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METHODOLOGIES AND APPLICATION

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Dealing with the problem of null weights and scores in Fuzzy Analytic Hierarchy Process

Francisco Rodrigues Lima-Junior[1] · **Luiz Cesar Ribeiro Carpinetti**[2]

Abstract

Fuzzy Analytic Hierarchy Process (Fuzzy AHP) has been widely adopted to support decision making problems. The Fuzzy AHP approach based on the synthetic extent analysis is the most applied approach to calculate the values of the criteria weights from fuzzy comparative matrices. The min operator is used to calculate the weights based on values of degree of possibility. If any of the degrees of possibilities is zero, the output of this operator will also be zero. Thus, the criterion weight or alternative score will be set to zero. If not prevented, this problem may lead to a distorted rank. Despite the fact that there are other propositions based on synthetic extent analysis method, none of the studies found in the literature investigate how the problem of null weights and scores can be avoided. This paper investigates different approaches of the Fuzzy AHP method to evaluate whether they can avoid the problem of null weights and scores without affecting the consistency of the results. Five different approaches based on synthetic extent analysis method were implemented and evaluated. Tests were performed considering 12 decision problems. The results indicated that the Fuzzy AHP approach proposed by Ahmed and Kilic is the most appropriate to overcome the problem of null weight of criteria and scores of alternatives without affecting the consistency of the results. Other benefits of using this approach are the simplicity of the computational implementation and better ability to differentiate the importance of the criteria when the weight values are very close.

Keywords Fuzzy sets · Fuzzy AHP · Synthetic extent analysis · Null weights · Multicriteria decision making

1 Introduction

Multicriteria decision making methods (MCDM) are largely used to aid researchers and practitioners in the process of decision making in several fields of application (Roy and Dutta 2018; Banaeian et al. 2018; Ho and Ma 2018; Shaygan and Testik 2017; Wang 2018). These techniques are adequate to deal with decision problems that involve evaluation of given alternatives taking into account multiple decision criteria with the main goal of selecting, ordering or categorizing the alternatives. Since each criterion may lead to a different order of preference of alternatives, these techniques aim to yield a global classification based on the scores of each alternative when simultaneously considering all the multiple criteria (Kahraman 2008; Chai et al. 2013; Marttunen et al. 2017).

Modeling decision making problems usually requires specialists in the problem domain, acting as decision makers, who evaluate the importance of the criteria and the scores of the alternatives according to their own judgments. Since the judgments are influenced by intuition and experience of the decision makers, the input values of the decision model are subjective. Moreover, numerical values used to quantify the relative importance of the criteria and the scores of alternatives with respect to qualitative criteria are arbitrarily chosen (De Boer et al. 1998; Kahraman 2008; Chai et al. 2013). Therefore, it brings extra difficulty to the decision process.

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✉ Francisco Rodrigues Lima-Junior
frjunior@utfpr.edu.br

Luiz Cesar Ribeiro Carpinetti
carpinet@sc.usp.br

1 Department of Management and Economics, Federal Technological University of Paraná, Av. Sete de Setembro, 3165, Rebouças, Curitiba, Paraná 80230-901, Brazil

2 Production Engineering Department, São Carlos School of Engineering– University of São Paulo, Av. Trabalhador São-Carlense, 400, São Carlos, São Paulo 13566-590, Brazil

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This is the case of Fuzzy Analytic Hierarchy Process—Fuzzy AHP (Ho and Ma 2018), Fuzzy Analytic Network Process—Fuzzy ANP (Vinodh et al. 2011), Fuzzy Technique for Order Preference by Similarity to Ideal Solution—Fuzzy TOPSIS (Banaeian et al. 2018), Fuzzy Quality Function Deployment—Fuzzy QFD (Lima-Junior and Carpinetti 2016), Fuzzy Making Trial and Evaluation Laboratory—Fuzzy DEMATEL (Büyüközkan and Cifçi 2011), Fuzzy Elimination and Choice Translating Reality—Fuzzy ELECTRE (Hatami-Marbini and Tavana 2011), among others. In these techniques, fuzzy numbers allow quantification of imprecise data. The vertex values of the fuzzy sets are chosen so as to better model the linguistic terms given by each decision maker to assess the alternatives on different decision criteria (Lima-Junior et al. 2014). Thus, fuzzy-based methods provide a suitable language to handle uncertainty and they are able to integrate the analysis of qualitative and quantitative factors (Kahraman 2008).

The Fuzzy AHP method deals with imprecision and lack of information by using linguistic variables to carry out pairwise comparisons. The literature presents several techniques to calculate the relative weights of the criteria and the scores of alternatives of the fuzzy comparison matrices (Chang 1996; Fattahi and Khalilzadeh 2018; Goyal and Kaushal 2018; Ho and Ma 2018). The simplicity and ease of use of the synthetic extent analysis (Chang 1996) make it one of the most applied techniques (Kubler et al. 2016; Yadegaridehkordi et al. 2018). A study presented by Kubler et al. (2016) analyzed 190 publications and identified that 57% of the Fuzzy AHP applications are based on the synthetic extent analysis proposed by Chang (1996). In the field of operations management, this method is applied in problems such as supplier selection (Lima-Junior et al. 2014, Ho and Ma 2018), equipment selection (Yazdani-Chamzini and Yakhchali 2012), project selection (Taylan et al. 2014), supply chain performance measurement (Gou et al. 2013), inventory classification (Kabir and Hasin 2013), failure mode and effects analysis (Kutlu and Ekmekçioglu 2012), customization of sustainability assessment tools (Zarghami et al. 2018), among others.

Despite the increasing application of Fuzzy AHP to model uncertainty in decision making, this technique uses a mathematical procedure that may lead to null values for the criteria weights (Wang et al. 2008, Lima-Junior et al. 2014). In this procedure, the min non-compensatory aggregation operator is used to calculate the eigenvectors of each object of the hierarchical structure regarding each objective (Chang 1996). The output of the min operator is the minimum value of a list of values of degree of possibility, which represents, in the calculation of the criteria weights, the preference of one criterion over another. In case any of the degrees of possibilities is zero, the output of the min operator will also be zero. Consequently, the weight of the criterion will be set to zero and will not contribute to the global evaluation of the alternatives. Likewise, this problem may happen in the evaluation of the performance of each alternative (score) regarding each criterion (Lima-Junior et al. 2014). This problem can impair the decision process and therefore should be prevented. Despite the fact that there are other propositions of the Fuzzy AHP method based on the synthetic extent analysis (Wang et al. 2008; Bulut et al. 2012; Duru et al. 2012; Ahmed and Kilic 2015), none of the studies found in the literature investigate how the problem of null weight of criteria and scores of alternatives can be avoided.

Therefore, this paper proposes to investigate different approaches of the Fuzzy AHP method based on Chang (1996) so as to evaluate whether they can avoid the problem of null weights and scores without affecting the consistency of the results. Five different Fuzzy AHP approaches based on the synthetic extent analysis were implemented and evaluated. Computational implementation was carried out using Matlab©. Tests were performed considering 12 generic multicriteria decision problems. Evaluation of the Fuzzy AHP approaches was based on two factors: the ability to prevent the problem of null weight of criteria; the consistency of weights calculated for each criterion, which is quantified by the root mean square error (RMSE).

The paper is organized as follows: Sects. 2 and 3 revise some fundamental concepts regarding Fuzzy Set Theory and the Fuzzy AHP method. Section 4 details an illustrative case that highlights the problem of null weights and scores in the conventional Fuzzy AHP method proposed by Chang (1996). Section 5 presents the results of the comparative analyses of the different Fuzzy AHP approaches studied in this piece of work. Finally, conclusions about this research work are presented in Sect. 6.

2 Fuzzy Set Theory

Fuzzy Set Theory (Zadeh 1965) has been used for modeling decision making processes based on linguistic variables to deal with imprecise, qualitative or vague information. The values of a linguistic variable are not numbers but words or sentences, closer to the way that humans express their knowledge (Pedrycz and Gomide 2007). Such characteristics have contributed to the widespread use of fuzzy set theory and its extensions in several studies related to group decision making. In one of these studies, Ureña et al. (2019) propose a consensus model that uses intuitionistic fuzzy preference relations to allow decision makers to judge with the possibility of allocating uncertainty. In addition, the model is able to identify and

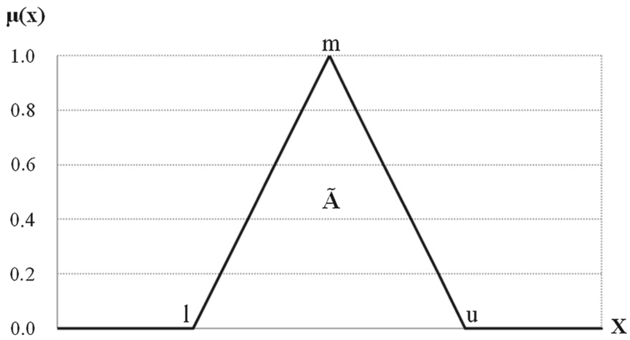

Fig. 1 Triangular fuzzy number. *Source*: Based on Pedrycz and Gomide (2007)

isolate decision makers with malicious behavior. Another interesting study related to group decision making was developed by Ureña et al. (2015) who propose the use of a new aggregation operator that takes into account the experts' consistency as well as their confidence degree on the provided opinion.

In the Fuzzy Set Theory, a linguistic variable is expressed qualitatively by linguistic terms and quantitatively by a fuzzy set in the universe of discourse and respective membership function (Pedrycz and Gomide 2007). A fuzzy set $\tilde{A}$ in universe of discourse X is defined by:

$$\tilde{A} = \{x, \mu_{\tilde{A}}(x) | x \in X\} \tag{1}$$

where $\mu_A(x) : X \to [0, 1]$ is the membership function of $\tilde{A}$ and $\mu_A(x)$ is the degree of membership of x in $\tilde{A}$. A fuzzy number is a fuzzy set in which the membership function satisfies the conditions of normality and convexity (Zimmermann 1991).

The triangular fuzzy number has a membership function, $\mu_A(x)$ as illustrated in Fig. 1 in which l, m and u are real numbers with $l < m < u$ (Pedrycz and Gomide 2007).

Algebraic operations between fuzzy numbers such as addition, subtraction, multiplication and division are well described in the literature (Zimmermann 1991; Pedrycz and Gomide 2007).

3 The Fuzzy AHP method

Chang (1996) proposed a Fuzzy AHP approach based on the extent analysis method, which is widely used in a variety of multicriteria decision problems concerning selection, ordering and categorization of alternatives. This method uses linguistic variables to express the comparative judgments given by decision makers. Table 1 presents a brief review of applications of the Fuzzy AHP approach based on the extent analysis method. It also presents some applications of the Fuzzy AHP in combination with other techniques in which Fuzzy AHP is used simply to define the weights of the criteria. This is the case of Fuzzy AHP combined with TOPSIS, Fuzzy TOPSIS, two-tuple, Artificial Neural Network, DEA among others.

In the method proposed by Chang (1996), each object x_i pertaining to $X = \{x_1, x_i, \ldots, x_n\}$ is taken and extent analysis is performed for each goal, u_j, where $U = \{u_1, u_j.., u_m\}$. For each object x_i, m extent analysis values are obtained with the signs:

$$M^1_{x_i}, M^j_{x_i}, \ldots, M^m_{x_i}, \quad i = 1, 2, \ldots, n \tag{2}$$

where $M^j_{x_i}$ ($j = 1, 2, \ldots, m$) are triangular fuzzy numbers.

The method proposed by Chang (1996) comprises the following steps (Lima-Junior et al. 2014):

(i) Calculate the value of the fuzzy synthetic extent in regard to the ith object using Eq. 3.

$$S_i = \sum\nolimits_{j=1}^{m} M^j_{x_i} \left[\sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{m} M^j_{x_i}\right]^{-1} \tag{3}$$

where $\sum_{j=1}^{m} M^j_{x_i}$ is the fuzzy addition operation of m extent analysis values for a matrix such that

$$\sum\nolimits_{j=1}^{m} M^j_{x_i} = \left(\sum\nolimits_{j=1}^{m} l_j, \sum\nolimits_{j=1}^{m} m_j, \sum\nolimits_{j=1}^{m} u_j\right) \tag{4}$$

and $\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M^j_{x_i}\right]^{-1}$ is given by

$$\left[\sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{m} M^j_{x_i}\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{m} u_{ij}}, \frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{m} m_{ij}}, \frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{m} l_{ij}}\right) \tag{5}$$

(ii) Compute the degree of possibility of $S_2 = (l_2, m_2, u_2) \geq S_1 = (l_1, m_1, u_1)$, where S_2 and S_1 are calculated based on Eq. 3. The degree of possibility between two fuzzy synthetic extents is given by Eq. 6, which is equivalently expressed as in Eqs. 7 and 8.

$$V(S_2 \geq S_1) = \sup_{y \geq x}\left[\min\left(\mu_{S_2}(y), \mu_{S_1}(x)\right)\right] \tag{6}$$

$$V(S_2 \geq S_1) = hgt(S_1 \cap S_2) = \mu_{S_2}(d) \tag{7}$$

$$\mu_{S_2}(d) = \left\{\begin{array}{ll} 1, & \text{if } m_2 \geq m_1 \\ 0, & \text{if } l_1 \geq u_2 \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise} \end{array}\right\} \tag{8}$$

In Eqs. 7 and 8, d represents the ordinate of the highest intersection point D between μ_{S_1} and μ_{S_2}, as illustrated in Fig. 2. The comparison between M_{x_1} and M_{x_2} requires the values of $V(S_2 \geq S_1)$ and $V(S_1 \geq S_2)$.

Table 1 Single and combined applications of the Fuzzy AHP method proposed by Chang (1996)

Approach	Proposed by	Technique(s) used	Scope
Single method	Calabrese et al. (2013)	Fuzzy AHP	Management of intellectual capital assets
	Gou et al. (2013)	Fuzzy AHP	Performance evaluation of service-oriented catering supply chain
	Heo et al. (2012)	Fuzzy AHP	Selection of hydrogen production methods
	Kabir and Hasin (2011)	Fuzzy AHP	Inventory classification based on multiple criteria
	Larimian et al. (2013)	Fuzzy AHP	Evaluation of environmental sustainability from the perspective of secured by design scheme
	Lee et al. (2011)	Fuzzy AHP	Prioritization of hydrogen energy technologies
	Mosadeghi et al. (2015)	Fuzzy AHP	A multicriteria decision making model for urban land-use planning
	Shaygan and Testik (2017)	Fuzzy AHP	A methodology for project prioritization and selection
	Wang et al. (2012)	Fuzzy AHP	Evaluation of strategic environmental assessment effectiveness
	Zarghami et al. (2018)	Fuzzy AHP	Customization of sustainability assessment tools
Combined method	Chen et al. (2015)	Fuzzy AHP and fuzzy comprehensive evaluation method	A framework for teaching performance evaluation
	Cho and Lee (2013)	Fuzzy AHP and Delphi method	Prioritization of success factors for commercialization of new products
	Choudhary and Shankar (2012)	Fuzzy AHP and TOPSIS	Evaluation and selection of thermal power plant location
	Das et al. (2012)	Fuzzy AHP and COPRAS method	Performance measurement of Indian technical institution
	Chen et al. (2015)	Fuzzy AHP and 2-tuple	Evaluation of emergency response capacity
	Cho and Lee (2013)	Fuzzy AHP and Artificial Neural Network	Inventory classification based on multiple criteria
	Choudhary and Shankar (2012)	Fuzzy AHP and fuzzy TOPSIS	Failure modes and effects analysis
	Das et al. (2012)	Fuzzy AHP and DEA (Data Envelopment Analysis)	Allocation of energy R&D resources
	Mandic et al. (2014)	Fuzzy AHP and TOPSIS	Analysis of the financial parameters of Serbian banks
	Paksoy et al. (2012)	Fuzzy AHP and fuzzy TOPSIS	Organizational strategy development in distribution channel management
	Rostamzadeh and Sofian (2011)	Fuzzy AHP and fuzzy TOPSIS	Prioritizing effective 7Ms to improve production systems performance
	Yazdani-Chamzini and Yakhchali (2012)	Fuzzy AHP and fuzzy TOPSIS	Selection of Tunnel Boring Machine
	Taylan et al. (2014)	Fuzzy AHP and fuzzy TOPSIS	Selection of construction projects and risk assessment
	Yadegaridehkordi et al. (2018)	Fuzzy AHP and structural equation modeling	Prediction of the adoption of cloud-based technology
	Wang et al. (2015)	Fuzzy AHP and TOPSIS	An integrated fuzzy methodology for green product development

(iii) Calculate the degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers S_i (i = 1,…, k). This is worked out according to Eq. 9.

$$\begin{aligned} V(S \geq S_1, S_2, \ldots, S_k) \\ = V[(S \geq S_1) \text{ and } (S \geq S_2) \text{ and} \ldots \text{and } (S \geq S_k)] \\ = \min V(S \geq S_i), i = 1, 2, \ldots, k. \end{aligned} \tag{9}$$

(iv) Calculate the vector W', according to Eq. 10.

$$W' = \left(d'(A_1), d'(A_2), \ldots, d'(A_k)\right)^{\mathrm{T}} \tag{10}$$

considering that

$$\begin{aligned} d'(A_i) = \min V\left(S_i \geq S_j\right), \\ \text{for } i = 1, 2, .., k, j = 1, 2, \ldots, k, i \neq j \end{aligned} \tag{11}$$

The normalized vector W is given by

$$W = (d(A_1), d(A_2), \ldots, d(A_k))^{\mathrm{T}} \tag{12}$$

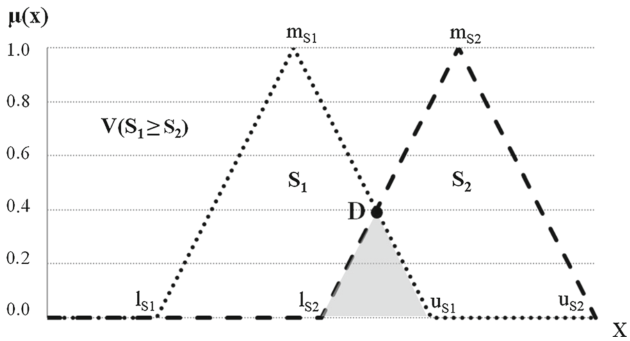

Fig. 2 The intersection between S_1 and S_2. *Source*: Based on Chang (1996)

where W is a non-fuzzy number worked out for each comparison matrix.

Despite the simplicity of implementation, the Fuzzy AHP method proposed by Chang (1996) has been criticized by several authors. Wang et al. (2008) discuss the problem of the null weight by using numerical examples. They argue that the extent analysis should not be used for estimating priorities from a fuzzy pairwise comparison matrix. The authors present a new normalization procedure; however, they do not propose a solution to overcome the problem of null weight. Zhü (2014) discusses some points related to the mathematical logic of the Fuzzy AHP. He argues that the use of a continuous membership function violates the discrete nature of the method; the operations of fuzzy numbers in the method do not consider the membership grade; the reciprocal condition cannot be guaranteed in the judgment matrices; there is a lack of a generally accepted method for evaluating inconsistency of judgements. On the other hand, Fedrizzi and Krejcí (2015) argue that the evidences and reasoning presented by Zhü (2014) are "very poor and far from proving the fallacy of the Fuzzy AHP." Fedrizzi and Krejcí (2015) conclude that "the Fuzzy AHP method still remains an interesting research topic with challenging development possibilities both in theory and in application."

The criticisms as well as the wide application of the Fuzzy AHP method proposed by Chang (1996) have led to the development of other propositions based on the synthetic extent analysis. Bulut et al. (2012) propose a method to differentiate expertise in the decision group in which crisp numbers are used to weigh the importance of the judgments of different decision makers according to their experiences. An important contribution of this paper in regard to the Fuzzy AHP method is the proposition of a procedure to control the consistency of comparative judgment matrices, called the centric consistency index—CCI. Duru et al. (2012) propose a procedure to deal with decision making problems in which the priorities of the criteria or alternatives vary according to a specific choice of alternatives. The technique of prioritization of the judgments of the decision makers and the centric consistency index proposed by Bulut et al. (2012) are also used in this study. However, in these studies, the technique of synthetic extent analysis proposed by Chang (1996), using the min operator, was adopted to calculate the eigenvectors from the comparative matrices, which also led to the problem of null weight of criteria and null scores of alternatives.

Another approach of the Fuzzy AHP method based on the synthetic extent analysis was proposed by Ahmed and Kilic (2015). This approach includes two parts. First it calculates the synthetic extent following the same procedure proposed by Chang (1996). Next, instead of using the degrees of possibility to work out the ordering of weights, they apply the center of area operator for defuzzification of the values of synthetic extent. The min operator is not used in this procedure and therefore it may be a way to overcome this problem of null weight. However, the authors limit the discussion to the consistency of the procedure, not analyzing the effect of this procedure on the null weight problem.

In a comparative study involving Fuzzy AHP and Fuzzy TOPSIS, Lima-Junior et al. (2014) suggest to test other fuzzy operators such as the arithmetic mean as alternatives to the min operator in order to avoid null weights and scores in Fuzzy AHP. Therefore, although there are some possible solutions to overcome the problem of null weights in Fuzzy AHP, none of the studies found in the literature discuss how this problem can be avoided.

4 The null weight problem in the conventional Fuzzy AHP method

To illustrate the problem of null weight of criteria in the application of the Fuzzy AHP proposed by Chang (1996), a generic multicriteria decision problem is considered, with five criteria, C_j ($j = 1,\ldots, 5$), and five alternatives, A_i ($i = 1,\ldots, 5$). The set of linguistic terms shown in Table 2 were adopted by a decision maker to assess comparatively the weight of the criteria and the scores of the alternatives. Based on Chang (1996), triangular fuzzy numbers (TFN) were used to modeling the linguistic values of these variables.

Table 2 Comparative linguistic scale

Linguistic terms	Fuzzy triangular number
Equally preferable	(1.00, 1.00, 3.00)
Slightly preferable	(1.00, 3.00, 5.00)
Fairly preferable	(3.00, 5.00, 7.00)
Extremely preferable	(5.00, 7.00, 9.00)
Absolutely preferable	(7.00, 9.00, 9.00)

Table 3 Fuzzy numbers of comparative judgments of the weights of the criteria

	C_1	C_2	C_3	C_4	C_5
C_1	(1.00, 1.00, 1.00)	(0.20, 0.33, 1.00)	(1.00, 3.00, 5.00)	(3.00, 5.00, 7.00)	(7.00, 9.00, 9.00)
C_2	(1.00, 3.00, 5.00)	(1.00, 1.00, 1.00)	(3.00, 5.00, 7.00)	(3.00, 5.00, 7.00)	(5.00, 7.00, 9.00)
C_3	(0.20, 0.33, 1.00)	(0.14, 0.20, 0.33)	(1.00, 1.00, 1.00)	(1.00, 3.00, 5.00)	(1.00, 3.00, 5.00)
C_4	(0.14, 0.20, 0.33)	(0.14, 0.20, 0.33)	(0.20, 0.33, 1.00)	(1.00, 1.00, 1.00)	(1.00, 3.00, 5.00)
C_5	(0.11, 0.11, 0.14)	(0.11, 0.14, 0.20)	(0.20, 0.33, 1.00)	(0.20, 0.33, 1.00)	(1.00, 1.00, 1.00)

Table 3 shows the TFN of the comparative judgments about the weights of the criteria given by the decision maker. Similarly, the fuzzy values of the comparative judgments about the alternative scores for each criterion are shown in Tables 4, 5, 6, 7 and 8.

The centric consistency index (CCI) as proposed by Bulut et al. (2012) for the comparative matrices of Fuzzy AHP was used to evaluate the consistency of the judgments. Table 9 presents the index values obtained for the matrices in Tables 3, 4, 5, 6, 7 and 8. According to Bulut et al. (2012), matrices of size $n > 4$ are sufficiently consistent if the CCI is lower than or equal to 0.37. As presented in Table 9, the values of the CCI are well below 0.37, confirming the consistency of the judgments.

For the criteria matrix, the values of the fuzzy synthetic extent are:

$$S_{C1} = (12.20, 18.33, 23.00) \otimes \left(\frac{1}{75.33}, \frac{1}{53.51}, \frac{1}{32.65}\right) = (0.16, 0.34, 0.70)$$

$$S_{C2} = (13.00, 21.00, 29.00) \otimes \left(\frac{1}{75.33}, \frac{1}{53.51}, \frac{1}{32.65}\right) = (0.17, 0.39, 0.88)$$

$$S_{C3} = (3.34, 7.53, 12.33) \otimes \left(\frac{1}{75.33}, \frac{1}{53.51}, \frac{1}{32.65}\right) = (0.04, 0.14, 0.38)$$

$$S_{C4} = (2.48, 4.73, 7.66) \otimes \left(\frac{1}{75.33}, \frac{1}{53.51}, \frac{1}{32.65}\right) = (0.03, 0.08, 0.23)$$

$$S_{C5} = (1.62, 1.92, 3.34) \otimes \left(\frac{1}{75.33}, \frac{1}{53.51}, \frac{1}{32.65}\right) = (0.02, 0.03, 0.10)$$

Table 4 Fuzzy numbers of comparative judgments of the alternative ratings related to criterion C_1

	A_1	A_2	A_3	A_4	A_5
A_1	(1.00, 1.00, 1.00)	(1.00, 1.00, 3.00)	(0.14, 0.20, 0.33)	(0.14, 0.20, 0.33)	(0.20, 0.33, 1.00)
A_2	(0.33, 1.00, 1.00)	(1.00, 1.00, 1.00)	(0.14, 0.20, 0.33)	(0.20, 0.33, 1.00)	(0.20, 0.33, 1.00)
A_3	(3.00, 5.00, 7.00)	(3.00, 5.00, 7.00)	(1.00, 1.00, 1.00)	(1.00, 1.00, 3.00)	(1.00, 3.00, 5.00)
A_4	(3.00, 5.00, 7.00)	(1.00, 3.00, 5.00)	(0.33, 1.00, 1.00)	(1.00, 1.00, 1.00)	(3.00, 5.00, 7.00)
A_5	(1.00, 3.00, 5.00)	(1.00, 3.00, 5.00)	(0.20, 0.33, 1.00)	(0.14, 0.20, 0.33)	(1.00, 1.00, 1.00)

Table 5 Fuzzy numbers of comparative judgments of the alternative ratings related to criterion C_2

	A_1	A_2	A_3	A_4	A_5
A_1	(1.00, 1.00, 1.00)	(1.00, 3.00, 5.00)	(1.00, 3.00, 5.00)	(1.00, 1.00, 3.00)	(5.00, 7.00, 9.00)
A_2	(0.20, 0.33, 1.00)	(1.00, 1.00, 1.00)	(1.00, 1.00, 3.00)	(0.20, 0.33, 1.00)	(1.00, 3.00, 5.00)
A_3	(0.20, 0.33, 1.00)	(0.33, 1.00, 1.00)	(1.00, 1.00, 1.00)	(0.14, 0.20, 0.33)	(3.00, 5.00, 7.00)
A_4	(0.33, 1.00, 1.00)	(1.00, 3.00, 5.00)	(3.00, 5.00, 7.00)	(1.00, 1.00, 1.00)	(5.00, 7.00, 9.00)
A_5	(0.11, 0.14, 0.20)	(0.20, 0.33, 1.00)	(0.14, 0.20, 0.33)	(0.11, 0.14, 0.20)	(1.00, 1.00, 1.00)

Table 6 Fuzzy numbers of comparative judgments of the alternative ratings related to criterion C_3

	A_1	A_2	A_3	A_4	A_5
A_1	(1.00, 1.00, 1.00)	(3.00, 5.00, 7.00)	(0.20, 0.33, 1.00)	(7.00, 9.00, 9.00)	(1.00, 3.00, 5.00)
A_2	(0.14, 0.50, 0.33)	(1.00, 1.00, 1.00)	(0.14, 0.20, 0.33)	(3.00, 5.00, 7.00)	(1.00, 3.00, 5.00)
A_3	(1.00, 3.00, 5.00)	(3.00, 5.00, 7.00)	(1.00, 1.00, 1.00)	(7.00, 9.00, 9.00)	(3.00, 5.00, 7.00)
A_4	(0.11, 0.11, 0.14)	(0.14, 0.20, 0.33)	(0.11, 0.11, 0.14)	(1.00, 1.00, 1.00)	(5.00, 7.00, 9.00)
A_5	(0.20, 0.33, 1.00)	(0.20, 0.33, 1.00)	(0.14, 0.20, 0.33)	(0.11, 0.14, 0.20)	(1.00, 1.00, 1.00)

Table 7 Fuzzy numbers of comparative judgments of the alternative ratings related to criterion C_4

	A_1	A_2	A_3	A_4	A_5
A_1	(1.00, 1.00, 1.00)	(0.11, 0.14, 0.20)	(0.11, 0.11, 0.14)	(0.14, 0.20, 0.33)	(0.11, 0.14, 0.20)
A_2	(5.00, 7.00, 9.00)	(1.00, 1.00, 1.00)	(1.00, 1.00, 3.00)	(1.00, 3.00, 5.00)	(3.00, 5.00, 7.00)
A_3	(7.00, 9.00, 9.00)	(0.33, 1.00, 1.00)	(1.00, 1.00, 1.00)	(3.00, 5.00, 7.00)	(5.00, 7.00, 9.00)
A_4	(3.00, 5.00, 7.00)	(0.20, 0.33, 1.00)	(0.14, 0.20, 0.33)	(1.00, 1.00, 1.00)	(1.00, 1.00, 3.00)
A_5	(5.00, 7.00, 9.00)	(0.14, 0.20, 0.33)	(0.11, 0.14, 0.20)	(0.33, 1.00, 1.00)	(1.00, 1.00, 1.00)

Table 8 Fuzzy numbers of comparative judgments of the alternative ratings related to criterion C_5

	A_1	A_2	A_3	A_4	A_5
A_1	(1.00, 1.00, 1.00)	(3.00, 5.00, 7.00)	(5.00, 7.00, 9.00)	(1.00, 3.00, 5.00)	(3.00, 5.00, 7.00)
A_2	(0.14, 0.20, 0.33)	(1.00, 1.00, 1.00)	(3.00, 5.00, 7.00)	(0.20, 0.33, 1.00)	(1.00, 3.00, 5.00)
A_3	(0.11, 0.14, 0.20)	(0.14, 0.20, 0.33)	(1.00, 1.00, 1.00)	(0.20, 0.33, 1.00)	(1.00, 1.00, 3.00)
A_4	(0.20, 0.33, 1.00)	(1.00, 3.00, 5.00)	(1.00, 3.00, 5.00)	(1.00, 1.00, 1.00)	(5.00, 7.00, 9.00)
A_5	(0.14, 0.20, 0.33)	(0.20, 0.33, 1.00)	(0.33, 1.00, 1.00)	(0.11, 0.14, 0.20)	(1.00, 1.00, 1.00)

Table 9 Centric consistency indices (CCI) of comparative matrices

C_1	C_2	C_3	C_4	C_5	Weight
0.051	0.033	0.050	0.066	0.071	0.037

The degrees of possibility of these fuzzy values are (based on Eqs. 7 and 8):

$V(S_{C1} \geq S_{C2}) = 0.91$
$V(S_{C1} \geq S_{C3}) = 1.00$
$V(S_{C1} \geq S_{C4}) = 1.00$
$V(S_{C1} \geq S_{C5}) = 1.00$
$V(S_{C2} \geq S_{C1}) = 1.00$
$V(S_{C2} \geq S_{C3}) = 1.00$
$V(S_{C2} \geq S_{C4}) = 1.00$
$V(S_{C2} \geq S_{C5}) = 1.00$
$V(S_{C3} \geq S_{C1}) = 0.52$
$V(S_{C3} \geq S_{C2}) = 0.45$
$V(S_{C3} \geq S_{C4}) = 1.00$
$V(S_{C3} \geq S_{C5}) = 1.00$
$V(S_{C4} \geq S_{C1}) = 0.22$
$V(S_{C4} \geq S_{C2}) = 0.17$
$V(S_{C4} \geq S_{C3}) = 0.78$
$V(S_{C4} \geq S_{C5}) = 1.00$
$V(S_{C5} \geq S_{C1}) = 0.00$
$V(S_{C5} \geq S_{C2}) = 0.00$
$V(S_{C5} \geq S_{C3}) = 0.36$
$V(S_{C5} \geq S_{C4}) = 0.57$

Consequently, the weight vector W', calculated as in Eqs. 9 and 10, is:

$$d'(C_1) = V[(S_{C1} \geq S_{C2}) \text{ and } (S_{C1} \geq S_{C3}) \text{ and } (S_{C1} \geq S_{C4}) \text{ and } (S_{C1} \geq S_{C5})] = \min(0.91, 1.00, 1.00, 1.00) = 0.91$$
$$d'(C_2) = V[(S_{C2} \geq S_{C1}) \text{ and } (S_{C2} \geq S_{C3}) \text{ and } (S_{C2} \geq S_{C4}) \text{ and } (S_{C2} \geq S_{C5})] = \min(1.00, 1.00, 1.00, 1.00) = 1.00$$
$$d'(C_3) = V[(S_{C3} \geq S_{C1}) \text{ and } (S_{C3} \geq S_{C2}) \text{ and } (S_{C3} \geq S_{C4}) \text{ and } (S_{C3} \geq S_{C5})] = \min(0.52, 0.45, 1.00, 1.00) = 0.45$$
$$d'(C_4) = V[(S_{C4} \geq S_{C1}) \text{ and } (S_{C4} \geq S_{C2}) \text{ and } (S_{C4} \geq S_{C3}) \text{ and } (S_{C4} \geq S_{C5})] = \min(0.22, 0.17, 0.78, 1.00) = 0.17$$
$$d'(C_5) = V[(S_{C5} \geq S_{C1}) \text{ and } (S_{C5} \geq S_{C2}) \text{ and } (S_{C5} \geq S_{C3}) \text{ and } (S_{C5} \geq S_{C4})] = \min(0.00, 0.00, 0.36, 0.57) = 0.00$$
$$W' = (0.91, 1.00, 0.45, 0.17, 0.00)$$

The weight vector after normalization is $(0.36, 0.40, 0.18, 0.06, 0.00)$.

Calculation of the scores for the alternative evaluation matrices followed the same procedure. Table 10 summarizes the weight vectors of the criteria and alternatives. The normalized weight vectors from Tables 4, 5, 6, 7 and 8 are, respectively, (0.08, 0.04, 0.33, 0.33, 0.22), (0.32, 0.17, 0.17, 0.34, 0.00), (0.36, 0.18, 0.42, 0.00, 0.04), (0.00, 0.35, 0.43, 0.11, 0.10) and (0.43, 0.23, 0.00, 0.34, 0.00).

Table 10 Weight vectors of the criteria and alternatives using min operator

	C_1	C_2	C_3	C_4	C_5
A_1	0.24	0.94	0.84	0.00	1.00
A_2	0.12	0.49	0.43	0.80	0.55
A_3	1.00	0.51	1.00	1.00	0.01
A_4	1.00	1.00	0.00	0.26	0.79
A_5	0.67	0.00	0.09	0.24	0.00
Weights of criteria	0.91	1.00	0.45	0.17	0.00

Table 11 Global performance of alternatives and outranking using min operator

Alternative	Normalized global scores	Rank
A_1	0.21	3rd
A_2	0.13	4th
A_3	0.28	1st
A_4	0.27	2nd
A_5	0.10	5th

Table 11 shows the global performance and ranking position for all the alternatives.

Following this procedure, alternative A_3 is the best evaluated one, followed by A_4, A_1, A_2 and A_5, in this order. Note that this preference order may have been affected by the null values resulting from the aggregation of the degrees of possibilities using the min operator in Eq. 9. This was the case the weight of C_5 and the scores of A_5 regarding C_2 and C_5, the score of A_4 on C_3 and A_1 on C_4. In the following sections, other Fuzzy AHP approaches are tested so as to evaluate whether they can avoid the problem of null weight.

5 Tests and discussion

5.1 Evaluated approaches of the Fuzzy AHP method

In order to investigate how the problem of null weight can be overcome, the following Fuzzy AHP approaches were tested:

- *Approach #1* it has been proposed by Ahmed and Kilic (2015) and combines the synthetic extent with the center of area defuzzification operator to ordering the weights. In this approach, the normalization of the decision matrix, to calculate the synthetic extent, follows the procedure presented by Chang (1996);
- *Approach #2* it combines the normalization procedure proposed by Wang et al. (2008) to calculate the synthetic extent with the center of area defuzzification operator proposed by Ahmed and Kilic (2015) to ordering the weights;
- *Approach #3* it is based on the procedure proposed by Chang (1996) but following the suggestion made by Lima-Junior et al. (2014), it uses the arithmetic mean instead of the min operator. It is important to note that the arithmetic mean operator was used as an alternative to the min operator only to model the logic connectives of Eq. 9; otherwise, to calculate the degrees of possibilities as in Eqs. 7 and 8, the min operator was still used, since in this particular case the objective is to compute the height of the intersection between two fuzzy sets;
- *Approach #4* it is based on the method proposed by Chang (1996) but using the normalization procedure proposed by Wang et al. (2008) to calculate the synthetic extent and the arithmetic mean operator, suggested by Lima-Junior et al. (2014), to model the logic connectives of Eq. 9;
- *Approach #5* it is based entirely on the method proposed by Chang (1996) and was used so as to compare the consistency of the previous approaches as well as to evaluate the ability of the other approaches to avoid the problem of null weight.

The Fuzzy AHP approaches proposed by Bulut et al. (2012) and Duru et al. (2012) were not considered since they also use the min operator and also cause the problem of null weight.

5.2 Data generation

The procedure to generate the fuzzy comparison matrices for the weights of criteria was similar to the one used by Wang et al. (2008) and Ahmed and Kilic (2015). First, crisp weights $w_1, w_2, \ldots, w_n$ were defined and normalized in a way that $\sum_{i=1}^{n} w_i = 1$. Following that, each of its elements was converted to a triangular fuzzy number (l, m, u). The vertex points of the fuzzy numbers were set as $m = w_i$, $l = w_i - \alpha$ and $u = w_i + \alpha$. A fuzzy comparison matrix was then defined as:

$$\begin{pmatrix} \frac{\tilde{w}_1}{\tilde{w}_1} & \frac{\tilde{w}_1}{\tilde{w}_i} & \frac{\tilde{w}_1}{\tilde{w}_n} \\ \frac{\tilde{w}_i}{\tilde{w}_1} & \frac{\tilde{w}_i}{\tilde{w}_2} & \frac{\tilde{w}_i}{\tilde{w}_n} \\ \frac{\tilde{w}_n}{\tilde{w}_1} & \frac{\tilde{w}_n}{\tilde{w}_2} & \frac{\tilde{w}_n}{\tilde{w}_n} \end{pmatrix}$$

Tests were carried out with matrices of size 2, 3, 4 and 5, and with values of α equal to 0.025, 0.05 and 0.10. The values of the weights $\tilde{w}_i$ were chosen so as to evaluate the tested approaches under 3 different circumstances as follows:

- Different weights of all the criteria with no intersection between the fuzzy numbers as in Fig. 3a–d and 3 l (cases 1, 2, 3, 4 and 12, respectively);
- Different weights of all the criteria but with some degree of intersection between the fuzzy numbers as in Fig. 3e–h (cases 5, 6, 7 and 8, respectively);
- Some criteria with equal weights as in Fig. 3i–k (cases 9, 10 and 11, respectively).

Table 12 presents the values of w_i and α for the 12 cases, which were used to generate the comparative matrices shown in Tables 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23

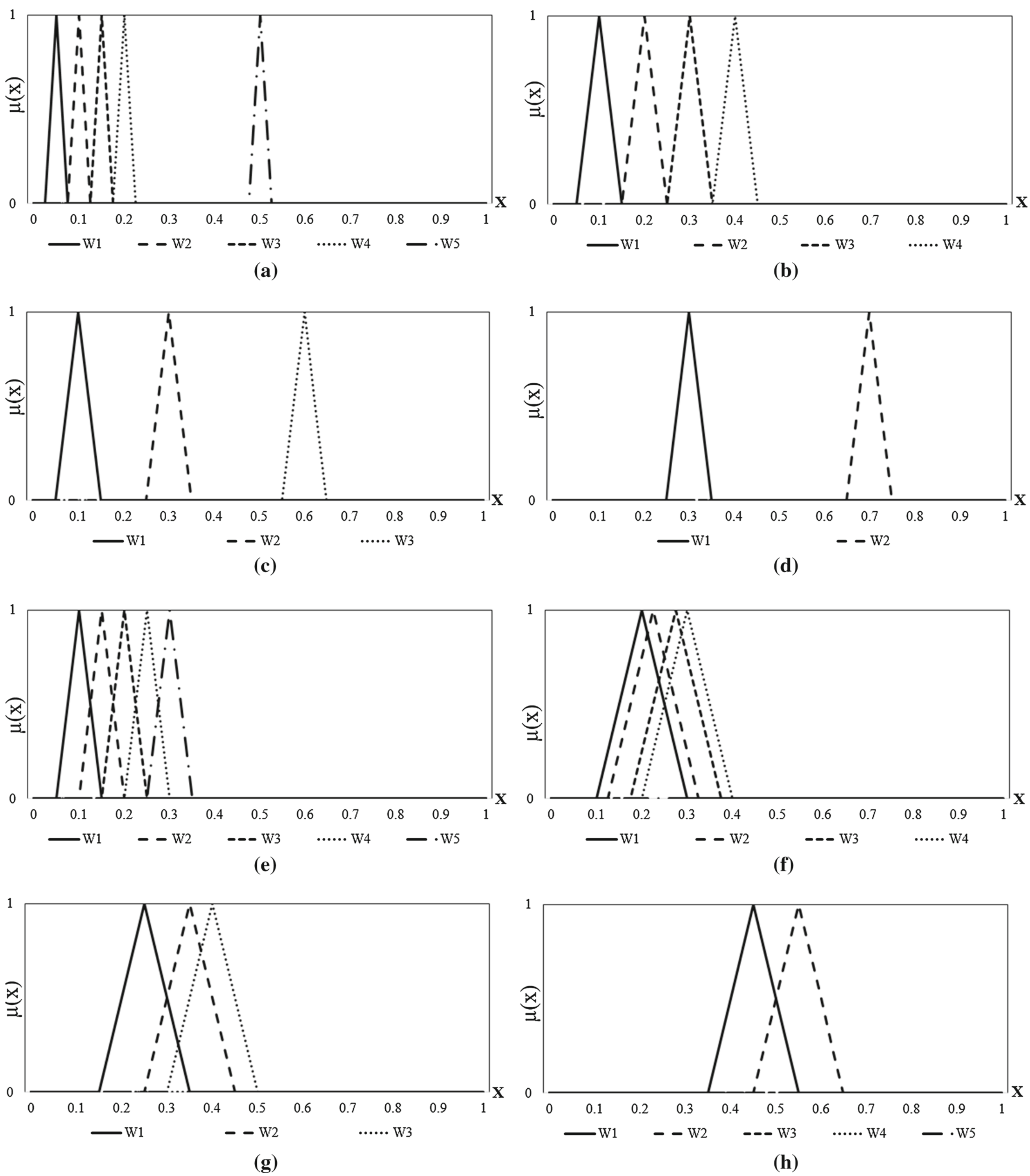

Fig. 3 Generated fuzzy weights for the cases 1 (**a**), 2 (**b**), 3 (**c**), 4 (**d**), 5 (**e**), 6 (**f**), 7 (**g**), 8 (**h**), 9 (**i**), 10 (**j**), 11 (**k**) and 12 (**l**)

and 24. Tables 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23 and 24 also present the values of centric consistency index (CCI) calculated for each comparative matrix according Bulut et al. (2012). All the computed values of CCI are lower than the values of thresholds of the geometric consistency index (GCI = 0.31 for $n = 3$; GCI = 0.35 for $n = 4$ and; GCI = 0.37 for $n > 4$), confirming the consistency of the generated comparative matrices. Each Fuzzy AHP approach was tested separately using the same comparative matrix of the 12 illustrative cases.

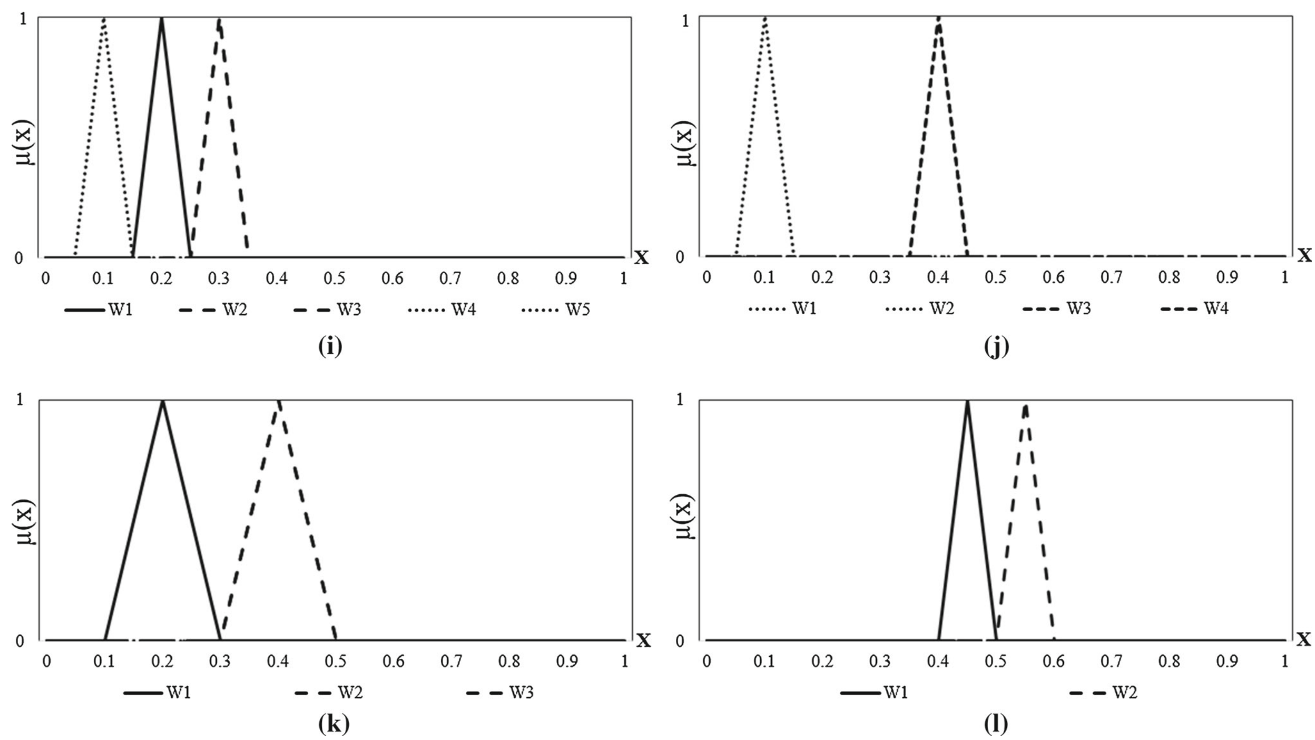

Fig. 3 continued

Table 12 Triangular fuzzy weights generated for each case

	Matrix size	*A*	C_1	C_2	C_3	C_4	C_5
Case 1	5	0.025	(0.025, 0.05, 0.075)	(0.075, 0.1, 0.125)	(0.125, 0.15, 0.175)	(0.175, 0.2, 0.225)	(0.475, 0.5, 0.525)
Case 2	4	0.05	(0.05, 0.1, 0.15)	(0.15, 0.2, 0.25)	(0.25, 0.3, 0.35)	(0.35, 0.4, 0.45)	–
Case 3	3	0.05	(0.05, 0.1, 0.15)	(0.25, 0.3, 0.35)	(0.55, 0.6, 0.65)	–	–
Case 4	2	0.05	(0.65, 0.7, 0.75)	(0.25, 0.3, 0.35)	–	–	–
Case 5	5	0.05	(0.05, 0.1, 0.15)	(0.1, 0.15, 0.2)	(0.15, 0.2, 0.25)	(0.2, 0.25, 0.3)	(0.25, 0.3, 0.35)
Case 6	4	0.1	(0.1, 0.2, 0.3)	(0.125, 0.225, 0.325)	(0.175, 0.275, 0.375)	(0.2, 0.3, 0.4)	–
Case 7	3	0.1	(0.15, 0.25, 0.35)	(0.25, 0.35, 0.45)	(0.3, 0.4, 0.5)	–	–
Case 8	2	0.1	(0.35, 0.45, 0.55)	(0.45, 0.55, 0.65)	–	–	–
Case 9	5	0.05	(0.15, 0.2, 0.25)	(0.25, 0.3, 0.35)	(0.25, 0.3, 0.35)	(0.05, 0.1, 0.15)	(0.05, 0.1, 0.15)
Case 10	4	0.05	(0.05, 0.1, 0.15)	(0.05, 0.1, 0.15)	(0.35, 0.4, 0.45)	(0.35, 0.4, 0.45)	–
Case 11	3	0.1	(0.1, 0.2, 0.3)	(0.3, 0.4, 0.5)	(0.3, 0.4, 0.5)	–	–
Case 12	2	0.05	(0.4, 0.45, 0.5)	(0.5, 0.55, 0.6)	–	–	–

Table 13 Fuzzy comparative matrix of the weights of the criteria for case 1

CCI = 0.0025	C_1	C_2	C_3	C_4	C_5
C_1	(1, 1, 1)	(0.2, 0.5, 1)	(0.1429, 0.3333, 0.6)	(0.1111, 0.25, 0.4286)	(0.0476, 0.1, 0.1579)
C_2	(1, 2, 5)	(1, 1, 1)	(0.4286, 0.6667, 1)	(0.3333, 0.5, 0.7143)	(0.1429, 0.2, 0.2632)
C_3	(1.6667, 3, 7)	(1, 1.5, 2.3333)	(1, 1, 1)	(0.5556, 0.75, 1)	(0.2381, 0.3, 0.3684)
C_4	(2.3333, 4, 9)	(1.4, 2, 3)	(1, 1.3333, 1.8)	(1, 1, 1)	(0.3333, 0.4, 0.4737)
C_5	(6.3333, 10, 21)	(3.8, 5, 7)	(2.7143, 3.3333, 4.2)	(2.1111, 2.5, 3)	(1, 1, 1)

Table 14 Fuzzy comparative matrix of the weights of the criteria for case 2

CCI = 0.0051	C_1	C_2	C_3	C_4
C_1	(1, 1, 1)	(0.2, 0.5, 1)	(0.1429, 0.3333, 0.6)	(0.1111, 0.25, 0.4286)
C_2	(1, 2, 5)	(1, 1, 1)	(0.4286, 0.6667, 1)	(0.3333, 0.5, 0.7143)
C_3	(1.6667, 3, 7)	(1, 1.5, 2.3333)	(1, 1, 1)	(0.5556, 0.75, 1)
C_4	(2.3333, 4, 9)	(1.4, 2, 3)	(1, 1.3333, 1.8)	(1, 1, 1)

Table 15 Fuzzy comparative matrix of the weights of the criteria for case 3

CCI = 0.0071	C_1	C_2	C_3
C_1	(1, 1, 1)	(0.1429, 0.3333, 0.6)	(0.0769, 0.1667, 0.2727)
C_2	(1.6667, 3, 7)	(1, 1, 1)	(0.3846, 0.5, 0.6364)
C_3	(3.6667, 6, 13)	(1.5714, 2, 2.6)	(1, 1, 1)

Table 16 Fuzzy comparative matrix of the weights of the criteria for case 4

CCI = 0	C_1	C_2
C_1	(1, 1, 1)	(1.8571, 2.3333, 3)
C_2	(0.3333, 0.4286, 0.5385)	(1, 1, 1)

5.3 Results

Table 25 presents the values of the synthetic extent for approaches 1, 3 and 5, calculated according to Chang's normalization procedure (Chang 1996), and for approaches 2 and 4, calculated following Wang et al. (2008). Table 26 presents the values of the weights and normalized weights. Table 27 shows the values of the root mean square error (RMSE), which were based on the difference between the normalized weight and the value set as the middle vertex *m* of the triangular fuzzy weight generated for each criterion in each case (as shown in Table 12).

Table 28 presents for each approach and each case the number of criteria with null weight. Table 29 shows the number of criteria for which the calculated normalized weight fell outside the interval $[l, u]$ of $\tilde{w}_i$. Table 30 presents the mean value of the RMSE values according to the

Table 17 Fuzzy comparative matrix of the weights of the criteria for case 5

CCI = 0.0057	C_1	C_2	C_3	C_4	C_5
C_1	(1, 1, 1)	(0.25, 0.6667, 1.5)	(0.2, 0.5, 1)	(0.1667, 0.4, 0.75)	(0.1429, 0.3333, 0.6)
C_2	(0.6667, 1.5, 4)	(1, 1, 1)	(0.4, 0.75, 1.3333)	(0.3333, 0.6, 1)	(0.2857, 0.5, 0.8)
C_3	(1, 2, 5)	(0.75, 1.3333, 2.5)	(1, 1, 1)	(0.5, 0.8, 1.25)	(0.4286, 0.6667, 1)
C_4	(1.3333, 2.5, 6)	(1, 1.6667, 3)	(0.8, 1.25, 2)	(1, 1, 1)	(0.5714, 0.8333, 1.2)
C_5	(1.6667, 3, 7)	(1.25, 2, 3.5)	(1, 1.5, 2.3333)	(0.8333, 1.2, 1.75)	(1, 1, 1)

Table 18 Fuzzy comparative matrix of the weights of the criteria for case 6

CCI = 0.0225	C_1	C_2	C_3	C_4
C_1	(1, 1, 1)	(0.3077, 0.8889, 2.4)	(0.2667, 0.7273, 1.7143)	(0.25, 0.6667, 1.5)
C_2	(0.4167, 1.125, 3.25)	(1, 1, 1)	(0.3333, 0.8182, 1.8571)	(0.3125, 0.75, 1.625)
C_3	(0.5833, 1.375, 3.75)	(0.5385, 1.2222, 3)	(1, 1, 1)	(0.4375, 0.9167, 1.875)
C_4	(0.6667, 1.5, 4)	(0.6154, 1.3333, 3.2)	(0.5333, 1.0909, 2.2857)	(1, 1, 1)

Table 19 Fuzzy comparative matrix of the weights of the criteria for case 7

CCI = 0.0108	C_1	C_2	C_3
C_1	(1, 1, 1)	(0.3333, 0.7143, 1.4)	(0.3, 0.625, 1.1667)
C_2	(0.7143, 1.4, 3)	(1, 1, 1)	(0.5, 0.875, 1.5)
C_3	(0.8571, 1.6, 3.3333)	(0.6667, 1.1429, 2)	(1, 1, 1)

Table 20 Fuzzy comparative matrix of the weights of the criteria for case 8

CCI = 0	C_1	C_2
C_1	(1, 1, 1)	(0.5385, 0.8182, 1.2222)
C_2	(0.8182, 1.2222, 1.8571)	(1, 1, 1)

three types of evaluated situations: cases in which there is no intersection between the fuzzy weights of criteria (cases 1, 2, 3, 4 and 12); (2) the weights of the criteria are different, but there is intersection between the triangular fuzzy numbers (cases 5, 6, 7 and 8); and (3) there are some weights with equal values (cases 9, 10 and 11).

As shown in Tables 27 and 30, approach 1 yielded the lower values of the RMSE and therefore the results for criteria weights are the most consistent ones. Approach 2 also produced low values for the RMSE. In cases 4 and 11, the RMSE obtained by approach 2 is lower than that using approach 1. The worst values of the RMSE were obtained for approaches 3 and 4, which uses arithmetic mean as an alternative to the min operator.

As shown in Table 28, only approaches 1 and 2 were able to avoid the problem of null weight in all different decision situations. Approaches 3 and 4 present better results than approach 5, but they could not completely avoid the problem of null weight. This is in part explained by the fact that when all the aggregated degrees of possibilities are zero the calculated weight will also be zero. However, on top of yielding the worst values of RMSE, approaches 3 and 4 also presented the largest number of criteria for which the calculated normalized weight fell outside the interval [l, u] of $\tilde{w}_i$, as presented in Table 29. It is important to note that the problem of null weight happens when the values of the generated fuzzy weights are spread and without intersections as in Fig. 3*a*, *c*, *d* and *l* (cases 1, 3, 4 and 12).

Table 23 Fuzzy comparative matrix of the weights of the criteria for case 11

CCI = 0.0160	C_1	C_2	C_3
C_1	(1, 1, 1)	(0.2, 0.5, 1)	(0.2, 0.5, 1)
C_2	(1, 2, 5)	(1, 1, 1)	(0.6, 1, 1.6667)
C_3	(1, 2, 5)	(0.6, 1, 1.6667)	(1, 1, 1)

Table 24 Fuzzy comparative matrix of the weights of the criteria for case 12

CCI = 0	C_1	C_2
C_1	(1, 1, 1)	(0.6667, 0.8182, 1)
C_2	(1, 1.2222, 1.5)	(1, 1, 1)

Regarding the results of the traditional Fuzzy AHP method proposed by Chang (1996), it presented the third better performance in relation to the value of RMSE and produced the highest number of null weights (approach 5, Tables 27, 28 and 30). The worst results yielded by this approach occurred in cases 1, 3 and 12, when there is no intersection between the fuzzy weights generated for the criteria.

Another point is that the normalization procedure proposed by Wang et al. (2008) led to similar values for the calculated weights of criteria when the values of generated weight $\tilde{w}_i$ are different. For instance, in case 1, using approach 2, for criteria 3 and 4, $\tilde{w}_3 = (0.125, 0.15, 0.175)$ and $\tilde{w}_4 = (0.175, 0.20, 0.225)$; however, for both criteria, the calculated value of the normalized criterion weight was 0.2002. Another example refers to case 6, when $\tilde{w}_2 = (0.125, 0.225, 0.325)$ and $\tilde{w}_3 = (0.175, 0.225, 0.375)$, but

Table 21 Fuzzy comparative matrix of the weights of the criteria for case 9

CCI = 0.0119	C_1	C_2	C_3	C_4	C_5
C_1	(1, 1, 1)	(0.4286, 0.6667, 1)	(0.4286, 0.6667, 1)	(1, 2, 5)	(1, 2, 5)
C_2	(1, 1.5, 2.3333)	(1, 1, 1)	(0.7143, 1, 1.4)	(1.6667, 3, 7)	(1.6667, 3, 7)
C_3	(1, 1.5, 2.3333)	(0.7143, 1, 1.4)	(1, 1, 1)	(1.6667, 3, 7)	(1.6667, 3, 7)
C_4	(0.2, 0.5, 1)	(0.1429, 0.3333, 0.6)	(0.1429, 0.3333, 0.6)	(1, 1, 1)	(0.3333, 1, 3)
C_5	(0.2, 0.5, 1)	(0.1429, 0.3333, 0.6)	(0.1429, 0.3333, 0.6)	(0.3333, 1, 3)	(1, 1, 1)

Table 22 Fuzzy comparative matrix of the weights of the criteria for case 10

CCI = 0.0129	C_1	C_2	C_3	C_4
C_1	(1, 1, 1)	(0.3333, 1, 3)	(0.1111, 0.25, 0.4286)	(0.1111, 0.25, 0.4286)
C_2	(0.3333, 1, 3)	(1, 1, 1)	(0.1111, 0.25, 0.4286)	(0.1111, 0.25, 0.4286)
C_3	(2.3333, 4, 9)	(2.3333, 4, 9)	(1, 1, 1)	(0.7778, 1, 1.2857)
C_4	(2.3333, 4, 9)	(2.3333, 4, 9)	(0.7778, 1, 1.2857)	(1, 1, 1)

Table 25 Values of synthetic extent of the criteria according Chang (1996) and Wang et al. (2008)

Case	*Criteria*	Synthetic extent (according Chang 1996)	Synthetic extent (according Wang et al. 2008)
Case 1	C_1	(0.0202, 0.05, 0.1031)	(0.0207, 0.05, 0.0978)
	C_2	(0.0391, 0.1, 0.2582)	(0.0419, 0.1, 0.2218)
	C_3	(0.06, 0.15, 0.3788)	(0.0665, 0.15, 0.3069)
	C_4	(0.0816, 0.2, 0.4944)	(0.0931, 0.2, 0.3809)
	C_5	(0.2147, 0.5, 1.1718)	(0.295, 0.5, 0.708)
Case 2	C_1	(0.0394, 0.1, 0.2137)	(0.0412, 0.1, 0.1923)
	C_2	(0.0749, 0.2, 0.5444)	(0.0865, 0.2, 0.4034)
	C_3	(0.1145, 0.3, 0.7997)	(0.1419, 0.3, 0.5325)
	C_4	(0.1555, 0.4, 1.0444)	(0.2062, 0.4, 0.6369)
Case 3	C_1	(0.045, 0.1, 0.1782)	(0.0461, 0.1, 0.1678)
	C_2	(0.0873, 0.2328, 0.8218)	(0.1418, 0.3, 0.5366)
	C_3	(0.1786, 0.4655, 1.5796)	(0.3725, 0.6, 0.7954)
Case 4	C_1	(0.5159, 0.7, 0.9545)	(0.65, 0.7, 0.75)
	C_2	(0.2407, 0.3, 0.3671)	(0.25, 0.3, 0.35)
Case 5	C_1	(0.0335, 0.1, 0.2611)	(0.0356, 0.1, 0.2238)
	C_2	(0.0511, 0.15, 0.4378)	(0.0571, 0.15, 0.3385)
	C_3	(0.07, 0.2, 0.5786)	(0.0809, 0.2, 0.4191)
	C_4	(0.0896, 0.25, 0.7105)	(0.1069, 0.25, 0.4876)
	C_5	(0.1095, 0.3, 0.8388)	(0.1347, 0.3, 0.5485)
Case 6	C_1	(0.0529, 0.2, 0.7142)	(0.0615, 0.2, 0.4707)
	C_2	(0.0599, 0.225, 0.8349)	(0.0716, 0.225, 0.5179)
	C_3	(0.0743, 0.275, 1.0392)	(0.0934, 0.275, 0.5895)
	C_4	(0.0817, 0.3, 1.1322)	(0.1051, 0.3, 0.6193)
Case 7	C_1	(0.1061, 0.25, 0.5598)	(0.1213, 0.25, 0.4295)
	C_2	(0.1438, 0.35, 0.8632)	(0.1828, 0.35, 0.5695)
	C_3	(0.1639, 0.4, 0.994)	(0.2177, 0.4, 0.6221)
Case 8	C_1	(0.3029, 0.45, 0.662)	(0.35, 0.45, 0.55)
	C_2	(0.358, 0.55, 0.8512)	(0.45, 0.55, 0.65)
Case 9	C_1	(0.0614, 0.2, 0.6636)	(0.0718, 0.2, 0.4524)
	C_2	(0.0962, 0.3, 0.9562)	(0.1205, 0.3, 0.5804)
	C_3	(0.0962, 0.3, 0.9562)	(0.1205, 0.3, 0.5804)
	C_4	(0.0289, 0.1, 0.3165)	(0.0311, 0.1, 0.2586)
	C_5	(0.0289, 0.1, 0.3165)	(0.0311, 0.1, 0.2586)
Case 10	C_1	(0.0309, 0.1, 0.3036)	(0.0331, 0.1, 0.2516)
	C_2	(0.0309, 0.1, 0.3036)	(0.0331, 0.1, 0.2516)
	C_3	(0.1282, 0.4, 1.2679)	(0.1768, 0.4, 0.6798)
	C_4	(0.1282, 0.4, 1.2679)	(0.1768, 0.4, 0.6798)
Case 11	C_1	(0.0764, 0.2, 0.4545)	(0.0837, 0.2, 0.3659)
	C_2	(0.1418, 0.4, 1.1616)	(0.196, 0.4, 0.6571)
	C_3	(0.1418, 0.4, 1.1616)	(0.196, 0.4, 0.6571)
Case 12	C_1	(0.3704, 0.45, 0.5455)	(0.4, 0.45, 0.5)
	C_2	(0.4444, 0.55, 0.6818)	(0.5, 0.55, 0.6)

the calculated value of the normalized weight for both criteria was 0.25. This indicates that in some cases this approach presents a poor sensitivity to differentiate the relative importance of a set of objects (alternatives or criteria) in regard to an objective. In cases 9, 10 and 11, when some of the generated fuzzy weights are equal, all the approach tested computed the weights consistently. For instance, in case 10, $\tilde{w}_1$ and $\tilde{w}_2 = (0.05, 0.1, 0.15)$ and the calculated normalized weights using a particular approach

Table 26 Values of weights and normalized weights for each Fuzzy AHP approach

Cases	Criteria	Approach 1		Approach 2		Approach 3		Approach 4		Approach 5	
		Weight	Normalized weight	Weight	Normalized weight	Weight	Normalized weight	Weight	Normalized weight	Weight	Normalized weight
Case 1	C_1	0.1001	0.0769	0.1001	0.0882	0.2472	0.0697	0.1992	0.0604	0	0
	C_2	0.1335	0.1026	0.1335	0.1177	0.6338	0.1788	0.5798	0.1759	0.0982	0.0517
	C_3	0.2002	0.1538	0.2002	0.1765	0.7938	0.2239	0.7108	0.2157	0.3192	0.1680
	C_4	0.2336	0.1795	0.2002	0.1765	0.8706	0.2456	0.8056	0.2445	0.4825	0.2540
	C_5	0.6340	0.4872	0.5005	0.4412	1.0000	0.2821	1.0000	0.3035	1.0000	0.5263
Case 2	C_1	0.1335	0.1000	0.1001	0.0909	0.3585	0.1143	0.2385	0.0822	0.1626	0.0605
	C_2	0.2669	0.2000	0.2336	0.2121	0.8239	0.2626	0.7400	0.2551	0.6604	0.2456
	C_3	0.4004	0.3000	0.3337	0.3030	0.9552	0.3044	0.9218	0.3178	0.8656	0.3220
	C_4	0.5339	0.4000	0.4338	0.3939	1.0000	0.3187	1.0000	0.3448	1.0000	0.3719
Case 3	C_1	0.1001	0.0789	0.1001	0.1000	0.1236	0.0632	0.0575	0.0332	0	0
	C_2	0.4004	0.3158	0.3337	0.3334	0.8318	0.4254	0.6768	0.3902	0.6636	0.3989
	C_3	0.7674	0.6053	0.5672	0.5666	1.0000	0.5114	1.0000	0.5766	1.0000	0.6011
Case 4	C_1	0.7341	0.7097	0.7070	0.7000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	C_2	0.3003	0.2903	0.3030	0.3000	0	0	0	0	0	0
Case 5	C_1	0.1335	0.0930	0.1335	0.1111	0.6071	0.1393	0.5259	0.1260	0.4311	0.1115
	C_2	0.2336	0.1628	0.2002	0.1667	0.8359	0.1919	0.7780	0.1864	0.6864	0.1776
	C_3	0.2669	0.1860	0.2336	0.1945	0.9329	0.2141	0.9005	0.2158	0.8243	0.2133
	C_4	0.3670	0.2558	0.3003	0.2500	0.9808	0.2251	0.9690	0.2322	0.9232	0.2389
	C_5	0.4338	0.3023	0.3337	0.2778	1.0000	0.2295	1.0000	0.2396	1.0000	0.2587
Case 6	C_1	0.3337	0.1961	0.2336	0.1945	0.9073	0.2358	0.8535	0.2276	0.8635	0.2304
	C_2	0.4004	0.2353	0.3003	0.2500	0.9493	0.2467	0.9136	0.2436	0.9094	0.2427
	C_3	0.4671	0.2745	0.3003	0.2500	0.9915	0.2577	0.9836	0.2622	0.9746	0.2601
	C_4	0.5005	0.2941	0.3670	0.3055	1.0000	0.2599	1.0000	0.2666	1.0000	0.2668
Case 7	C_1	0.3337	0.2564	0.3003	0.2727	0.7657	0.2802	0.6485	0.2507	0.7252	0.2728
	C_2	0.4671	0.3589	0.3670	0.3333	0.9666	0.3538	0.9378	0.3626	0.9333	0.3511
	C_3	0.5005	0.3846	0.4338	0.3940	1.0000	0.3660	1.0000	0.3867	1.0000	0.3762
Case 8	C_1	0.5005	0.4545	0.4671	0.4516	0.7525	0.4294	0.5000	0.3333	0.7525	0.4294
	C_2	0.6006	0.5455	0.5672	0.5484	1.0000	0.5706	1.0000	0.6667	1.0000	0.5706
Case 9	C_1	0.3003	0.2000	0.2336	0.2000	0.9251	0.2147	0.8842	0.2147	0.8502	0.2181
	C_2	0.4338	0.2889	0.3337	0.2857	1.0000	0.2321	1.0000	0.2428	1.000	0.2565
	C_3	0.4338	0.2889	0.3337	0.2857	1.0000	0.2321	1.0000	0.2428	1.000	0.2565
	C_4	0.1668	0.1111	0.1335	0.1143	0.6917	0.1605	0.6171	0.1498	0.5241	0.1344
	C_5	0.1668	0.1111	0.1335	0.1143	0.6917	0.1605	0.6171	0.1498	0.5241	0.1344
Case 10	C_1	0.1668	0.1087	0.1335	0.1250	0.5793	0.1834	0.4664	0.1590	0.3690	0.1348
	C_2	0.1668	0.1087	0.1335	0.1250	0.5793	0.1834	0.4664	0.1590	0.3690	0.1348
	C_3	0.6006	0.3913	0.4004	0.3750	1.0000	0.3166	1.0000	0.3410	1.0000	0.3652
	C_4	0.6006	0.3913	0.4004	0.3750	1.0000	0.3166	1.0000	0.3410	1.0000	0.3652
Case 11	C_1	0.2439	0.1761	0.2169	0.2064	0.6099	0.2337	0.4593	0.1868	0.6099	0.2337
	C_2	0.5706	0.4120	0.4171	0.3968	1.0000	0.3832	1.0000	0.4066	1.0000	0.3832
	C_3	0.5706	0.4120	0.4171	0.3968	1.0000	0.3832	1.0000	0.4066	1.0000	0.3832
Case 12	C_1	0.4671	0.4516	0.4671	0.4516	0.5025	0.3344	0	0	0.5025	0.3344
	C_2	0.5672	0.5484	0.5672	0.5484	1.0000	0.6656	1.0000	1.0000	1.0000	0.6656

are the same (0.1087 when using approach 1; 0.125 when using approach 2).

Therefore, the results indicate that although approaches 1 and 2 can overcome the problem of null weights and scores, approach 1 is the most adequate so as it produced

Table 27 Values of the RMSE obtained by each Fuzzy AHP approach

	Approach 1	Approach 2	Approach 3	Approach 4	Approach 5
Case 1	0.0191	0.0360	0.1110	0.1008	0.0419
Case 2	2.05×10^{-5}	0.0083	0.0518	0.0410	0.0350
Case 3	0.0155	0.0272	0.0911	0.0662	0.0812
Case 4	0.0097	0	0.3000	0.3000	0.3000
Case 5	0.0095	0.0136	0.0426	0.0353	0.0241
Case 6	0.0062	0.0181	0.0303	0.0244	0.0253
Case 7	0.0109	0.0167	0.0264	0.0106	0.0191
Case 8	0.0045	0.0051	0.0206	0.1167	0.0206
Case 9	0.0099	0.0128	0.0579	0.0484	0.0360
Case 10	0.0087	0.0250	0.0834	0.0590	0.0348
Case 11	0.0169	0.0045	0.0238	0.0094	0.0238
Case 12	0.0016	0.0016	0.1156	0.4500	0.1156

Table 28 Number of null weights yielded by each evaluated Fuzzy AHP approach

Fuzzy AHP approach	Different weights (without intersection)	Different weights (with intersection)	Some equal weights	Total
1	0	0	0	0
2	0	0	0	0
3	1	0	0	1
4	2	0	0	2
5	3	0	0	3

Table 29 Number of weights that fell outside the fuzzy interval of $\tilde{W}_i$ for each Fuzzy AHP approach

Fuzzy AHP approach	Different weights (no intersection)	Different weights (with intersection)	Some equal weights	Total
1	0	0	0	0
2	0	0	0	0
3	12	0	8	20
4	12	3	6	21
5	10	0	1	11

Table 30 Mean of the RMSE for each Fuzzy AHP approach

Fuzzy AHP approach	Different weights (no intersection)	Different weights (with intersection)	Some equal weights	All the cases
1	0.0092	0.0078	0.0118	0.0094
2	0.0146	0.0125	0.0141	0.0138
3	0.1339	0.0300	0.0550	0.0795
4	0.1916	0.0467	0.0389	0.1051
5	0.1147	0.0223	0.0315	0.0631

lower values of RMSE. Other benefits of using this approach are: better ability to differentiate the relative importance of the criteria when values of the fuzzy weights are very close; the simplicity of the computational implementation of both Chang's normalization procedure and the center of area defuzzification operator; and the simplicity of calculations, which contributes to a better understanding by the decision makers about how the results are generated.

6 Conclusion

This paper presented a new study comparing different Fuzzy AHP approaches based on extent analysis method aiming at identifying which are able to overcome the problem of null weights of criteria and scores of alternatives without affecting the consistency of results. Although there are other propositions of the Fuzzy AHP method based on the synthetic extent analysis, none of the studies found in the literature presents a discussion on how to avoid or minimize the problem of null weight of criteria and null scores of alternatives. Therefore, it is the first study to investigate alternatives to prevent the problem of null weight of criteria and scores of alternatives in the Fuzzy AHP method proposed by Chang (1996). In addition, it is the first study to analyze the consistency of the values of the criteria weights calculated by different Fuzzy AHP approaches based on Chang (1996).

With each approach tested separately in 12 generic multicriteria decision problems, it was possible to evaluate the behavior of the different Fuzzy AHP approaches under different situations. The test results indicated that the Fuzzy AHP approach proposed by Ahmed and Kilic (2015) is the most appropriate to overcome the problem of null weight of criteria and scores of alternatives. Moreover, this approach yielded the lower values of RMSE. The results also indicate that when using the Fuzzy AHP approaches 3, 4 and 5, the problem of null weights can occur if one or more criteria are much more important than the others, especially in situations in which the values of the generated fuzzy weights are spread and with no intersections between them.

Although the number of evaluated cases is limited, the obtained results can be extrapolated to other decision making situations. Since the calculated values of the synthetic extent will always be positive, the value yielded by the centroid deffuzification operator will also be positive. Therefore, the use of Fuzzy AHP approach proposed by Ahmed and Kilic (2015) will never produce null weights.

Further applications of the Fuzzy AHP method can use the approach proposed by Ahmed and Kilic (2015) rather than the Chang's approach. Also, further researches may also assess whether there are other methods for ordering of the synthetic measures able to overcome the problem of null weights without affecting the consistency of the results.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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