ELSEVIER European Journal of Operational Research 115 (1999) 596–607

EUROPEAN JOURNAL OF OPERATIONAL RESEARCH

Theory and Methodology

An enhanced DEA Russell graph efficiency measure

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Received 29 September 1997; accepted 17 February 1998

Abstract

The measurement of productive efficiency is an issue of great interest. Since Farrell (Farrell, M.J., 1957. Journal of Royal Statistical Society, Series A 120, 253) implemented the first measure of technical efficiency, many researchers have developed new measures or have extended the already existing ones. The beginning of Data Envelopment Analysis (DEA) meant a new way of empirically measuring productive efficiency. Under some specific technologies, Farrell's measure was implemented giving rise to the first DEA models, CCR (Charnes, A., Cooper, W.W., Rhodes, E., 1978. European Journal of Operational Research 2, 429) and BCC (Banker, R.D., Charnes, A., Cooper, W.W., 1984. Management Science, 1078). The fact that these measures only account for radial inefficiency has motivated the development of the so-called Global Efficiency Measures (GEMs) (Cooper, W.W., Pastor, J.T., 1995. Working Paper, Departamento de Estadística e Investigación Operativa, Universidad de Alicante, Alicante, Spain). In this paper we propose a new GEM inspired by the Russell Graph Measure of Technical Efficiency which avoids the computational and interpretative difficulties with this latter measure. Additionally, the new measure satisfies some other desirable properties.

Keywords: Data envelopment analysis; Efficiency measures; Fractional programming; Linear programming

1. Introduction

The measurement of technical efficiency started with the works of Debreu (1951) and Koopmans (1951). Following them, Farrell (1957) implemented the first measure of technical efficiency. Later, Färe and Lovell (1978) pointed out some difficulties with this measure which motivated the development of new measures of technical efficiency. In their work of 1978, these authors axiomatically approached this issue by suggesting some desirable properties that an ideal technical efficiency measure should satisfy, and then proposed a measure which satisfied them (it was later, in Färe et al. (1983), when it was noted that this measure does not satisfy homogeneity of degree -1 in inputs). This measure was called the Russell Input Measure of Technical Efficiency and was extended to the multiple output case by Färe et al. (1983). An output version, the Russell Output Measure of Technical Efficiency, was similarly

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defined by Färe et al. (1985). They also defined the Russell Graph Measure of Technical Efficiency which extends the two previous ones in the sense that it simultaneously accounts for the inefficiency in both inputs and outputs. There are also some graph versions of the Farrell measure. Färe et al. (1985) defined two of them: the hyperbolic and the generalized hyperbolic graph efficiency measures. Recently, Briec (1997) has also proposed a new graph-type extension of the Farrell measure. The main difference between Farrell and Russell measures is that Farrell measures are radial, whereas Russell ones are not, so they do not necessarily agree in classifying the same subset of units as efficient (in the particular case of DEA, they disagree when a DMU on the frontier has nonzero slacks). A comparative study of the performance of these measures which also includes two other measures can be found in Ferrier et al. (1994) and De Borger and Kerstens (1996).

The development of measures of efficiency has also been approached from the particular perspective of DEA. Initially, Farrell's measure was implemented in the LP problems which gave rise to the first DEA models, the CCR (Charnes, Cooper and Rhodes, 1978) and the BCC (Banker, Charnes and Cooper, 1984). Due to their radial nature, the efficiency scores obtained from these models overstate efficiency when nonzero slacks are present because they do not account for the nonradial inefficiency of the slacks. In contrast to these radial models, the additive model (Charnes et al., 1985) accounts for all sources of inefficiency, i.e., radial and nonradial inefficiency, both in inputs and in outputs. However, it does not directly provide an efficiency measure. To sort out these problems, several measures which consider all types of inefficiency detected by a given DEA model have been designed in the last few years, and it is still an issue of great interest. In Cooper and Pastor (1995) a complete revision with new proposal of these measures, which they call "Global Efficiency Measures" (GEM), can be found. Besides this, the authors list four basic properties that such a measure should satisfy.

GEMs can be defined both for radial and for nonradial DEA models. In this paper, we focus on the latter possibility. Next, we refer to two GEMs of this kind existing in the literature: the "Measure of Efficiency Proportions" (MEP) developed by Banker and Cooper (1994) and the "Range Adjusted Measure" (RAM) of Cooper et al. (1998) (see Appendix A for the expression of these measures). These two measures, together with the TDT measure (Thompson and Thrall, 1994) which is not a GEM, are the new approaches to inefficiency measurement in DEA explained in Cooper and Tone (1997).

MEP should be used after an optimal solution of the additive or the invariant additive model is obtained. Therefore, we may have different values of this measure for the different alternate optima (if any). This also happens to all GEMs not included in the DEA model from which they are computed. A way of avoiding this problem is to include these GEMs as the objective of the models used for their computation. The difficulty with this including is that it usually gives rise to nonlinear programming problems which are complicated to solve, as in the case of MEP.

With these considerations in mind, we set two main goals for the GEM we are going to develop: (1) that it is well defined and (2) easy to compute. Additionally, we want our measure to satisfy some desirable properties, like the four basic ones listed by Cooper and Pastor (1995) and, in addition, that it is readily understood. RAM is an example of a measure meeting all these requirements, so we will take it as a reference to evaluate the behavior of our measure.

Aside from the mentioned approaches, the efficiency measurement with DEA models has been extended and enhanced in other directions. Some of these developments involve incorporating judgement or prior knowledge by restricting the range for the multipliers: see, for instance, Charnes et al. (1990) for the cone ratio model, Thompson et al. (1990) for the assurance region approach and Dyson and Thanassoulis (1988) which impose bounds on individual multipliers. In other extensions stochastic elements are introduced into the DEA models: see, for example, Sengupta (1987) for efficiency measurement in the stochastic case, Banker (1993) for maximum likelihood estimation of inefficiency and hypothesis testing and Land et al. (1993), Olesen and Petersen (1995) and Cooper et al. (1996) for the chance-constrained DEA approach.

The paper unfolds as follows. In Section 2 we define the new measure and show the way to compute it by means of an LP problem. Section 3 contains a set of desirable properties that the new measure satisfies. In Section 4 we include an example to illustrate the performance of the measure. Section 5 concludes.

2. A new DEA global efficiency measure

In this section we develop a new DEA efficiency measure which is closely related to the Russell measures. Assume that we have a set of n DMUs with m inputs and s outputs,

$$\{(X_j, Y_j) = (x_{1j}, \ldots, x_{mj}, y_{1j}, \ldots, y_{sj}), j = 1, \ldots, n\},$$

where all inputs and outputs are positive. Let us also assume that the production possibility set $\mathbf{T} = \{(X, Y)/Y \text{ can be produced from } X\}$ satisfies the usual postulates of convexity, free disposability, constant returns to scale and minimum extrapolation (see Banker et al., 1984), as in the CCR model.

The Russell Graph Measure of Technical Efficiency was defined as a combination of the Input and Output Russell Measures of Technical Efficiency (see Färe et al. (1985), pp. 160, 161), for the corresponding formulations). For a given DMU_0, (X_0, Y_0), the value of this measure can be obtained from the following DEA formulation:

$$\begin{aligned}
\min \quad & R_{\mathrm{g}}(X_0, Y_0) = \frac{1}{m+s}\left(\sum_{i=1}^{m}\theta_i + \sum_{r=1}^{s}\frac{1}{\phi_r}\right) \\
\text{s.t.} \quad & \sum_{j=1}^{n}\lambda_j x_{ij} \leqslant \theta_i x_{i0}, \quad i = 1, \ldots, m, \\
& \sum_{j=1}^{n}\lambda_j y_{rj} \geqslant \phi_r y_{r0}, \quad r = 1, \ldots, s, \\
& 0 < \theta_i \leqslant 1, \quad \phi_r \geqslant 1 \ \forall i, r, \\
& \lambda_j \geqslant 0, \quad j = 1, \ldots, n.
\end{aligned} \tag{1}$$

In the formulation above the constraints $0 < \theta_i \leqslant 1$ and $\phi_r \geqslant 1$ are the requirements for dominance. In addition, the convexity constraint $\sum_{j=1}^{n}\lambda_j = 1$ would be included if $\mathbf{T}$ were not assumed to satisfy constant returns to scale.

Although R_{g} is well defined in the sense we explained in Section 1 and it also satisfies the four basic properties listed by Cooper and Pastor, there are some difficulties with this measure. First, it must be computed from a nonlinear programming problem whose solution is not easily obtained. And, secondly, it is not readily understood because, as Cooper et al. (1998) note, R_{g} is a weighted average of arithmetic and harmonic means. Moreover, this measure fails to satisfy other properties we study in Section 3. Therefore, we propose an alternative to this measure which, although closely related, avoids the mentioned difficulties.

2.1. Definition of the new measure

Instead of combining the input and output Russell measures in an additive way, as in Eq. (1), we define our measure as the ratio between them. That is, we separately average the input and the output efficiency and then combine these two efficiency components in a ratio form. The result is the following model:

$$\begin{aligned}
\min \quad & R_{\mathrm{e}}(X_0, Y_0) = \frac{\frac{1}{m}\sum_{i=1}^{m}\theta_i}{\frac{1}{s}\sum_{r=1}^{s}\phi_r} \\
\text{s.t.} \quad & \sum_{j=1}^{n}\lambda_j x_{ij} \leqslant \theta_i x_{i0}, \quad i = 1, \ldots, m, \\
& \sum_{j=1}^{n}\lambda_j y_{rj} \geqslant \phi_r y_{r0}, \quad r = 1, \ldots, s, \\
& \theta_i \leqslant 1, \quad \phi_r \geqslant 1 \ \forall i, r, \\
& \lambda_j \geqslant 0, \quad j = 1, \ldots, n.
\end{aligned} \tag{2}$$

On the analogy of Russell measures, we will call R_{e} the Enhanced Russell Graph Efficiency Measure. It can be interpreted as the ratio between the average efficiency of inputs and the average efficiency of outputs, which is a more straightforward interpretation than that of R_{g}. Moreover, R_{e} may be decomposed into an input component of average efficiency and an output one to better explain the efficiency of the DMU being evaluated. If $(\theta_1^*, \ldots, \theta_m^*, \phi_1^*, \ldots, \phi_s^*)$ is an optimal solution of Eq. (2), these components of efficiency are, respectively, the numerator $(1/m)\sum_{i=1}^{m}\theta_i^*$ and the denominator $(1/s)\sum_{r=1}^{s}\phi_r^*$ of R_e, which represent, respectively, the proportion with respect to DMU_0 of used inputs and produced outputs (on average) of a DMU (or virtual) on the efficient frontier. There may be alternate optima of Eq. (2) which would give rise to different decompositions of R_e into the input and output components. Obviously, they all have the same associated R_e value.

On the other hand, by means of the following change of variables:

$$\begin{aligned}\theta_i &= \frac{x_{i0}-s_{i0}^-}{x_{i0}} = 1-\frac{s_{i0}^-}{x_{i0}}, \quad i=1,\ldots,m,\\ \phi_r &= \frac{y_{r0}+s_{r0}^+}{y_{r0}} = 1+\frac{s_{r0}^+}{y_{r0}}, \quad r=1,\ldots,s,\end{aligned} \tag{3}$$

it is easy to reexpress formulation (2) of R_e in terms of total slacks. The result is this new problem which provides an alternative expression of the Enhanced Russell Measure connecting R_e with the usual GEMs:

$$\begin{aligned}\min \quad & R_e(X_0,Y_0) = \frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_{i0}^-}{x_{i0}}}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_{r0}^+}{y_{r0}}}\\ \text{s.t.} \quad & \sum_{j=1}^{n}\lambda_j x_{ij} = x_{i0}-s_{i0}^-, \quad i=1,\ldots,m,\\ & \sum_{j=1}^{n}\lambda_j y_{rj} = y_{r0}+s_{r0}^+, \quad r=1,\ldots,s,\\ & s_{i0}^-, s_{r0}^+ \geqslant 0 \ \forall i,r,\\ & \lambda_j \geqslant 0, \quad j=1,\ldots,n.\end{aligned} \tag{4}$$

In a similar fashion, by means of Eq. (3) formulation (1) can also be reexpressed as a problem having the MEP as objective and the same set of constraints as in Eq. (4)

Again, the convexity constraint would appear in Eqs. (2) and (4) if **T** were not assumed to satisfy constant returns to scale.

Concerning the first main goal we set for our measure in Section 1, we can say that R_e is clearly well defined in the sense we explained there, because it is the optimal value of the used DEA model. Besides, it can be readily understood.

We remark that the objective in Eq. (4) was proposed as a new global efficiency measure in Cooper and Pastor (1995), although it was not included as the objective of any model there. Besides, the model used by Lovell et al. (1995) for their macroeconomic evaluation of the OECD countries coincides with Eq. (4) in the case of having a unique constant input, as happened in that analysis.

2.2. Computational aspects

One of the most important advantages of R_e over R_g is that the value of R_e can be computed more easily than that of R_g. As explained before, the implementation of both measures for the usual DEA technologies gives rise to Eq. (1) for R_g and Eq. (2) or Eq. (4) for R_e. Although these three problems are nonlinear, Eq. (1) is more complicated to solve than Eq. (2) or Eq. (4) because the two latter are ordinary linear fractional programming problems whose solution can be found through a linear programming problem.

Following Charnes and Cooper (1962), let

$$\begin{aligned}\beta &= \left(1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_{r0}^+}{y_{r0}}\right)^{-1},\\ t_{i0}^- &= \beta s_{i0}^-, \quad i=1,\ldots,m,\\ t_{r0}^+ &= \beta s_{r0}^+, \quad r=1,\ldots,s,\\ \mu_j &= \beta\lambda_j, \quad j=1,\ldots,n.\end{aligned} \tag{5}$$

Then, an optimal solution of the following linear programming problem:

$$\begin{aligned}\min \quad & \beta-\frac{1}{m}\sum_{i=1}^{m}\frac{t_{i0}^-}{x_{i0}}\\ \text{s.t.} \quad & \beta+\frac{1}{s}\sum_{r=1}^{s}\frac{t_{r0}^+}{y_{r0}} = 1,\\ & -\beta x_{i0}+\sum_{j=1}^{n}\mu_j x_{ij}+t_{i0}^- = 0, \quad i=1,\ldots,m,\\ & -\beta y_{r0}+\sum_{j=1}^{n}\mu_j y_{rj}-t_{r0}^+ = 0, \quad r=1,\ldots,s,\\ & \beta \geqslant 0,\\ & t_{i0}^-, t_{r0}^+ \geqslant 0, \quad \forall i,r,\\ & \mu_j \geqslant 0, \quad j=1,\ldots,n\end{aligned} \tag{6}$$

(which would include the constraint $-\beta + \sum_{j=1}^{n} \mu_j = 0$ associated to the convexity constraint in Eq. (4) if VRS over the reference technology were assumed) gives rise to an optimal solution of Eq. (4). To be precise, we know that from any optimal solution of Eq. (6) with $\beta > 0$ we can obtain an optimal solution of Eq. (4) through the change of variables (5). Moreover, the associated optima are equal (see Charnes and Cooper, 1962). Note, in addition, that no feasible solution of Eq. (6) satisfies $\beta = 0$, so we can use Eq. (6) to solve Eq. (4) and, in particular, to obtain the R_e values as the optimal values of Eq. (6). Thus, if we are only interested in these efficiency scores and not in the efficient projection of the DMUs being evaluated, we do not even need to transform the optimal solutions of Eq. (6) through Eq. (5).

Hence, we have a well-defined measure which, as intended, improves the Russell Graph one with respect to the computational and interpretative difficulties. In the next section we study some desirable properties which are satisfied by R_e.

3. Properties of R_e

Färe and Lovell (1978) were the first ones who proposed a set of desirable properties that an ideal efficiency measure should satisfy, although these were enunciated for the particular case of an input oriented measure. Recently, Cooper and Pastor (1995) listed similar requirements for the DEA context and suggested some others. Next, we study the properties which the proposed Enhanced Russell Measure satisfies.

Theorem 1. *The following is true for* R_e*:*

(i) $0 < R_e \leqslant 1$.

(ii) $R_e = 1 \iff \mathrm{DMU}_0$ *being evaluated is Koopmans-efficient.*

(iii) R_e *is units invariant.*

(iv) R_e *is strongly monotonic in inputs and in outputs.*

(v) R_e *satisfies the following relationships*:

(v.1) If $\theta > (<)1$ *then*

$$R_e(\theta X_0, Y_0) \leqslant (\geqslant) \frac{1}{\theta} R_e(X_0, Y_0).$$

(v.2) If $\phi < (>)1$ *then*

$$R_e(X_0, \phi Y_0) \leqslant (\geqslant) \phi R_e(X_0, Y_0).$$

(v.3) If $\lambda > (<)1$ *then*

$$R_e(\lambda X_0, \frac{1}{\lambda} Y_0) \leqslant (\geqslant) \frac{1}{\lambda^2} R_e(X_0, Y_0).$$

(vi) R_e *satisfies the following relationships with respect to the radial efficiency measures* θ *and* ϕ:

(vi.1) $R_e \leqslant \theta$.

(vi.2) $R_e \leqslant 1/\phi$.

Proof. See Appendix B.

Remark 1. Relations in parentheses in (v) are true provided the resulting point belongs to the production possibility set.

(i) to (iv) are the four basic properties listed by Cooper and Pastor (1995). The first two mean that the Enhanced Russell Measure is bounded by 0 and 1, reaching the top value of 1 if, and only if, DMU_0 is Koopmans-efficient. Property (iii) guarantees that the values of R_e are independent of the units of measurement of the considered inputs and outputs. Property (iv) requires sensitivity of input usage and output production in any single dimension: if we rate two units which have the same values for all their inputs and outputs except one, the more inefficient unit gets a smaller R_e value. That is, if the two units differ in one input, the one with the smaller input value gets an R_e value greater than the one of the other unit. Analogously, if they differ in one output, the unit having a higher output value gets a greater value of R_e. Note that (iv) is not a usual property since, as Cooper and Pastor (1995) assert, it is very difficult to achieve. This leads some authors to consider instead some weaker property as weak-monotonicity or decreasing in the relative values of the slacks. Concerning other existing measures, we remark that R_g and RAM also satisfy these four basic properties. In particular, (i) and (ii) imply that the three measures determine the same set of efficient DMUs, so they only differ in the assigned value to the inefficient units. We should finally clarify that both (ii) and an input version of

(iv) were previously proposed by Färe and Lovell (1978) in a more general context.

Färe and Lovell (1978) also suggested that a good efficiency measure should satisfy homogeneity of degree -1 in inputs. The Russell input measure failed to meet this requirement, but it satisfied the weaker property of subhomogeneity of degree -1 in inputs stated in (v.1) (see Färe et al., 1983). In Färe et al. (1985), relations between the scaling of an input vector and/or an output vector and the resulting Russell graph efficiency measure can be found. The three properties gathered in (v) include similar, but more satisfying relations, referring to R_e measure. (v.1) means that the scaling of the input vector by a factor larger (smaller) than unity leads to an efficiency measure smaller (larger) than or equal to the inverse scaling of the efficiency measure by the same factor. (v.2) studies the effect on R_e of the scaling of an output vector, whereas (v.3) considers the simultaneous scaling of the input and output vectors by inverse factors. Hence, this property in some way quantifies the sensitivity of the Enhanced Russell Measure guaranteed by property (iv). We remark that, concerning the scaling of an input or an output vector, for R_g and RAM it can only be asserted that

$$R_g(\theta X_0, Y_0) < R_g(X_0, Y_0) \text{ and } \mathrm{RAM}(\theta X_0, Y_0) < \mathrm{RAM}(X_0, Y_0),\ if\ \theta > 1,$$

and

$$R_g(X_0, \phi Y_0) < R_g(X_0, Y_0) \text{ and } \mathrm{RAM}(X_0, \phi Y_0) < \mathrm{RAM}(X_0, Y_0),\ if\ \phi < 1.$$

As for the simultaneous scaling of the two vectors by inverse factors,

$$\mathrm{RAM}(\lambda X_0, \frac{1}{\lambda} Y_0) < \mathrm{RAM}(X_0, Y_0) \text{ and } R_g(\lambda X_0, \frac{1}{\lambda} Y_0) \leqslant \frac{1}{\lambda} R_g(X_0, Y_0),\ if\ \lambda > 1.$$

However, all these relationships (except the last one), in contrast to those in (v), are not sensitive to the scaling factor magnitude hence they add no information to the one provided by the monotonicity property.

Finally, since, like any GEM, our global efficiency measure R_e accounts for radial and nonradial inefficiency both in inputs and outputs, property (vi) reflects the expected relationship between R_e and the usual radial efficiency scores θ and ϕ. This property asserts that R_e can never score a given DMU as more efficient than θ and ϕ do, which are oriented measures and only account for radial inefficiency. The meaning of (vi.1) is clear, but the difficulty with relating the output-oriented radial efficiency score ϕ and R_e leads us to consider in (vi.2) the inverse of ϕ (which equals θ if CRS in the efficient frontier were considered). Moreover, R_e and θ (ϕ) are closely related as Eq. (2) extends the radial DEA models in the sense that any feasible solution of the input oriented (output oriented) model $(\lambda_1, \lambda_2, \ldots, \lambda_n, \theta)$($\phi$ instead of θ for the output oriented case), whose objective function value equals θ (ϕ) gives rise to a feasible solution of Eq. (2) $(\lambda_1, \ldots, \lambda_n, \theta, \overset{(m)}{\ldots}, \theta, 1, \overset{(s)}{\ldots}, 1)$ $((\lambda_1, \ldots, \lambda_n, 1, \overset{(m)}{\ldots}, 1, \phi, \overset{(s)}{\ldots}, \phi))$ whose corresponding objective function value also equals θ ($1/\phi$ for the output oriented case). In contrast to R_e, neither R_g or RAM satisfy either (vi.1) or (vi.2). Relations in (vi) were also listed by Cooper and Pastor (1995) as desirable properties, whereas Färe et al. (1985) only related, in a more general context, Russell and Farrell measures according to their orientation (input, output or graph).

Finally, we refer to the "translation invariance" property, an additional desirable property proposed by Cooper and Pastor. This property allows us to deal with inputs and outputs unrestricted in sign and is satisfied by RAM, but not by R_g and R_e.

4. Example

In order to illustrate the performance of our new GEM, we have used the data relative to the agencies engaged in supplying water and related services in the Kanto region of Japan analyzed in Aida et al. (1998). These data contain 108 observations on five inputs (Number of Employees, Operating Expenses before Depreciation, Net Plant and Equipment, Population and Length of Pipes) and two outputs (Operating Revenues and

Table 1
Computation and decomposition of R_e

DMU	Enhanced Russell measure			Slacks						
				Inputs					Outputs	
	R_e	Input	Output	Employee	Expenses	Plants	Population	Pipes	Water	Revenue
1	0.84930	0.84930	1.00000	41.73	0.00	6994270.00	0.00	232.68	0.00	0.00
2	0.83189	0.83189	1.00000	33.20	0.00	981442.87	37945.48	163.64	0.00	0.00
4	0.76751	0.97709	1.27307	1.31	0.00	178487.80	1189.01	0.00	3369.17	0.00
5	0.85247	0.87821	1.03020	1.83	0.00	0.00	14557.88	99.36	319.01	0.00
6	0.70358	0.96015	1.36466	0.00	0.00	0.00	1704.30	47.80	3217.97	18222.47
10	0.55681	0.86749	1.55795	5.50	23579.97	298492.90	0.00	72.55	3825.11	43897.76
11	0.78772	0.93006	1.18070	4.70	0.00	2276747.30	9529.89	0.00	4119.69	0.00
13	0.72127	0.92228	1.27869	12.14	0.00	515978.24	0.00	0.00	3262.88	0.00
14	0.47117	0.85589	1.81653	0.00	99351.94	989729.34	0.00	19.02	1450.84	109830.79
16	0.33429	0.85323	2.55240	0.00	0.00	628074.07	91519.21	0.00	5917.03	318084.98
19	0.69759	0.89684	1.28563	3.65	0.00	0.00	27398.35	9.85	3540.09	0.00
21	0.68144	0.82958	1.21739	9.97	0.00	720070.03	30539.09	12.07	3167.82	0.00
23	0.77267	0.84297	1.09097	5.23	0.00	34386.76	10075.53	91.84	981.10	5534.02
24	0.79986	0.97123	1.21425	0.00	0.00	0.00	11588.96	38.15	5123.05	0.00
25	0.58087	0.92245	1.58804	0.00	26682.33	618495.12	6349.62	30.26	4651.39	46115.34
26	0.65214	0.87836	1.34690	0.00	229221.57	0.00	1842.67	56.87	2614.24	0.00
27	0.69756	0.83690	1.19975	0.00	0.00	2502975.84	0.00	106.87	1769.02	0.00
28	0.65515	0.88045	1.34388	0.00	0.00	101098.90	0.00	327.43	3665.09	0.00
31	0.90776	0.91203	1.00471	15.16	0.00	1718578.82	0.00	27.31	0.00	18045.53
32	0.87817	0.87817	1.00000	15.69	0.00	0.00	11379.40	144.78	0.00	0.00
33	0.90682	0.91434	1.00829	11.25	0.00	1481689.18	0.00	134.70	323.44	0.00
34	0.86910	0.90371	1.03982	5.50	0.00	52627.53	7484.98	0.00	286.00	0.00
36	0.73202	0.85670	1.17033	10.55	7316.51	1054403.75	0.00	28.85	1056.45	112192.97
39	0.82122	0.89308	1.08751	10.46	0.00	954190.43	5230.75	473.56	0.00	882470.90
42	0.80514	0.89571	1.11248	0.00	0.00	1002426.81	8114.18	121.53	2163.91	0.00
44	0.83981	0.91844	1.09363	24.12	0.00	2540339.78	6537.04	70.19	0.00	1023502.72
45	0.68765	0.82201	1.19539	13.69	0.00	7234111.49	3041.12	7.82	3531.06	0.00
46	0.79388	0.94995	1.19658	0.00	0.00	0.00	3162.73	53.50	427.82	326228.26
49	0.84417	0.87215	1.03314	14.05	0.00	53428.09	6278.01	93.53	0.00	137381.01
50	0.86877	0.94407	1.08668	0.00	0.00	977538.08	16664.50	65.21	0.00	496841.07
51	0.90223	0.90709	1.00538	0.00	0.00	3273286.93	21583.78	39.25	0.00	30609.48
53	0.87079	0.93540	1.07420	5.09	0.00	30870.65	4212.87	17.16	1341.20	14577.38
59	0.80107	0.84167	1.05069	3.48	0.00	2936854.48	23972.59	113.11	0.00	220520.96
65	0.83004	0.91669	1.10440	0.46	0.00	1092569.29	0.00	76.39	1865.83	0.00
66	0.85531	0.86374	1.00985	8.13	0.00	984484.79	2127.95	65.65	203.95	0.00
67	0.82111	0.85326	1.03916	8.38	0.00	1722870.97	17926.75	13.95	724.70	11539.66
69	0.85406	0.86669	1.01478	7.20	0.00	579998.25	19249.30	120.75	0.00	68427.69
70	0.75351	0.96507	1.28076	0.00	0.00	611484.50	0.00	15.28	3496.08	0.00
72	0.73984	0.85083	1.15003	0.00	0.00	4184844.87	11458.64	0.00	1749.33	0.00

Table 1 (Continued)

DMU	Enhanced Russell measure			Slacks						
				Inputs					Outputs	
	R_e	Input	Output	Employee	Expenses	Plants	Population	Pipes	Water	Revenue
74	0.83133	0.88909	1.06947	18.93	0.00	0.00	0.00	101.74	1671.82	0.00
75	0.56641	0.89027	1.57177	7.46	67504.75	140505.84	3747.25	0.00	4680.52	0.00
77	0.61736	0.72020	1.16658	5.04	0.00	2198477.78	364312.1	23.19	2544.91	45046.06
78	0.79566	0.96774	1.21627	0.00	0.00	2062407.64	0.00	4.80	4509.76	0.00
79	0.51243	0.83019	1.62012	6.96	0.00	3422098.25	5013.76	0.00	3720.71	0.00
80	0.68466	0.84645	1.23631	0.00	0.00	2465332.00	32014.92	0.00	1963.21	65750.38
81	0.76490	0.78510	1.02641	0.00	174384.67	5967689.65	34527.45	211.97	0.00	122775.45
82	0.58739	0.79684	1.35659	0.00	298907.58	594946.94	0.00	81.65	1709.10	43985.80
83	0.80854	0.84525	1.04540	2.54	0.00	1300387.14	51213.42	47.23	0.00	136725.11
84	0.86219	0.86219	1.00000	0.00	0.00	5195915.50	50786.68	272.88	0.00	0.00
86	0.29861	0.47528	1.59164	18.58	232353.15	9691556.19	203795.0	216.28	6012.23	0.00
87	0.82001	0.83585	1.01932	5.79	0.00	722091.86	20977.32	246.82	578.24	0.00
88	0.79388	0.85456	1.07644	9.72	0.00	3478913.12	17547.23	116.91	2408.39	0.00
89	0.64894	0.66462	1.02416	9.00	542864.19	5889811.94	32654.01	166.58	415.00	24673.01
91	0.74960	0.84022	1.12089	0.00	0.00	1585651.48	0.00	548.45	2556.17	0.00
92	0.52442	0.71847	1.37001	3.73	142955.15	6458172.76	0.00	228.99	3996.90	0.00
93	0.74912	0.84633	1.12975	0.00	84303.58	1785713.06	12623.59	76.74	2132.88	0.00
94	0.74790	0.78456	1.04901	0.00	210339.92	2248498.60	0.00	238.11	717.03	0.00
104	0.91973	0.91973	1.00000	15.91	0.00	1931213.75	17967.99	10.72	0.00	0.00
105	0.78094	0.84798	1.08584	0.93	0.00	3090680.00	7184.66	29.21	1098.7	23220.29

Water Billed). Variable returns to scale on the efficient frontier were assumed.

Applying Eq. (4) to the data reveals 49 efficient units. Results for the 59 remaining DMUs are shown in Table 1. The first column records the value of R_e. The second and third, respectively, contain the average efficiency components of inputs and outputs. The remaining columns contain the optimal slacks or inefficiencies for each of the inputs and outputs.

For example, $R_e = 0.84930$ for DMU1 indicates that the ratio between the average efficiency of inputs and of outputs for this DMU equals 0.84930. Decomposition of the Enhanced Russell Measure into the input and output components reveals the existence of an efficient DMU or a linear combination of efficient DMUs which uses, on average, 84.93% of the inputs used by DMU 1 maintaining the same level of output production. The most inefficient observation detected by the Enhanced Russell Measure is DMU 86, with $R_e = 0.29861$. In this case the decomposition of R_e indicates that there exists an efficient DMU or a linear combination of efficient DMUs which produces, on average, 59.16% more outputs than DMU 86 by using, on average, 47.53% of the inputs used by this DMU.

Next, we compare results obtained for R_e to those of RAM. Obviously, both measures agree in the classification of the efficient DMUs. Table 2 records R_e and RAM values for the inefficient observations. We immediately notice the great difference between the magnitude of R_e values and those of RAM, highlighting the large values taken by the latter measure. Values of R_e for the inefficient units go from 0.29861, for DMU 86, to 0.91974, for DMU 104. In contrast, the minimum for RAM is 0.97759 and only four DMUs score less than 0.99. This shows that the discriminating capability of R_e over a given set of DMUs is much stronger than that of RAM measure. Such a large magnitude of RAM values is due to the use of the range inverse as weights of the slacks in the linear combination which defines this measure (see Appendix A).

To illustrate the fulfilment of property (vi.1) by R_e, Table 2 also includes a third column with the radial input-oriented score θ. It can be seen that θ is always greater than R_e. As for RAM, it is never lower than θ. Very recently, a new version of RAM has been proposed so as to broaden the range of its values.

5. Conclusions

This paper is concerned with the measurement of efficiency from a DEA perspective. We have defined a new nonradial nonoriented efficiency measure. Because of the analogy to the Russell measures, we have called it the Enhanced Russell Measure. First of all, it represents a solution for the problem of nonzero slacks when measuring efficiency by means of DEA models. However, other interesting goals have been achieved: the measure is well defined and can be easily computed by solving an LP problem. In addition, the interpretation is straightforward in opposition to the usual Russell Graph Measure. The new measure represents the ratio between average efficiency in inputs and in outputs. These two average efficiency components are helpful to interpret the efficiency of the DMU under evaluation.

To sum up, we have defined an efficiency measure that is well behaved since, apart from the above mentioned: (1) it is bounded by 0 and 1, attaining the maximum value of unity if and only if the units being rated are Koopmans-efficient; (2) in computing the efficiency of an inefficient unit, the DMU being evaluated is compared to efficient units; (3) it is units invariant; (4) it monotonically declines for any increase in input usage or any reduction in output production and does it at least equiproportionately for any proportionate increase in inputs usage or any proportionate reduction in output production; (5) it does not exceed the value of the radial efficiency scores, and, finally, as said before, (6) it is well-defined because it is the optimal value of a mathematical programming problem and can be easily computed and interpreted.

Finally, we would like to stress that in the future it could be interesting to study the behavior of our measure for other kind of technologies.

For instance, some aspects of global efficiency measurement in FDH (Deprins et al., 1984) are

Table 2
Comparison of R_e, RAM and θ

DMU	R_e	RAM	θ
1	0.84930	0.99032	0.90355
2	0.83189	0.99228	0.89862
4	0.76750	0.99776	0.86967
5	0.85247	0.99728	0.98278
6	0.70358	0.99730	0.88164
10	0.55681	0.99571	0.76881
11	0.78772	0.99523	0.89129
13	0.72127	0.99713	0.89514
14	0.47117	0.99829	0.98691
16	0.33428	0.99093	0.56904
19	0.69759	0.99615	0.84046
21	0.68144	0.99535	0.86315
23	0.77267	0.99677	0.94924
24	0.79986	0.99632	0.97778
25	0.58087	0.99587	0.76737
26	0.65213	0.99683	0.86182
27	0.69755	0.99635	0.90223
28	0.65515	0.99235	0.88284
31	0.90776	0.99787	0.97943
32	0.87817	0.99567	0.96410
33	0.90682	0.99598	0.97944
34	0.86910	0.99908	0.97738
36	0.73202	0.99714	0.89499
39	0.82122	0.98865	0.94726
42	0.80514	0.99577	0.87221
44	0.83981	0.99332	0.91951
45	0.68765	0.99341	0.82711
46	0.79388	0.99729	0.90825
49	0.84417	0.99634	0.90088
50	0.86877	0.99670	0.95455
51	0.90223	0.99701	0.97128
53	0.87079	0.99771	0.90522
59	0.80107	0.99521	0.83600
65	0.83004	0.99732	0.91375
66	0.85531	0.99711	0.91124
67	0.82111	0.99721	0.88614
69	0.85406	0.99616	0.89374
70	0.75351	0.99796	0.90899
72	0.73984	0.99708	0.89360
74	0.83133	0.99635	0.96093
75	0.56641	0.99624	0.75070
77	0.61736	0.98081	0.84261
78	0.79566	0.99593	0.93431
79	0.51243	0.99610	0.82798
80	0.68466	0.99611	0.85388
81	0.76490	0.99173	0.86237
82	0.58739	0.99620	0.84663
83	0.80854	0.99576	0.88111
84	0.86220	0.99114	0.91666
86	0.29861	0.97759	0.48215
87	0.82001	0.99351	0.91635
88	0.79388	0.99342	0.86389
89	0.64894	0.99110	0.71821
91	0.74960	0.98825	0.91180
92	0.52442	0.99145	0.79104
93	0.74913	0.99554	0.81298
94	0.74790	0.99415	0.94925
104	0.91974	0.99669	0.94034
105	0.78094	0.99730	0.86975

discussed in Pastor et al. (1998). On the other hand, it is also interesting to study the dual formulation of Eq. (6) (see Tone, 1997).

Acknowledgements

We are grateful to two anonymous referees for their comments and to Generalitat Valenciana (GV-C-CN-10-068-96) for its financial support.

Appendix A. Other global efficiency measures

In this appendix we include the expression of some other global efficiency measures. The expression of the MEP is

$$1-\frac{1}{m+s}\left(\sum_{i=1}^{m}\frac{s_{i0}^{-}}{x_{i0}}+\sum_{r=1}^{s}\frac{s_{r0}^{+}}{y_{r0}+s_{r0}^{+}}\right).$$

The expression of the RAM is

$$1-\frac{1}{m+s}\left(\sum_{i=1}^{m}\frac{s_{i0}^{-}}{R_i^{-}}+\sum_{r=1}^{s}\frac{s_{r0}^{+}}{R_r^{+}}\right).$$

R_i^- and R_r^+ being the range for input i and output r, respectively.

Appendix B. Proof of Theorem 1

Proofs can equivalently be done either for Eq. (2) or Eq. (4). As (i)–(iv) have been introduced as the four basic properties of a GEM, we resort to expression (4) to prove them:

(i) and (ii) are immediate as a consequence of the definition of R_e. (iii) is also a consequence of the definition of R_e, since the ratios considered in the objective function of Eq. (4) are dimensionless and the constraints are lineal. (iv) First, we are going to rate two units differing only in one input. Consider an observation DMU_0 with vector of inputs and outputs $(x_{10}, \ldots, x_{m0}, y_{10}, \ldots, y_{s0})$, and another observation, DMU_a, with the same values for all inputs and outputs but input k, which has the value $x_{ka} = x_{k0} + a$, $a > 0$. We have to show that the value of R_e for the second observation, $R_\mathrm{e}(X_a, Y_a)$, is smaller than $R_\mathrm{e}(X_0, Y_0)$, the value of R_e for the first unit. Throughout this proof let us call problems (P_a) and (P_0) the fractional problems (4) evaluating DMU_a and DMU_0, respectively, and let $R_\mathrm{e}(X_a, Y_a)$ and $R_\mathrm{e}(X_0, Y_0)$ be the corresponding optima. Let $(\hat{\lambda}_{10}, \hat{\lambda}_{20}, \ldots, \hat{\lambda}_{n0}, \hat{s}^-_{10}, \ldots, \hat{s}^-_{m0}, \hat{s}^+_{10}, \ldots, \hat{s}^+_{s0})$ be an optimal solution of problem (P_0). Then, it is easy to check that $(\hat{\lambda}_{10}, \hat{\lambda}_{20}, \ldots, \hat{\lambda}_{n0}, h^-_{1a}, \ldots, h^-_{ma}, \hat{s}^+_{10}, \ldots, \hat{s}^+_{s0})$ where $h^-_{ia} = \hat{s}^-_{i0}, i \neq k$, and $h^-_{ka} = \hat{s}^-_{k0} + a$, is a feasible solution of (P_a), with an associated value of the objective function (which is greater than or equal to $R_\mathrm{e}(X_a, Y_a)$) being less than $R_\mathrm{e}(X_0, Y_0)$. So, we can state $R_\mathrm{e}(X_a, Y_a) < R_\mathrm{e}(X_0, Y_0)$.

Following the notation above, let us now consider DMU_a equal to DMU_0 except for the output p, taking the value $y_{pa} = y_{p0} + a$, $a > 0$, for DMU_a. Now, we have to prove $R_\mathrm{e}(X_a, Y_a) > R_\mathrm{e}(X_0, Y_0)$. Let us start the proof by showing that any solution of the problem (P_a) gives a feasible solution of the problem (P_0) with a smaller value of the objective function than the one associated with the first solution. Let $(\lambda_{1a}, \lambda_{2a}, \ldots, \lambda_{na}, s^-_{1a}, \ldots, s^-_{ma}, s^+_{1a}, \ldots, s^+_{sa})$ be a solution of (P_a). Then, we can see that $(\lambda_{1a}, \lambda_{2a}, \ldots, \lambda_{na}, s^-_{1a}, \ldots, s^-_{ma}, h^+_{10}, \ldots, h^+_{s0})$, where $h^+_{r0} = s^+_{ra}, r \neq p$, and $h^+_{p0} = s^+_{pa} + a$, is a feasible solution for (P_0) verifying the above requirement. In particular, if the starting solution of (P_a) is an optimum, we find a solution of problem (P_0) with an associated value of the objective function less than $R_\mathrm{e}(X_a, Y_a)$, so we can conclude $R_\mathrm{e}(X_a, Y_a) > R_\mathrm{e}(X_0, Y_0)$.

Property (v) is more naturally related formulation (2) of R_e, because it deals with equiproportionate scalings of the observations. Therefore, we use Eq. (2) to prove (v):

(v.1) If $\theta > 1$ and $(\theta^*_i, \phi^*_r, \lambda^*_j)$ is an optimal solution of Eq. (2) when DMU_0 is being evaluated, then $(\theta^*_i/\theta, \phi^*_r, \lambda^*_j)$ is a feasible solution of Eq. (2) when $(\theta X_0, Y_0)$ is under evaluation, because the constraints for inputs and outputs are clearly satisfied and $\theta^*_i/\theta < \theta^*_i \leqslant 1$. Therefore,

$$R_\mathrm{e}(\theta X_0, Y_0) \leqslant \frac{\frac{1}{m}\sum_{i=1}^{m} \frac{\theta^*_i}{\theta}}{\frac{1}{s}\sum_{r=1}^{s} \phi^*_r} = \frac{1}{\theta} R_\mathrm{e}(X_0, Y_0).$$

Moreover, it is easy to find examples in which both the given bound is reached and $R_\mathrm{e}(\theta X_0, Y_0)$ is lower than $(1/\theta) R_\mathrm{e}(X_0, Y_0)$. Therefore, a less general relationship between both values of R_e cannot be stated.

(v.2) Identical to (v.1), but taking $(\theta^*_i, \phi\phi^*_r, \lambda^*_j)$ as a feasible solution of Eq. (2) for $(X_0, \phi Y_0)$.

(v.3) This is also similar to the two previous proofs. Now, we only need to consider that for an optimal solution of Eq. (2) when DMU_0 is being evaluated, $(\theta^*_i, \phi^*_r, \lambda^*_j)$, vector $(\theta^*_i/\lambda, \lambda\phi^*_r, \lambda^*_j)$ is a feasible solution of Eq. (2) when $(\lambda X_0, (1/\lambda) Y_0)$, $\lambda > 1$, is under evaluation. Therefore,

$$R_\mathrm{e}\left(\lambda X_0, \frac{1}{\lambda} Y_0\right) \leqslant \frac{\frac{1}{m}\sum_{i=1}^{m} \frac{\theta^*_i}{\lambda}}{\frac{1}{s}\sum_{r=1}^{s} \lambda\phi^*_r} = \frac{1}{\lambda^2} R_\mathrm{e}(X_0, Y_0).$$

The same remark about the given bound as in (v.1) can be made here.

(vi.1) To get the desired relation $R_\mathrm{e} \leqslant \theta$, we have only to check that the total inefficiencies in the input-oriented radial model together with the optimal values of the scalars λ_j, give a feasible solution of Eq. (4) with an associated value of the fractional objective function less than or equal to θ.

(vi.2) The proof of this property is similar to the previous one, but replacing the input-oriented radial model by the output-oriented one, and checking that the value of the fractional objective function in the output radial model associated with the feasible solution of Eq. (4) given by the total inefficiencies and the optimal values of λ_j is less than or equal to $1/\phi$.

Finally, it should be noted that if we include the convexity constraint in the formulation of R_e the above proof remains valid. □

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