Applied Soft Computing 21 (2014) 194–209

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

A comparison between Fuzzy AHP and Fuzzy TOPSIS methods to supplier selection

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ARTICLE INFO

Article history:
Received 8 July 2013
Received in revised form 13 January 2014
Accepted 20 March 2014
Available online 13 April 2014

Keywords:
Supplier selection
Fuzzy TOPSIS
Fuzzy AHP
Multicriteria decision making

ABSTRACT

Supplier selection has become a very critical activity to the performance of organizations and supply chains. Studies presented in the literature propose the use of the methods Fuzzy TOPSIS (Fuzzy Technique for Order of Preference by Similarity to Ideal Solution) and Fuzzy AHP (Fuzzy Analytic Hierarchy Process) to aid the supplier selection decision process. However, there are no comparative studies of these two methods when applied to the problem of supplier selection. Thus, this paper presents a comparative analysis of these two methods in the context of supplier selection decision making. The comparison was made based on the factors: adequacy to changes of alternatives or criteria; agility in the decision process; computational complexity; adequacy to support group decision making; the number of alternative suppliers and criteria; and modeling of uncertainty. As an illustrative example, both methods were applied to the selection of suppliers of a company in the automotive production chain. In addition, computational tests were performed considering several scenarios of supplier selection. The results have shown that both methods are suitable for the problem of supplier selection, particularly to supporting group decision making and modeling of uncertainty. However, the comparative analysis has shown that the Fuzzy TOPSIS method is better suited to the problem of supplier selection in regard to changes of alternatives and criteria, agility and number of criteria and alternative suppliers. Thus, this comparative study contributes to helping researchers and practitioners to choose more effective approaches for supplier selection. Suggestions of further work are also proposed so as to make these methods more adequate to the problem of supplier selection.

1. Introduction

Supplier selection (SS) is one of the most important activities of acquisition as its results have a great impact on the quality of goods and performance of organizations and supply chains [1–3]. Through SS it is also possible to anticipate evaluation of the potential of suppliers to establish a collaborative relationship [4].

Essentially, supplier selection is a decision process with the aim of reducing the initial set of potential suppliers to the final choices [5,6]. Decisions are based on evaluation of suppliers on multiple quantitative as well as qualitative criteria. Depending on the situation at hand, selecting suppliers may require searching for new suppliers or choosing suppliers from the existing pool of suppliers. In any case there is a degree of uncertainty in the decision process, which is caused by subjective evaluation of qualitative or quantitative criteria, by multiple decision makers, with no previous data to rely on [2,5,7,8].

Fuzzy set theory combined with multicriteria decision making (MCDM) methods has been extensively used to deal with uncertainty in the supplier selection decision process [9], since it provides a suitable language to handle imprecise criteria, being able to integrate the analysis of qualitative and quantitative factors. This is the case of Fuzzy AHP – Fuzzy Analytic Hierarchy Process [10–16], Fuzzy TOPSIS – Fuzzy Technique for Order Preference by Similarity to Ideal Solution [1,17–23], among others.

Despite the large number of articles proposing the use of Fuzzy AHP and Fuzzy TOPSIS, there are no comparative studies of these two methods when applied to the problem of supplier selection. Ertugrul and Karakasoglu [24] report a comparison of Fuzzy AHP and Fuzzy TOPSIS methods applied to facility location decision making. However, as the authors point out, there is still a need for a comparative evaluation of both methods in the context of

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http://dx.doi.org/10.1016/j.asoc.2014.03.014
1568-4946/

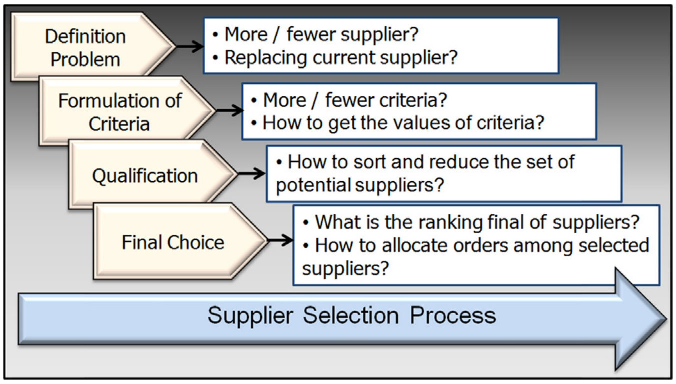

Fig. 1. The supplier selection framework [5].

supplier selection, since the relative advantages of both methods also depend on the characteristics of the problem domain. To fill this gap, this paper presents a comparative analysis of the methods Fuzzy TOPSIS and Fuzzy AHP applied to the problem of supplier selection. Comparison of the methods was made considering the factors: adequacy to changes of alternatives or criteria; agility in the decision process; computational complexity; adequacy to support group decision making; the amount of alternative suppliers and criteria; and modeling of uncertainty.

A descriptive quantitative approach was adopted as the research method [25]. Algorithms of the methods Fuzzy TOPSIS and Fuzzy AHP were developed in Matlab© and applied to the selection of suppliers of a company. Comparison of both methods was made based on the analysis of mathematical procedures considering the structure of the problem depicted by the illustrative application case.

The paper is organized as follows: Section 2 briefly revises the subject of supplier selection and the main requirements of multi-criteria decision making methods used in this context. Section 3 presents some fundamental concepts regarding fuzzy set theory and the methods Fuzzy TOPSIS and Fuzzy AHP. Section 4 presents the results of using both methods in a real case application. Section 5 presents the comparative analyses of both methods. Finally, conclusions about this research work and suggestions for further research are made in Section 6.

2. Supplier selection process

Supplier selection is a decision-making process comprising several steps. Based on the studies of Faris et al. [26] and Kraljic [27], De Boer et al. [5] propose a framework for supplier selection that consists of four steps: problem definition, formulation of criteria, qualification and final choice, as illustrated in Fig. 1. The first step aims at clearly defining the problem at hand, which may mean searching for new suppliers for a completely new product, replacing current suppliers, or choosing suppliers for new products from the existing pool of suppliers. Especially in the case of selecting new suppliers, depending on the item to be purchased, the number of alternative suppliers may be very large. This situation demands decision making techniques that are able to simultaneously evaluate several alternatives.

In the next step, the buyer should convert its requirements into decision criteria so as to guide the choices. There are several criteria that must be considered in the selection process, both quantitative and qualitative. Table 1 lists some important criteria for supplier selection. On top of traditional quantitative measures of performance, such as quality of conformance, delivery time or cost, other measures of subjective evaluation, for example supplier profile and relationship, are gaining importance [28]. Therefore, the techniques

Table 1
Supplier performance criteria according to selected authors.

Criteria	Proposed by													
	Kannan and Tan [29]	Kahraman et al. [11]	Katsikeas et al. [30]	Chan and Kumar [12]	Kirytopoulos et al. [31]	Guneri et al. [32]	Shen and Yu [33]	Ordoobadi [34]	Boran et al. [35]	Amin and Razmi [36]	Ku et al. [37]	Wang [38]	Lin et al. [39]	Büyüközkan and Çifçi [40]
Technical capability	X		X	X	X		X			X	X	X	X	X
Commitment to quality	X	X	X	X		X					X		X	X
Quality of conformance	X			X	X		X		X		X		X	X
Flexibility (response to change)	X		X	X	X			X			X	X		
Cost/price	X		X		X			X	X		X	X	X	X
Financial situation	X	X		X	X					X	X	X		X
Easy of communication	X		X	X	X			X				X	X	
On-time delivery	X			X	X			X	X		X	X	X	
Reputation			X	X	X	X				X			X	
Relationship	X					X		X	X	X			X	
Product performance								X		X		X	X	X
Delivery reliability			X		X	X		X		X				
After sale/warranty		X	X				X	X				X	X	
Geographic location	X			X							X		X	
End use		X						X						X
Social factors														X
Environmental factor		X												
Logistic costs								X						
Innovation							X							

Table 2
Decision making approaches applied to supplier selection.

Approach	Method(s)	Proposed by
Single method	Analytic Hierarchy Process (AHP)	Tam and Tummala [43]
		Hudymáčová et al. [44]
	Analytic Network Process (ANP)	Gencer and Gürpinar [45]
		Kirytopoulos et al. [31]
	Data Envelopment Analysis (DEA)	Saen [46]
	Fuzzy Inference	Carrera and Mayorga [47]
	Fuzzy Preference Relation	Hsu et al. [48]
	Genetic Algorithm	Liao and Rittscher [49]
Combined method	Fuzzy Adaptive Resonance Theory (Fuzzy ART)	Keskin et al. [50]
	Fuzzy AHP	Kahraman et al. [11]
		Chan and Kumar [12]
		Lee [13]
		Chamodrakas et al. [14]
		Zeydan et al. [15]
		Kilinci and Onal [16]
	Fuzzy ANP	Önut et al. [51]
		Vinodh et al. [52]
	Fuzzy c-means and Rough Set Theory	Omurca [53]
	Fuzzy Deployment Quality Function (Fuzzy QFD)	Bevilacqua et al. [54]
		Amin and Razmi [36]
		Dursun and Karsak [55]
	Fuzzy DEMATEL	Büyüközkan and Çifçi [40]
	Fuzzy Inference and Fuzzy Algebraic Operations	Amindoust et al. [8]
	Fuzzy Multiobjective Linear Programming	Amid et al. [56]
		Arikan [57]
	Fuzzy TOPSIS	Chen et al. [1]
		Chen [17]
		Dagdeviren et al. [18]
		Büyüközkan and Ersoy [19]
		Awasthi et al. [20]
		Liao and Kao [21]
		Jolai et al. [22]
		Zouggari and Benyoucef [23]
		Shahanaghi and Yazdian [58]
		Bottani and RIZZI [59]
	Fuzzy Two-Tupple	Wang [38]
	Fuzzy VIKOR	Shemshadi et al. [60]

used in the decision process must be able to process several criteria of both qualitative and quantitative nature [7].

In the qualification step, the main objective is to reduce the initial set of suppliers by sorting potential suppliers from the initial set of suppliers based on qualifying criteria. The last step aims to rank the potential suppliers so as to make the final choice. Based on this framework, Wu and Barnes [6] proposed a further step with the purpose of giving potential suppliers feedback on their performance in the selection process.

2.1. Multicriteria decision making methods in supplier selection

Multicriteria decision making (MCDM) methods for supplier selection include multi-attribute techniques, mathematical programming, stochastic programming and artificial intelligence techniques [6,9,41]. There are several different MCDM methods used mostly for outranking.

As shown in Table 2, the combination between techniques is usually adopted to deal with the problem of supplier selection. Fuzzy set theory (FST) [42] has been extensively used for modeling decision making processes based on imprecise and vague information such as judgment of decision makers. The use of appropriate techniques can bring effectiveness and efficiency to the selection process [7]. To decide which techniques to use one must take into account the alignment of the particularities of the problem at hand with the characteristics of the techniques [24]. For instance, when selecting a new supplier of a routine item with many potential suppliers [5], techniques that do not limit analysis to only a few alternatives are more adequate than others.

Other aspects to be considered to align techniques to particularities of supplier selection are as follows:

- Adequacy to support group decision making: purchasing decisions are influenced by several requirements from different functional areas within an organization. This implies that multiple actors from different functional areas are involved in the decision making process [7]. Therefore, it is desirable that the techniques used in supplier selection be adequate to combine different judgments of multiple decision makers.
- Adequacy to changes of alternatives or criteria: in the case of modified rebuy, one may be interested in purchasing existing products from new suppliers. This may lead to inclusion or exclusion of supply alternatives in the evaluation process. Alternatively, when modified rebuy refers to purchasing new products from current suppliers, this may imply inclusion or exclusion of decision criteria [5,26]. In both cases, the outranking techniques should be robust enough not to cause inconsistencies in the ordering of alternative suppliers.
- Agility in the decision process: this factor relates mainly to the required amount of judgments of the decision makers in data collection. Depending on the MCDM technique and the number of criteria and alternatives, the quantity of judgments needed to collect all the data can make the supplier selection process very time consuming [61].
- Computational complexity: this factor may be related to either time or space complexity. The main concern in the supplier selection decision process is related to time complexity, which refers to the time in which the algorithm is accomplished [10]. Time complexity varies from technique to technique as a function of the number of input variables, which in the case of supplier selection refers to the number of alternative suppliers and criteria.
- Uncertainty: in supplier selection, the uncertainty in decision making may refer to the lack of precision of the scores of the alternatives as well as the relative importance of different criteria. This imprecision may be due to: subjective evaluation by multiple decision makers; inexistence of previous data on the performance of potential suppliers and; difficulty of assessing intangible aspects of supplier performance [7,62].

3. Fuzzy set theory

Fuzzy set theory [42] has been used for modeling decision making processes based on imprecise and vague information such as judgment of decision makers. Qualitative aspects are represented by means of linguistic variables, which are expressed qualitatively by linguistic terms and quantitatively by a fuzzy set in the universe of discourse and respective membership function [63]. Operations between linguistic variables involve the concepts presented next.

3.1. Fundamental definitions

3.1.1. Definition 1: fuzzy set

A fuzzy set $\tilde{A}$ in X is defined by:

$$\tilde{A} = \{x, \mu_A(x)\}, \quad x \in X \tag{1}$$

in which $\mu_A(x) : X \to [0, 1]$ is the membership function of $\tilde{A}$ and $\mu_A(x)$ is the degree of pertinence of x in $\tilde{A}$ If $\mu_A(x)$ equals zero, x does not belong to the fuzzy set $\tilde{A}$. If $\mu_A(x)$ equals 1, x completely belongs to the fuzzy set $\tilde{A}$. However, unlike the classical set theory, if $\mu_A(x)$

has a value between zero and 1, x partially belongs to the fuzzy set $\tilde{A}$. That is, the pertinence of x is true with degree of membership given by $\mu_A(x)$ [42,62].

3.1.2. Definition 2: fuzzy numbers

A fuzzy number is a fuzzy set in which the membership function satisfies the conditions of normality

$$\sup \tilde{A}(\mathrm{x})_{x \in X} = 1 \quad (2)$$

and of convexity

$$\begin{array}{cc} & \begin{array}{cccc} C_1 & C_2 & C_j & C_m \end{array} \\ \tilde{D} = \begin{array}{c} A_1 \\ A_i \\ A_n \end{array} & \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \tilde{x}_{1j} & \tilde{x}_{1m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{n1} & \tilde{x}_{n2} & \tilde{x}_{nj} & \tilde{x}_{nm} \end{bmatrix} \end{array} \quad (3)$$

for all x_1, $x_1 \in X$ and all $\lambda \in [0,1]$. The triangular fuzzy number is commonly used in decision making due to its intuitive membership function, $\tilde{W} = [\tilde{w}_1 + \tilde{w}_2 + \cdots + \tilde{w}_m]$, given by:

$$\mu_A(x) = \begin{cases} 0 & \text{for}\, x < l, \\ \dfrac{x-l}{m-l} & \text{for}\, l \le x \le m, \\ \dfrac{u-x}{u-m} & \text{for}\, m \le x \le u, \\ 0 & \text{for}\, x > u, \end{cases} \quad (4)$$

in which l, m and u are real numbers with $l < m < u$. Outside the interval $[l, u]$, the pertinence degree is null, and m represents the point in which the pertinence degree is maximum. Trapezoidal fuzzy numbers are also frequently used in decision making processes [62,64].

3.1.3. Algebraic operations with fuzzy numbers

Given any real number K and two fuzzy triangular numbers $\tilde{A} = (l_1, m_1, u_1)$ and $\tilde{B} = (l_2, m_2, u_2)$, the main algebraic operations are expressed as follows [62,64]:

(1) Addition of two triangular fuzzy numbers

$$\tilde{A}(+)\tilde{B} = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \quad l_1 \ge 0,\ l_2 \ge 0 \quad (5)$$

(2) Multiplication of two triangular fuzzy numbers

$$\tilde{A}(\times)\tilde{B} = (l_1 \times l_2, m_1 \times m_2, u_1 \times u_2) \quad l_1 \ge 0,\ l_2 \ge 0 \quad (6)$$

(3) Subtraction of two triangular fuzzy numbers

$$\tilde{A}(-)\tilde{B} = (l_1 - l_2, m_1 - m_2, u_1 - u_2) \quad l_1 \ge 0,\ l_2 \ge 0 \quad (7)$$

(4) Division of two triangular fuzzy numbers

$$\tilde{A}(\div)\tilde{B} = (l_1 \div l_2, m_1 \div m_2, u_1 \div u_2) \quad l_1 \ge 0,\ l_2 \ge 0 \quad (8)$$

(5) Inverse of a triangular fuzzy number

$$\tilde{A}^{-1} = \left(\frac{1}{u_1}, \frac{1}{m_1}, \frac{1}{l_1}\right) \ge 0 \quad (9)$$

(6) Multiplication of a triangular fuzzy number by a constant

$$k \times \tilde{A} = (k \times l_1, k \times m_1, k \times u_1) \quad l_1 \ge 0,\ k \ge 0 \quad (10)$$

(7) Division of a triangular fuzzy number by a constant

$$\frac{\tilde{A}}{k} = \left(\frac{l_1}{k}, \frac{m_1}{k}, \frac{u_1}{k}\right) \quad l_1 \ge 0,\ k \ge 0 \quad (11)$$

3.2. Fuzzy TOPSIS

The Fuzzy TOPSIS method was proposed by Chen [17] to solve multicriteria decision making problems under uncertainty. Linguistic variables are used by the decision makers, D_r ($r = 1, \ldots, k$), to assess the weights of the criteria and the ratings of the alternatives. Thus, $\tilde{W}_r^j$ describes the weight of the jth criterion, C_j ($j = 1, \ldots, m$), given by the rth decision maker. Similarly, $\tilde{x}_{ij}^r$ describes the rating of the ith alternative, A_i ($i = 1, \ldots, n$), with respect to criterion j, given by the rth decision maker. Given that, the method comprises the following steps:

(i) Aggregate the weights of criteria and ratings of alternatives given by k decision makers, as expressed in Eqs. (12) and (13) respectively:

$$\tilde{w}_j = \frac{1}{k}[\tilde{w}_j^1 + \tilde{w}_j^2 + \cdots + \tilde{w}_j^k] \quad (12)$$

$$\tilde{x}_{ij} = \frac{1}{k}[\tilde{x}_{ij}^1 + \tilde{x}_{ij}^r + \cdots + \tilde{x}_j^k] \quad (13)$$

(ii) Assemble the fuzzy decision matrix of the alternatives ($\tilde{D}$) and the criteria ($\tilde{W}$), according to Eqs. (14) and (15):

$$\begin{array}{cc} & \begin{array}{cccc} C_1 & C_2 & C_j & C_m \end{array} \\ \tilde{D} = \begin{array}{c} A_1 \\ A_i \\ A_n \end{array} & \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \tilde{x}_{1j} & \tilde{x}_{1m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{n1} & \tilde{x}_{n2} & \tilde{x}_{nj} & \tilde{x}_{nm} \end{bmatrix} \end{array} \quad (14)$$

$$\tilde{W} = [\tilde{w}_1 + \tilde{w}_2 + \cdots + \tilde{w}_m] \quad (15)$$

(iii) Normalize the fuzzy decision matrix of the alternatives ($\tilde{D}$) using linear scale transformation. The normalized fuzzy decision matrix $\tilde{R}$ is given by:

$$\tilde{R} = [\tilde{r}_{ij}]_{m \times n} \quad (16)$$

$$\tilde{r}_{ij} = \left(\frac{l_{ij}}{u_j^+}, \frac{m_{ij}}{u_j^+}, \frac{u_{ij}}{u_j^+}\right) \text{ and } u_j^+ = \max_i u_{ij}\, (\text{benefit criteria}) \quad (17)$$

$$\tilde{r}_{ij} = \left(\frac{l_j^-}{u_{ij}}, \frac{l_j^-}{m_{ij}}, \frac{l_j^-}{l_{ij}}\right) \text{ and } l_j^- = \max_i l_{ij}\, (\text{cost criteria}) \quad (18)$$

(iv) Compute the weighted normalized decision matrix, $\tilde{V}$, by multiplying the weights of the evaluation criteria, $\tilde{w}_j$, by the elements $\tilde{r}_{ij}$ of the normalized fuzzy decision matrix

$$\tilde{V} = [\tilde{v}_{ij}]_{m \times n} \quad (19)$$

where $\tilde{v}_{ij}$ is given by Eq. (20).

$$\tilde{v}_{ij} = \tilde{x}_{ij} \times \tilde{w}_j \quad (20)$$

(v) Define the Fuzzy Positive Ideal Solution (FPIS, A^+) and the Fuzzy Negative Ideal Solution (FNIS, A^-), according to Eqs. (21) and (22).

$$A^+ = \{\tilde{v}_1^+, \tilde{v}_j^+, \ldots, \tilde{v}_m^+\} \quad (21)$$

$$A^- = \{\tilde{v}_1^-, \tilde{v}_j^-, \ldots, \tilde{v}_m^-\} \quad (22)$$

where $\tilde{v}_j^+ = (1, 1, 1)$ and $\tilde{v}_j^- = (0, 0, 0)$.

(vi) Compute the distances d_j^+ and d_i^- of each alternative from respectively $\tilde{v}_j^+$ and $\tilde{v}_j^-$ according to Eqs. (23) and (24)

$$d_i^+ = \sum\nolimits_{j=1}^{n} d_v(\tilde{v}_{ij}, \tilde{v}_j^+) \quad (23)$$

$$d_i^- = \sum_{j=1}^{n} d_v(\tilde{v}_{ij}, \tilde{v}_j^-) \quad (24)$$

where $d(.,.)$ represents the distance between two fuzzy numbers according to the vertex method. For triangular fuzzy numbers, this is expressed as in Eq. (25).

$$d(\tilde{x}, \tilde{z}) = \sqrt{\frac{1}{3}[(l_x - l_z)^2 + (m_x - m_z)^2 + (u_x - u_z)^2]} \quad (25)$$

(vii) Compute the closeness coefficient, CC_i, according to Eq. (26).

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad (26)$$

(viii) Define the ranking of the alternatives according to the closeness coefficient, CC_i, in decreasing order. The best alternative is closest to the FPIS and farthest to the FNIS.

3.3. Fuzzy AHP

Chang [10] proposed a Fuzzy AHP approach based on the extent analysis method, which is widely used in supplier selection problems [16,24]. This method uses linguistic variables to express the comparative judgments given by decision makers. Let $X = \{x_1, x_i, \ldots, x_n\}$ represent an object set and $G = \{g_1, g_j, \ldots, g_m\}$ a goal set. In the method proposed by Chang [10], each object, x_i, is taken and extent analysis is performed for each goal, g_j. Thus, m extent analysis values for each object can be obtained, with the following signs:

$$M_{gi}^1, M_{gi}^j, \ldots, M_{gi}^m, \quad i = 1, 2, \ldots, n \quad (27)$$

where all the M_{gi}^j (j = 1, 2, . . .,m) are triangular fuzzy numbers.

The method follows the steps described next.

(i) Compute the value of the fuzzy synthetic extent with respect to the ith object according to Eq. (28).

$$S_i = \sum_{j=1}^{m} M_{gi}^i \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^i\right]^{-1} \quad (28)$$

where $\sum_{j=1}^{m} M_{gi}^i$ is obtained by performing the fuzzy addition operation of m extent analysis values for a particular matrix such that

$$\sum_{j=1}^{m} M_{gi}^i = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right) \quad (29)$$

and $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^i\right]^{-1}$ is given by

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^i\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{m} u_i}, \frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{m} m_i}, \frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{m} l_i}\right) \quad (30)$$

(ii) Compute the degree of possibility of $S_2(l_2, m_2, u_2) \geq S_1 = (l_1, m_1, u_1)$, where S_2 and S_1 are given by Eq. (28). The degree of possibility between two fuzzy synthetic extents is defined as in Eq. (31)

$$V(S_2 \geq S_1) = \sup_{y \geq x}[\min(\mu_{S_2}(y), \mu_{S_1}(x))] \quad (31)$$

which can be equivalently expressed as in Eqs. (32) and (33).

$$V(S_2 \geq S_1) = hgt(S_1 \cap S_2) = \mu_{S_2}(d) \quad (32)$$

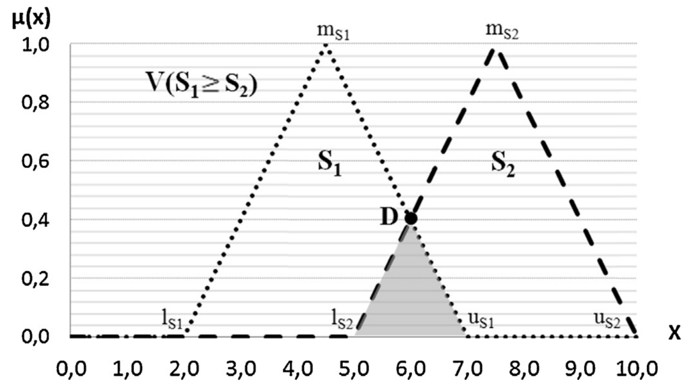

Fig. 2. The intersection between S_1 and S_2.

$$\mu_{S_2}(d) = \begin{Bmatrix} 1, & \text{if } m_2 \geq m_1 \\ 0, & \text{if } l_1 \geq u_2 \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise} \end{Bmatrix} \quad (33)$$

In Eqs. (32) and (33), d represents the ordinate of the highest intersection point D between μ_{S_1} and μ_{S_2}, as it can be seen in Fig. 2. The comparison between M_1 and M_2 requires the values of $V(S_2 \geq S_1)$ and $V(S_1 \geq S_2)$.

(iii) Compute the degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers S_i (i = 1, . . ., k). This is calculated according to Eq. (34).

$$\begin{aligned} &V(S \geq S_1, S_2, \ldots, S_k) \\ &= V[(S \geq S_1) \text{ and } (S \geq S_2) \text{ and } \cdots \text{ and } (S \geq S_k)] \\ &= \min V(S \geq S_i), \quad i = 1, 2, \ldots, k. \end{aligned} \quad (34)$$

(iv) Compute the vector W', which is given by Eq. (35).

$$W' = (d'(A_1), d'(A_2), \ldots, d'(A_k))^T \quad (35)$$

assuming that

$$d'(A_i) = \min V(S_i \geq S_j), \quad \text{for } i = 1, 2, .., k, \; j = 1, 2, \ldots, k, \; k \neq j \quad (36)$$

The normalized vector is indicated by

$$W = (d(A_1), d(A_2), \ldots, d(A_k))^T \quad (37)$$

where W is a non-fuzzy number calculated for each comparison matrix.

4. Application case in the automotive industry

A manufacturer of transmission cables for motorcycles needs to select a supplier of metallic components used in a variety of transmission cables. To select the best alternative, five potential suppliers were evaluated against five decision criteria. The evaluation of the potential suppliers in each criterion was made based on linguistic judgments given by the decision makers, a group of employees from the quality and purchase areas of the company. The criteria were defined by the decision makers, as follows:

- Quality (C_1): related to quality of conformance, quality management and after sale service quality.
- Price (C_2): related to the acquisition cost.
- Delivery (C_3): related to delivery time and reliability.
- Supplier profile (C_4): related to supplier reputation and financial health.
- Supplier relationship (C_5): related to the degree of cooperation and trust in the buyer–supplier relationship.

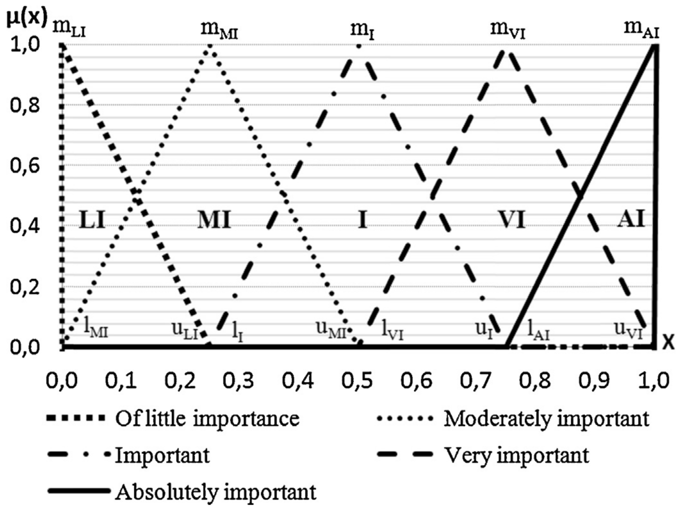

Fig. 3. Linguistic scale of the weights of the criteria.

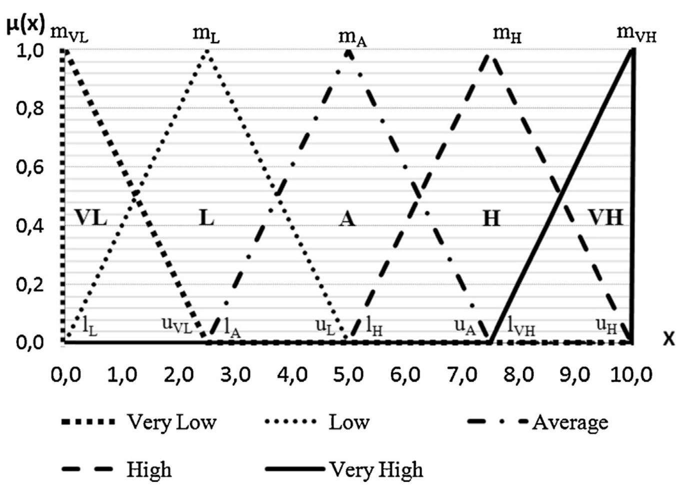

Fig. 4. Linguistic scale of the ratings of the alternatives.

The Fuzzy TOPSIS and Fuzzy AHP methods were applied to this case, as described next.

4.1. Fuzzy TOPSIS application

Evaluations of the weight of the criteria and the ratings of the alternatives were made by the decision makers according to the linguistic terms depicted in Figs. 3 and 4 respectively. Based on Chen [17], triangular fuzzy numbers (TFN) were used to specify the linguistic values of these variables, as presented in Tables 3 and 4.

Table 5 presents the linguistic judgments of the weights of the criteria and the ratings of the alternatives for the three decision makers involved in the selection process. The linguistic variables shown in Table 5 are converted into TFN. Table 6 presents the parameters of the TFN resulting from aggregation of the judgments presented in Table 5, which represents the fuzzy decision matrix. The normalized fuzzy decision matrix and the weighted normalized fuzzy decision matrix are represented respectively in Tables 7 and 8.

According to Chen [17], the Fuzzy Positive Ideal Solution (FPIS, A^+) and the Fuzzy Negative Ideal Solution (FNIS, A^-) were defined as

$$A^+ = [(1, 1, 1), (1, 1, 1), (1, 1, 1), (1, 1, 1), (1, 1, 1)]$$
$$A^- = [(0, 0, 0), (0, 0, 0), (0, 0, 0), (0, 0, 0), (0, 0, 0)]$$

The distances d_i^+ and d_i^- of the ratings of each alternative from A^+ and A^-, calculated according to Eqs. (23)–(25), are presented in Tables 9 and 10 respectively.

The global performance of each supplier alternative is given by the closeness coefficient, CC_i, calculated as in Eq. (26) and presented

Table 3
Linguistic scale to evaluate the weight of the criteria.

Linguistic terms	Fuzzy triangular number
Of little importance (VL)	(0.0, 0.0, 0.25)
Moderately important (MI)	(0.0, 0.25, 0.50)
Important (I)	(0.25, 0.50, 0.75)
Very important (VI)	(0.50, 0.75, 1.0)
Absolutely important (AI)	(0.75, 1.0, 1.0)

Table 4
Linguistic scale to evaluate the ratings of the alternative suppliers.

Linguistic terms	Fuzzy triangular number
Very low (VL)	(0.0, 0.0, 2.5)
Low (L)	(0.0, 2.5, 5.0)
Good (G)	(2.5, 5.0, 7.5)
High (H)	(5.0, 7.5, 10.0)
Excellent (EX)	(7.5, 10.0, 10.0)

Table 5
Linguistic ratings of the alternative suppliers by different decision makers.

	C_1	C_2	C_3	C_4	C_5
DM1					
A_1	G	H	G	G	L
A_2	VH	VH	VH	VH	H
A_3	H	H	VH	G	H
A_4	G	H	G	H	H
A_5	G	H	H	G	G
Weights of criteria	VI	AI	VI	I	VI
DM2					
A_1	G	G	G	H	G
A_2	VH	VH	VH	VH	VH
A_3	VH	H	VH	H	VH
A_4	H	VH	H	H	H
A_5	H	H	H	H	H
Weights of criteria	AI	AI	VI	I	I
DM3					
A_1	H	G	G	G	H
A_2	VH	VH	VH	VH	VH
A_3	VH	H	VH	G	VH
A_4	G	VH	H	H	G
A_5	L	G	G	G	G
Weights of criteria	AI	AI	AI	VI	I

Table 6
Fuzzy numbers of the aggregated ratings of the alternative suppliers.

	C_1	C_2	C_3	C_4	C_5
A_1	(3.33, 5.83, 8.33)	(3.33, 5.83, 8.33)	(2.50, 5.00, 7.50)	(3.33, 5.83, 8.33)	(0.25, 5.00, 7.50)
A_2	(7.50, 10.0, 10.0)	(7.50, 10.0, 10.0)	(7.50, 10.0, 10.0)	(7.50, 10.0, 10.0)	(6.67, 9.17, 10.0)
A_3	(6.67, 9.17, 10.0)	(5.00, 7.50, 10.0)	(7.50, 10.0, 10.0)	(3.33, 5.83, 8.33)	(6.67, 9.17, 10.0)
A_4	(3.33, 5.83, 8.33)	(6.67, 9.17, 10.0)	(4.17, 6.67, 9.17)	(5.00, 7.50, 10.0)	(4.17, 6.67, 9.17)
A_5	(2.50, 5.00, 7.50)	(4.17, 6.67, 9.17)	(4.17, 6.67, 9.17)	(3.33, 5.83, 8.33)	(3.33, 5.83, 8.33)
Weights of criteria	(0.67, 0.92, 1.00)	(0.75, 1.00, 1.00)	(0.67, 92, 1.00)	(0.42, 0.67, 0.92)	(0.33, 0.58, 0.83)

Table 7
Normalized fuzzy decision matrix.

	C_1	C_2	C_3	C_4	C_5
A_1	(0.33, 0.58, 0.83)	(0.33, 0.58, 0.83)	(0.25, 0.50, 0.75)	(0.33, 0.58, 0.83)	(0.25, 0.50, 0.75)
A_2	(0.75, 1.00, 1.00)	(0.75, 1.00, 1.00)	(0.75, 1.00, 1.00)	(0.75, 1.00, 1.00)	(0.67, 0.92, 1.00)
A_3	(0.67, 0.92, 1.00)	(0.50, 0.75, 1.00)	(0.75, 1.00, 1.00)	(0.33, 0.58, 0.83)	(0.67, 0.92, 1.00)
A_4	(0.33, 0.58, 0.83)	(0.67, 0.92, 1.00)	(0.42, 0.67, 0.92)	(0.50, 0.75, 1.00)	(0.42, 0.67, 0.92)
A_5	(0.25, 0.50, 0.75)	(0.42, 0.67, 0.92)	(0.42, 0.67, 0.92)	(0.33, 0.58, 0.83)	(0.33, 0.58, 0.83)

Table 8
Weighted normalized fuzzy decision matrix.

	C_1	C_2	C_3	C_4	C_5
A_1	(0.22, 0.53, 0.83)	(0.25, 0.58, 0.83)	(0.17, 0.45, 0.75)	(0.13, 0.39, 0.76)	(0.08, 0.29, 0.63)
A_2	(0.50, 0.92, 1.00)	(0.56, 1.00, 1.00)	(0.50, 0.92, 1.00)	(0.31, 0.67, 0.92)	(0.22, 0.53, 0.83)
A_3	(0.44, 0.84, 1.00)	(0.38, 0.75, 1.00)	(0.50, 0.92, 1.00)	(0.14, 0.39, 0.76)	(0.22, 0.53, 0.83)
A_4	(0.22, 0.53, 0.83)	(0.50, 0.92, 1.00)	(0.28, 0.61, 0.92)	(0.21, 0.50, 0.92)	(0.14, 0.39, 0.76)
A_5	(0.16, 0.46, 0.75)	(0.31, 0.67, 0.92)	(0.28, 0.61, 0.92)	(0.14, 0.39, 0.76)	(0.11, 0.34, 0.69)

Table 9
Distances of the ratings of each alternative from A^+ with respect to each criterion.

	C_1	C_2	C_3	C_4	C_5	d_i^+
$d(A_1, A^+)$	0.53	0.50	0.59	0.62	0.70	2.95
$d(A_2, A^+)$	0.29	0.25	0.29	0.44	0.53	1.81
$d(A_3, A^+)$	0.33	0.39	0.29	0.62	0.53	2.17
$d(A_4, A^+)$	0.53	0.29	0.48	0.54	0.62	2.47
$d(A_5, A^+)$	0.59	0.44	0.48	0.62	0.66	2.80

Table 10
Distances of the ratings of each alternative from A^- with respect to each criterion.

	C_1	C_2	C_3	C_4	C_5	d_i^-
$d(A_1, A^-)$	0.59	0.60	0.52	0.50	0.40	2.61
$d(A_2, A^-)$	0.83	0.88	0.83	0.68	0.59	3.81
$d(A_3, A^-)$	0.80	0.75	0.83	0.50	0.59	3.47
$d(A_4, A^-)$	0.59	0.83	0.66	0.61	0.50	3.19
$d(A_5, A^-)$	0.52	0.69	0.66	0.50	0.45	2.80

in Table 11. Finally, this calculation led to the outranking presented in Table 11, meaning that supplier A_5 is the best alternative, followed by A_1, A_2, A_3 and A_4, in this order.

4.2. Fuzzy AHP application

The linguistic terms presented in Fig. 5 were used by the decision makers to comparatively evaluate the weight of the criteria and the ratings of the alternatives. Following Chang [10], TFN were used to specify the linguistic values of these variables, as presented in Table 12.

Table 11
Outranking of alternative suppliers according to Fuzzy TOPSIS.

Suppliers	CC_i	Rank
A_1	0.47	5th
A_2	0.68	1st
A_3	0.62	2nd
A_4	0.56	3rd
A_5	0.50	4th

Table 12
Comparative linguistic scale for ratings of alternatives and weights of criteria.

Linguistic terms	Fuzzy triangular number
Equally preferable (EQ)	(1.0, 1.0, 3.0)
Slightly preferable (SP)	(1.0, 3.0, 5.0)
Fairly preferable (FP)	(3.0, 5.0, 7.0)
Extremely preferable (XP)	(5.0, 7.0, 9.0)
Absolutely preferable (AP)	(7.0, 9.0, 9.0)

Table 13 presents the comparative judgments of the weights of the criteria made by the three decision makers involved already converted into TFN. The results of aggregation of these fuzzy values are presented in Table 14 and were obtained by the arithmetic mean of the judgments.

Likewise, the fuzzy values of the aggregated comparative judgments of the alternative suppliers for each criterion made by the three decision makers are presented in Tables 15–19.

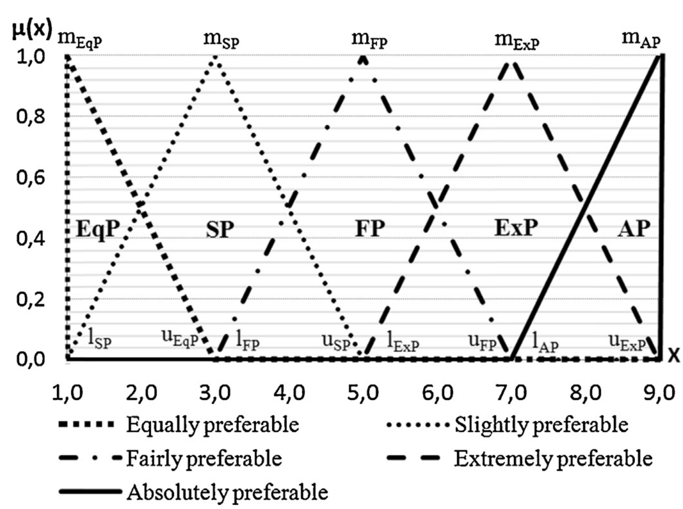

Fig. 5. Comparative linguistic scale of the weights of the criteria and ratings of the alternatives.

Table 13
Comparative judgments of the weights of the criteria made by decision makers.

	C_1	C_2	C_3	C_4	C_5
DM1					
C_1	(1.0, 1.0, 1.0)	(0.14, 0.20, 0.33)	(1.0, 1.0, 3.0)	(7.0, 9.0, 9.0)	(1.0, 3.0, 5.0)
C_2	(3.0, 5.0, 7.0)	(1.0, 1.0, 1.0)	(5.0, 7.0, 9.0)	(7.0, 9.0, 9.0)	(3.0, 5.0, 7.0)
C_3	(0.33, 1.0, 1.0)	(0.11, 0.14, 0.20)	(1.0, 1.0, 1.0)	(3.0, 5.0, 7.0)	(1.0, 3.0, 5.0)
C_4	(0.11, 0.11, 0.14)	(0.11, 0.11, 0.14)	(0.14, 0.20, 0.33)	(1.0, 1.0, 1.0)	(0.20, 0.33, 1.0)
C_5	(0.20, 0.33, 1.0)	(0.14, 0.20, 0.33)	(0.20, 0.33, 1.0)	(1.0, 3.0, 5.0)	(1.0, 1.0, 1.0)
DM2					
C_1	(1.0, 1.0, 1.0)	(1.0, 3.0, 5.0)	(3.0, 5.0, 7.0)	(3.0, 5.0, 7.0)	(3.0, 5.0, 7.0)
C_2	(0.20, 0.33, 1.0)	(1.0, 1.0, 1.0)	(3.0, 5.0, 7.0)	(1.0, 3.0, 5.0)	(1.0, 3.0, 5.0)
C_3	(0.14, 0.20, 0.33)	(0.14, 0.20, 0.33)	(1.0, 1.0, 1.0)	(1.0, 1.0, 3.0)	(1.0, 1.0, 3.0)
C_4	(0.14, 0.20, 0.33)	(0.20, 0.33, 1.0)	(0.33, 1.0, 1.0)	(1.0, 1.0, 1.0)	(1.0, 1.0, 3.0)
C_5	(0.14, 0.20, 0.33)	(0.20, 0.33, 1.0)	(0.33, 1.0, 1.0)	(0.33, 1.0, 1.0)	(1.0, 1.0, 1.0)
DM3					
C_1	(1.0, 1.0, 1.0)	(1.0, 1.0, 3.0)	(1.0, 1.0, 3.0)	(1.0, 3.0, 5.0)	(1.0, 3.0, 5.0)
C_2	(0.33, 1.0, 1.0)	(1.0, 1.0, 1.0)	(1.0, 1.0, 3.0)	(1.0, 3.0, 5.0)	(1.0, 3.0, 5.0)
C_3	(0.33, 1.0, 1.0)	(0.33, 1.0, 1.0)	(1.0, 1.0, 1.0)	(1.0, 1.0, 3.0)	(1.0, 3.0, 5.0)
C_4	(0.20, 0.33, 1.0)	(0.20, 0.33, 1.0)	(0.33, 1.0, 1.0)	(1.0, 1.0, 1.0)	(1.0, 1.0, 3.0)
C_5	(0.20, 0.33, 1.0)	(0.20, 0.33, 1.0)	(0.20, 0.33, 1.0)	(0.33, 1.0, 1.0)	(1.0, 1.0, 1.0)

Table 14
Fuzzy numbers of the aggregated weights of the criteria.

	C_1	C_2	C_3	C_4	C_5
C_1	(1.0, 1.0, 1.0)	(0.71, 1.40, 2.78)	(1.67, 2.33, 4.33)	(3.67, 5.67, 7.0)	(1.67, 3.67, 5.67)
C_2	(1.18, 2.11, 3.00)	(1.0, 1.0, 1.0)	(3.00, 4.33, 6.33)	(3.0, 5.0, 6.33)	(1.67, 3.67, 5.67)
C_3	(0.27, 0.73, 0.78)	(0.19, 0.45, 0.51)	(1.0, 1.0, 1.0)	(1.67, 3.0, 5.0)	(1.00, 2.33, 4.33)
C_4	(0.19, 0.44, 0.49)	(0.17, 0.26, 0.71)	(0.23, 0.51, 0.78)	(1.0, 1.0, 1.0)	(0.73, 0.78, 2.33)
C_5	(0.18, 0.29, 0.78)	(0.18, 0.29, 0.78)	(0.24, 0.56, 1.00)	(0.56, 1.67, 2.33)	(1.0, 1.0, 1.0)

Table 15
Fuzzy numbers of the supplier alternative ratings related to criterion C_1.

	A_1	A_2	A_3	A_4	A_5
A_1	(1.00, 1.00, 1.00)	(0.73, 1.44, 3.00)	(0.18, 0.29, 0.78)	(0.73, 1.44, 3.00)	(0.73, 0.78, 2.33)
A_2	(0.51, 1.44, 2.33)	(1.00, 1.00, 1.00)	(1.40, 2.11, 3.67)	(1.67, 2.33, 4.33)	(1.40, 2.78, 4.33)
A_3	(1.67, 3.67, 5.67)	(0.49, 1.40, 2.11)	(1.00, 1.00, 1.00)	(1.40, 2.78, 4.33)	(1.40, 2.78, 4.33)
A_4	(0.51, 1.44, 2.33)	(0.27, 0.73, 0.78)	(0.45, 1.18, 2.11)	(1.00, 1.00, 1.00)	(1.13, 1.89, 3.00)
A_5	(0.56, 1.67, 2.33)	(0.45, 1.18, 2.11)	(0.45, 1.18, 2.11)	(0.71, 2.07, 3.44)	(1.00, 1.00, 1.00)

Table 16
Fuzzy numbers of the supplier alternative ratings related to criterion C_2.

	A_1	A_2	A_3	A_4	A_5
A_1	(1.00, 1.00, 1.00)	(0.43, 1.10, 1.78)	(0.47, 1.22, 2.33)	(0.45, 0.51, 1.44)	(0.73, 0.78, 2.33)
A_2	(4.07, 5.44, 6.33)	(1.00, 1.00, 1.00)	(1.00, 1.67, 3.67)	(0.73, 0.78, 2.33)	(1.40, 2.78, 4.33)
A_3	(0.73, 2.11, 3.67)	(0.29, 0.78, 1.00)	(1.00, 1.00, 1.00)	(0.73, 0.78, 2.33)	(1.67, 3.00, 5.00)
A_4	(1.44, 3.00, 4.33)	(0.56, 1.67, 2.33)	(0.56, 1.67, 2.33)	(1.00, 1.00, 1.00)	(1.67, 3.00, 5.00)
A_5	(0.78, 1.67, 3.00)	(0.45, 1.18, 2.11)	(0.23, 0.51, 0.78)	(0.23, 0.51, 0.78)	(1.00, 1.00, 1.00)

Table 17
Fuzzy numbers of the supplier alternative ratings related to criterion C_3.

	A_1	A_2	A_3	A_4	A_5
A_1	(1.00, 1.00, 1.00)	(1.67, 3.00, 5.00)	(1.00, 1.67, 3.67)	(1.00, 1.67, 3.67)	(0.73, 1.44, 3.00)
A_2	(0.22, 0.51, 0.78)	(1.00, 1.00, 1.00)	(1.00, 1.00, 3.00)	(0.73, 0.78, 2.33)	(0.73, 1.44, 3.00)
A_3	(0.28, 0.78, 1.00)	(0.33, 1.00, 1.00)	(1.00, 1.00, 1.00)	(1.00, 1.67, 3.67)	(0.72, 0.73, 2.11)
A_4	(0.28, 0.78, 1.00)	(0.56, 1.67, 2.33)	(0.29, 0.78, 1.00)	(1.00, 1.00, 1.00)	(0.73, 1.44, 3.00)
A_5	(0.51, 1.44, 2.33)	(0.51, 1.44, 2.33)	(1.22, 2.33, 3.00)	(0.51, 1.44, 2.33)	(1.00, 1.00, 1.00)

Table 18
Fuzzy numbers of the supplier alternative ratings related to criterion C_4.

	A_1	A_2	A_3	A_4	A_5
A_1	(1.00, 1.00, 1.00)	(0.73, 0.78, 2.33)	(1.67, 3.66, 5.67)	(0.73, 1.44, 3.00)	(1.78, 2.51, 3.44)
A_2	(0.56, 1.66, 2.33)	(1.00, 1.00, 1.00)	(3.67, 5.00, 7.00)	(1.40, 2.77, 4.33)	(0.71, 2.06, 3.44)
A_3	(0.18, 0.29, 0.78)	(0.20, 0.43, 0.47)	(1.00, 1.00, 1.00)	(0.73, 0.78, 2.33)	(0.47, 0.56, 1.67)
A_4	(0.51, 1.44, 2.33)	(0.45, 1.18, 2.11)	(0.56, 1.67, 2.33)	(1.00, 1.00, 1.00)	(0.47, 1.22, 2.33)
A_5	(0.44, 1.11, 1.84)	(1.13, 1.89, 3.00)	(0.78, 2.33, 3.67)	(0.73, 2.11, 3.67)	(1.00, 1.00, 1.00)

Table 19
Fuzzy numbers of the supplier alternative ratings related to criterion C_5.

	A_1	A_2	A_3	A_4	A_5
A_1	(1.00, 1.00, 1.00)	(1.40, 2.11, 3.67)	(1.40, 2.11, 3.67)	(0.73, 1.44, 3.00)	(0.45, 0.51, 1.44)
A_2	(0.49, 1.40, 2.11)	(1.00, 1.00, 1.00)	(1.00, 1.67, 3.67)	(2.07, 2.77, 4.33)	(0.46, 0.56, 1.67)
A_3	(0.49, 1.40, 2.11)	(0.29, 0.78, 1.00)	(1.00, 1.00, 1.00)	(0.73, 1.44, 3.00)	(0.46, 0.56, 1.67)
A_4	(0.51, 1.44, 2.33)	(0.48, 1.38, 2.07)	(0.51, 1.44, 2.33)	(1.00, 1.00, 1.00)	(0.70, 0.71, 2.07)
A_5	(1.44, 3.00, 4.33)	(0.78, 2.33, 3.67)	(0.78, 2.33, 3.67)	(1.89, 3.00, 3.67)	(1.00, 1.00, 1.00)

Table 20
Consistency Ratios of comparative matrices.

	C_1	C_2	C_3	C_4	C_5	Weight
DM1	0.16	0.10	0.08	0.13	0.15	0.12
DM2	0.18	0.18	0.19	0.15	0.17	0.09
DM3	0.15	0.16	0.16	0.14	0.17	0.15

The consistency ratios (CR) for each comparative matrix were calculated according to Saaty [65] and Facchinetti et al. [66] and are presented in Table 20. As it can be seen, all the values of CR are below 0.20, which confirms the consistency of the comparative judgments.

The values of the fuzzy synthetic extent for the criteria matrix are:

$$S_{C1} = (8.71, 14.07, 20.78) \otimes \left(\frac{1}{65.94}, \frac{1}{44.48}, \frac{1}{27.17}\right) = (0.13, 0.32, 0.76)$$

$$S_{C2} = (9.84, 16.11, 22.33) \otimes \left(\frac{1}{65.94}, \frac{1}{44.48}, \frac{1}{27.17}\right) = (0.15, 0.36, 0.82)$$

$$S_{C3} = (4.13, 7.51, 11.62) \otimes \left(\frac{1}{65.94}, \frac{1}{44.48}, \frac{1}{27.17}\right) = (0.06, 0.17, 0.42)$$

$$S_{C4} = (2.32, 2.99, 5.31) \otimes \left(\frac{1}{65.94}, \frac{1}{44.48}, \frac{1}{27.17}\right) = (0.04, 0.07, 0.20)$$

$$S_{C5} = (2.16, 3.80, 5.89) \otimes \left(\frac{1}{65.94}, \frac{1}{44.48}, \frac{1}{27.17}\right) = (0.03, 0.08, 0.22)$$

The degrees of possibility of these fuzzy values, computed as in Eqs. (32) and (33) are:

$$V(S_{C1} \geq S_{C2}) = 0.93$$

$$V(S_{C1} \geq S_{C3}) = 1.00$$

$$V(S_{C1} \geq S_{C4}) = 1.00$$

$$V(S_{C1} \geq S_{C5}) = 1.00$$

$$V(S_{C2} \geq S_{C1}) = 1.00$$

$$V(S_{C2} \geq S_{C3}) = 1.00$$

$$V(S_{C2} \geq S_{C4}) = 1.00$$

$$V(S_{C2} \geq S_{C5}) = 1.00$$

$$V(S_{C3} \geq S_{C1}) = 0.67$$

$$V(S_{C3} \geq S_{C2}) = 0.59$$

$$V(S_{C3} \geq S_{C4}) = 1.00$$

$$V(S_{C3} \geq S_{C5}) = 1.00$$

$$V(S_{C4} \geq S_{C1}) = 0.20$$

$$V(S_{C4} \geq S_{C2}) = 0.14$$

$$V(S_{C4} \geq S_{C3}) = 0.57$$

$$V(S_{C4} \geq S_{C5}) = 0.90$$

$$V(S_{C5} \geq S_{C1}) = 0.27$$

$$V(S_{C5} \geq S_{C2}) = 0.20$$

$$V(S_{C5} \geq S_{C3}) = 0.65$$

$$V(S_{C5} \geq S_{C4}) = 1.00$$

Therefore, the weight vector W', computed as in Eqs. (35) and (36), is:

$$d'(C_1) = V(S_{C1} \geq S_{C2}, S_{C3}, S_{C4}, S_{C5}) = \min(0.93, 1.00, 1.00, 1.00) = 0.93$$

$$d'(C_2) = V(S_{C2} \geq S_{C1}, S_{C3}, S_{C4}, S_{C5}) = \min(1.00, 1.00, 1.00, 1.00) = 1.00$$

$$d'(C_3) = V(S_{C3} \geq S_{C1}, S_{C2}, S_{C4}, S_{C5}) = \min(0.67, 0.59, 1.00, 1.00) = 0.59$$

$$d'(C_4) = V(S_{C4} \geq S_{C1}, S_{C2}, S_{C3}, S_{C5}) = \min(0.20, 0.14, 0.57, 0.90) = 0.14$$

Table 21
Weight vectors of the criteria and alternative suppliers.

	C_1	C_2	C_3	C_4	C_5
A_1	0.17	0.15	0.24	0.23	0.20
A_2	0.23	0.26	0.18	0.26	0.20
A_3	0.24	0.21	0.18	0.11	0.16
A_4	0.20	0.24	0.18	0.18	0.18
A_5	0.16	0.14	0.22	0.22	0.26
Weights of criteria	0.33	0.35	0.21	0.05	0.07

Table 22
Global performance of alternatives and outranking.

Supplier	Global performance	*Rank*
A_1	0.80	5th
A_2	1.00	1st
A_3	0.90	2nd
A_4	0.87	3rd
A_5	0.81	4th

$$d'(C_5) = V(S_{C5} \geq S_{C1}, S_{C2}, S_{C3}, S_{C4}) = \min(0.27, 0.20, 0.65, 1.00) = 0.20$$

$$W' = (0.93, 1.00, 0.59, 0.14, 0.20)$$

After normalization the weight vector is (0.33, 0.35, 0.21, 0.05, 0.07).

Calculation of the weight vectors for the alternative evaluation matrices followed the same procedure. The weight vectors from Tables 15–19 are respectively (0.71, 0.97, 1.00, 0.83, 0.69), (0.58, 1.00, 0.80, 0.94, 0.53), (1.00, 0.77, 0.76, 0.78, 0.93), (0.88, 1.00, 0.40, 0.68, 0.82) and (0.80, 0.80, 0.63, 0.69, 1.00).

Table 21 summarizes the normalized weight vectors of the criteria and alternative suppliers.

For supplier alternative A_1, the global performance was computed as:

$$D(A_1) = (d'(A_{1_{C1}}) \times d'(C_1) + d'(A_{1_{C2}}) \times d'(C_2) + d'(A_{1_{C3}}) \times d'(C_3) + d'(A_{1_{C4}}) \times d'(C_4) + d'(A_{1_{C5}}) \times d'(C_5)) = 0.60$$

The global performance for the other alternative suppliers was computed similarly. Table 22 presents the global performance for all the alternatives and their ranking position. Therefore, following this procedure, similarly to the application of Fuzzy TOPSIS, supplier A_5 is the best evaluated alternative, followed by A_1, A_2, A_3 and A_4, in this order.

5. Comparative analysis of Fuzzy TOPSIS and Fuzzy AHP methods

Comparison of both methods was based on a set of required characteristics of the techniques so as to adequately deal with the problem of supplier selection, as presented in Section 2. The following factors were considered: adequacy to changes of alternatives or criteria; agility in the decision process; computational complexity; adequacy to supporting group decision making; the amount of alternative suppliers and criteria; and modeling of uncertainty.

5.1. Adequacy to changes of alternatives

In the supplier selection process, the evaluation of a different set of supply alternatives may require the inclusion or exclusion of alternatives. In this case, the selection method must produce a consistent preference order of alternatives.

In the Fuzzy AHP application case, with five alternatives and equal weights for all the criteria, the outranking is $A_2 > A_5 > A_1 > A_4 > A_3$, as illustrated in Fig. 6a. To test the Fuzzy AHP method, an additional supply alternative (A_6) was evaluated. Five tests were performed, each one with an additional alternative with a rating equal to one of the five existing alternatives. For most of the tests, the results have shown no significant changes in the alternative ranking. However, when the additional supply alternative has a rating equal to the best alternative (A_2 in Fig. 6a), the resulting preference order changes considerably. In this case, what was the worst alternative, A_3, becomes the best one, as shown in Fig. 6b, which is not expected in supplier selection problems. This inversion of alternatives, known as ranking reversal, already pointed out by other studies as a flaw in the AHP method [67–69] also happens in the Fuzzy AHP. On the other hand, in the Fuzzy TOPSIS application case, the same sequence of tests has caused no change at all to the alternative final scores. The order of preferences remained the same in all the tests, with the additional alternative having the same ranking number as the equal rating alternative.

5.2. Adequacy to changes of criteria

In some purchasing situations, we may also need to change some of the criteria used to evaluate the suppliers. In this case, the criteria importance order produced by the selection method must be consistent as well.

In the Fuzzy AHP application case, with the five criteria and respective weights, the criteria importance order given by the method was $C_2 > C_1 > C_3 > C_5 > C_4$. To evaluate the effect of adding a new criterion, five tests were carried out, each one with the additional criterion with a weight equal to one of the five existing criteria. For most of the tests, the results have shown no significant changes in the importance order. Yet, when the additional criterion

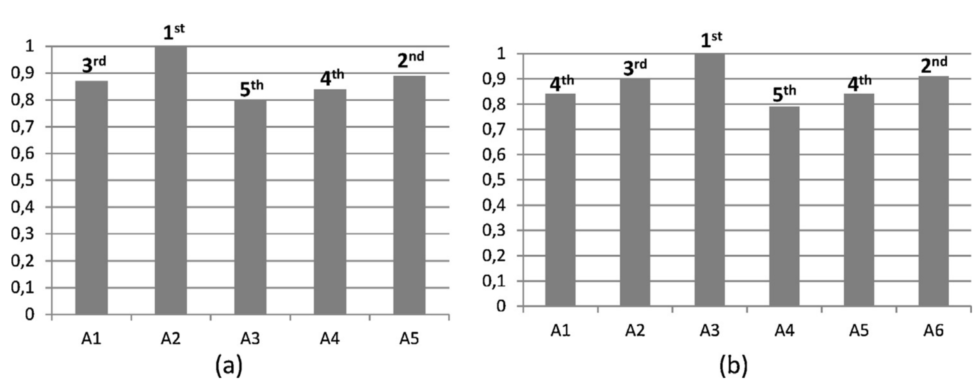

Fig. 6. Results of tests of changes of alternatives, Fuzzy AHP.

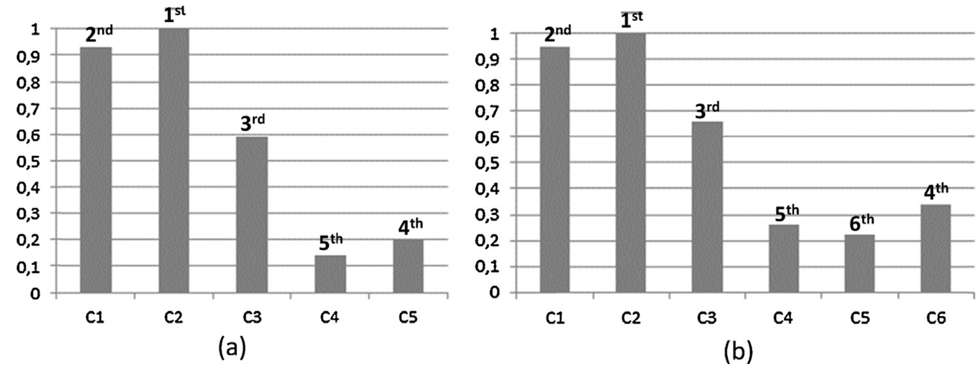

Fig. 7. Results of tests of changes of criteria, Fuzzy AHP.

has a weight equal to the one of C_5, then there is an inversion of the importance order. This indicates that the ranking reversal can also happen when there is a change of criteria. In the Fuzzy TOPSIS application case, adding a new criterion has caused no change at all to the criteria importance order.

Furthermore, a test was performed to evaluate the effect of excluding a criterion. The starting point was the criteria importance order given by the Fuzzy AHP application case ($C_2 > C_1 > C_3 > C_5 > C_4$), as illustrated in Fig. 7a. When the criterion C_5 was excluded, although the importance order was kept the same, the weight of criterion C_4 was reduced to zero. Because of the comparative judgment, the degree of possibility $V(S_{C4} \geq S_{C1})$ equals to zero. As the calculation of the weight vector uses the *MIN* operator, the resulting weight of criterion C_4 is null. Consequently this criterion does not count at all to the evaluation of the alternatives. The same effect was observed with the exclusion of a second criterion, C_4. It was observed that this problem of nulling the weight of a criterion will always happen when the difference between the synthetic extents of two criteria are large enough such that there is no intersection between them and consequently the degree of possibility is zero. Appendix A presents another example in detail in which 5 criteria are considered for weighting and the problem of null weight happens for 2 of them. On the other hand, in the Fuzzy TOPSIS application case, this problem did not happen. This is due to the fact that the criteria matrix ($\tilde{W}$) is computed using arithmetic mean between fuzzy numbers, which will never lead to a null weight.

5.3. Agility in the decision process

This factor evaluates the amount of judgments required from specialists in both methods. Considering n the number of suppliers and m the number of criteria, in the Fuzzy TOPSIS method, m judgments for each of the n alternatives are required, in addition to the m judgments related to the weight of the criteria. This can be expressed as in Eq. (38).

$$J_{n,m}^{\text{TOPSIS}} = m + nm = m(n+1) \tag{38}$$

In the case of the Fuzzy AHP method, the number of required judgments for a decision matrix A_{ixi} is:

$$J_{A_{i,i}} = i\frac{i-1}{2} \tag{39}$$

Since there are m matrices of size $n \times n$ (one for each decision criterion) in addition to the decision matrix of size $m \times m$ related to the weight of the criteria, the total number of required judgment is

$$J_{n,m}^{\text{AHP}} = m\frac{m-1}{2} + m\left[n\frac{n-1}{2}\right] \tag{40}$$

Based on Eqs. (38) and (40), Fig. 8 presents the number of judgments for both methods when the number of alternatives and criteria vary from 2 to 9. It can be seen that as the number of criteria and alternatives increase, the number of required judgment using Fuzzy AHP is in general greater than that using Fuzzy TOPSIS. In the application case, the Fuzzy TOPSIS required 30 judgments while the Fuzzy AHP required 60 judgments. If there are 9 alternatives and 9 criteria, the Fuzzy AHP requires four times more judgments than the Fuzzy TOPSIS. On the other hand, when there are few criteria and alternative ($J_{2\times2}, J_{2\times3}, J_{2\times4}, J_{3\times2}$), the required judgments using Fuzzy TOPSIS is greater than when using Fuzzy AHP. An exception is made to $J_{3\times3}$ and $J_{2\times5}$, when both methods require the same number of judgments. Therefore, it can be said that the Fuzzy TOPSIS method performs better than the Fuzzy AHP in regard to the level of interaction with decision makers to data collection. In this sense, Fuzzy TOPSIS provides greater agility in the decision process than Fuzzy AHP.

5.4. Computational complexity

The computational complexity of both methods was evaluated considering only the time complexity. Similarly to Chang [10], the time complexity, T, was appraised based on the number of times of multiplications within the algorithms. In this study, exponentiation and logical operations were additionally used as a measure of time complexity.

Considering there are n alternative suppliers and m criteria, the Fuzzy TOPSIS method requires $3nm$ operations to compute

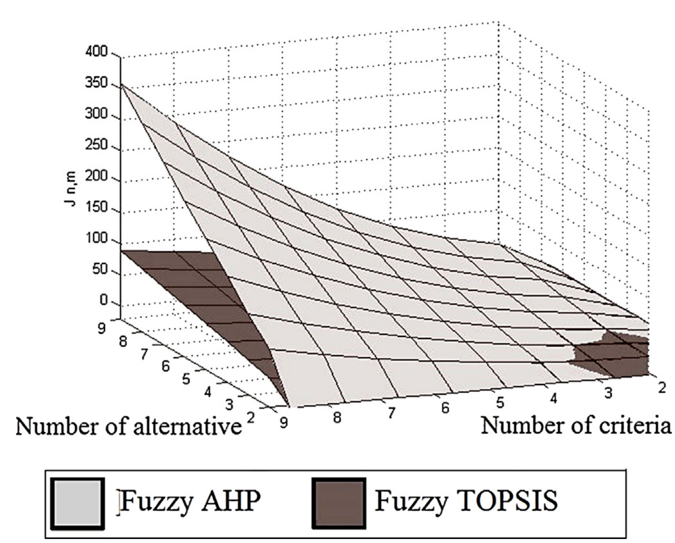

Fig. 8. results of tests of agility in the decision process.

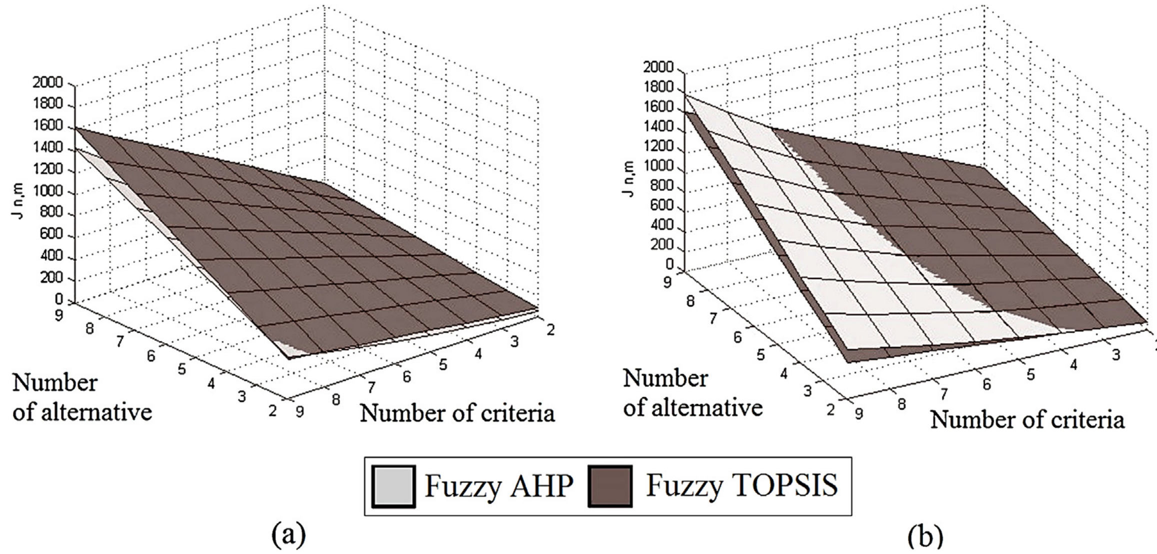

Fig. 9. Results of tests of time complexity.

the normalized decision matrix, $3nm$ operations to compute the weighted decision matrix and $14nm$ operations to compute the distances d_i^+ and d_i^-. Therefore, the time complexity, $T_{n,m}$, of the Fuzzy TOPSIS method is given by Eq. (41).

$$T_{n,m} = 3nm + 3nm + 7nm + 7nm = 20nm \quad (41)$$

Following the same approach, the Fuzzy AHP method requires $6m(n+1)$ operations to compute the fuzzy synthetic extent to all the decision matrices, $nm(n-1) + n(n-1)$ to compute the degrees of possibility, $n(m+1)$ to normalize the vector W' and finally nm operations to compute the global performance. Thus, the time complexity, $T_{n,m}$, of the Fuzzy AHP method is given by Eq. (42).

$$\begin{aligned} T'_{n,m} &= 6m(n+1) + nm(n-1) + n(n-1) + n(m+1) + nm \\ &= n^2(m+1) + m(7n+6) \end{aligned} \quad (42)$$

The graphics in Fig. 9a show the time complexity variation as a function of number of alternatives for different numbers of criteria for both methods. It can be seen that in general Fuzzy AHP performs better than Fuzzy TOPSIS. In the application case, the Fuzzy TOPSIS method required 500 operations while the Fuzzy AHP method required 355 operations. However, when the consistency tests of the judgment matrices are performed, the time complexity of the Fuzzy AHP method, $T'_{n,m}$, increases by a factor of $4n(m+1)$. In this case, the Fuzzy TOPSIS method performs slightly better than the Fuzzy AHP as it increases the number of alternatives, although in most cases Fuzzy AHP still performs better, as shown in Fig. 9b. In the application case, even with the consistency test, the Fuzzy AHP method required 475 operations, slightly less than the Fuzzy TOPSIS.

5.5. Adequacy to supporting group decision making

Both methods allow aggregation of judgments of more than one decision maker. In the case of the Fuzzy TOPSIS method, aggregation of different judgments is made according to Eqs. (12) and (13) for the weights of the criteria and the ratings of the alternative suppliers. In the case of the Fuzzy AHP, although this is not explicitly considered in the method proposed by Chang [10], he suggests that aggregation be made using the arithmetic mean of the judgments.

Since the amount of data required by the Fuzzy AHP method is greater than that required by the Fuzzy TOPSIS, increasing the number of decision makers will consequently cause a larger increase in the time complexity of the Fuzzy AHP when compared with the TOPSIS method. Therefore, although both methods support group decision making, due to the impact on time complexity, the Fuzzy TOPSIS method is preferable.

Even though both methods compute aggregation based on fuzzy arithmetic mean, an alternative approach would be to weight the judgments of the different decision makers and aggregate the data by computing a weighted mean. For instance, the procurement staff is better able to judge the performance of suppliers and therefore their judgments should be more relevant than the judgments of others not so involved with procurement.

5.6. The number of alternative suppliers and criteria

The Fuzzy TOPSIS method does not impose any restriction on the number of alternatives or criteria used in the selection process. On the other hand, the comparative analysis of the Fuzzy AHP method imposes some limitation on the number of criteria and alternatives. Saaty [65] suggests that the number criteria or alternatives to be compared using AHP be limited to nine so as not to compromise human judgment and its consistency. This suggestion applies equally to the Fuzzy AHP method. In the application case, with five criteria and five alternatives, the use of the Fuzzy AHP method was perfectly viable. Although the limitation of the number of criteria can be alleviated by deploying the criteria into the Fuzzy AHP hierarchy structure, the number of alternatives imposes a real limitation. Therefore, the choice of the method depends on the particularities of the circumstances at hand. For instance, when selecting a new supplier for a new product, with many potential suppliers, the Fuzzy TOPSIS is a better choice.

5.7. Modeling of uncertainty

Both methods utilize fuzzy set theory to deal with the inherent lack of precision of the data used in the supplier selection decision process. In both methods the fuzzy number morphology is the main resource for quantifying imprecision. Due to the vagueness of judgments of qualitative variables, the parameters of the triangular membership functions can be chosen so as to better represent the linguistic terms used by each decision maker to evaluate the alternatives regarding different decision criteria.

Table 23
Comparative linguistic judgments for replacing of supplier.

Regarding C_1	A_1	A_2	Regarding C_3	A_1	A_2
A_1	(1.0, 1.0, 1.0)	(0.2, 0.33, 1.0)	A_1	(1.0, 1.0, 1.0)	(1.0, 1.0, 3.0)
A_2	(1.0, 3.0, 5.0)	(1.0, 1.0, 1.0)	A_2	(0.33, 1.0, 1.0)	(1.0, 1.0, 1.0)
Regarding C_2	A_1	A_2	Regarding C_4	A_1	A_2
A_1	(1.0, 1.0, 1.0)	(0.11, 0.14, 0.14)	A_1	(1.0, 1.0, 3.0)	(0.2, 0.33, 1.0)
A_2	(7.0, 9.0, 9.0)	(1.0, 1.0, 1.0)	A_2	(1.0, 3.0, 5.0)	(1.0, 1.0, 1.0)

Table 24
Summarized comparative analysis of Fuzzy TOPSIS and Fuzzy AHP.

Parameters of comparison	Comparison of Fuzzy TOPSIS and Fuzzy AHP
Adequacy to changes of alternatives	Fuzzy AHP is prone to ranking reversal when including a new alternative, while Fuzzy TOPSIS produces consistent preference order
Adequacy to changes of criteria	In Fuzzy AHP ranking reversal also happens when a new criteria is included. Fuzzy TOPSIS produces consistent important order. Fuzzy AHP can yield null weight for selected criteria, while Fuzzy TOPSIS never leads to a null weight
Agility in the decision process	Fuzzy TOPSIS performs better than Fuzzy AHP in most cases except when there are very few criteria and suppliers
Time complexity	Fuzzy AHP performs better than Fuzzy TOPSIS in most cases. If Fuzzy AHP consistency test is included, as it increases the number of alternatives, then Fuzzy TOPSIS surpass Fuzzy AHP
Support to group decision making	Adequate for both methods. Aggregation based on fuzzy arithmetic mean. Alternative approach could be based on weighted mean
Number of criteria and alternative suppliers	No Limitation for Fuzzy TOPSIS. Fuzzy AHP limit the number of criteria and alternative. Fuzzy AHP allows inclusion of subcriteria into a hierarchy structure
Modeling of uncertainty	Both methods are adequate to deal with imprecision and subjectivity in supplier selection problems. But Fuzzy AHP is more appropriate when the purpose is to replace a supplier

In the case of Fuzzy AHP method, the use of pairwise comparisons by means of comparative linguistic variables is itself a way to deal with imprecision. This feature makes this method more appropriate than the Fuzzy TOPSIS when the purpose is to replace a supplier. As an example, Table 23 presents the judgments of two suppliers when compared in respect to four criteria of equal weights aiming at evaluating the benefit of replacing A_1 for A_2. In this case, the supplier global performance given as a relative measure facilitates the interpretation of the decision makers (equivalent to 0.32 for A_1 and 0.68 for A_2). Other advantages of the Fuzzy AHP compared to Fuzzy TOPSIS in this example are fewer judgments and less computational complexity.

6. Conclusion

This paper presented a new study comparing the Fuzzy AHP and the Fuzzy TOPSIS methods in regard to seven factors that are particularly relevant to the problem of supplier selection. This paper also presented the application of both methods to a case of supplier selection, in order to illustrate and clarify the use of these techniques for the problem of supplier selection. The comparative evaluation of the techniques in respect to changes of alternatives or criteria, agility and computational complexity was based on computational tests considering several scenarios of supplier selection. The performance of the methods concerning changes of alternative or criteria was evaluated through five tests based on inclusion and exclusion of alternative or criteria. As for agility and computational complexity, assessment was based on 64 tests considering supplier selection scenarios ranging from 2 to 9 alternatives and criteria. Comparison of the adequacy to supporting group decision making was based on the analysis of equations of both methods. For the other factors, comparison was based on qualitative analysis of the algorithms of both methods.

The comparative analysis of Fuzzy AHP and Fuzzy TOPSIS has shown some interesting outcomes that one should take into account so as to better align the technique to the particularities of the problem at hand. The obtained results concerning the analysis of the seven factors are valid for the context of supplier selection. For other decision making problems, changes of alternatives or criteria, agility and computational complexity may also be relevant and therefore the conclusions are also applicable to them.

Table 24 presents a summary of the findings. Regarding the factor adequacy to changes of alternatives and criteria, it can be seen that in some situations Fuzzy AHP causes the effect known as ranking reversal, changing the preference order of alternatives and the importance order of criteria. Ertugrul and Karakasoglu [24] point this effect when a non-optimal alternative is introduced. However, this study has shown by a numerical example that the ranking reversal in Fuzzy AHP also happens when an optimum alternative is introduced. On the other hand, Fuzzy TOPSIS produces very consistent results. Further research could explore alternative approaches to avoid the rank reversal in Fuzzy AHP. Another problem caused by Fuzzy AHP related to the criteria importance order is nulling the weight of the least important criterion, because of the MIN operator used in the computation of the degree of possibility.

Concerning the agility in the decision process, Fuzzy TOPSIS performs better than Fuzzy AHP in most cases except when there are very few criteria and suppliers. In addition, the increase in the number of supplier alternatives imposes some limitation to Fuzzy AHP. As for the Fuzzy TOPSIS, this is not a restriction to the use of the method. In the case of the number of criteria, the intrinsic limitation imposed by the Fuzzy AHP method can be overcome by deploying the criteria into the Fuzzy AHP hierarchy structure. At the same time that the Fuzzy TOPSIS does not constrain the number of criteria, it does not allow the deployment of the criteria into subcriteria, which can be understood as a weakness of the method when applied to the problem of supplier selection. A further study could focus on the adaptation of the Fuzzy TOPSIS so as to accommodate the criteria and subcriteria into the decision matrix.

As for the time complexity, it is in general lower for Fuzzy AHP than for Fuzzy TOPSIS. However, if the Fuzzy AHP decision matrix consistency test is performed, which is frequently needed, than the advantage of the Fuzzy AHP method is not so pronounced. This conclusion differs from that made by Ertugrul and Karakasoglu [24] who states with no further detail that Fuzzy AHP requires more complex computations than Fuzzy TOPSIS.

Both methods adequately support group decision making. It is worth to mention that weighted mean could be used to aggregate judgments instead of the arithmetic mean commonly used. By doing that, one could give different importance to different decision makers. Although both methods are equally adequate to deal with the lack of precision of scores of alternatives as well as the relative importance of different criteria, it is worth noting that the Fuzzy AHP is more appropriate than the Fuzzy TOPSIS when the purpose is to replace a supplier.

Finally, some genuine contributions of this study can be pointed out:

- It is the first study to analyze the adequability of MCDM methods to the problem of supplier selection taking into account the alignment of the particularities of the problem with the characteristics of the techniques. A study such as this can contribute to helping researchers and practitioners to choose more effective approaches to supplier selection;
- It complements the study by Ertugrul and Karakasoglu [24] not only by considering another problem domain but also by including numerical examples to comparatively test the techniques. It also includes other comparative criteria such as agility in the decision process, modeling of uncertainty and adequacy to supporting group decision making;
- Apart from the comparative analysis, another contribution of this study is the proposition of a set of seven factors for evaluation of MCDM methods. In this sense, this set of factors can be further used as a framework to assess the adequacy of other techniques to the problem of supplier selection;
- It is the first study to discuss and bring numeric examples of the problem of null weight of criteria of the Fuzzy AHP method. Further research could test other fuzzy operators such as the arithmetic mean and T-norms in order to avoid this problem.

Acknowledgments

To FAPESP (São Paulo State Science Foundation) and CAPES for supporting this research project. To the anonymous reviewers for their helpful contributions.

Appendix A. Example of the problem of null weight in Fuzzy AHP

Tables A1 and A2 present the comparative judgments of the importance of 5 criteria. Table A1 presents the judgments in linguistic terms and Table A2 shows them converted to the corresponding triangular fuzzy numbers, as presented in Table 12 of the paper. The consistency ratio for this matrix is 0.15.

The values of the fuzzy synthetic extent for the criteria matrix are:

$$S_{C1} = (10.14, 14.20, 18.33) \otimes \left(\frac{1}{75.48}, \frac{1}{57.96}, \frac{1}{37.69}\right) = (0.13, 0.24, 0.47)$$

$$S_{C2} = (19.0, 27.0, 33.0) \otimes \left(\frac{1}{75.48}, \frac{1}{57.96}, \frac{1}{37.69}\right) = (0.25, 0.46, 0.85)$$

$$S_{C3} = (5.44, 10.14, 14.20) \otimes \left(\frac{1}{75.48}, \frac{1}{57.96}, \frac{1}{37.69}\right) = (0.07, 0.17, 0.36)$$

$$S_{C4} = (1.56, 1.75, 2.62) \otimes \left(\frac{1}{75.48}, \frac{1}{57.96}, \frac{1}{37.69}\right) = (0.02, 0.03, 0.06)$$

$$S_{C5} = (1.54, 4.86, 7.33) \otimes \left(\frac{1}{75.48}, \frac{1}{57.96}, \frac{1}{37.69}\right) = (0.03, 0.08, 0.21)$$

The degrees of possibility for the criteria for the five criteria are:

$$V(S_{C1} \geq S_{C2}) = 0.50$$

$$V(S_{C1} \geq S_{C3}) = 1.00$$

$$V(S_{C1} \geq S_{C4}) = 1.00$$

$$V(S_{C1} \geq S_{C5}) = 1.00$$

$$V(S_{C2} \geq S_{C1}) = 1.00$$

$$V(S_{C2} \geq S_{C3}) = 1.00$$

$$V(S_{C2} \geq S_{C4}) = 1.00$$

$$V(S_{C2} \geq S_{C5}) = 1.00$$

$$V(S_{C3} \geq S_{C1}) = 0.77$$

$$V(S_{C3} \geq S_{C2}) = 0.29$$

$$V(S_{C3} \geq S_{C4}) = 1.00$$

$$V(S_{C3} \geq S_{C5}) = 1.00$$

$$V(S_{C4} \geq S_{C1}) = 0.00$$

$$V(S_{C4} \geq S_{C2}) = 0.00$$

$$V(S_{C4} \geq S_{C3}) = 0.00$$

$$V(S_{C4} \geq S_{C5}) = 0.39$$

Table A1
Matrix for comparative judgments of 5 criteria in linguistic terms.

	C_1	C_2	C_3	C_4	C_5
C_1		1/FP	EQ	AP	SP
C_2	–	–	XP	AP	FP
C_3	–	–		FP	SP
C_4	–	–			1/SP
C_5	–	–			

Table A2
Matrix for comparative judgments of 5 criteria in fuzzy numbers.

	C_1	C_2	C_3	C_4	C_5
C_1	(1.00, 1.00, 1.00)	(0.14, 0.20, 0.33)	(1.00, 1.00, 3.00)	(7.00, 9.00, 9.00)	(1.00, 3.00, 5.00)
C_2	(3.00, 5.00, 7.00)	(1.00, 1.00, 1.00)	(5.00, 7.00, 9.00)	(7.00, 9.00, 9.00)	(3.00, 5.00, 7.00)
C_3	(0.33, 1.00, 1.00)	(0.11, 0.14, 0.20)	(1.00, 1.00, 1.00)	(3.00, 5.00, 7.00)	(1.00, 3.00, 5.00)
C_4	(0.11, 0.11, 0.14)	(0.11, 0.11, 0.14)	(0.14, 0.20, 0.33)	(1.00, 1.00, 1.00)	(0.20, 0.33, 1.00)
C_5	(0.20, 0.33, 1.00)	(0.14, 0.20, 0.33)	(0.20, 0.33, 1.00)	(1.00, 3.00, 5.00)	(1.00, 1.00, 1.00)

$V(S_{C5} \geq S_{C1}) = 0.34$

$V(S_{C5} \geq S_{C2}) = 0.00$

$V(S_{C5} \geq S_{C3}) = 0.61$

$V(S_{C5} \geq S_{C4}) = 1.00$

Therefore, the weight vector W' is:

$$d'(C_1) = V(S_{C1} \geq S_{C2}, S_{C3}, S_{C4}, S_{C5}) = \min(0.50, 1.00, 1.00, 1.00) = 0.50$$

$$d'(C_2) = V(S_{C2} \geq S_{C1}, S_{C3}, S_{C4}, S_{C5}) = \min(1.00, 1.00, 1.00, 1.00) = 1.00$$

$$d'(C_3) = V(S_{C3} \geq S_{C1}, S_{C2}, S_{C4}, S_{C5}) = \min(0.77, 0.29, 1.00, 1.00) = 0.29$$

$$d'(C_4) = V(S_{C4} \geq S_{C1}, S_{C2}, S_{C4}, S_{C5}) = \min(0.00, 0.00, 0.00, 0.39) = 0.00$$

$$d'(C_5) = V(S_{C5} \geq S_{C1}, S_{C2}, S_{C3}, S_{C4}) = \min(0.34, 0.00, 0.61, 1.00) = 0.00$$

$W' = (0.50.1.00, 0.29, 0.00, 0.00)$

After normalization the weight vector is (0.28, 0.56, 0.16, 0.00, 0.0). This weight vector shows criteria C_4 and C_5 as having null weight which does not correspond to the comparative judgment.

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