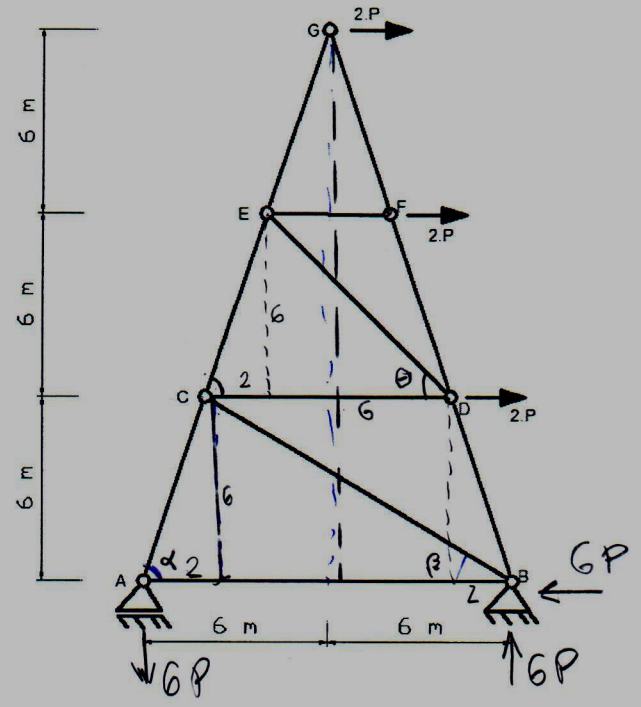
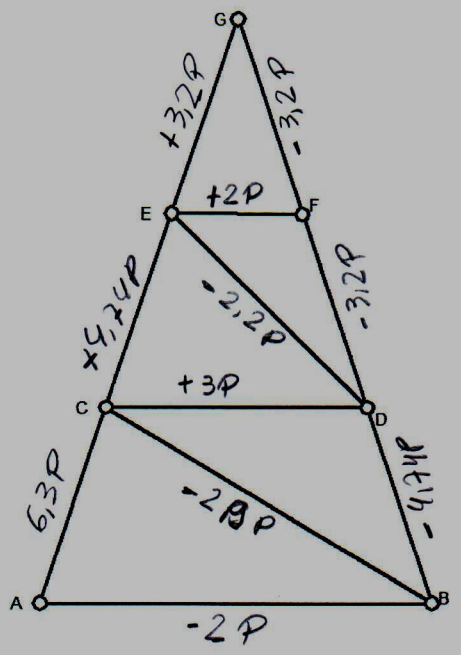
PEF2307 – 1° Semestre 2012 – 1ª. Prova – 10/09/2012

Nome: _____ N° USP: _____

1ª Questão (3,5 pontos) a) Determinar os esforços em todas as barras da treliça a seguir, em função de *P*. Indicar seus valores no desenho à direita.

b) Com base nos valores do item a) e sabendo-se que as barras são de mesmo material e área, e que resistem, no máximo, **a forças normais de 9,6 kN à tração e 6 kN à compressão**, obtenha o máximo valor de P que possa ser aplicado (P_{max}), para que nenhuma barra tenha valores maiores que os limites indicados.





P_{max} =

Resolução:

$\Sigma M_A = 0 : B_y \cdot 12 = 2P(18+12+6) \rightarrow B_y = 6P$

Nó A: $\cos\alpha = 0,3162 = 1/\sqrt{10}$; $\sin\alpha = 0,9487 = 3/\sqrt{10}$

$\Sigma F_y = 0 \quad N_{AC} \cdot 0,9487 = 6P$

$N_{AC} = 6,32P$

$\Sigma F_x = 0 : 0,3162 \cdot 6,32P + N_{AB} = 0$

$N_{AB} = -2P$

Nó B: $\cos\beta = 0,8575$; $\sin\beta = 0,5145$

$\Sigma F_y = 0 : N_{BD} \cdot 0,9487 + N_{BC} \cdot 0,5145 + 6P = 0$ (1)

$\Sigma F_x = 0 : N_{BC} \cdot 0,8575 + N_{BD} \cdot 0,3162 + 2P = 0$ (2)

$N_{BD} = -6,3244P - 0,5423 N_{BC}$

$N_{BD} = -12,6562P - 2,7119 N_{BC}$

$6,32P = -2,1696 N_{BC} \rightarrow N_{BC} = -2,91P$

$N_{BD} = -4,74P$

Nó C: $\Sigma F_y = 0 : N_{CE} \cdot 0,9487 + 2,91P \cdot 0,5145 = 6,32P \cdot 0,9487$

$N_{CE} = 6,32P - 1,5781P = 4,74P$

$\Sigma F_x = 0 : N_{CD} + 4,74P \cdot 0,3162 = 2,91P \cdot 0,8575 + 6,32P \cdot 0,3162$

$N_{CD} = 3,0P$

Nó D: $\Sigma F_y = 0 : N_{DE} \cdot 0,9487 + N_{DE} \cdot 0,7071 + 4,74P \cdot 0,5487 = 0$

$N_{DF} = -4,74P - 0,7453 N_{DE}$

$\Sigma F_x = 0 : 2P + N_{DF} \cdot 0,3162 + N_{DE} \cdot 0,7071 + 3P + 4,74P \cdot 0,7$

$-7,9P = 2,2362 N_{DE} = N_{DF}$

$\sin\theta = \text{use} = \sqrt{2}/2$

$3,16P = -1,19 N_{DE}$

$N_{DE} = -2,2P$

$N_{DF} = -3,2P$

Nó G: $\Sigma F_y = 0 : N_{GE} = -N_{GF}$

$\Sigma F_x = 0$

$N_{GE} \cdot \sin\gamma = N_{GF} \cdot \sin\gamma + 2P$

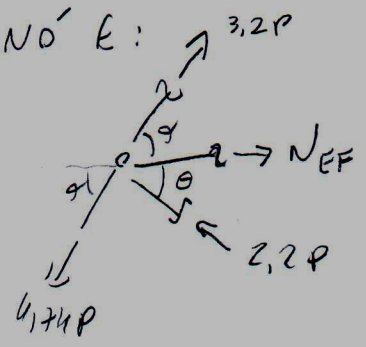
$\Sigma N_{GE} = 2P / \sin\gamma$

$\sin\gamma = 0,3162$

$N_{GE} = 3,2P$

$N_{GF} = -3,2P$

NÓ E:



$\Sigma F_x = 0$

$N_{EF} + 3{,}2P \cdot 0{,}3162 = 4{,}74P \cdot 0{,}3162 + 2{,}2P \cdot 0{,}7071$

$\boxed{N_{EF} = 2P}$

b) $MÁX\ (\text{compressão}) = -4{,}74P$ (barra BO) →

- Prova A: $4{,}74P \leq 6 \rightarrow P \leq 1{,}26$ k
- Prova B: $4{,}74P \leq 60 \rightarrow P \leq 12{,}6$ k

Máx (tração) $= 6{,}3P$ (barra AO) →

Prova A:
$6{,}3P \leq 9{,}6 \rightarrow P \leq 1{,}5$ kN

Prova B
$6{,}3P \leq 96 \rightarrow P \leq 15$ kN

a) Prova A: $\boxed{P_{máx} = 1{,}3\ kN}$

Prova B: $\boxed{P_{máx} = 13\ kN}$

0,3 → cada Normal

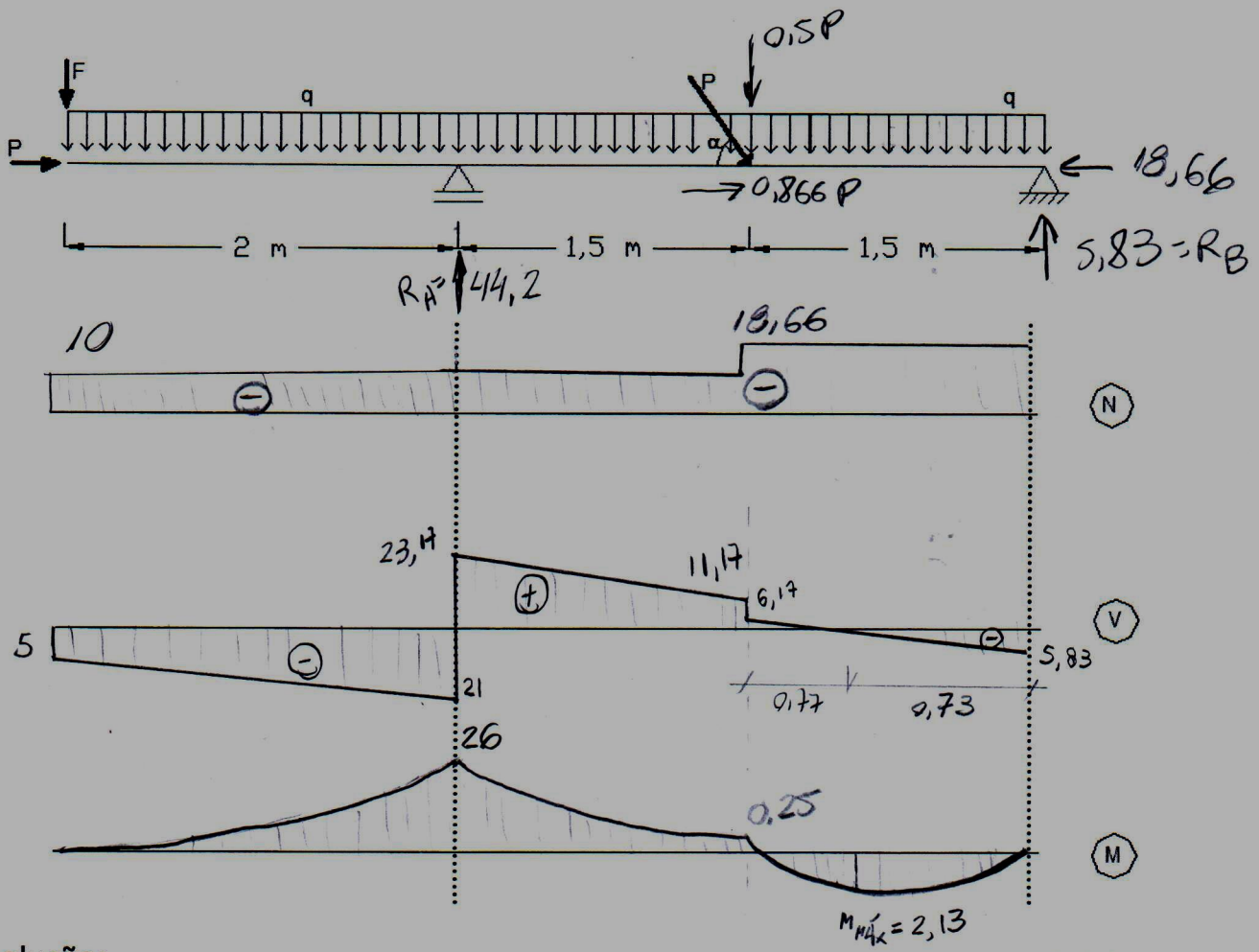
0,5 → item b

$\sqrt{10} = 3{,}16$

PEF2307 – 1° Semestre 2012 – 1ª. Prova – 10/09/2012

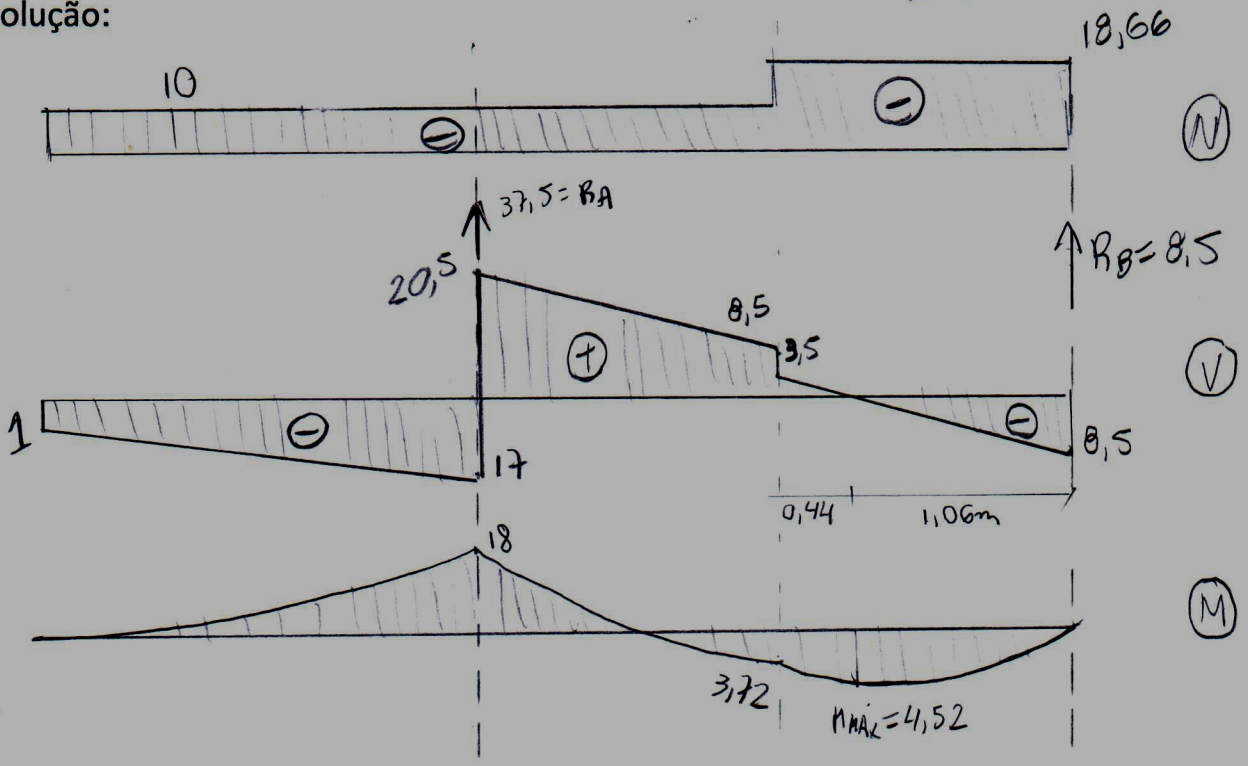
Nome: _____ N° USP: _____

2ª Questão (3,5 pontos) Determinar os esforços solicitantes (M,V e N) na estrutura esquematizada a seguir, sob a ações das cargas indicadas. Indique explicitamente os valores e os pontos de momentos extremos no desenhos em destaque. **Dados: P = 10 kN; F = P/2; q = 8 kN/m; α =30°**



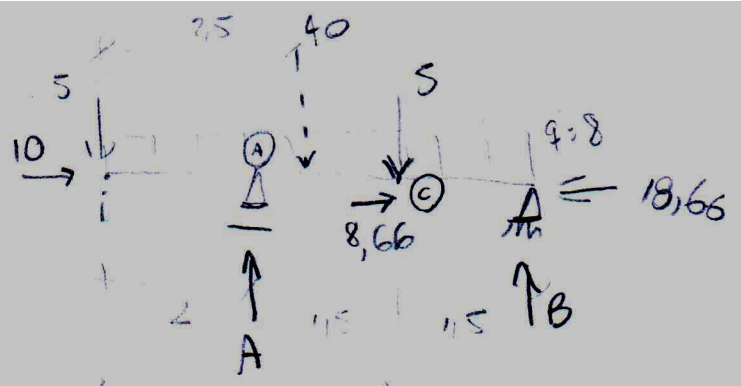
Prova A (F = 5 kN)

Resolução:



Prova B (F = 1 kN)

(N) → 1,0
(V) → 1,0
(M) → 1,5 pts



$\Sigma F_x = 0$ → $R_x = 18,66\,kN$

$\Sigma M_A = 0$: $3B + 5 \cdot 2 = 40 \cdot 0,5 + 5 \cdot$

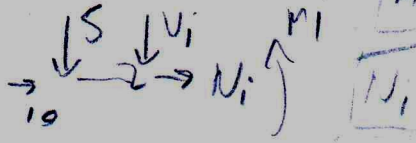
$B = 5,833\,kN\ (\uparrow)$

$\Sigma F_y = 0$ → $A = 50 - 5,833 = 44,167\,kN$

$A = 44,167\,kN\ (\uparrow)$

$B = 5,833\,kN\ (\uparrow)$

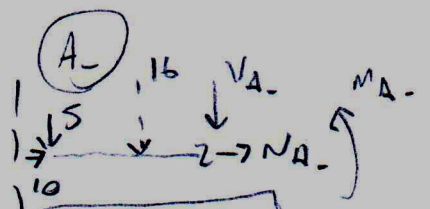
Corte i:



$M_i = 0$

$V_i = -10$

$V_i = -5\,kN$

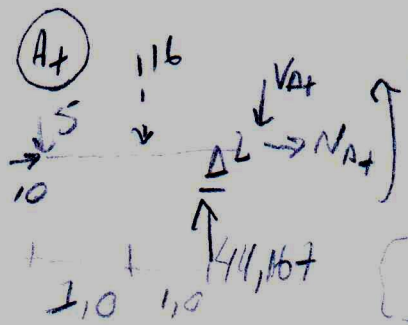


$N_{A-} = -10$

$M_{A-} + 16 \cdot 1 + 5 \cdot 2 = 0$

$V_{A-} = -21\,kN$

$M_{A-} = -26\,kN.m$

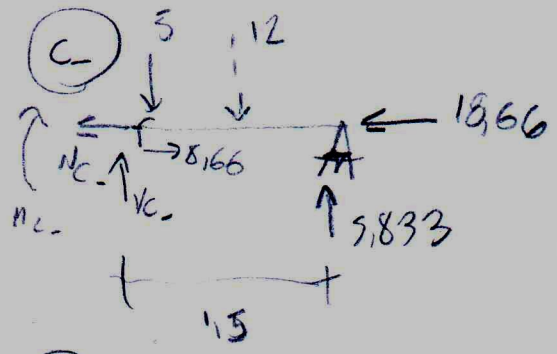


$N_{A+} = -10$

$M_{A+} = -26\,kN.m$

$V_{A+} + 21 = 11,167$

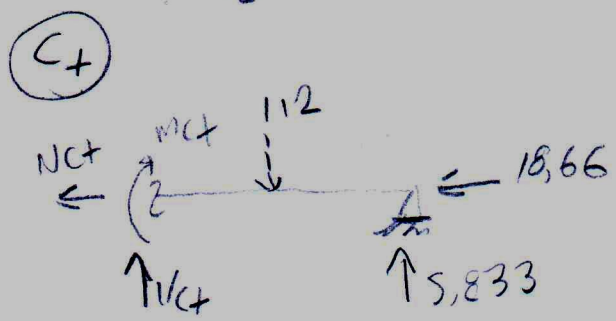
$V_{A+} = 23,167$



$N_{C-} + 18,66 - 8,66 = 0$ → $N_{C-} = -10\,kN$

$V_{C-} + 5,833 = 17$ → $V_{C-} = 11,167\,kN$

$M_{C-} + 12 \cdot \frac{1,5}{2} = 5,833 \cdot 1,5$ → $M_{C-} = -9,25$



$N_{C+} = -18,66$

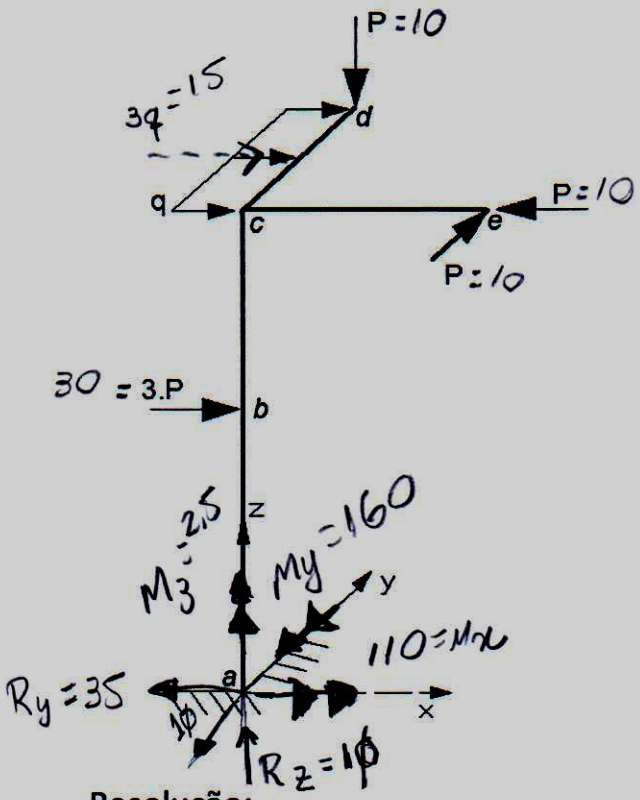
$V_{C+} = 12 - 5,833 = 6,16$

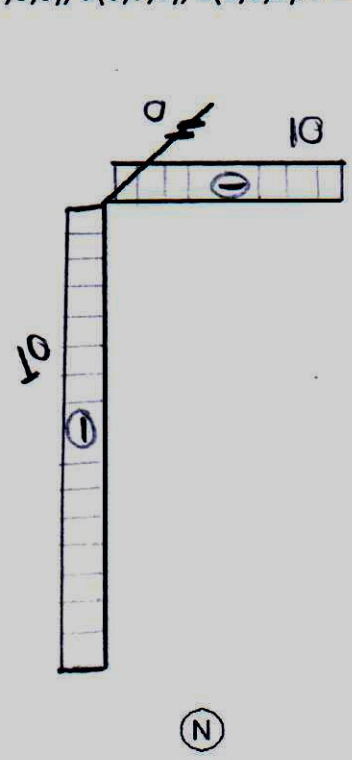
$M_{C+} = -0,25$

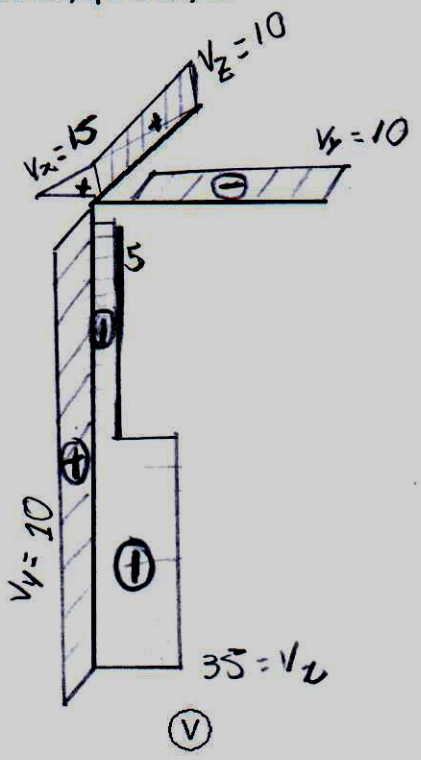
PEF2307 – 1° Semestre 2012 – 1ª. Prova – 10/09/2012

Nome: _____ N° USP: _____

3ª Questão (3 pontos) Determinar os esforços solicitantes (M,V, T e N) no pórtico tridimensional. As forças são paralelas aos eixos do sistema xyz, conforme indicado. Dados: As coordenadas dos pontos são, em metros: a(0;0;0), b(0;0;4), c(0;0;8), d(0;3;8), e(2;0;8). P = 10 kN; q = 5 kN/m.







Resolução:

$$\begin{cases} M_x = d_y \cdot F_z - d_z \cdot F_y \\ M_y = d_z \cdot F_x - d_x \cdot F_z \\ M_z = d_x \cdot F_y - d_y \cdot F_x \end{cases}$$

$$M_x + \left((-10)\cdot 3\right) - \left(10 \cdot 8\right) = 0$$

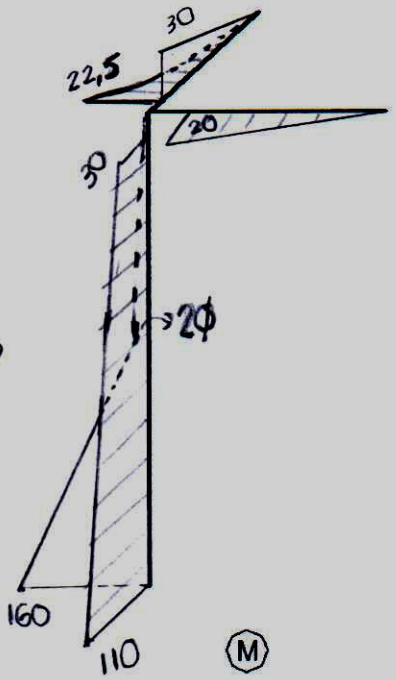
$$\boxed{M_x = 110\ KN.m}$$

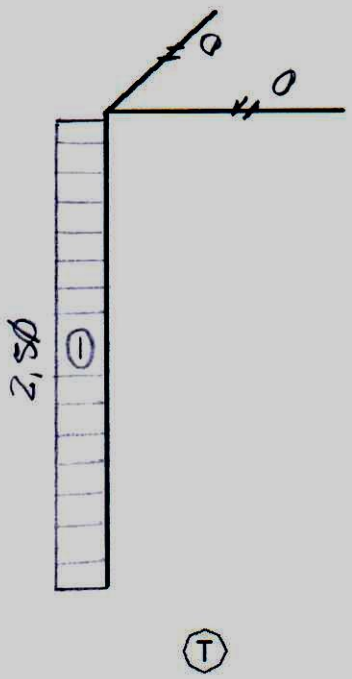
$$M_y + \left[15 \cdot 8 + 30 \cdot 4 - 10 \cdot 8\right] + 0 = 0$$

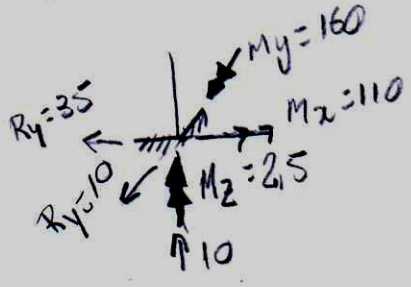
$$\boxed{M_y = -160\ KN.m}$$

$$M_z + \left[10 \cdot 2\right] - \left[15 \cdot 1{,}5\right] = 0$$

$$\boxed{M_z = 2{,}5\ KN.m}$$







(N) → 0,5
(V) → 0,5
(M) → 0,5
(T) → 0,5

(KN, m)