



ESCOLA POLITÉCNICA DA UNIVERSIDADE DE SÃO PAULO

Campo
Magnetostático
Lâmina
Solenóide
Toroide



Campo Magnético - Potencial Vetor Magnético

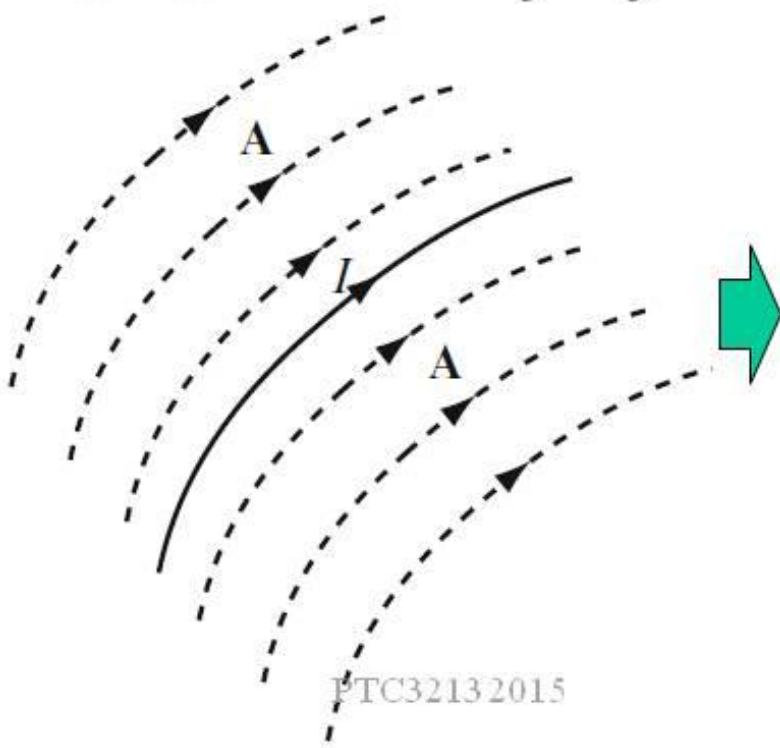
$$\nabla \times \vec{H} = \vec{J} \Rightarrow \vec{B} = \nabla \times \vec{A} \Rightarrow \nabla^2 \vec{A} = -\mu \vec{J}$$

$$\nabla^2 \vec{A} = \nabla^2 A_x \hat{u}_x + \nabla^2 A_y \hat{u}_y + \nabla^2 A_z \hat{u}_z$$

$$\nabla^2 A_x = -\mu J_x$$

$$\nabla^2 A_y = -\mu J_y$$

$$\nabla^2 A_z = -\mu J_z$$



$$\vec{H} = \frac{1}{\mu} \nabla \times \vec{A}$$

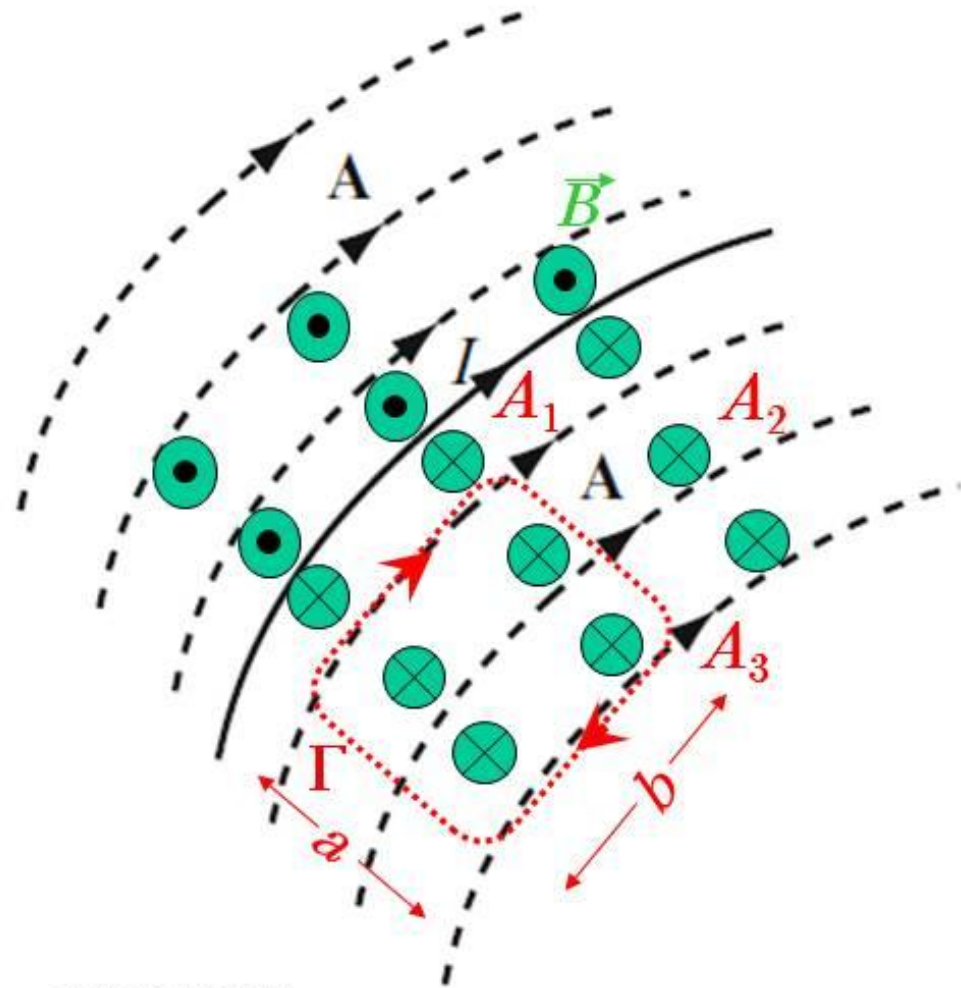
$$\psi = \oint_{\Gamma} \vec{A} \cdot d\vec{l}$$



Potencial Vetor Magnético

$$\psi = \oint_{\Gamma} \vec{A} \cdot d\vec{l}$$

$$\psi \approx b(A_1 - A_3)$$





Campo Magnetostático – Cálculo a partir de Fontes

$$I = \iint_S \vec{J} \cdot d\vec{S}$$

Lei de Ampère

$$\oint_{\Gamma} \vec{H} \cdot d\vec{l} = I$$

Lei de Biot-Savart

$$\vec{H}(\vec{r}) = \frac{I}{4\pi} \oint_{\Gamma} \frac{d\vec{l}' \times \hat{u}_R}{R^2}$$

$$\vec{A}(\vec{r}) = \frac{\mu I}{4\pi} \oint_{\Gamma} \frac{dl'}{R}$$



Lei de Ampère × Lei de Biot-Savart

Fonte

$$I = \iint_S \vec{J} \cdot d\vec{S}$$

Ampère:

Geometrias simples, com simetrias

Fios cilíndricos infinitos

Lâminas de Corrente

Toroide

Biot-Savart:

Mais geral

Fios finitos

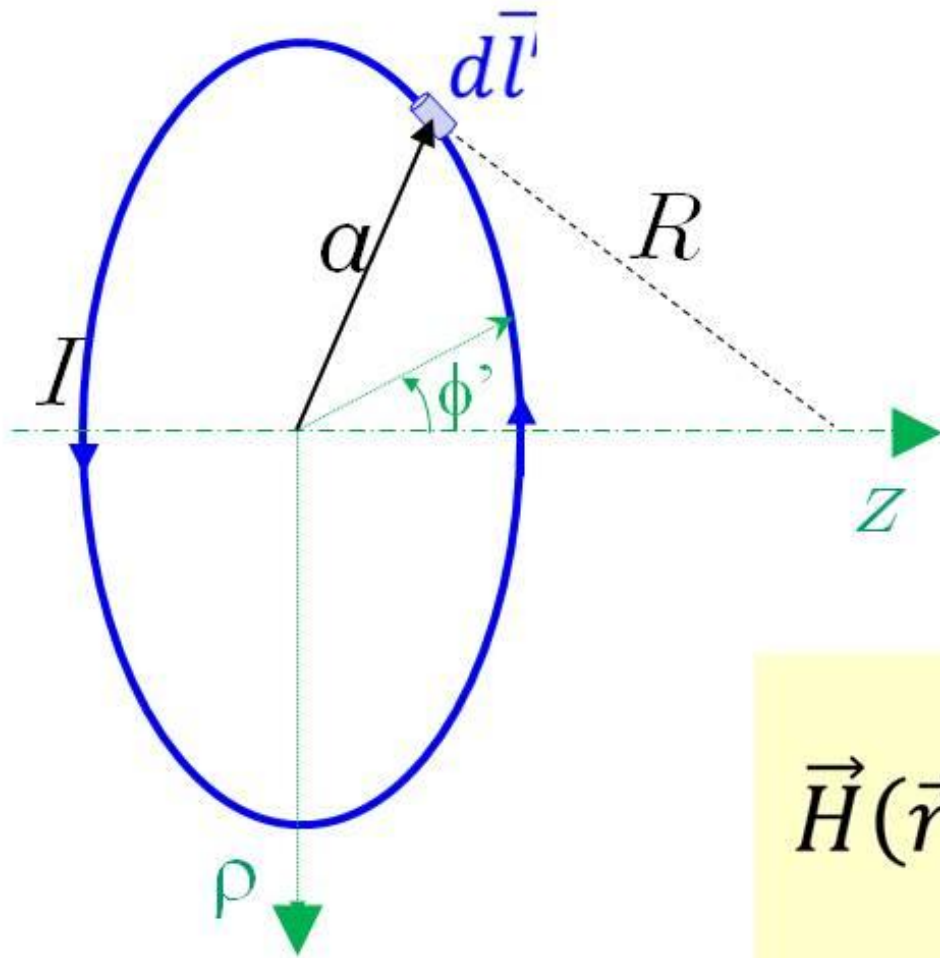
Espira

Solenóide



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Campo criado por Espira Circular



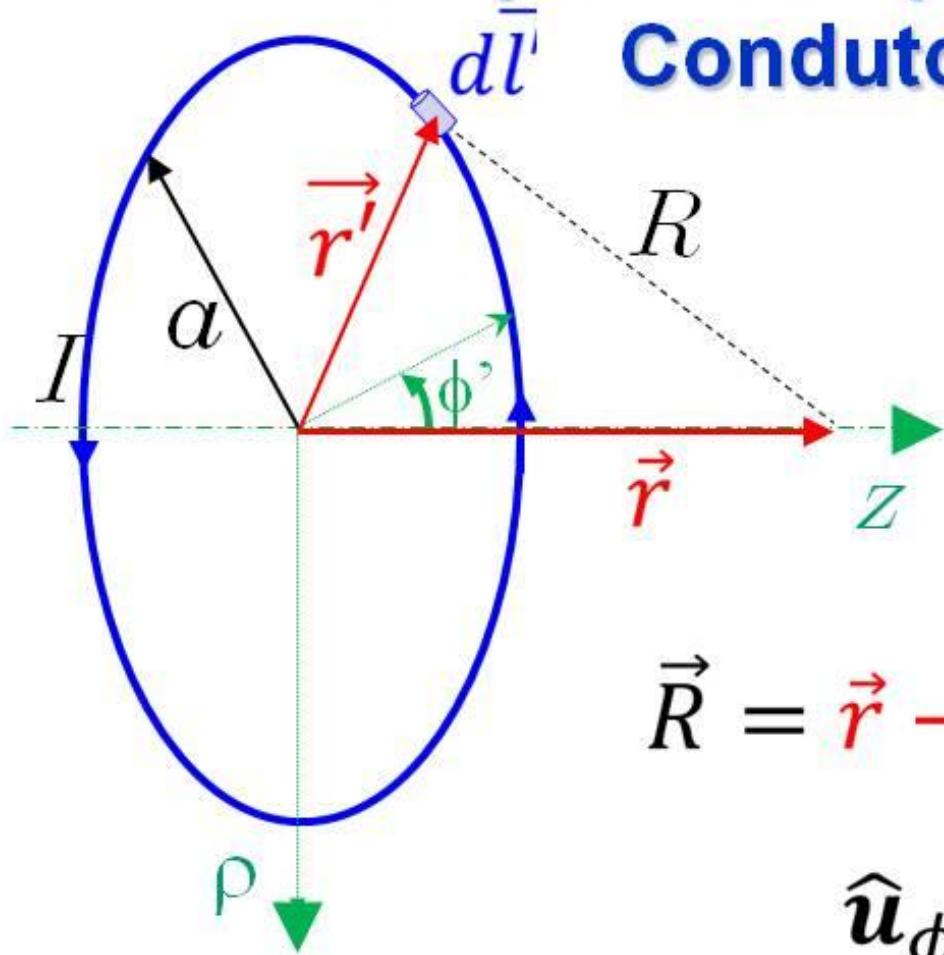
$$\vec{H}(\vec{r}) = \frac{I}{4\pi} \oint_{\Gamma} \frac{d\vec{l}' \times \vec{R}}{R^3}$$

$$d\vec{l}' = a d\phi' \hat{u}_{\phi}$$

$$\vec{H}(\vec{r}) = \frac{I}{4\pi} \oint_{\Gamma} \frac{a \hat{u}_{\phi} \times \vec{R}}{R^3} d\phi'$$



Campo criado por Espira Circular Condutor filiforme



$$\vec{H}(\vec{r}) = \frac{I}{4\pi} \oint_{\Gamma} \frac{a \hat{u}_{\phi} \times \vec{R}}{R^3} d\phi'$$

$$\vec{r} = z \hat{u}_z$$

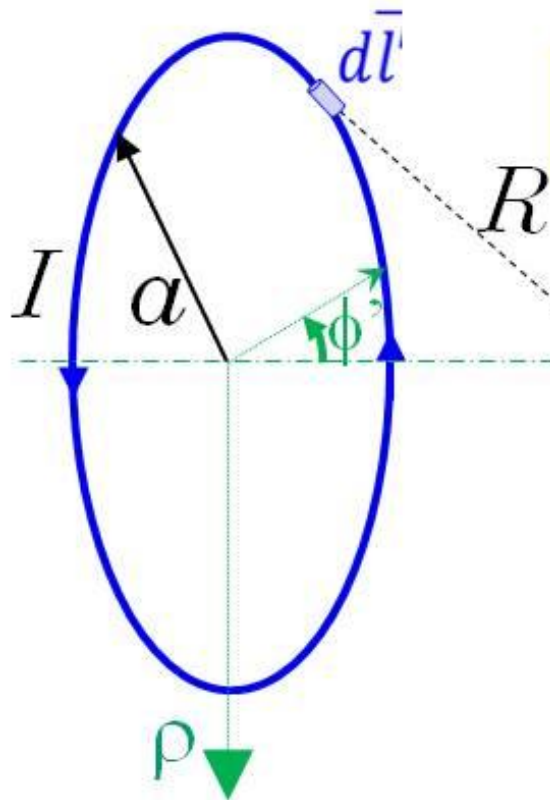
$$\vec{r}' = a \hat{u}_{\rho}$$

$$\vec{R} = \vec{r} - \vec{r}' \quad \Rightarrow \quad R = \sqrt{z^2 + a^2}$$

$$\hat{u}_{\phi} \times \vec{R} = z \hat{u}_{\rho} + a \hat{u}_z$$



Campo criado por Espira Circular



$$\vec{H}(\rho = 0, z) = \frac{I}{4\pi} \int_0^{2\pi} \frac{az \hat{u}_\rho + a^2 \hat{u}_z}{(z^2 + a^2)^{3/2}} d\phi'$$

$$= \frac{I}{4\pi} \frac{az}{(z^2 + a^2)^{3/2}} \int_0^{2\pi} \hat{u}_\rho d\phi' +$$

$\hat{u}_\rho(\phi') = \hat{u}_\rho(\phi' + \pi)$

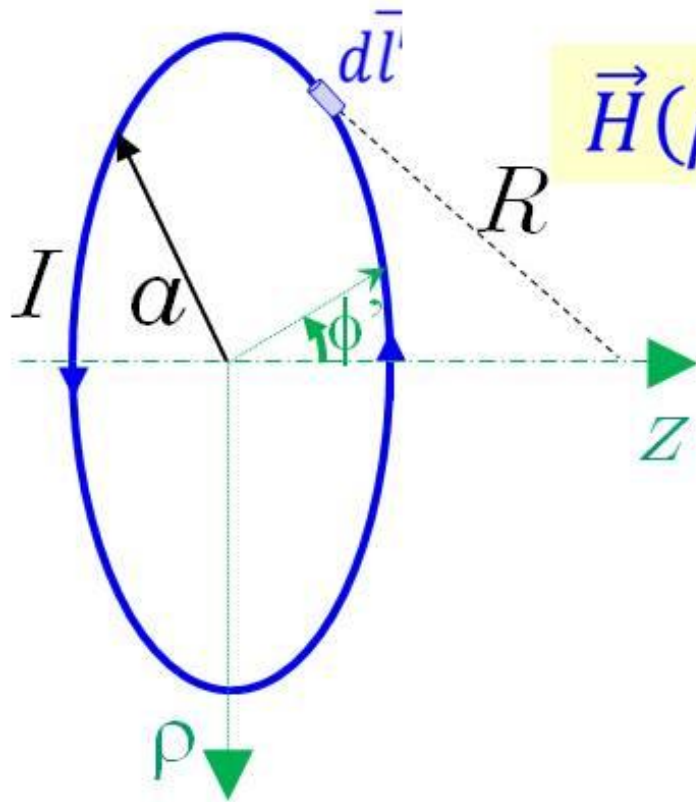
$\int_0^{2\pi} \hat{u}_\rho d\phi' = 0$

$$+ \frac{I}{4\pi} \frac{a^2 \hat{u}_z}{(z^2 + a^2)^{3/2}} \int_0^{2\pi} d\phi' = \frac{Ia^2}{2(z^2 + a^2)^{3/2}} \hat{u}_z$$



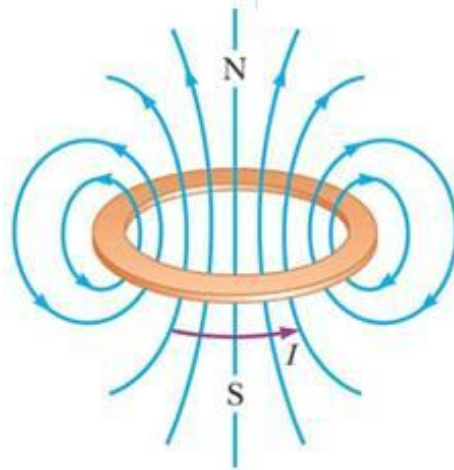
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Campo criado por Espira Circular



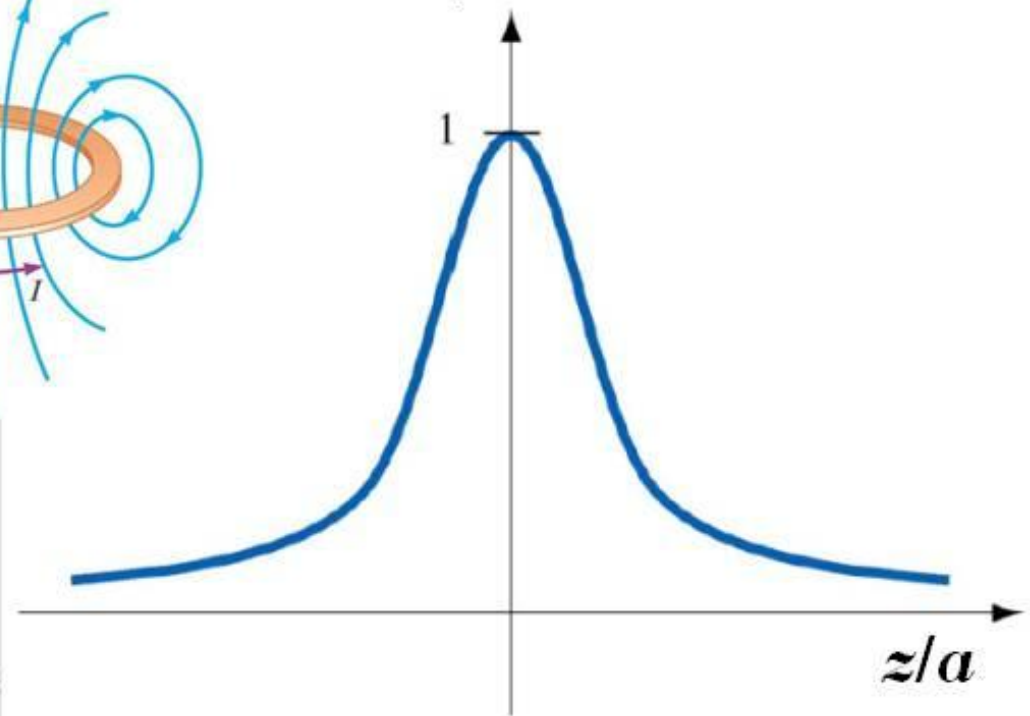
$$\vec{H}(\rho = 0, z) =$$

$$\frac{Ia^2}{2(z^2 + a^2)^{3/2}} \hat{u}_z$$



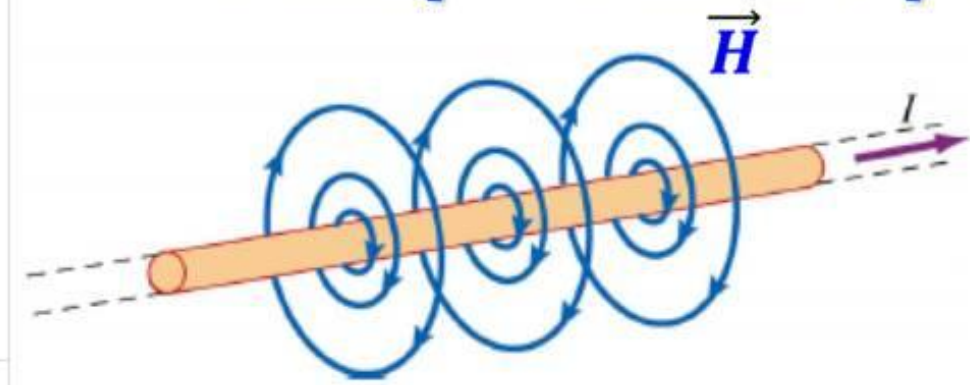
$$|\vec{H}(0,0)| = H_{max} = H_0$$

H_z/H_0

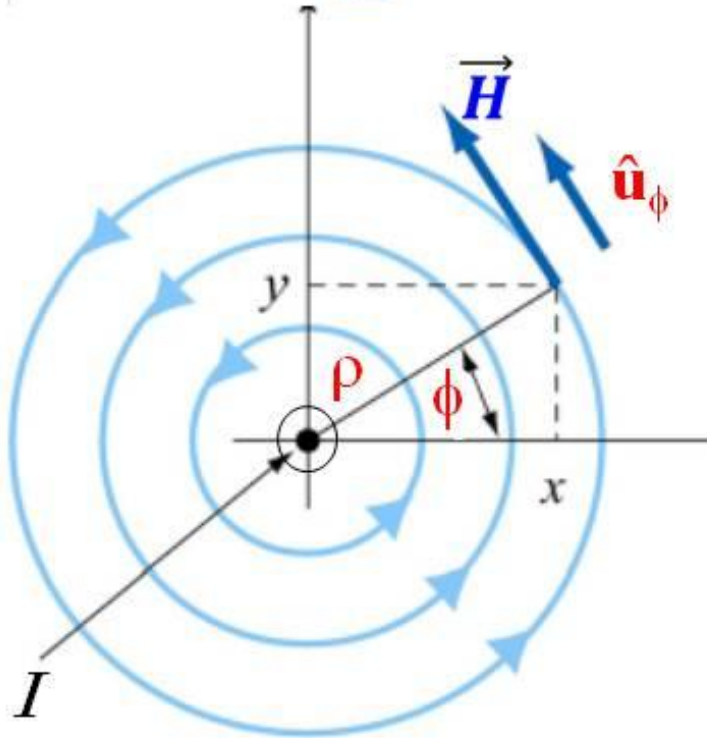




Campo Criado por Fio Reto Infinito



$$\oint_{\Gamma} \vec{H} \cdot d\vec{l} = \iint_S \vec{J} \cdot \vec{dS} = I$$



$$\oint_{\Gamma} \vec{H} \cdot d\vec{l} = \int_0^{2\pi} H_{\phi} \cdot \rho d\phi$$

$$= H_{\phi} \rho \int_0^{2\pi} d\phi$$



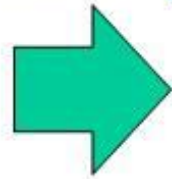
$$\vec{H} = \frac{I}{2\pi\rho} \hat{u}_{\phi}$$

$\rho > a$



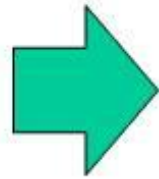
Campo Criado por Fio Reto Infinito

$$\rho > a$$



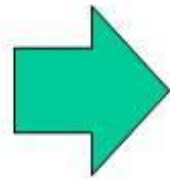
$$\vec{H} = \frac{I}{2\pi\rho} \hat{u}_\phi$$

$$I = \iint_S \vec{J} \cdot \vec{dS}$$

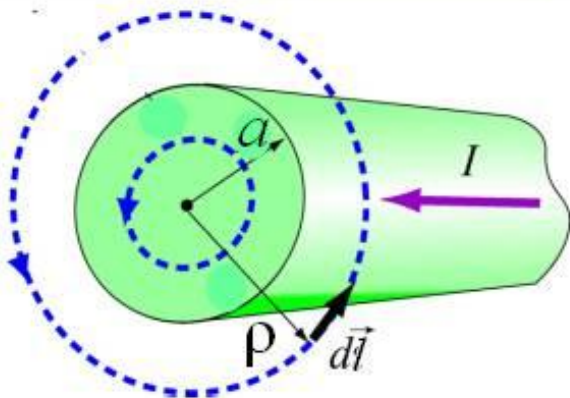


$$\vec{J} = \frac{I}{\pi a^2} \hat{u}_z$$

$$\rho < a$$



$$\iint_S \vec{J} \cdot \vec{dS} = \int_0^\rho \frac{I}{\pi a^2} 2\pi\rho' d\rho'$$



$$\oint_\Gamma \vec{H} \cdot d\vec{l} = \frac{I}{\pi a^2} \pi \rho^2 = \frac{I \rho^2}{a^2}$$

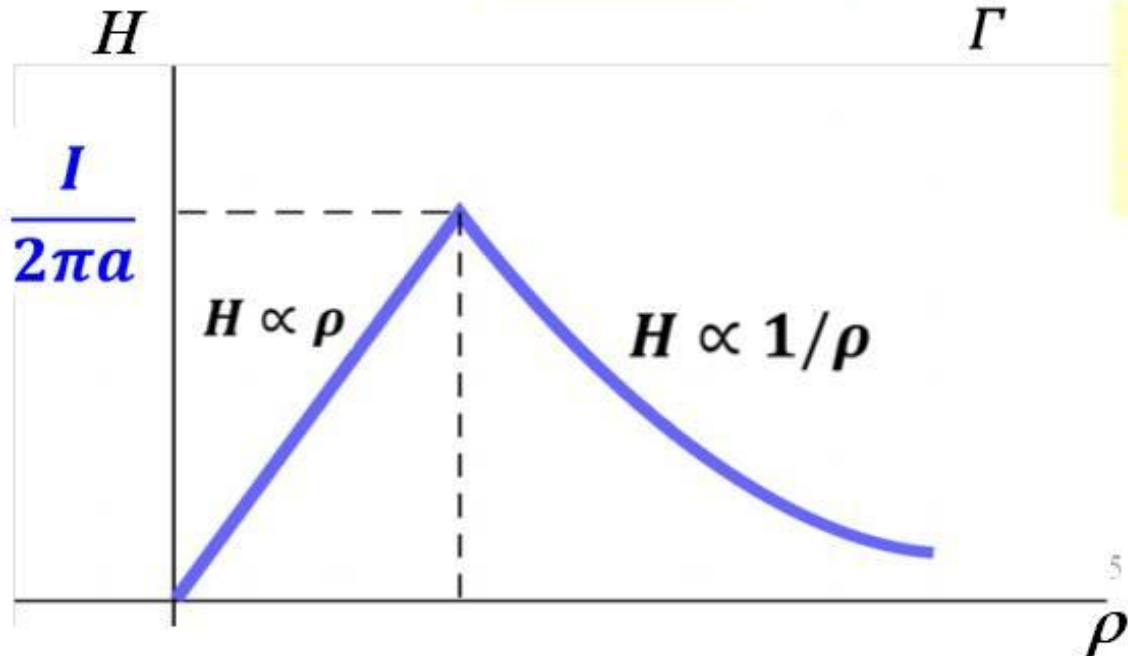


Campo Criado por Fio Reto Infinito

$\rho > a$ \rightarrow $\vec{H} = \frac{I}{2\pi\rho} \hat{u}_\phi$

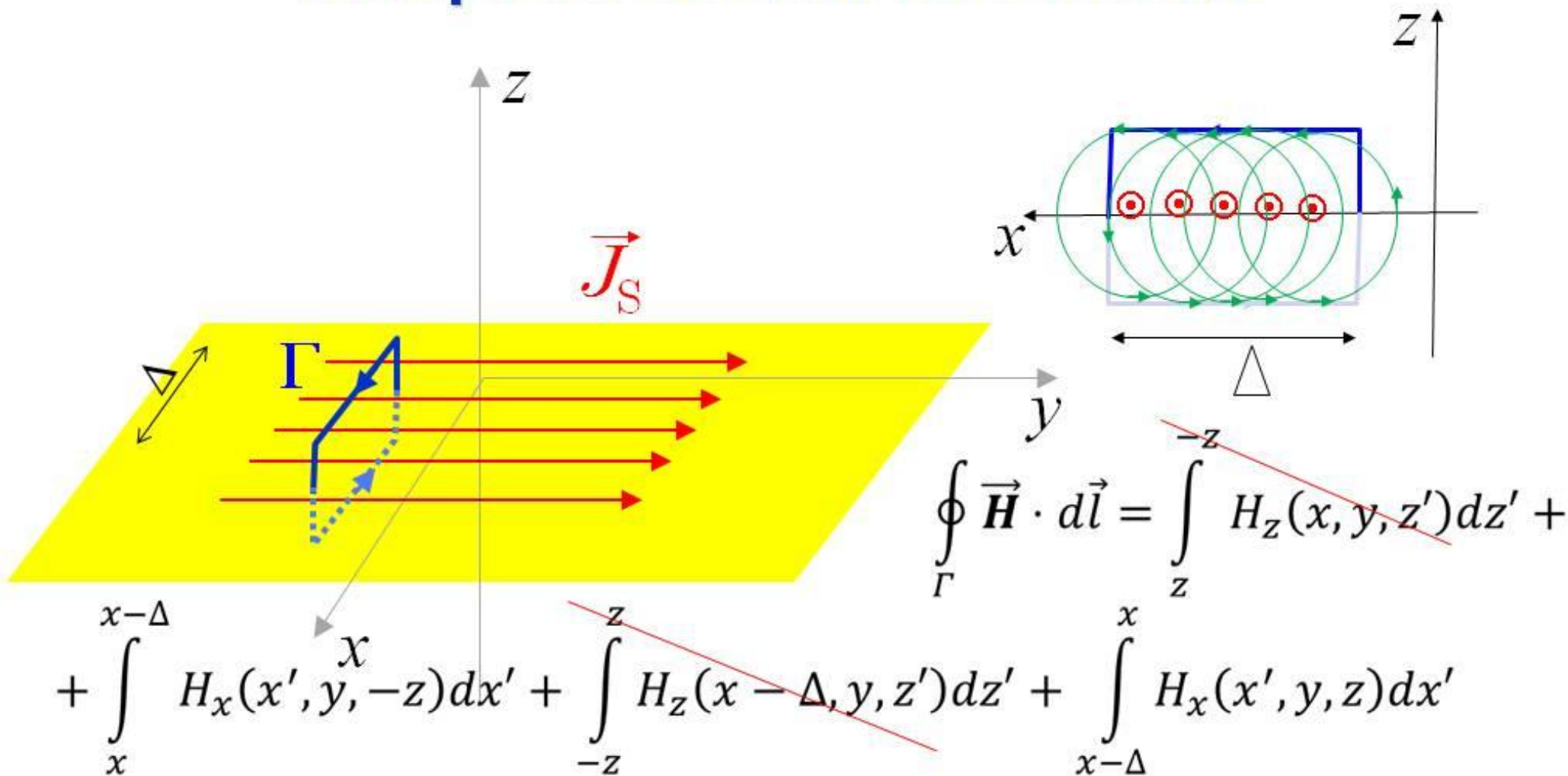
$\rho < a$ \rightarrow $\oint_{\Gamma} \vec{H} \cdot d\vec{l} = 2\pi\rho H_\phi = \frac{I\rho^2}{a^2}$

$\vec{H} = \frac{I}{2\pi a^2} \rho \hat{u}_\phi$



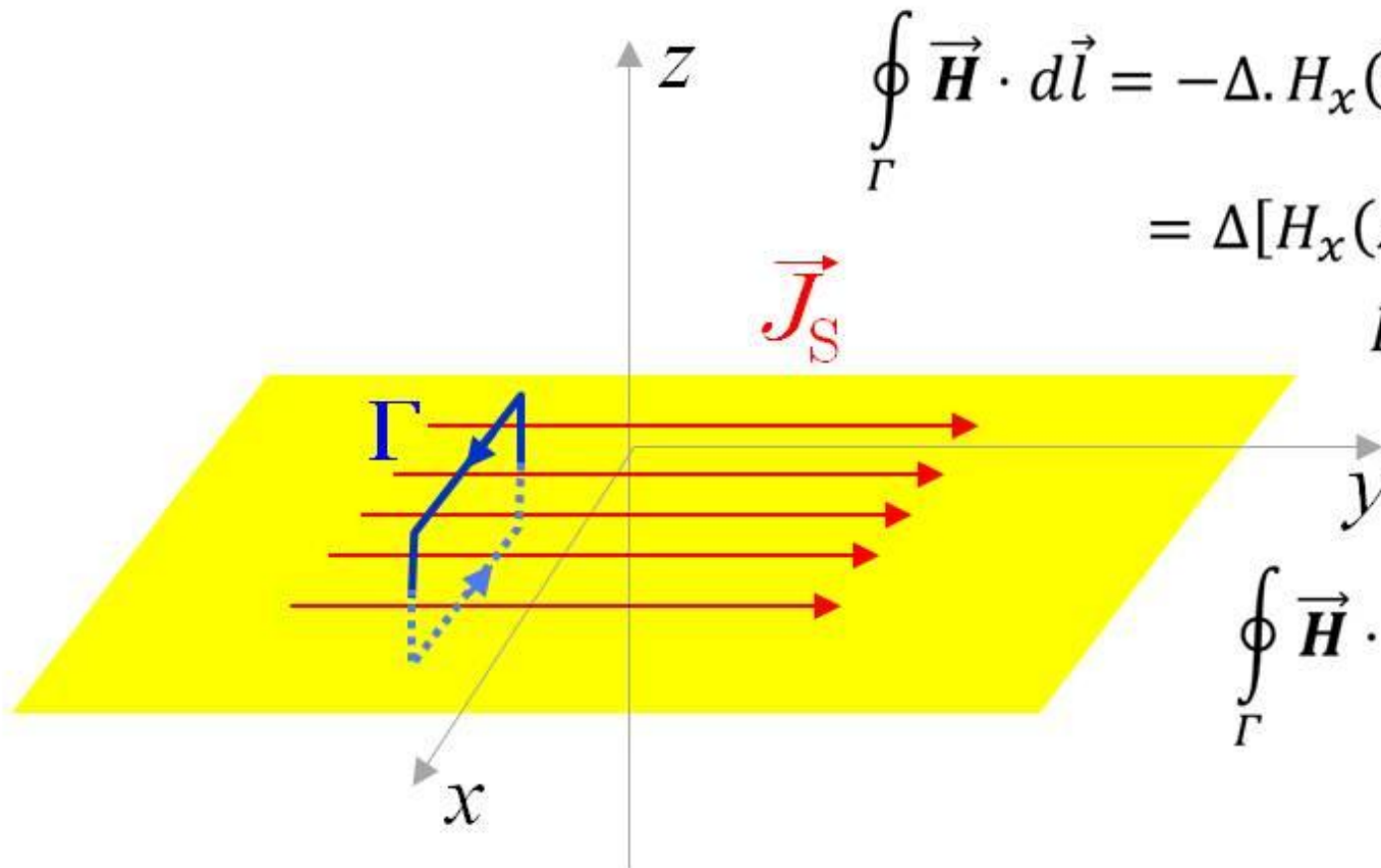


Campo de Lâmina de Corrente





Campo de Lâmina de Corrente



$$\begin{aligned} \oint_{\Gamma} \vec{H} \cdot d\vec{l} &= -\Delta \cdot H_x(-z) + \Delta \cdot H_x(z) = \\ &= \Delta[H_x(z) - H_x(-z)] = \\ &H_x(z) = -H_x(-z) \end{aligned}$$

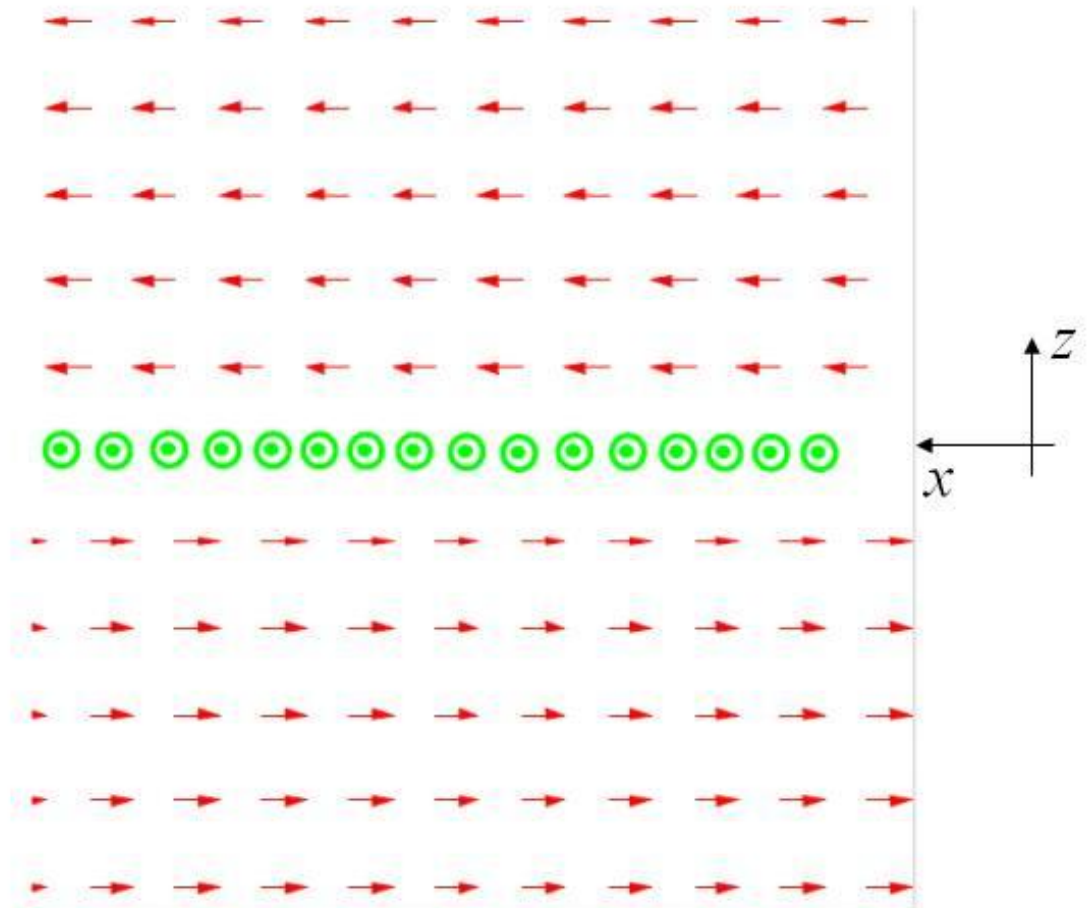
$$\oint_{\Gamma} \vec{H} \cdot d\vec{l} = 2 \cdot \Delta \cdot H_x(z) = J_S \Delta$$



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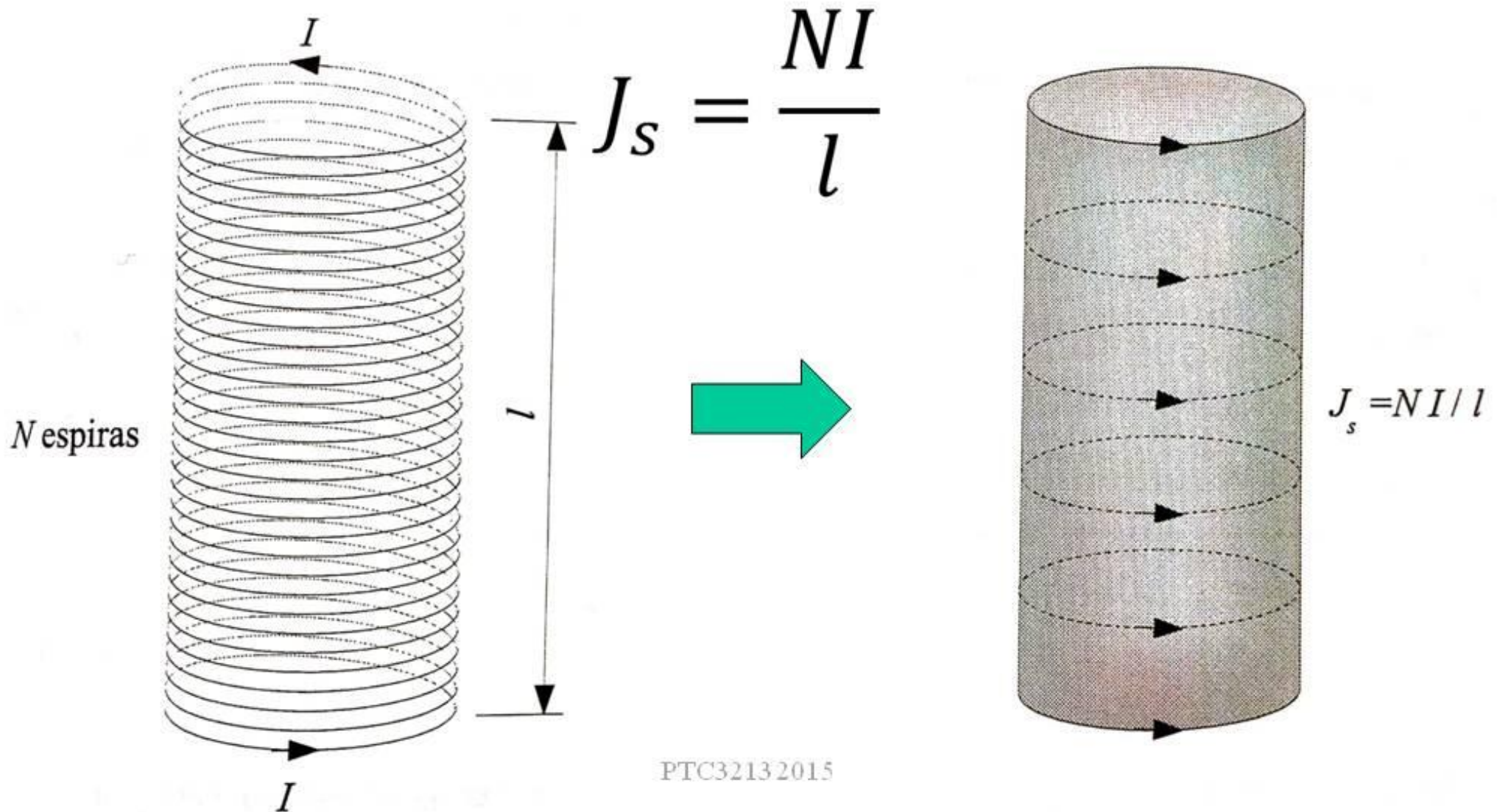
Campo de Lâmina de corrente

$$\vec{H} = \begin{cases} \frac{J_s}{2} \hat{u}_x, & z > 0 \\ -\frac{J_s}{2} \hat{u}_x, & z < 0 \end{cases}$$



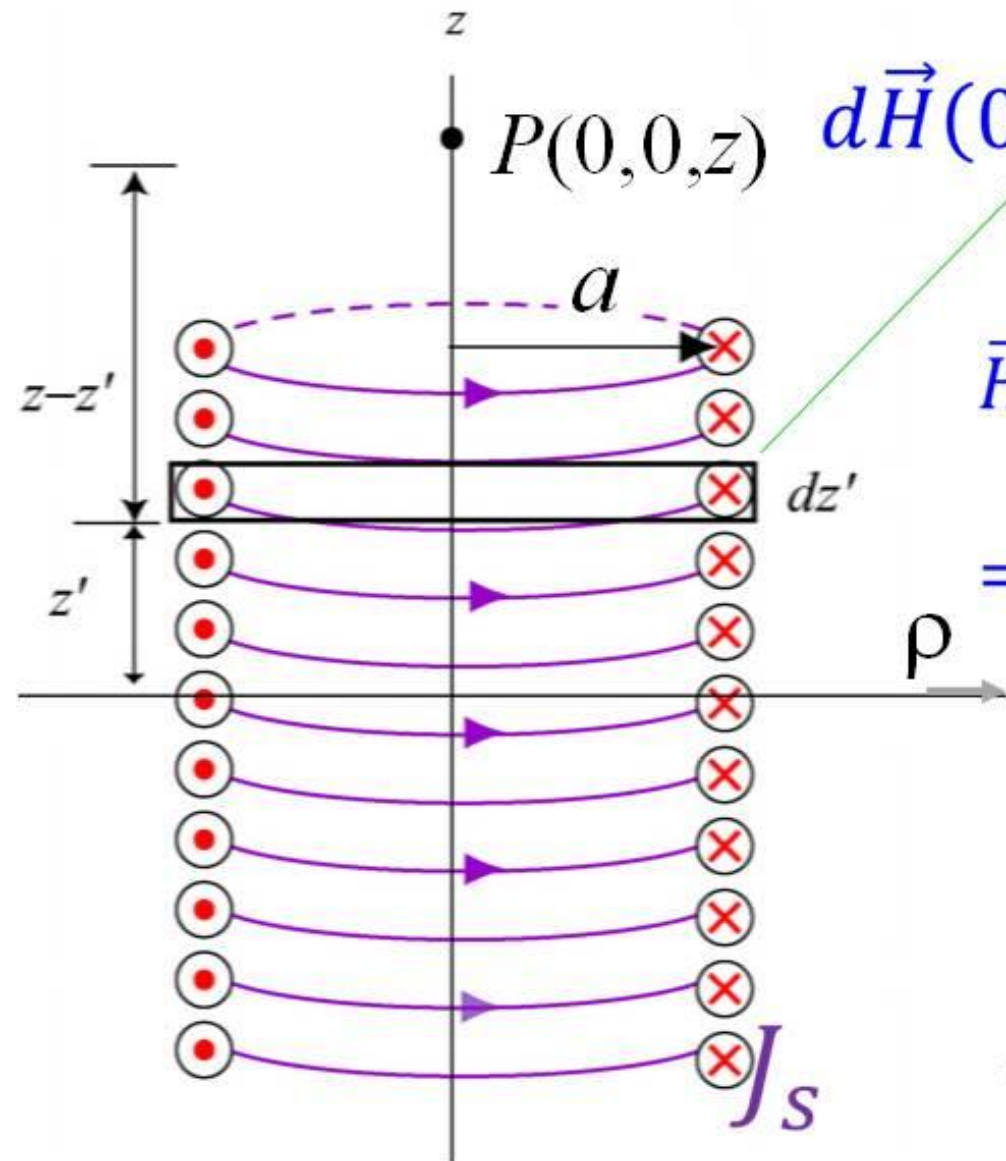


Campo de Solenoide





Campo de Solenoide



$$d\vec{H}(0, z) = \frac{J_s a^2 dz'}{2[(z - z')^2 + a^2]^{3/2}} \hat{u}_z$$

$$\begin{aligned} \vec{H}(0, z) &= \int_{-l/2}^{l/2} \frac{J_s a^2 dz'}{2[(z - z')^2 + a^2]^{3/2}} \hat{u}_z \\ &= J_s a^2 \int_{-l/2}^{l/2} \frac{dz'}{2[(z - z')^2 + a^2]^{3/2}} \hat{u}_z \end{aligned}$$



Campo de Solenoide

$P(0,0,z)$

$\vec{H}(0,z) =$

$$= \frac{NI}{2l} \left[\frac{z+l/2}{\sqrt{a^2+(z+l/2)^2}} - \frac{z-l/2}{\sqrt{a^2+(z-l/2)^2}} \right] \hat{u}_z$$

$\vec{H}(0,z) =$

$$\frac{NI}{2l} [\cos \theta_1 - \cos \theta_2] \hat{u}_z$$

J_s

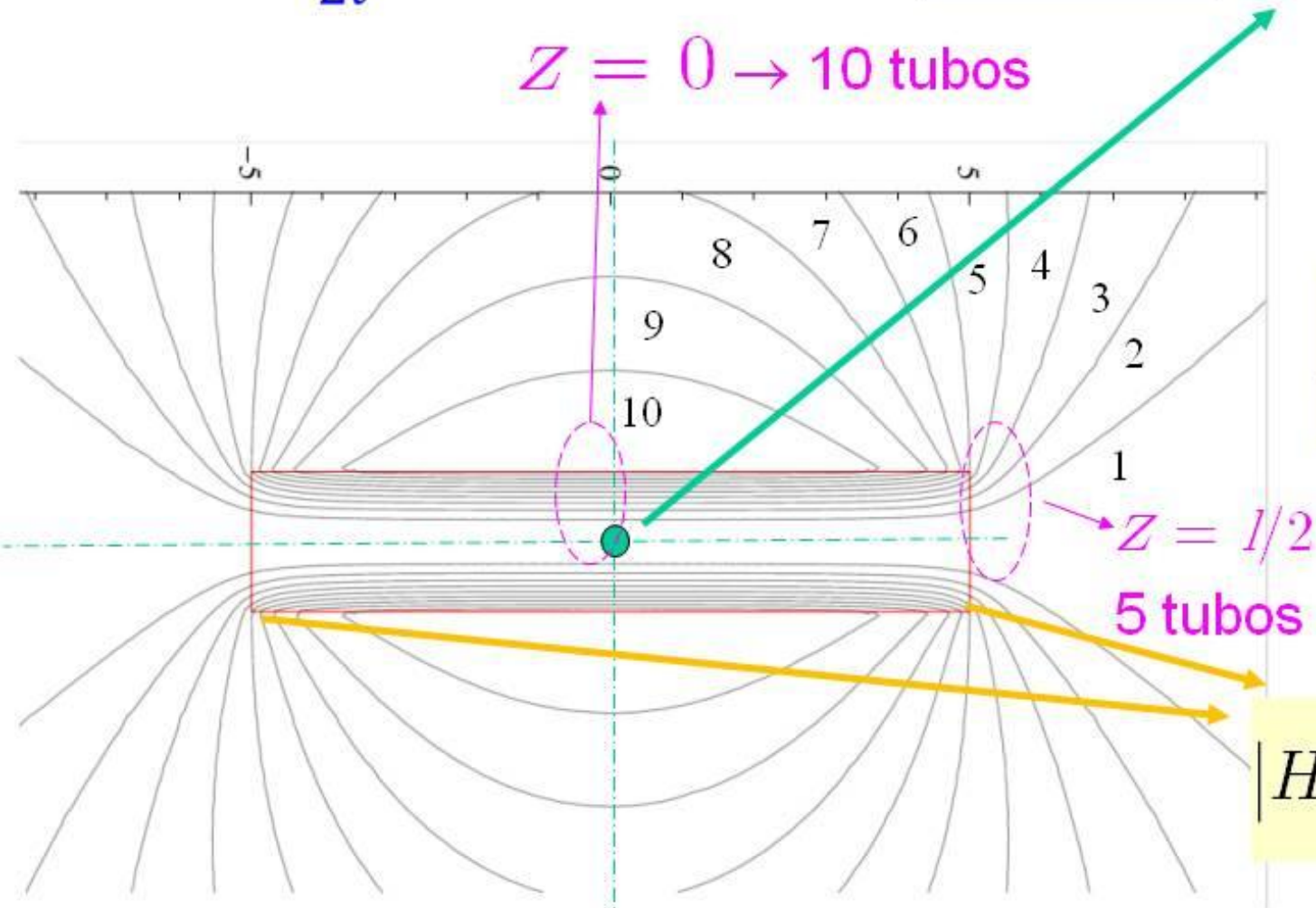


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Campo de Solenoide

$$\vec{H}(0, z) = \frac{NI}{2l} [\cos \theta_1 - \cos \theta_2] \hat{u}_z \quad \rho=0, z=0 \rightarrow \vec{H}(0,0) = \frac{NI}{\sqrt{4a^2 + l^2}} \hat{u}_z$$

$Z = 0 \rightarrow 10$ tubos



$$l \gg 2a$$

$$\vec{H}(0,0) \approx \frac{NI}{l} \hat{u}_z$$

$$z = \pm l/2$$

$$|H_z(0, z = \pm \frac{l}{2})| = \frac{NI}{2l}$$