Óptica ondulatória: feixes gaussianos

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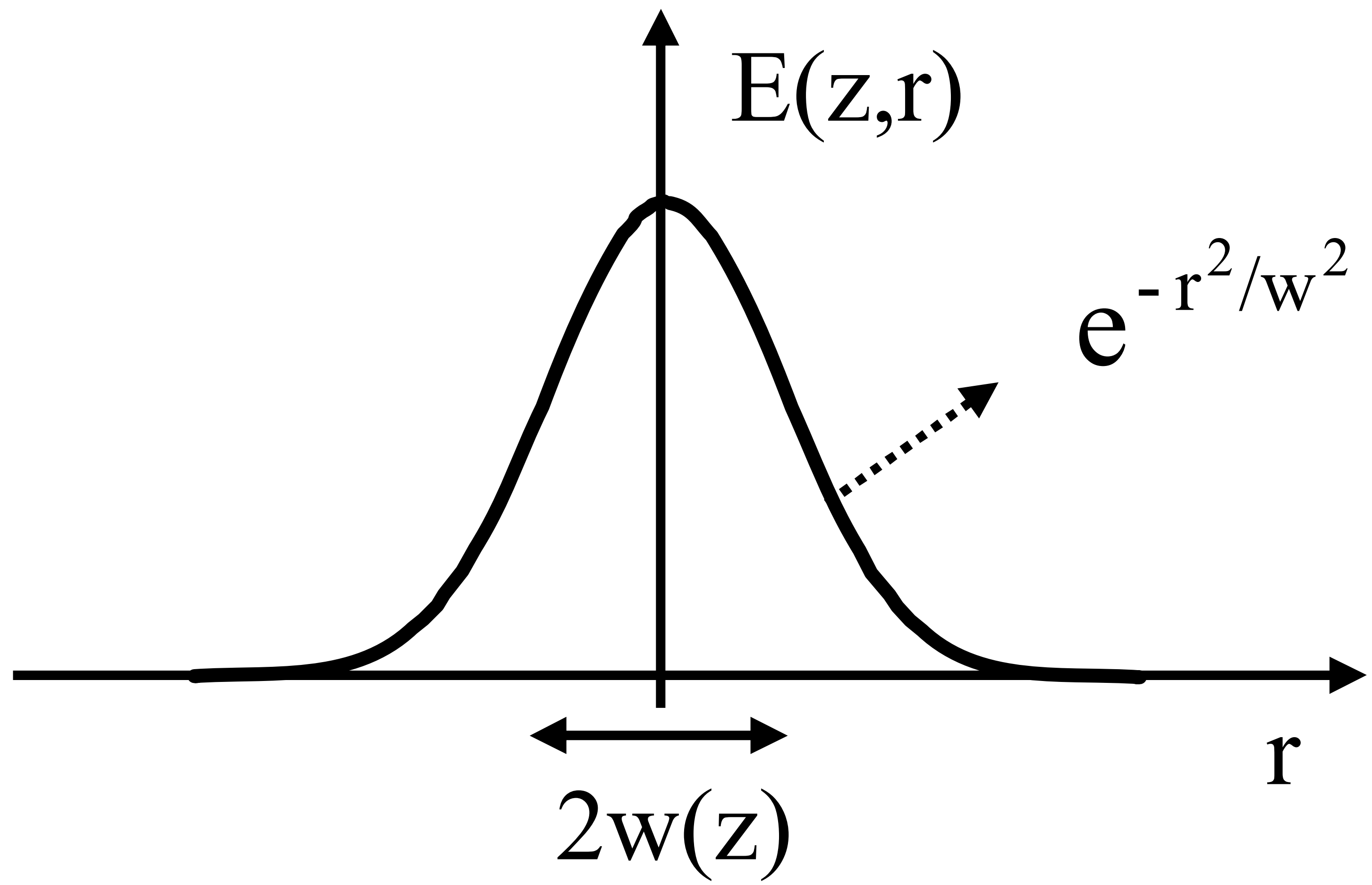


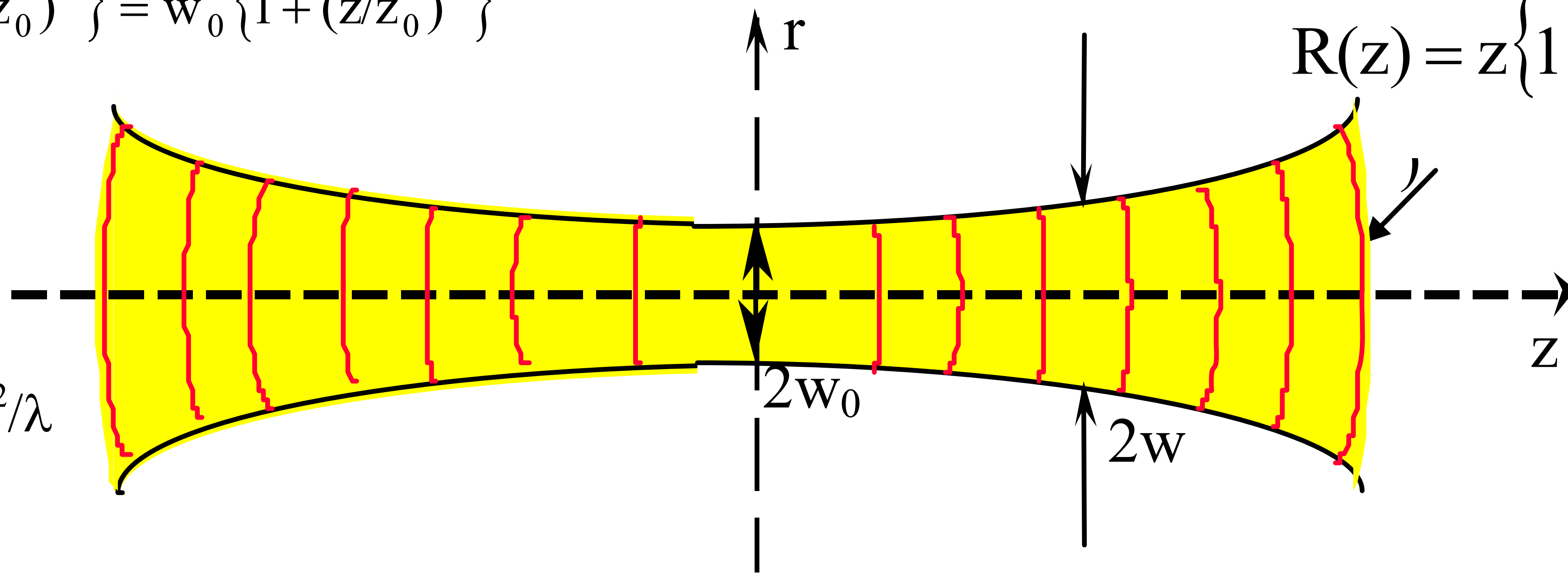
$$E(r,z) = E_0 \frac{\mathrm{w}_0}{\mathrm{w(z)}} \cdot \exp\left\{ -\left(\frac{r}{w(z)}\right)^2 \right\} \cdot \exp\left\{ -i\left[kz - \eta(z) + \frac{kr^2}{2R(z)} \right] \right\}$$

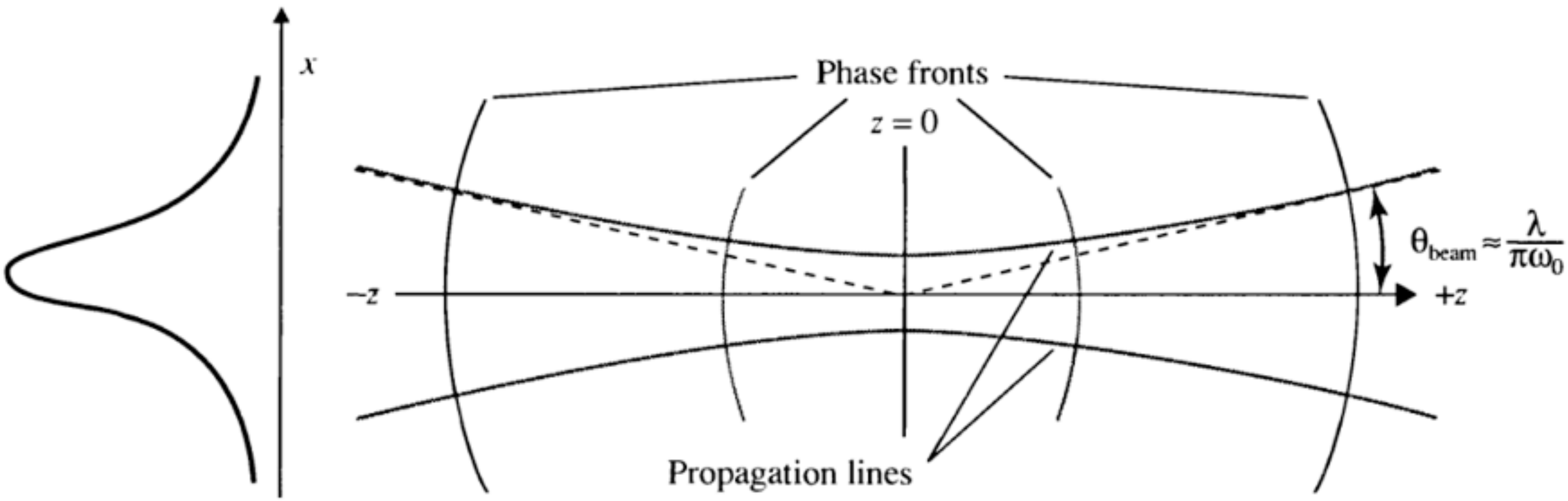
$$w^2(z) = \frac{2z_0}{k}\left\{1 + (z/z_0)^2\right\} = w_0^2\left\{1 + (z/z_0)^2\right\}$$

$$R(z) = z\left\{1 + (z_0/z)^2\right\}$$

$$w_0^2 = 2z_0/k$$

$$z_0 = kw_0^2/2 = \pi n w_0^2/\lambda$$





$$\eta(z) = tg^{-1}(z/z_0)$$

$$E(r,z) = E_0 \frac{w_0}{w(z)} \cdot \exp\left\{-\left(\frac{r}{w(z)}\right)^2\right\} \cdot \exp\left\{-i\left[kz - \eta(z) + \frac{kr^2}{2R(z)}\right]\right\}$$

Modos de ordem superior: *Hermite-Gauss* *(coordenadas cartesianas)*

$$E_{l,m}(x,y,z) = E_0 \frac{\omega_0}{\omega(z)} H_l\left(\sqrt{2}\frac{x}{\omega(z)}\right) H_m\left(\sqrt{2}\frac{y}{\omega(z)}\right) \times \exp\left(-ik\frac{x^2+y^2}{2q(z)} - ikz + i(l+m+1)\eta\right)$$

$$= E_0 \frac{\omega_0}{\omega(z)} H_l\left(\sqrt{2}\frac{x}{\omega(z)}\right) H_m\left(\sqrt{2}\frac{y}{\omega(z)}\right) \times \exp\left(-\frac{x^2+y^2}{\omega^2(z)} - ik\frac{x^2+y^2}{2R(z)} - ikz + i(l+m+1)\eta\right)$$

$$\theta = kz - (l+m+1)\tan^{-1}\left(\frac{z}{z_0}\right)$$

$$z_0 = \frac{\pi\omega_0^2 n}{\lambda}$$

$$H_l(\xi)\cdot\exp(-\xi^2/2)$$

$$\xi = \sqrt{2}\,x/\omega$$

$$u_l(\xi) = \frac{H_l(\xi)\,e^{-\xi^2/2}}{\sqrt{\pi^{1/2}\,l!\,2^l}}$$

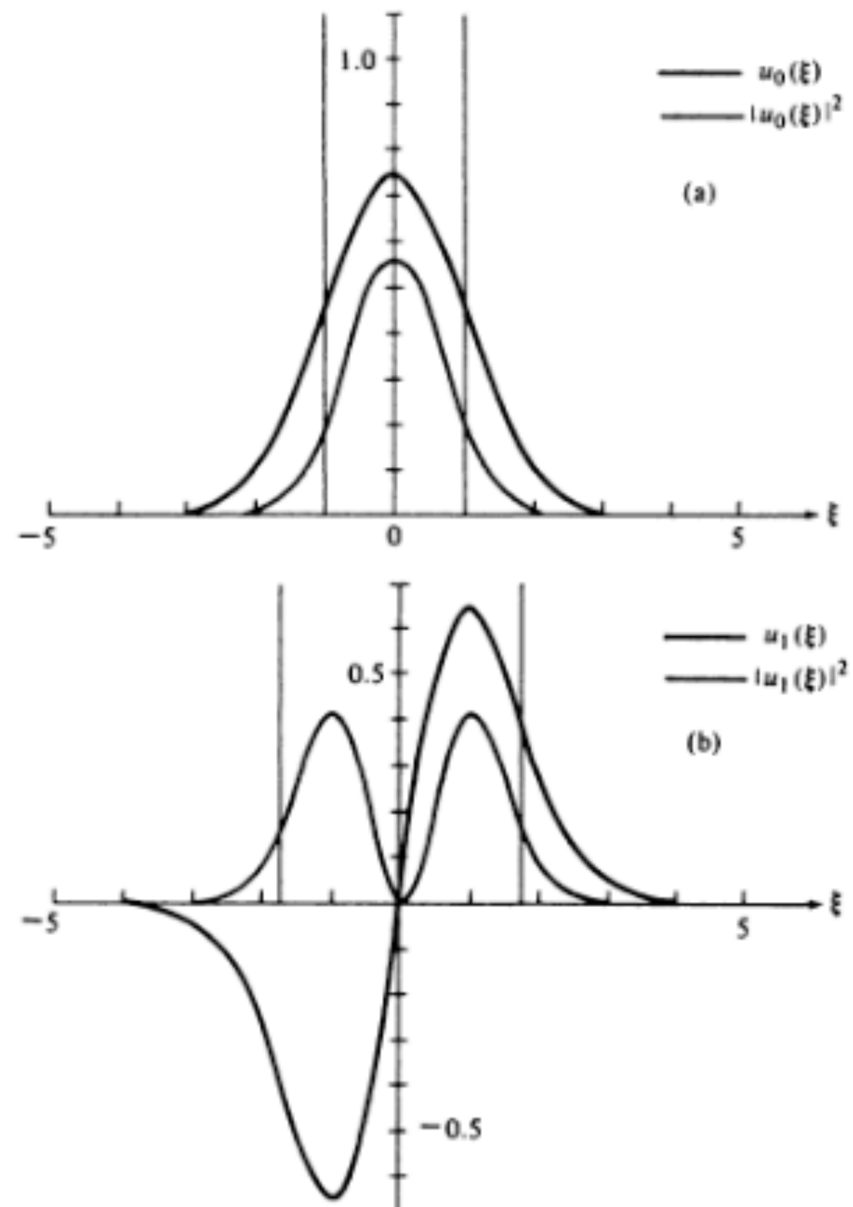
$$H_0(x) = 1$$

$$H_1(x) = 2x$$

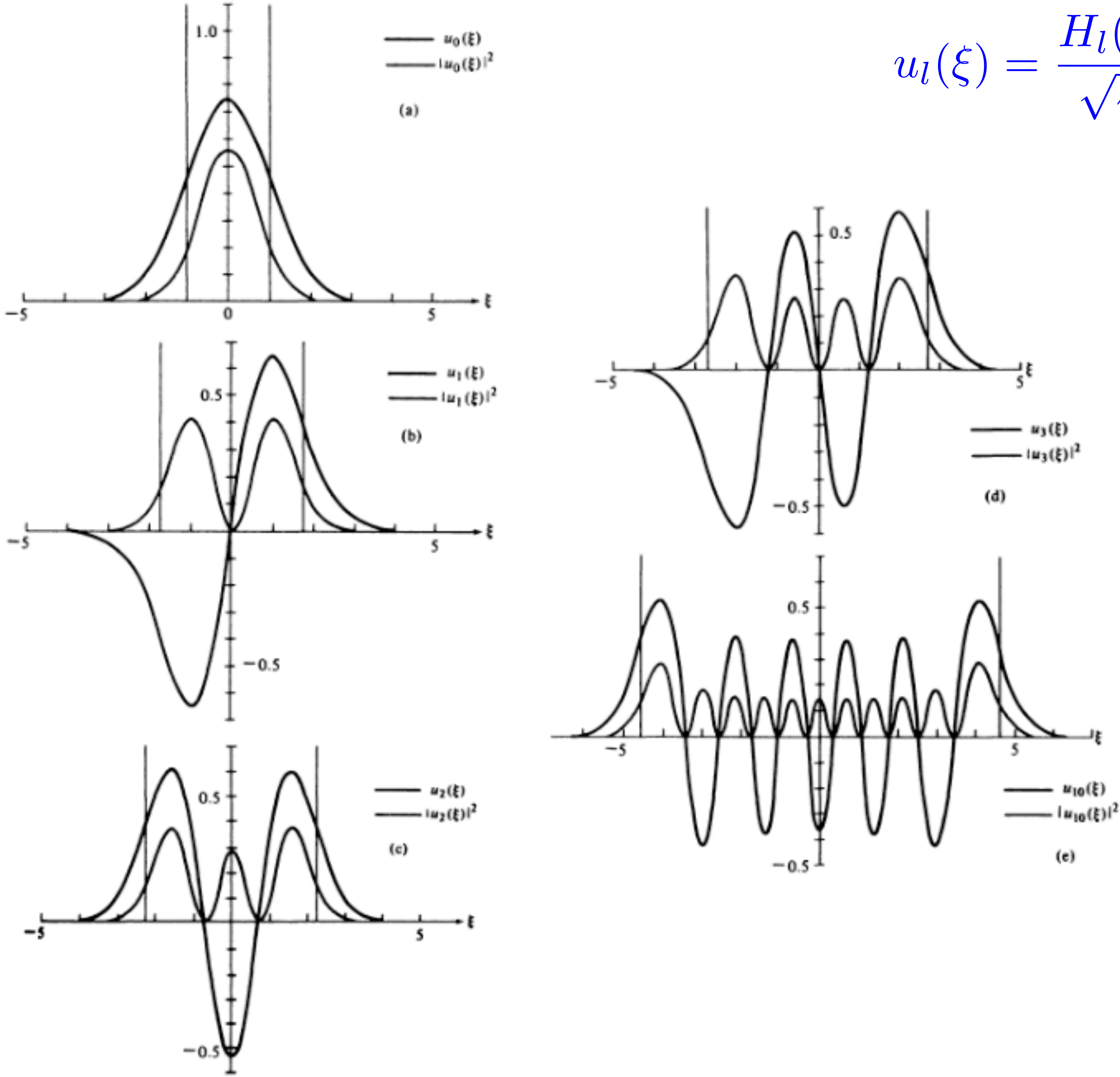
$$H_2(x) = 4x^2 - 2$$

$$H_3(x) = 8x^3 - 12x$$

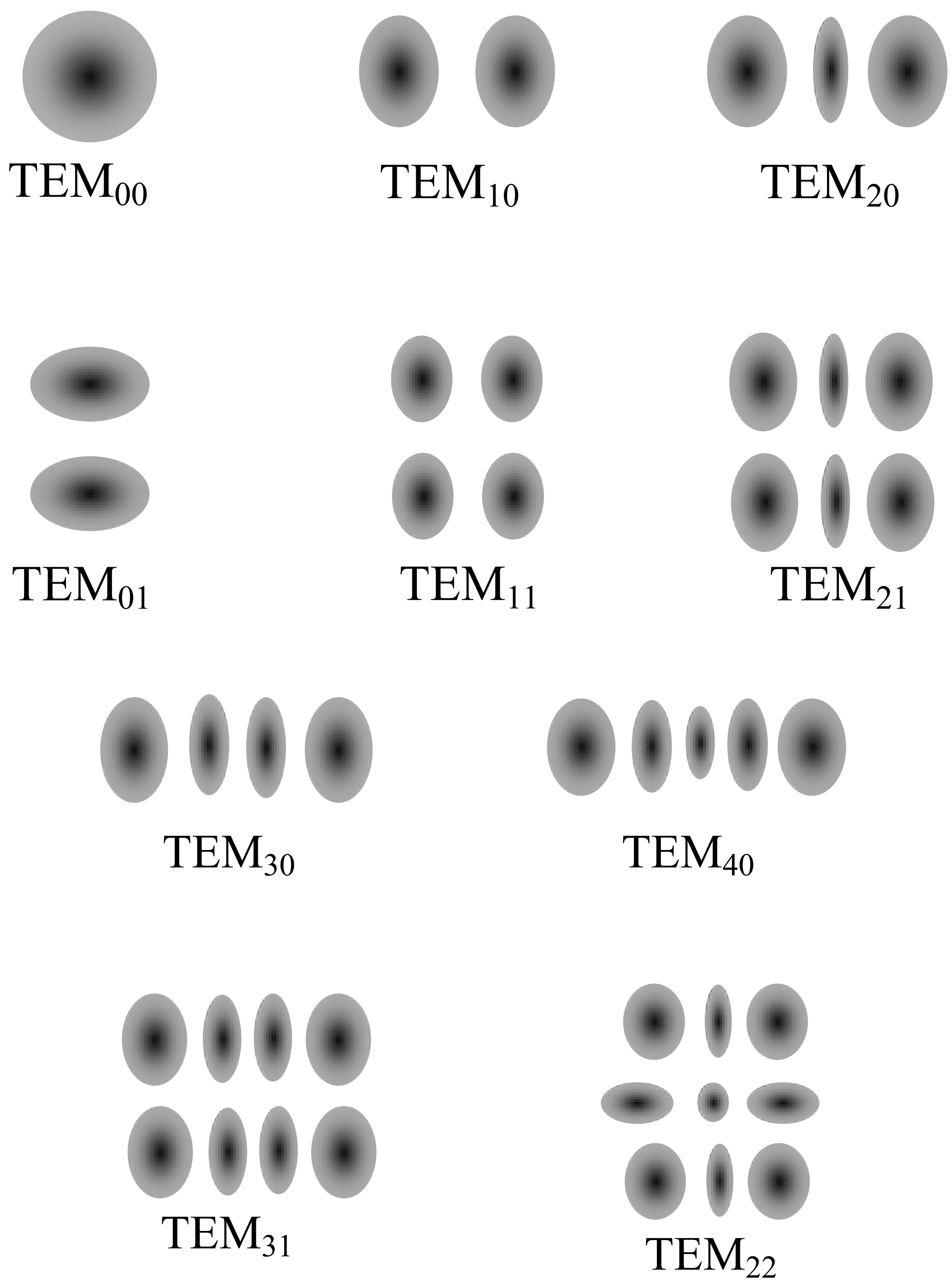
$$H_4(x) = 16x^4 - 48x^2 + 12$$

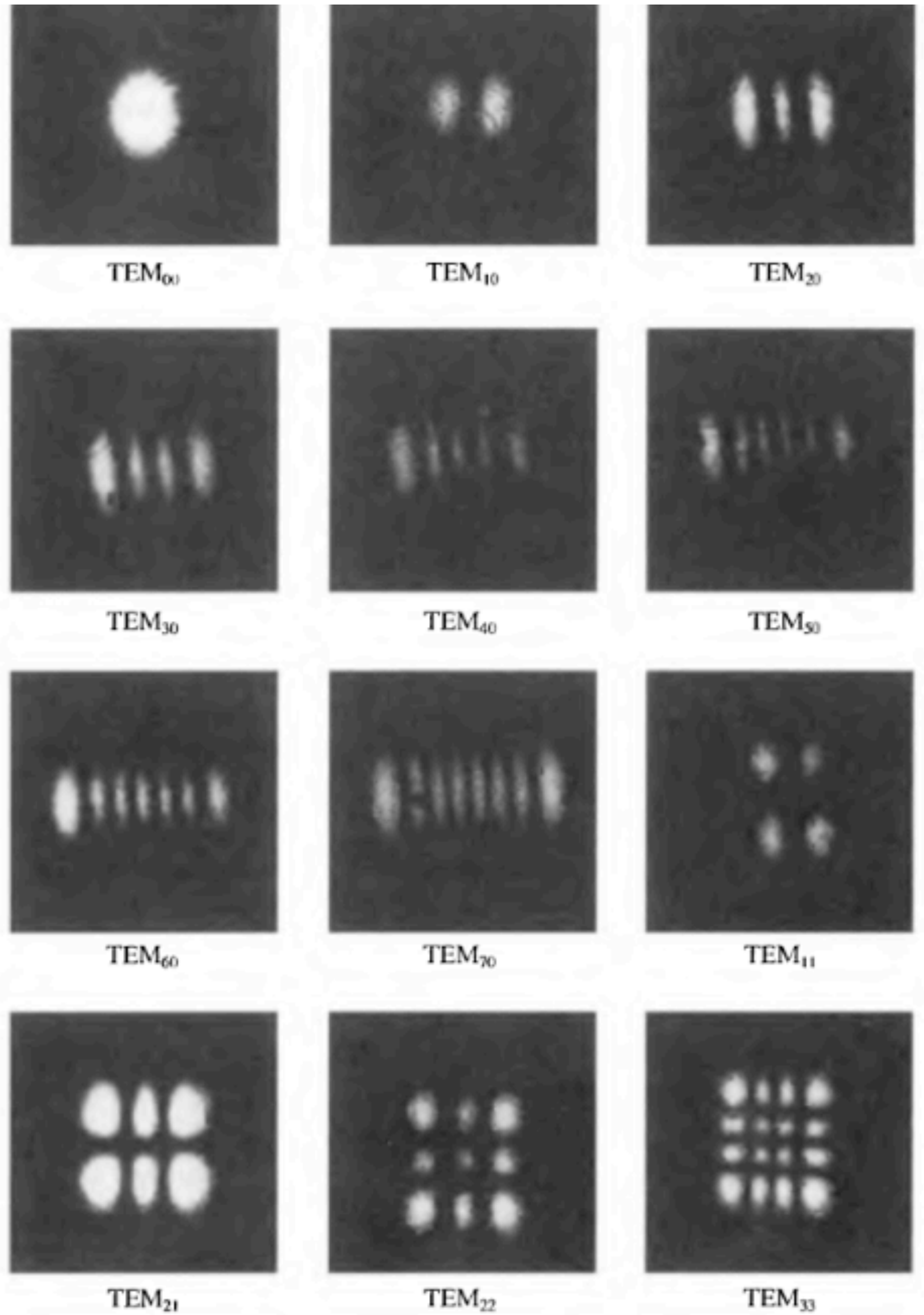


$$u_l(\xi) = \frac{H_l(\xi)\, e^{-\xi^2/2}}{\sqrt{\pi^{1/2}\, l!\, 2^l}}$$



Distribuição espacial dos modos
(*Hermite-Gauss*)

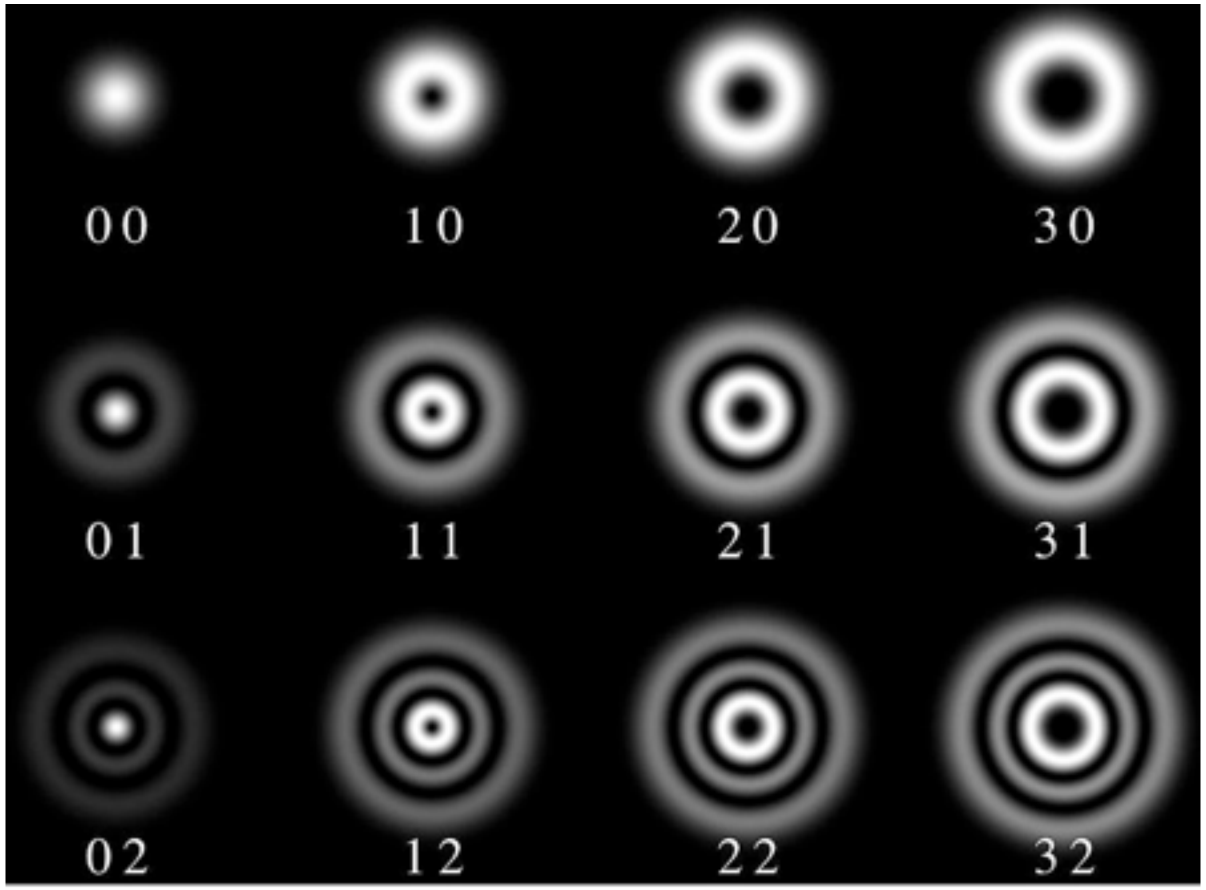




Modos de ordem superior: *Laguerre-Gauss* *(coordenadas cilindricas)*

$$E_{p,m}(r,\phi,z) = E_0 \frac{\omega_0}{\omega(z)} \left(\frac{\sqrt{2}r}{\omega(z)}\right)^{|m|} L_p^{|m|}\left(\frac{2r^2}{\omega^2(z)}\right) \exp\left[-ik\frac{r^2}{2q(z)} - ikz + im\phi + i(2p+|m|+1)\eta\right]$$

$$= E_0 \frac{\omega_0}{\omega(z)} \left(\frac{\sqrt{2}r}{\omega(z)}\right)^{|m|} L_p^{|m|}\left(\frac{2r^2}{\omega^2(z)}\right) \exp\left[-\frac{r^2}{\omega^2(z)} - ik\frac{r^2}{2R(z)} - ikz + im\phi + i(2p+|m|+1)\eta\right]$$



$$L_0^m(x) = 1$$

$$L_1^m(x) = -x + (m+1)$$

$$L_2^m(x) = \tfrac{1}{2}[x^2 - 2(m+2)x + (m+1)(m+2)]$$

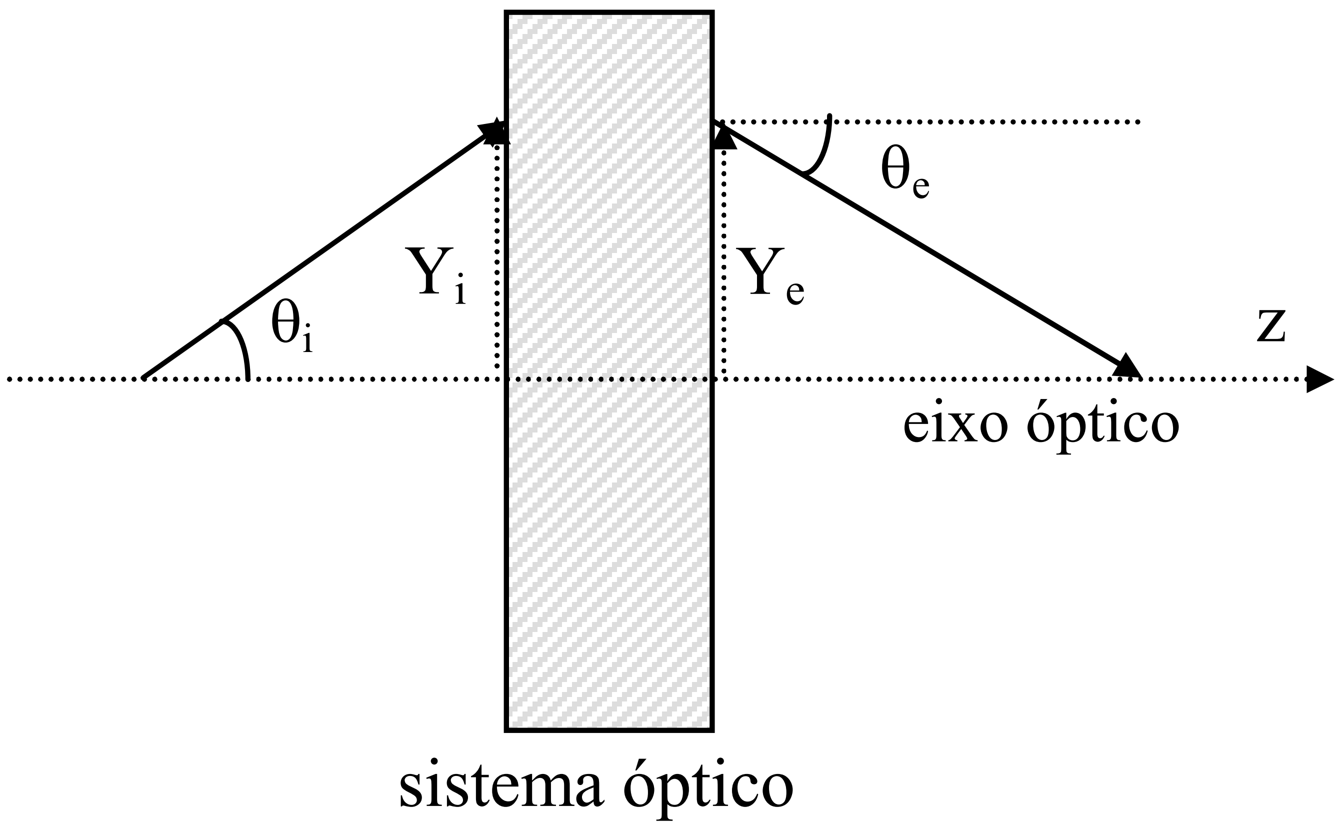
$$L_3^m(x) = \tfrac{1}{6}[-x^3 + 3(m+3)x^2 - 3(m+2)(m+3)x + (m+1)(m+2)(m+3)]$$

Propriedade importante...

$$\iint E^*_{l_1 m_1}(x, y, z)\, E_{l_2, m_2}(x, y, z)\, dx\, dy = \delta_{(l_1 m_1)(l_2 m_2)}$$

$$E(x, y, z) = \sum_l \sum_m c_{lm} E_{l,m}(x, y, z)$$

Propagação de raios: lei ABCD



$$\begin{pmatrix} Y_e \\ \theta_e \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} Y_i \\ \theta_i \end{pmatrix}$$

$$| R_e \rangle = S | R_i \rangle$$

$$| R_n \rangle = S_n S_{n-1} \ldots S_2 S_1 | R_1 \rangle$$

Propagação da luz: caso geral

$$E(x, y, z) = \iint A(k_x, k_y) \exp(-ik_x x - ik_y y) \exp(-ik_z z)\, dk_x\, dk_y$$

$$A(k_x, k_y) = \frac{1}{(2\pi)^2} \iint E(x, y) \exp(ik_x x + ik_y y)\, dx\, dy$$

$$E(x, y, L) = \frac{i}{\lambda L} e^{-ikL} \iint E(x', y') e^{-ik[(x-x')^2+(y-y')^2]/2L}\, dx'\, dy'$$

Integral de Difração de Fresnel-Kirchhoff