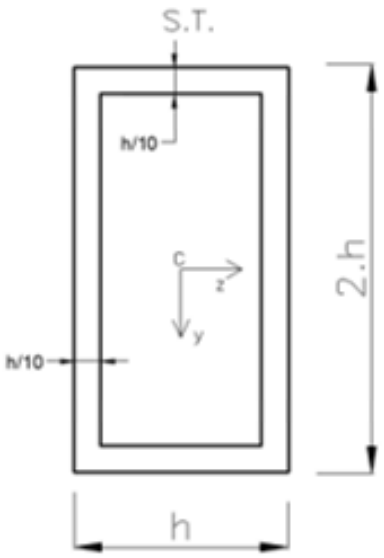


Área da seção, t =13 mm:

$$A = 100.75 - (100 - 26).(75 - 26) = 3874 \quad mm^2$$

Momento de inércia:

$$I_y = \frac{75.100^3}{12} - \frac{(75-26).(100-26)^3}{12} = 4.595.335{,}33 \quad mm^4$$



$$I_z = \frac{h.(2h)^3}{12} - \frac{4/5h.[\frac{9h}{5}]^3}{12} = \frac{521}{1875}h^4$$

$$I_z = \frac{\pi \cdot (d_e^{\,4} - d_i^4)}{64}$$