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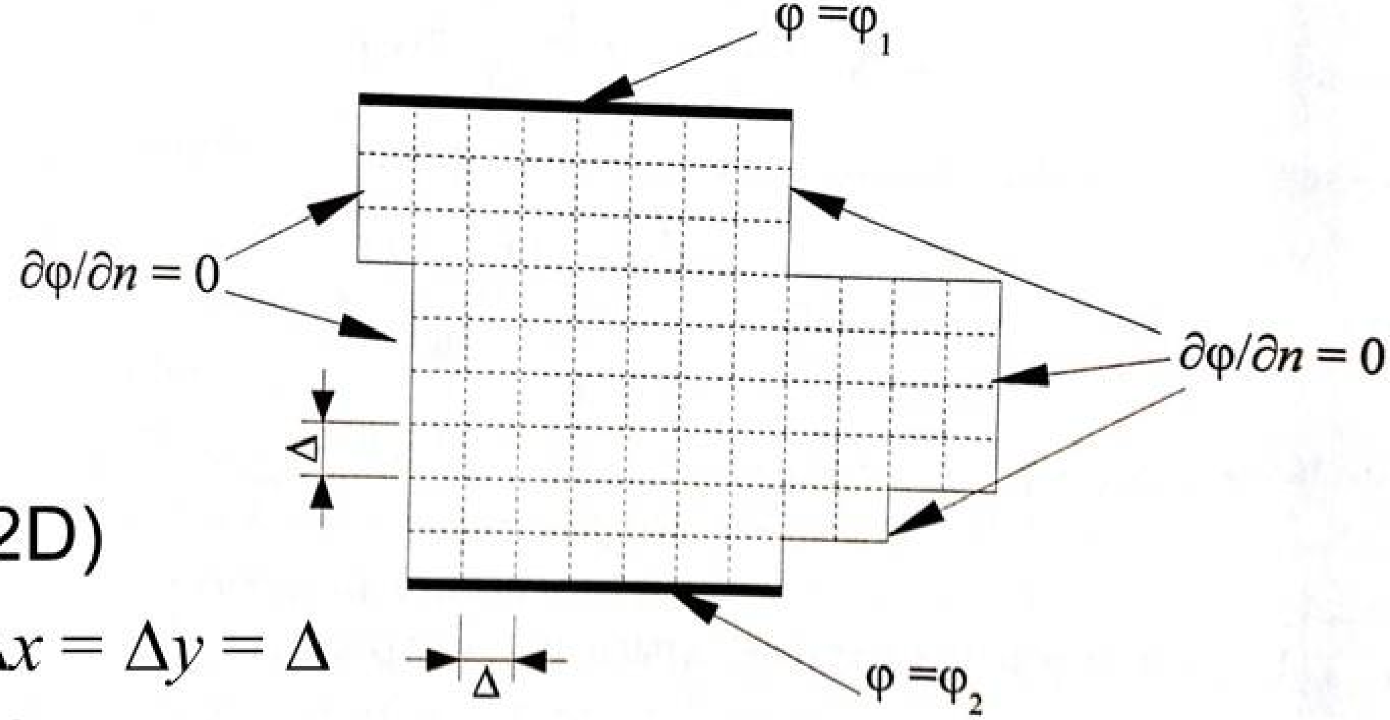
Solução da Equação de Laplace
Método das Diferenças Finitas em 2D

Mapa de Campo

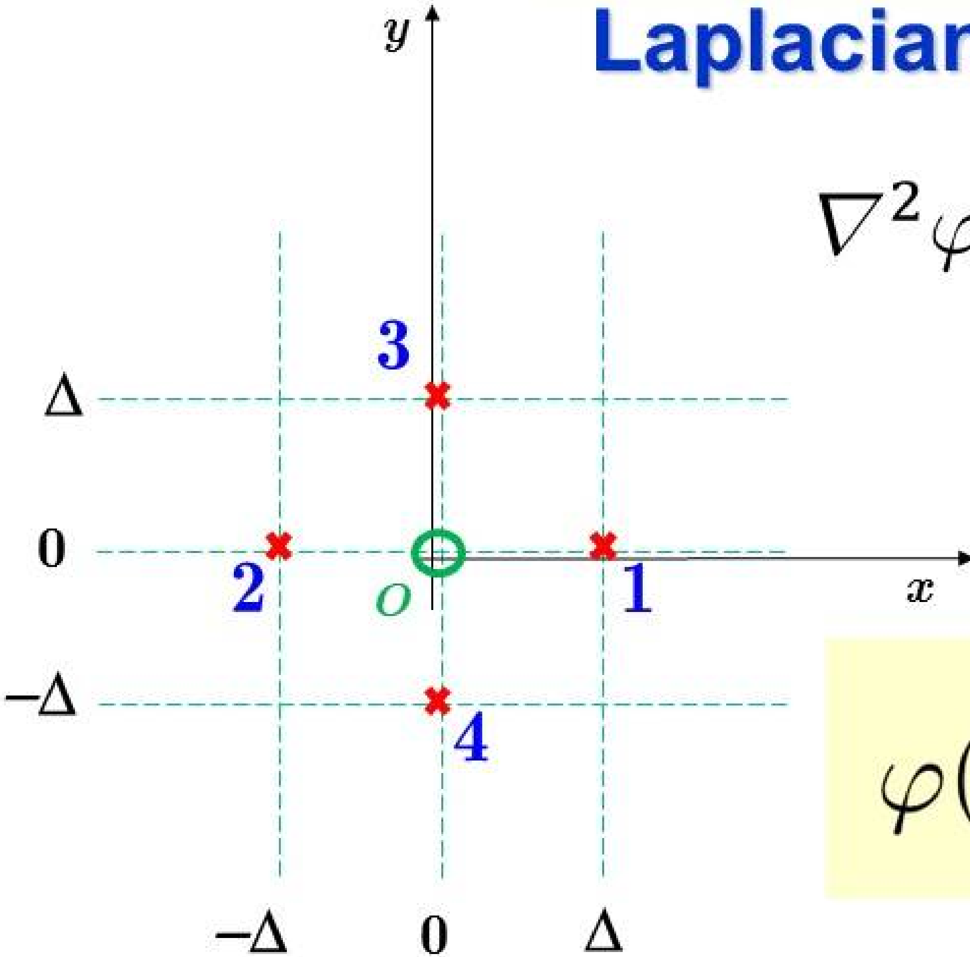
Dualidade

Método de Diferenças Finitas - Hipóteses



- Simetria Plana (2D)
- Grade Regular $\Delta x = \Delta y = \Delta$
- Contorno coincide com reticulado
- Sem interface entre materiais (material único)

Laplaciano aproximado em O



$$\nabla^2 \varphi = 0 \quad \Rightarrow \quad \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0$$

$$\varphi(0) = \frac{\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4}{4}$$

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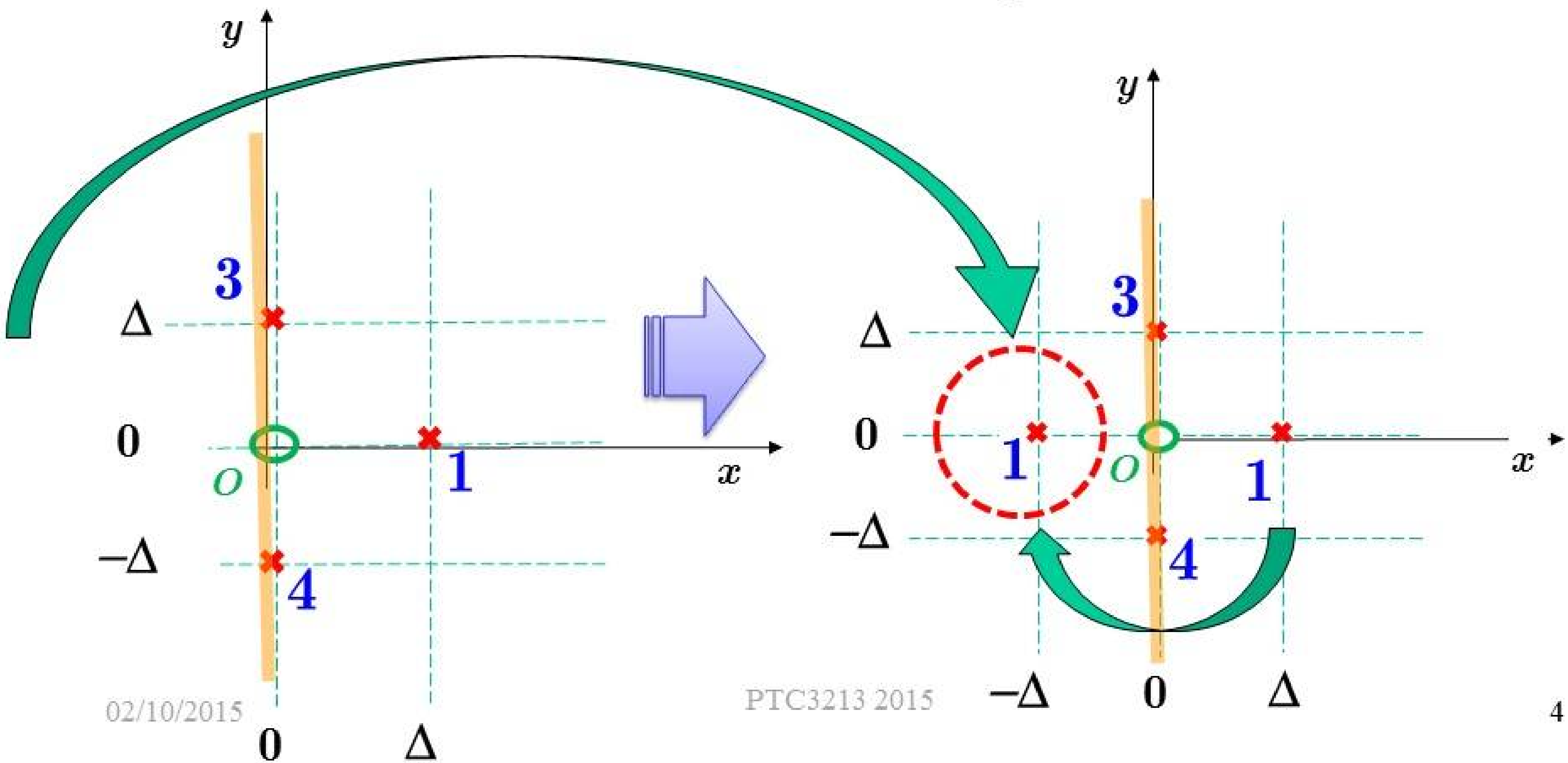
Pontos de Fronteira – Cond. Neumann

$$\varphi(O) = \varphi_O = \frac{2\varphi_1 + \varphi_3 + \varphi_4}{4}$$



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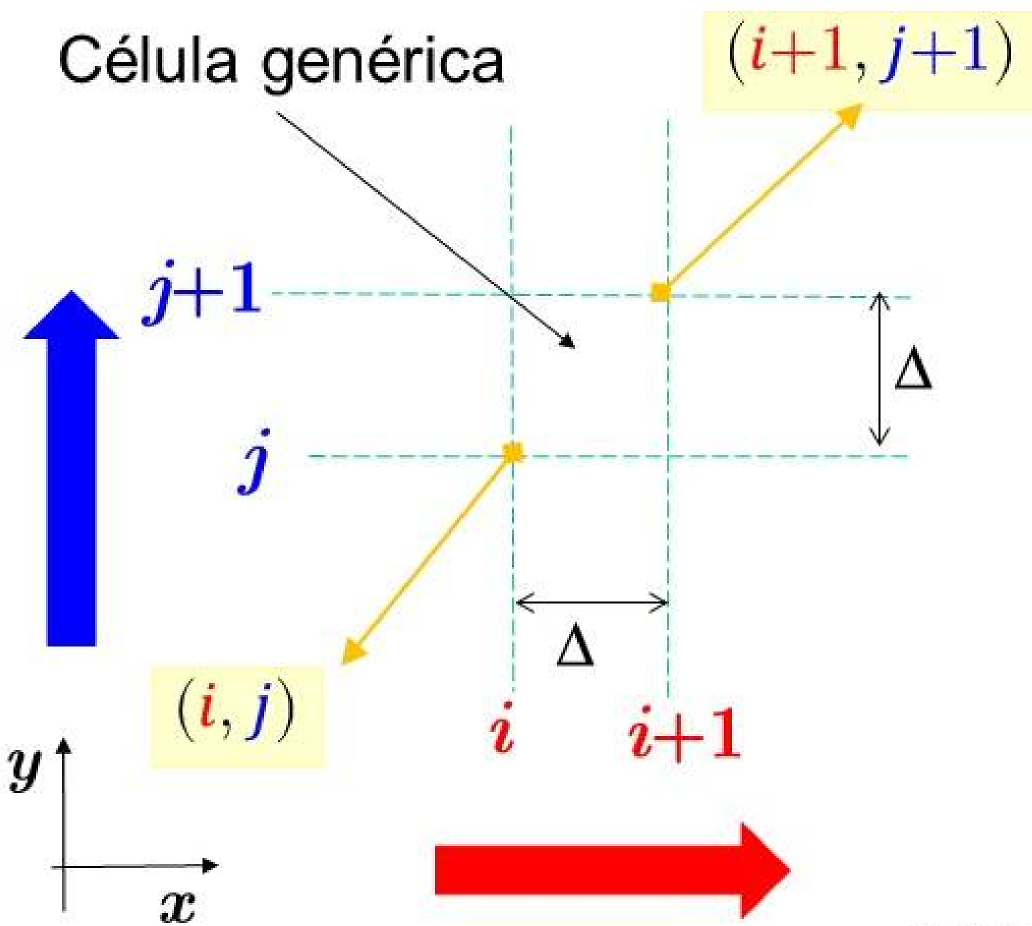
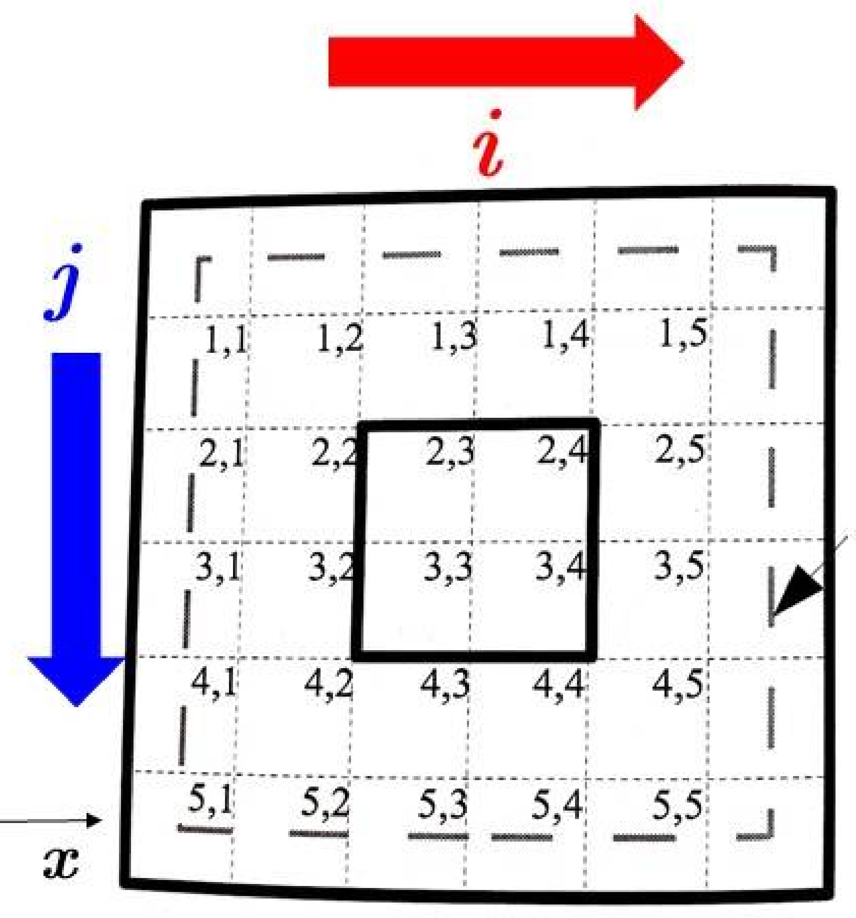
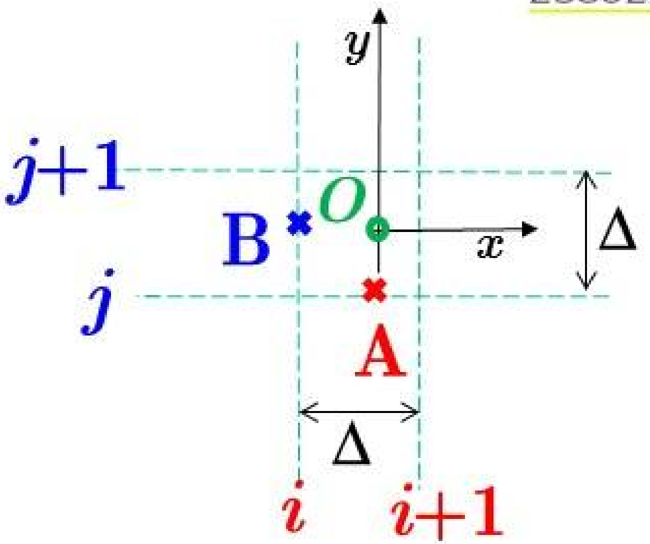
Indexação Local × Indexação Global



$$(j,i) = (\text{linha}, \text{coluna})$$





Cálculo do Campo Elétrico em uma célula genérica - Grandeza "Local"

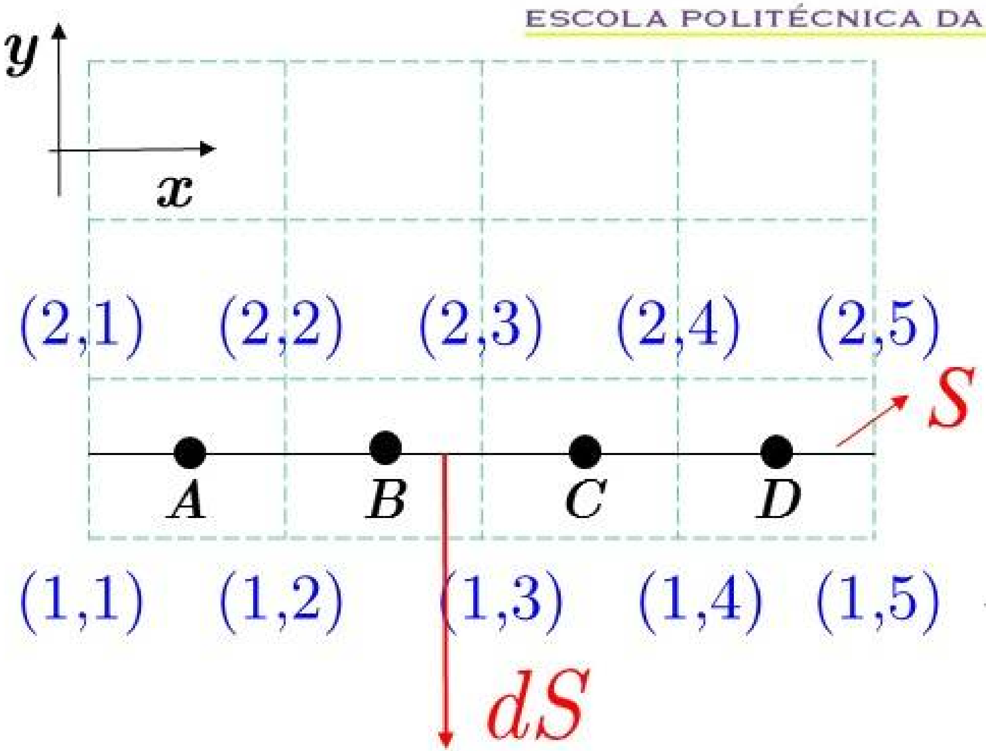
$$\overrightarrow{E} = -\nabla \varphi = -\frac{\partial \varphi}{\partial x}\widehat{u}_x - \frac{\partial \varphi}{\partial y}\widehat{u}_y$$

$$\varphi(i, j) = \varphi(x_{i,j}, y_{i,j}) = \varphi_{i,\, j}$$

$$E_x(O) = \frac{\varphi(i,j) + \varphi(i,j+1) - \varphi(i+1,j) - \varphi(i+1,j+1)}{2\Delta}$$

$$E_y(O) = \frac{\varphi(i,j) + \varphi(i+1,j) - \varphi(i,j+1) - \varphi(i+1,j+1)}{2\Delta}$$

Cálculo da Corrente - Grandeza "global"



$$\vec{J} = \sigma \vec{E} \Rightarrow I = \iint_S \vec{J} \cdot d\vec{S}$$

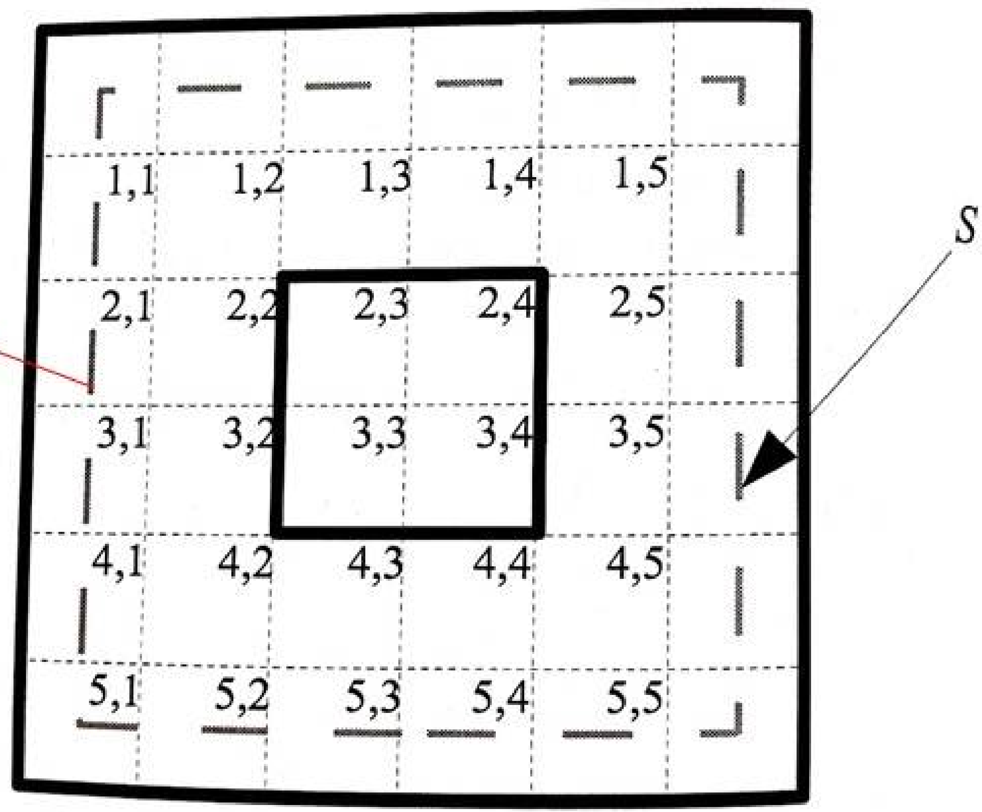
$(j,i) = $ (linha,coluna)

$$I_S = \sigma L \left[\frac{\varphi(2,1)}{2} + \varphi(2,2) + \varphi(2,3) + \varphi(2,4) + \frac{\varphi(2,5)}{2} - \right.$$
$$\left. - \frac{\varphi(1,1)}{2} - \varphi(1,2) - \varphi(1,3) - \varphi(1,4) - \frac{\varphi(1,5)}{2} \right]$$

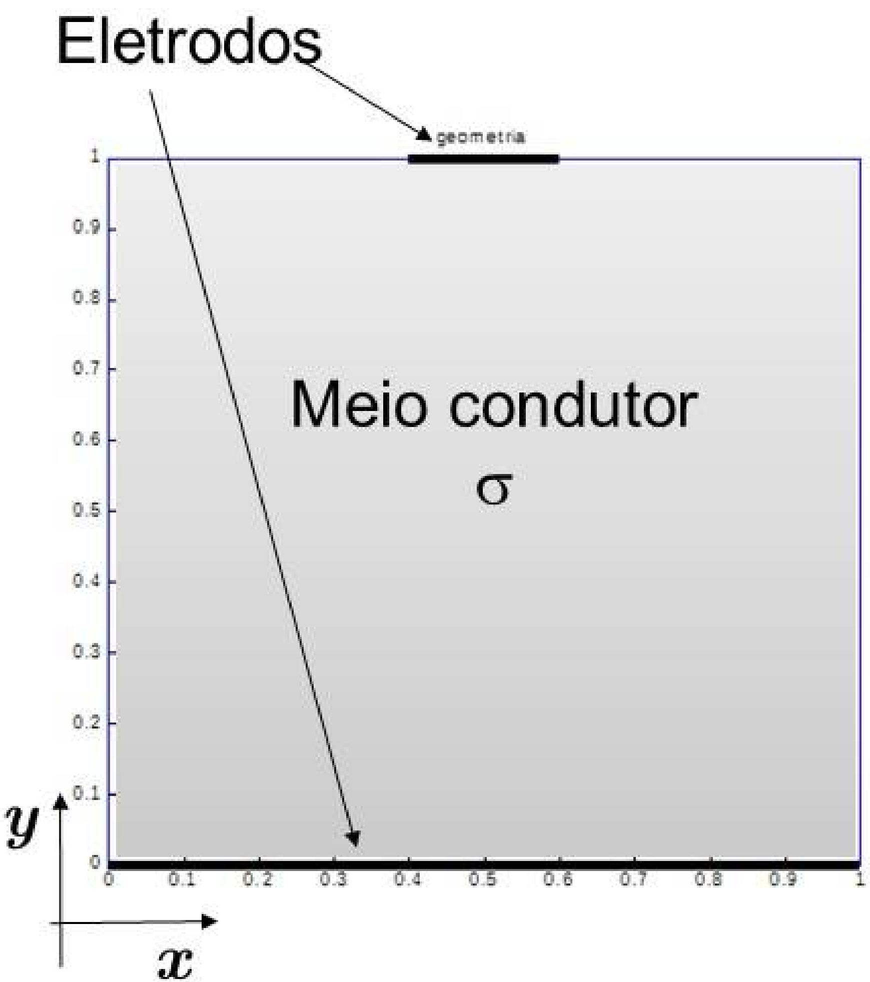
Cálculo da Corrente (ou Q)

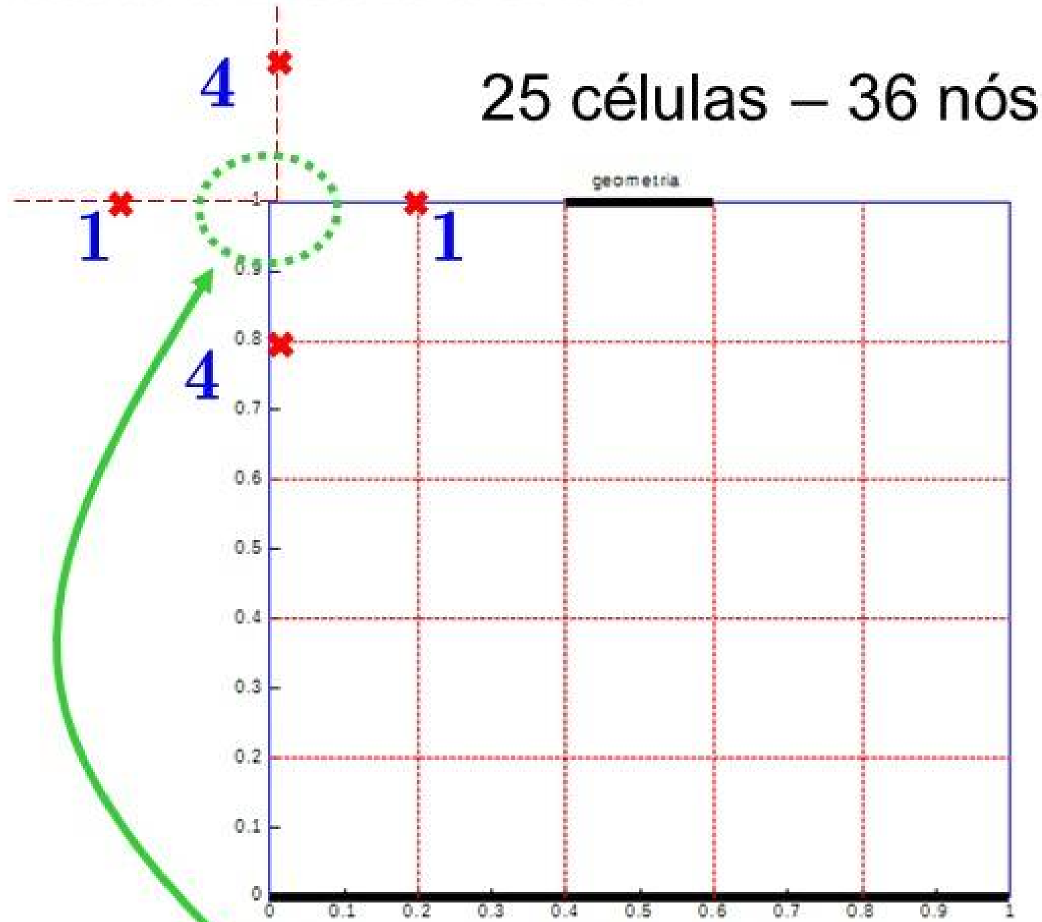
$$I_S = \sigma L \left[\sum_{i=1}^{5} \varphi(1,i) + \sum_{j=1}^{5} \varphi(j,5) + \sum_{i=1}^{5} \varphi(5,i) + \sum_{j=1}^{5} \varphi(j,1) \right]$$

S deve atravessar o > n° possível de células



Resistência entre Eletrodos





$$\sigma = 0{,}01 \text{ S/m} \qquad \Delta = 0{,}2 \text{ m} \quad L = 1 \text{ m}$$

$$\varphi_O = \frac{2\varphi_1 + 2\varphi_4}{4}$$

Equipotenciais

36a. Iteração

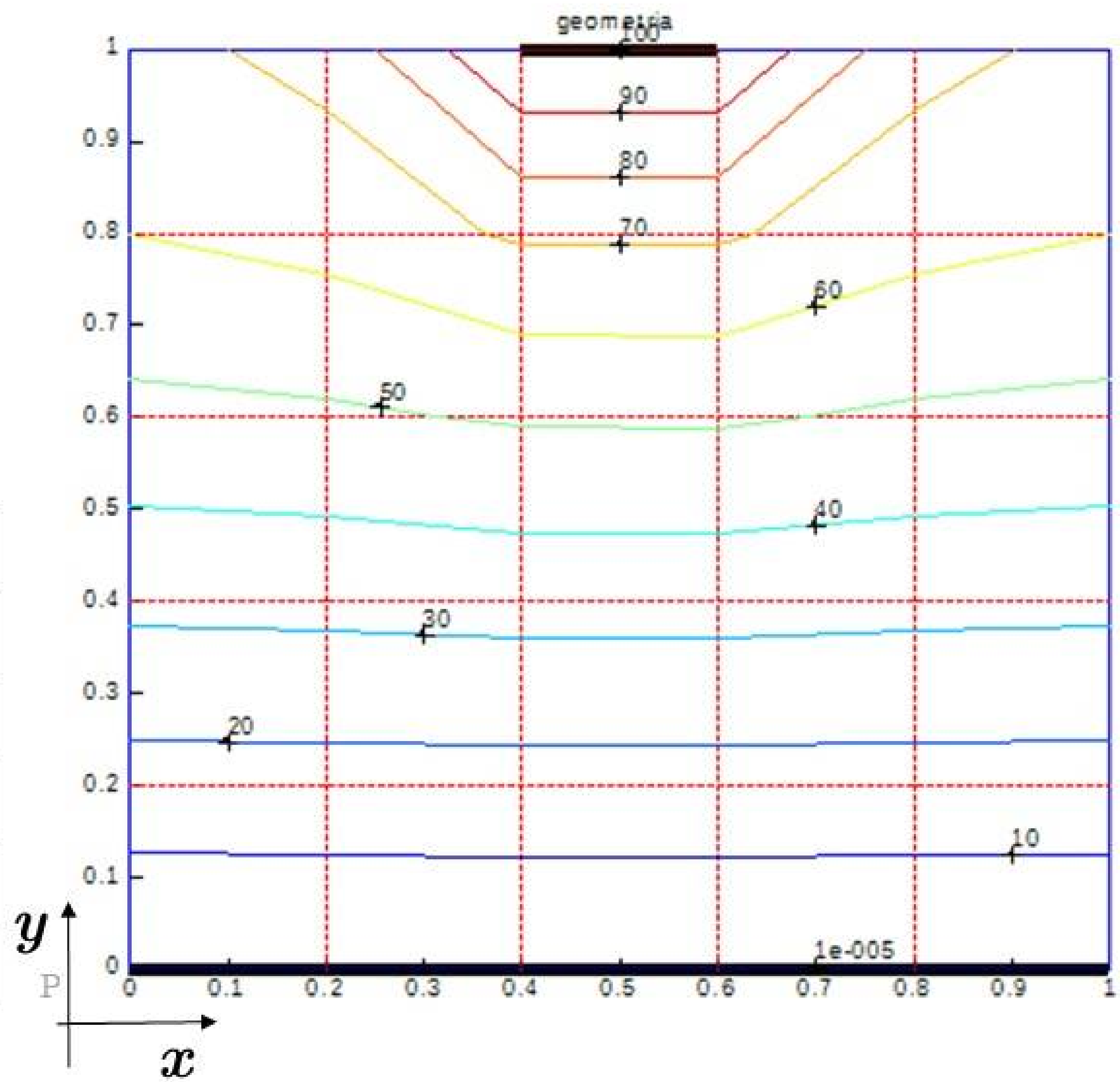
$$\varphi(j,i)$$

$(j,i) = (1,1),\ (1,2),\ \ldots(6,6)$

$i \rightarrow$, $j \downarrow$

$\varphi_{1,1}$ $\varphi_{1,2}$ …

67	73	100	100	73	67
60	63	71	71	63	60
47	49	51	51	49	47
32	33	33	33	33	32
16	16	17	17	16	16
0	0	0	0	0	0



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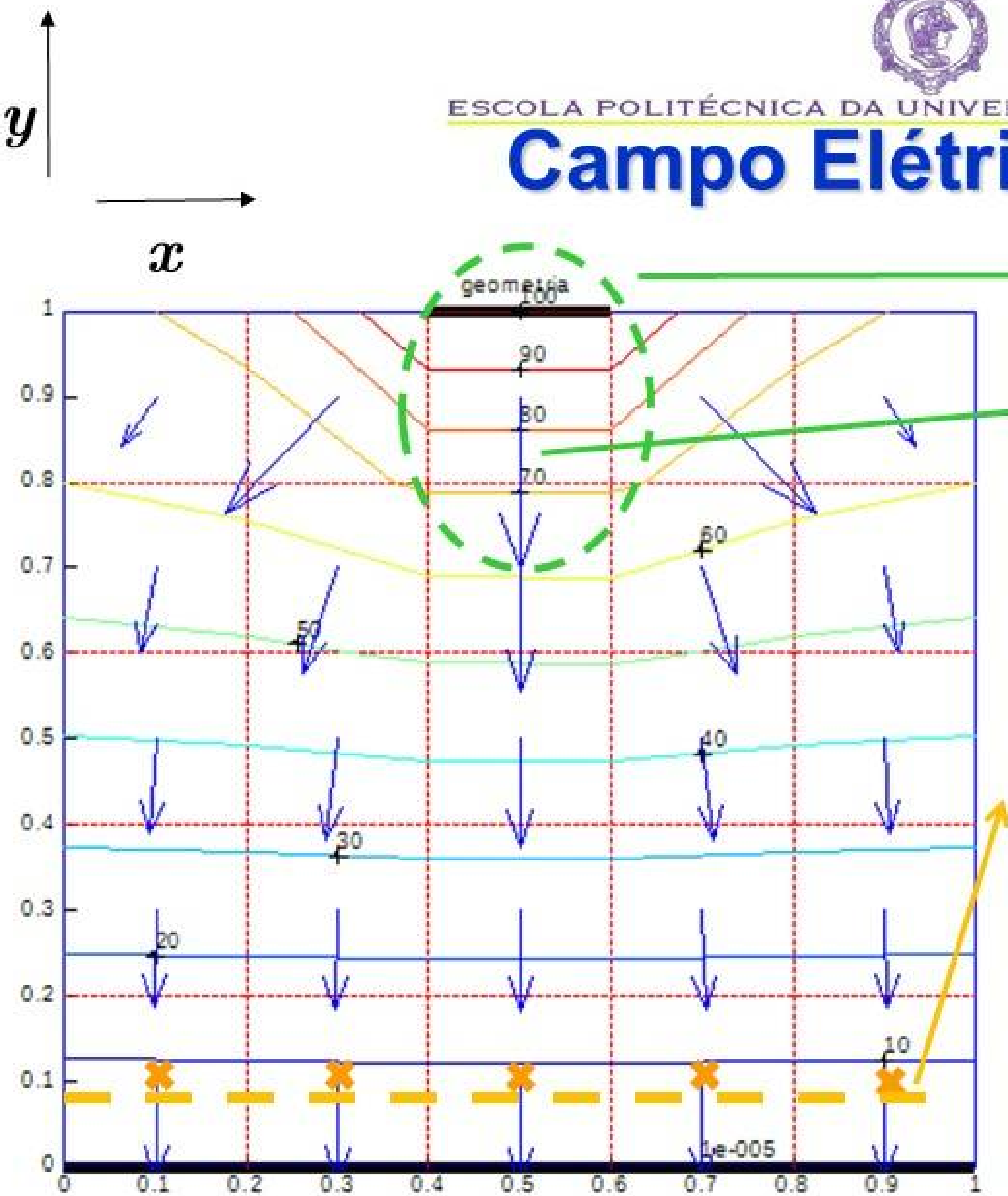
Campo Elétrico e R



$\mathbf{E}$ constante na célula

$$E = E_y = \frac{\Delta\varphi}{\Delta y} \cong \frac{100-72}{0{,}2}$$

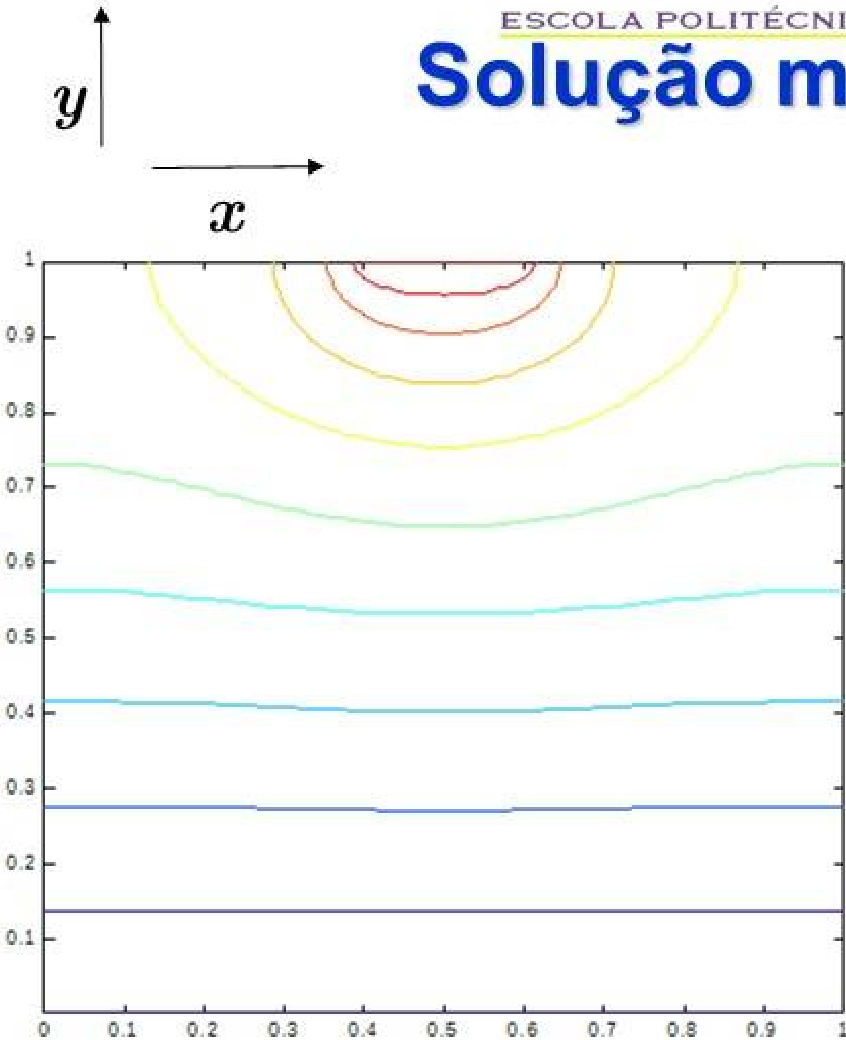
$$E_{\max} \cong 142\ \mathrm{V/m}$$

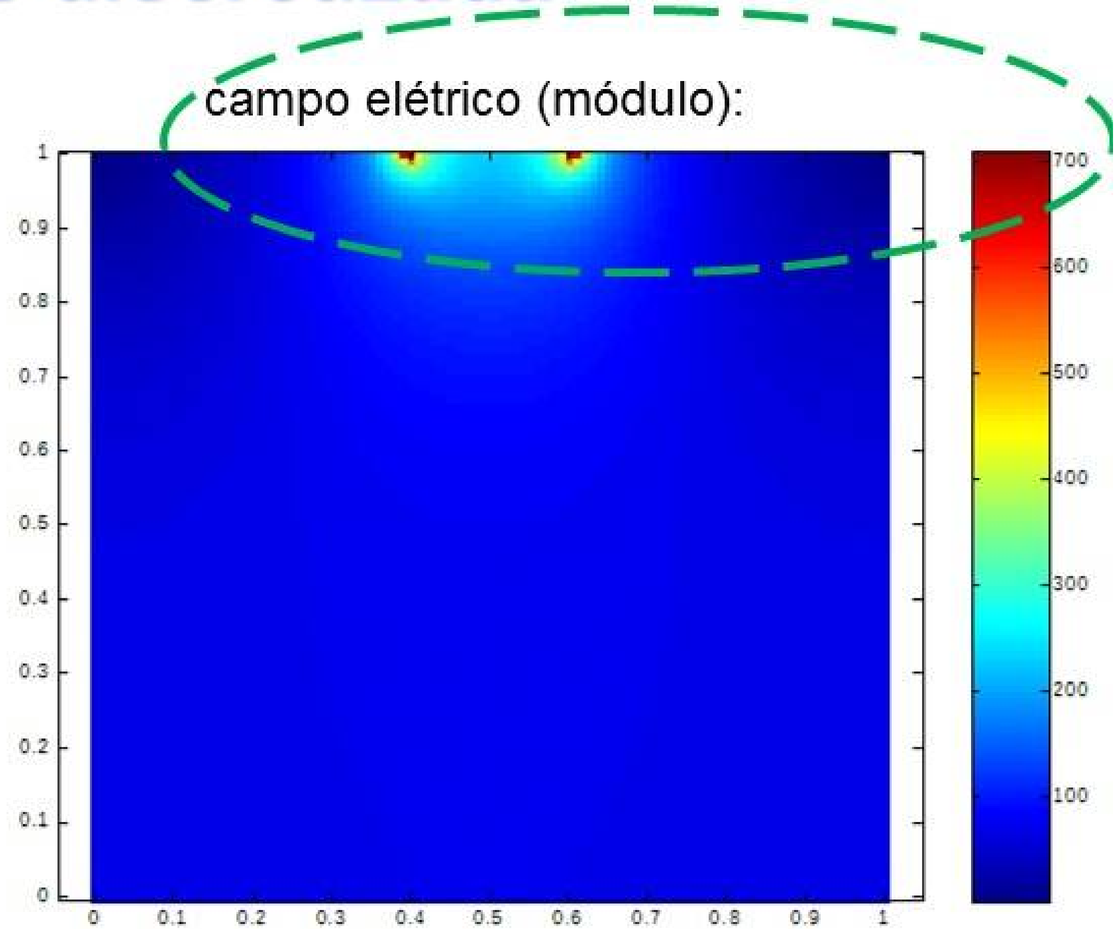
$$I = -\sigma\times(-81{,}33\ -82{,}31\ -82{,}94\ -82{,}41\ -81{,}54)\times L\times\Delta$$

I=0,8211 A

$$R = 121.7904\ \Omega$$

Solução mais discretizada





Δ=0.01, 10201 nós, 10 000 células

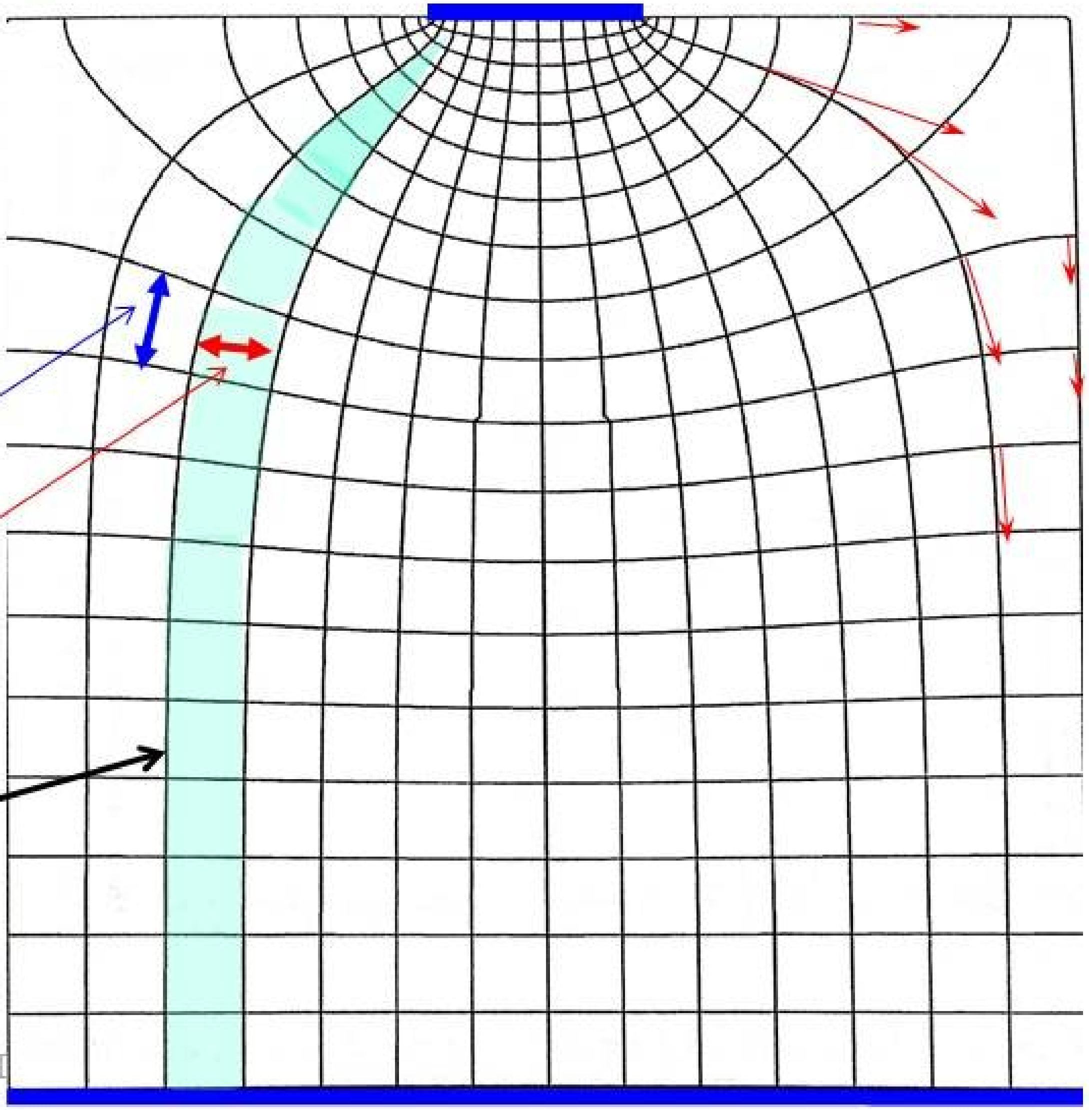
$$\mathrm{E}_{\mathbf{max}} = 700 \quad \mathrm{V/m} \qquad\qquad R = 136.3089\ \mathbf{\Omega}$$

Mapa de Campo - Quadrados Curvilíneos

Traçado de Equipotenciais e Linhas de Campo de modo que:

- $\Delta\varphi = \mathrm{cte}$
- $\Delta I = \mathrm{cte}$

•Tubo de corrente (ou campo)

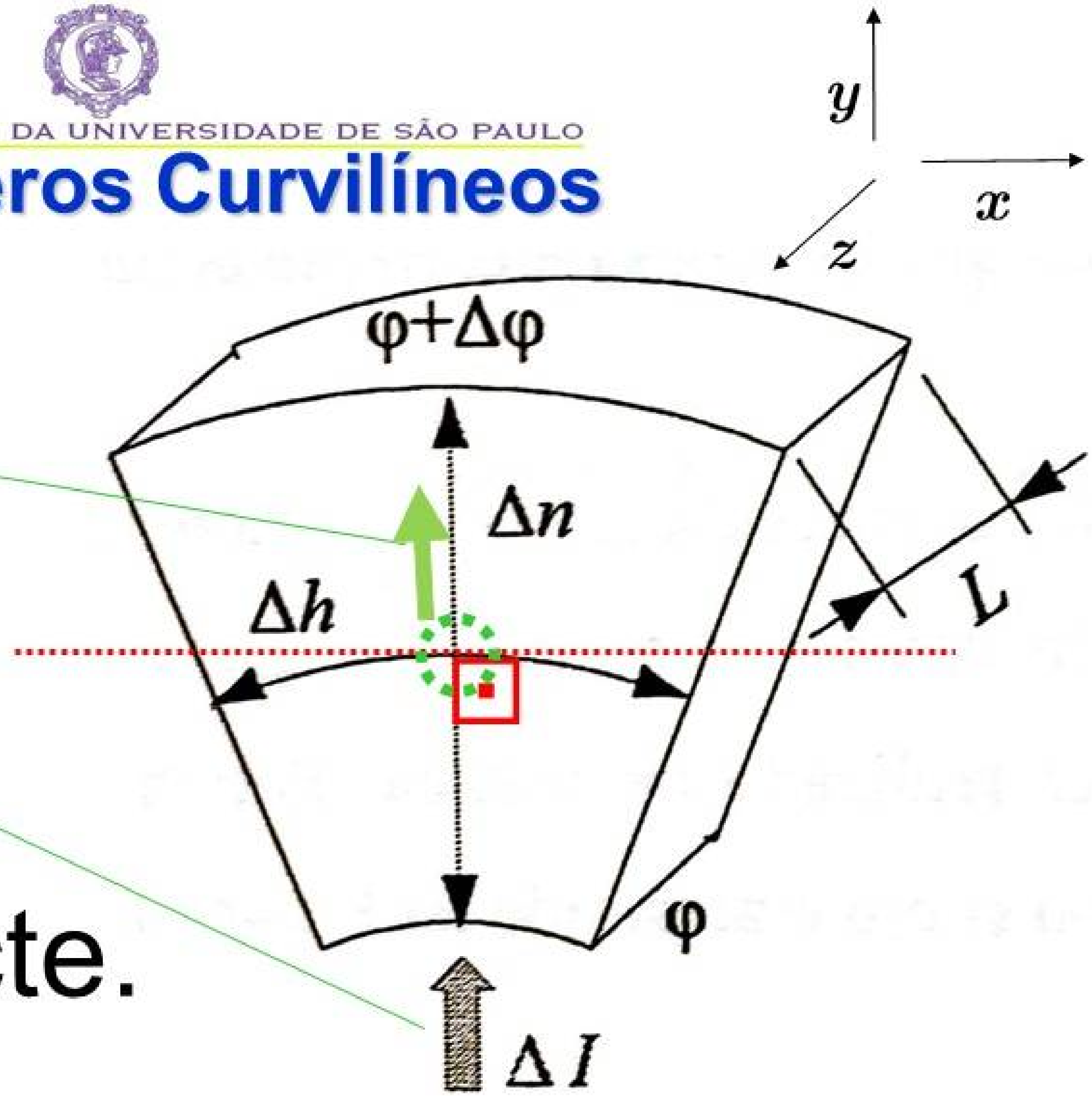


Quadriláteros Curvilíneos

$$E = \frac{\Delta\varphi}{\Delta n}$$

$$J = \frac{\Delta I}{\Delta h.\,L} = \sigma E$$

$$\frac{\Delta\varphi}{\Delta I} = \frac{\Delta n}{\sigma \Delta h.\,L}$$ =cte.



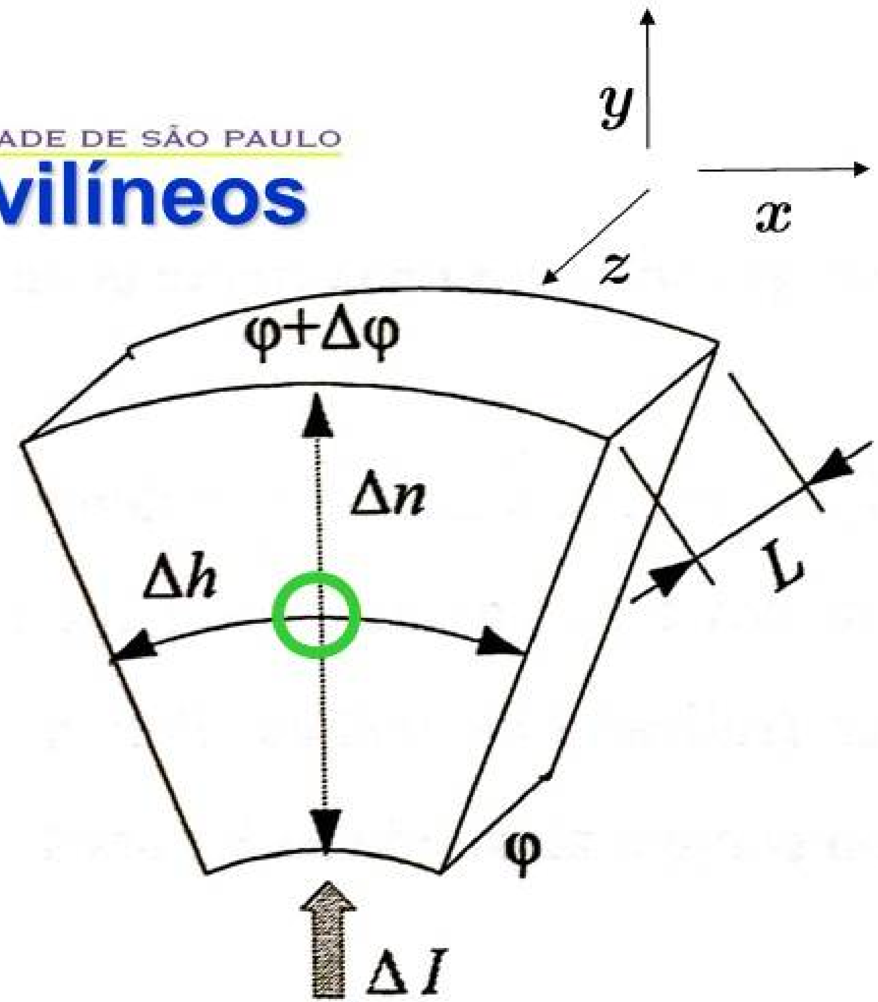
Se $\Delta \boldsymbol{n}/\Delta \boldsymbol{h}=1$ ⇒ Quadrados Curvilíneos

Quadrados Curvilíneos

$$\frac{\Delta \varphi}{\Delta I} = \frac{\Delta n}{\sigma \Delta h . L}$$

$$\Delta \boldsymbol{n} / \Delta \boldsymbol{h} = 1$$

$$\frac{\Delta \varphi}{\Delta I} = r = \frac{1}{\sigma L}$$



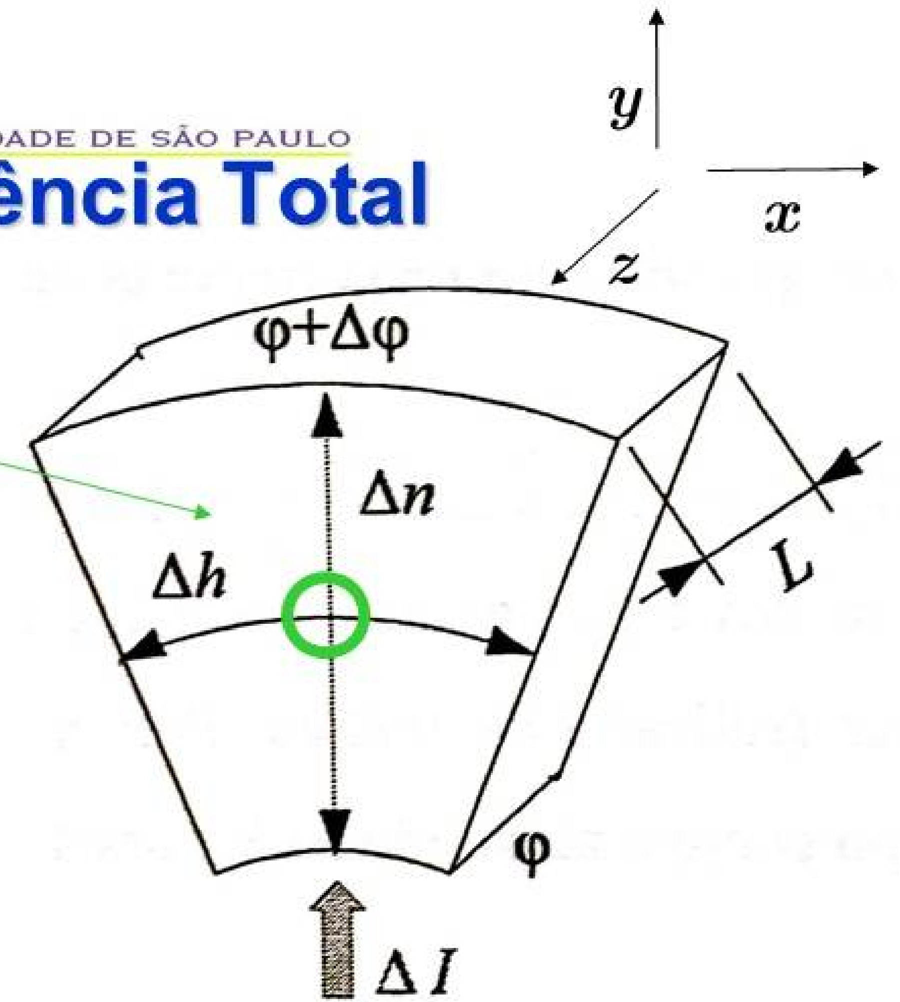
$$J = \sigma E = \sigma \frac{\Delta \varphi}{\Delta n}$$

$$E.J = \sigma \frac{\Delta \varphi}{\Delta n} . \frac{\Delta I}{L. \Delta h} \propto \frac{P}{S}$$

Densidade sup. potência

Cálculo da Resistência Total

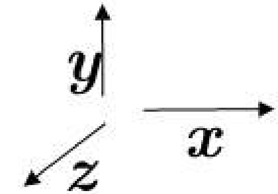
$$\frac{\Delta\varphi}{\Delta I} = r = \frac{1}{\sigma L}$$



Por construção, r é igual para todos os quadrados curvilíneos

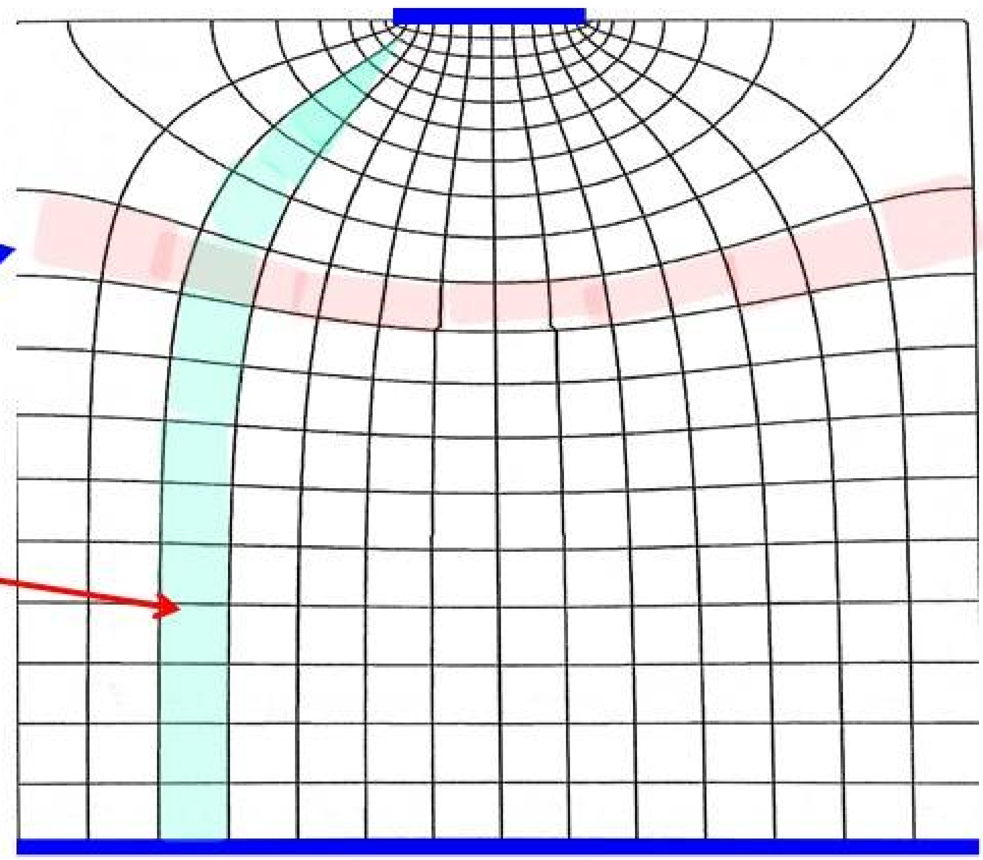
$$R = \frac{n_s}{n_p} r = \frac{n_s}{n_p}\frac{1}{\sigma L}$$

Quadrados Curvilíneos



$$R = \frac{n_s}{n_p} r = \frac{n_s}{n_p} \frac{1}{\sigma L}$$

$$\boldsymbol{n_s = 19} \quad \boldsymbol{n_p = 14}$$



$\sigma = 0{,}01$ S/m $\quad L = 1$ m

$$R = \frac{19}{14} \cdot \frac{1}{0{,}01.1}$$

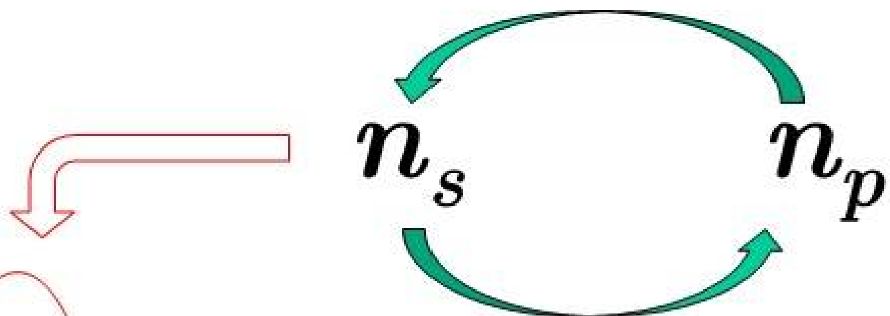
$$\boldsymbol{R} = 135{,}7\ \Omega$$

Dualidade

Problema Dual:

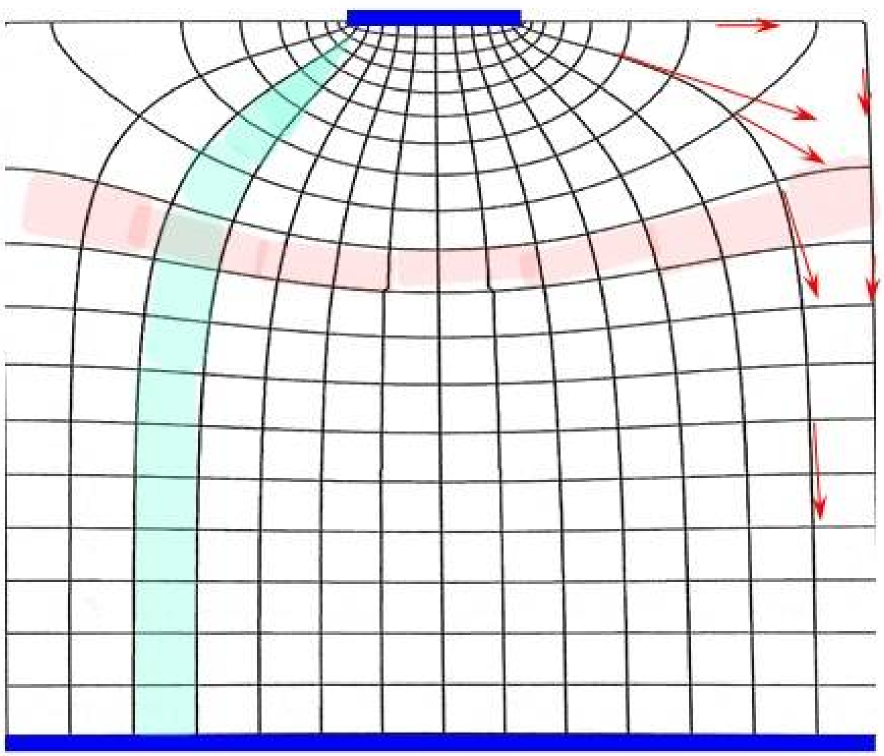
Invertem-se as famílias de linhas equipotenciais e linhas de corrente

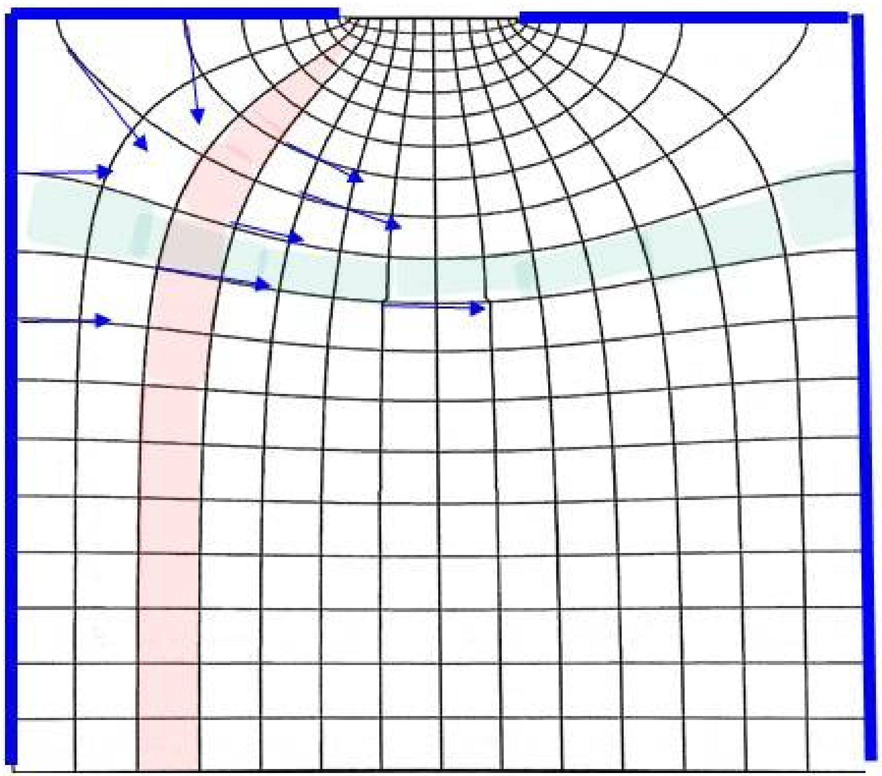
Relação entre $\boldsymbol{R}$ original e $\boldsymbol{R}$ do problema dual:



$$R_{dual} = \frac{n_p}{n_s}\frac{1}{\sigma L} = \frac{1}{R\sigma L}\frac{1}{\sigma L} \Rightarrow R_{dual} = \frac{1}{R}\frac{1}{\sigma^2 L^2}$$

Dualidade - Exemplo





$\boldsymbol{\sigma} = 0{,}01 \text{ S/m} \quad L = 1 \text{ m}$

$R = 136.3089 \ \boldsymbol{\Omega}$

$\boldsymbol{\sigma}_{\text{dual}} = 0{,}1 \text{ S/m} \quad L_{\text{dual}} = 3 \text{ m}$

$$R_{dual} = \frac{n_p}{n_s}\frac{1}{\sigma_{dual}\, L_{dual}} = \frac{1}{R\sigma L}\frac{1}{\sigma_{dual}\, L_{dual}}$$

$$R_{dual} = 244{,}54\ \Omega$$