

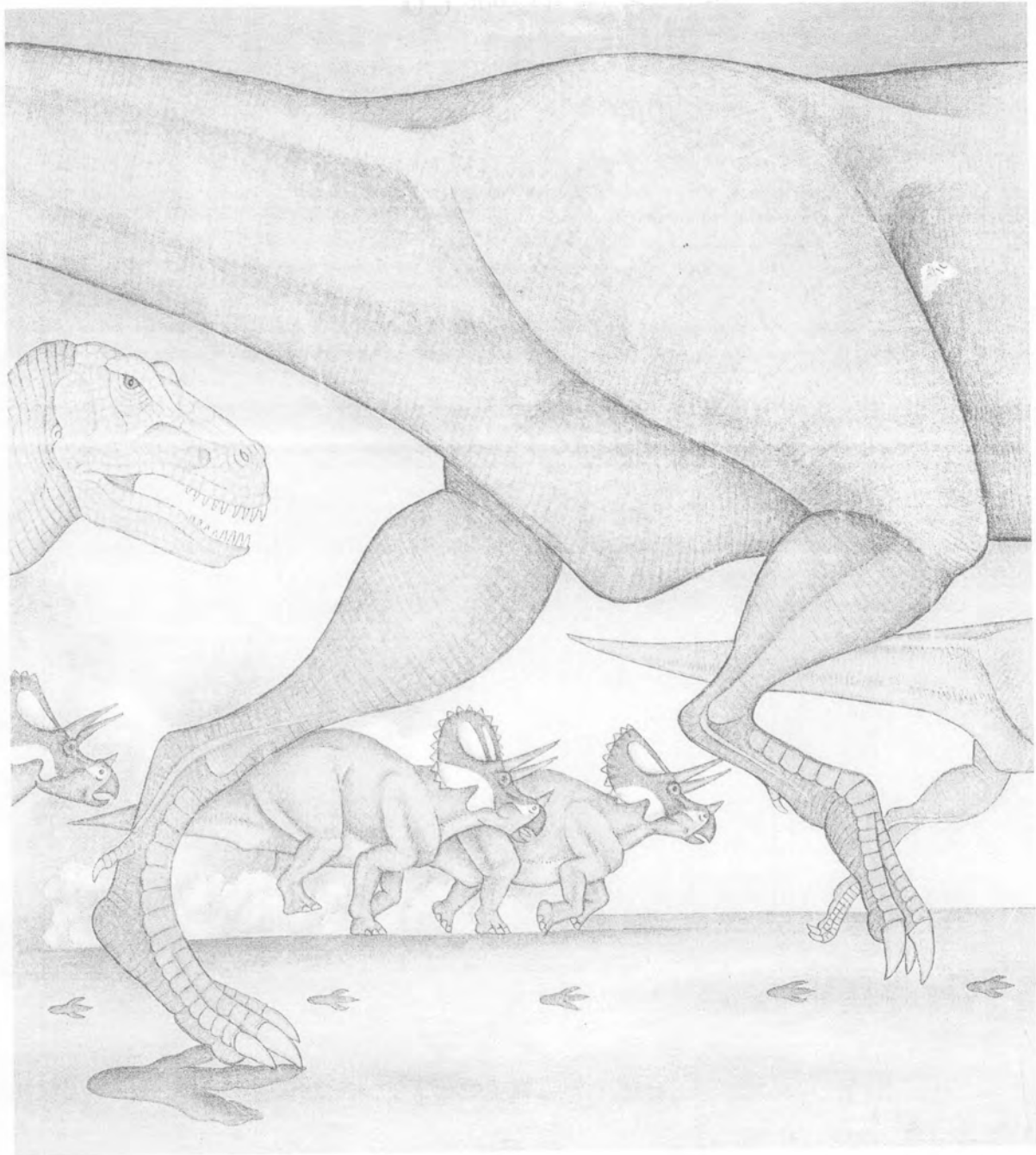
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Shock waves in the void of space.

The echoes of ancient languages.

What does energy really cost?



*If tyrannosaurs ran, how fast did they go?
The art of shipbuilding provides a clue.*

How Dinosaurs Ran

Did the giants of the Mesozoic period lumber sluggishly, or were they formidable running machines? Techniques borrowed from modern physics and engineering may give us the answer

by R. McNeill Alexander

Elephants do not prance and leap about like gazelles, and they cannot do so because of their size. Imagine how dinosaurs, some of them much more massive than any elephant, must have moved. Perhaps their legs were too weak to support their enormous weight, so they lived in lakes and depended on water to buoy them up, as some old pictures suggest. Or perhaps their limbs were strong enough to support them and they walked and ran like elephants—or some of the more athletic modern animals—despite their size.

Because the dinosaurs are extinct, we cannot confirm or refute any of these possibilities by direct observation, but fairly convincing answers can be obtained by applying methods taken from physics and engineering. Using the art of shipbuilding, the study of forces and stresses on structures and skeletons and the observation of wildlife, we can understand some of the physical rules that may have governed dinosaur movement.

We can combine this mechanical approach with what is known from the fossil record, both from bones and from dinosaur tracks, to describe what may have been the agility or sluggishness of these giant animals. The picture that emerges can tell us whether *Triceratops*—the horned quadruped—galloped or shuffled and whether *Ty-*

rannosaurus—the bipedal carnivorous king—could outmaneuver or outrun his *Triceratops* prey.

We know dinosaurs principally by their skeletons, from which we can measure their heights and lengths. But this information does not provide the weight that these skeletons, particularly their limbs, had to support. Nor do the measurements of length and height help us make comparisons between dinosaurs of different shapes. Unfortunately, we lack the most generally useful measure of size: body mass.

Most of the weight of living dinosaurs was skin, flesh and guts that decayed long ago, so today their body mass can only be estimated. Such calculations can be done by measuring scale models of the dinosaurs as they are thought to have been in life—suitable plastic models can be bought in many museums.

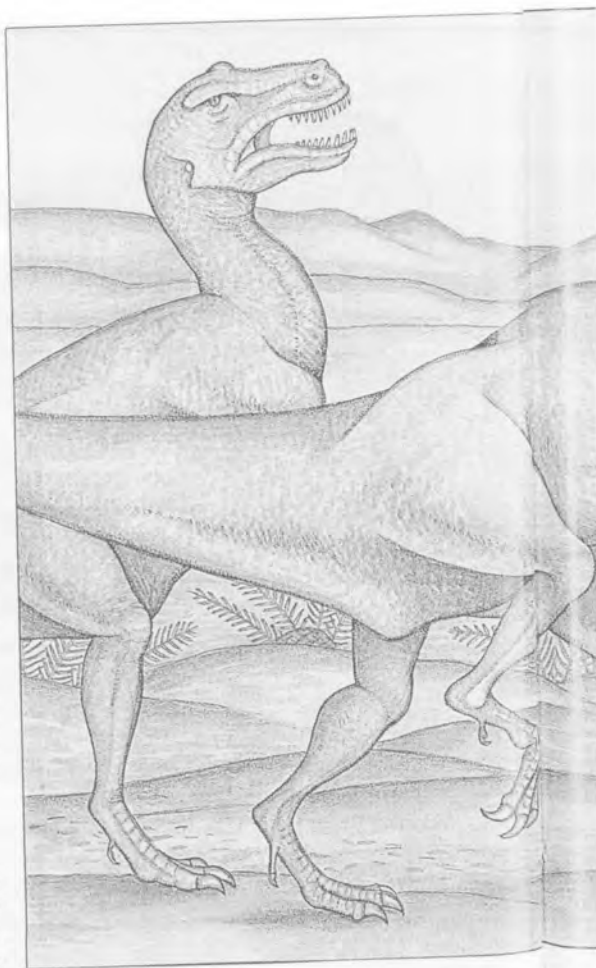
Using Archimedes' principle (immersing a model in water and measuring the amount of water displaced), we can determine the volume of plastic replicas. This measure can then be scaled up to approximate the volumes of the real creatures. Some models are made to a scale of one fortieth, so their volumes have to be multiplied by $40 \times 40 \times 40$ (length times height times width), or 64,000, to get the volumes of the actual dinosaurs. Multiplying this staggering volume, in turn, by 1,000 kilograms per cubic meter (the density of water and, approximately, of crocodiles and mammals) gives the dinosaurs' likely masses.

These measurements suggest a mass of more than seven metric tons for *Tyrannosaurus*, the largest-known flesh-eating dinosaur. (A metric ton is 2,205

pounds.) This weight is 10 times that of a fully grown male polar bear, the biggest modern land-living predator. *Brachiosaurus*, the largest plant-eating dinosaur of which there exists a reasonably complete skeleton, was even more gargantuan. It seems to have weighed about 50 metric tons, 10 times the mass of a mature male African elephant and about equivalent to that of an average sperm whale. Although *Brachiosaurus* was indeed a magnificent animal, standing 13 meters high, or well over twice as tall as an adult giraffe, it may not have been the biggest dinosaur. *Supersaurus* and *Ultrasaurus*, known only by a few bones each, may have been even more impressive.

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TYRANNOSAURUS, a carnivore, pursues the three-horned plant-eating *Triceratops*. Was the chase quick and lively, or did these two dinosaurs plod along?



The problem of support faced by very large land animals was first addressed by Galileo in the early 1600s, when he theorized about the relation of size to strength and structure. Consider two animals of different sizes that are geometrically similar—meaning that if the larger is, for example, twice as long as the smaller animal, it is also twice as wide and twice as high. In that case, the bigger animal has two (length) times two (height) times two (width), or eight, times the volume of the smaller one ($2 \times 2 \times 2 = 8$). And assuming that they are made of the same substances, the larger creature outweighs its smaller counterpart eight times.

But there is a complication. Although the volume of the larger animal is eight times greater, the strength of its legs increases only by a factor of four. Because leg strength is proportional to the area of the cross section of the limb, one leg would be only two (length) times two (width), or four, times stronger. (An increase in height would have no effect on the cross section.) In other words, eight times the weight would have to be carried by only four times the strength. So, as Galileo noted, if an animal becomes progressively bigger without changing its shape, it must eventually reach a size at which it is incapable of supporting itself.

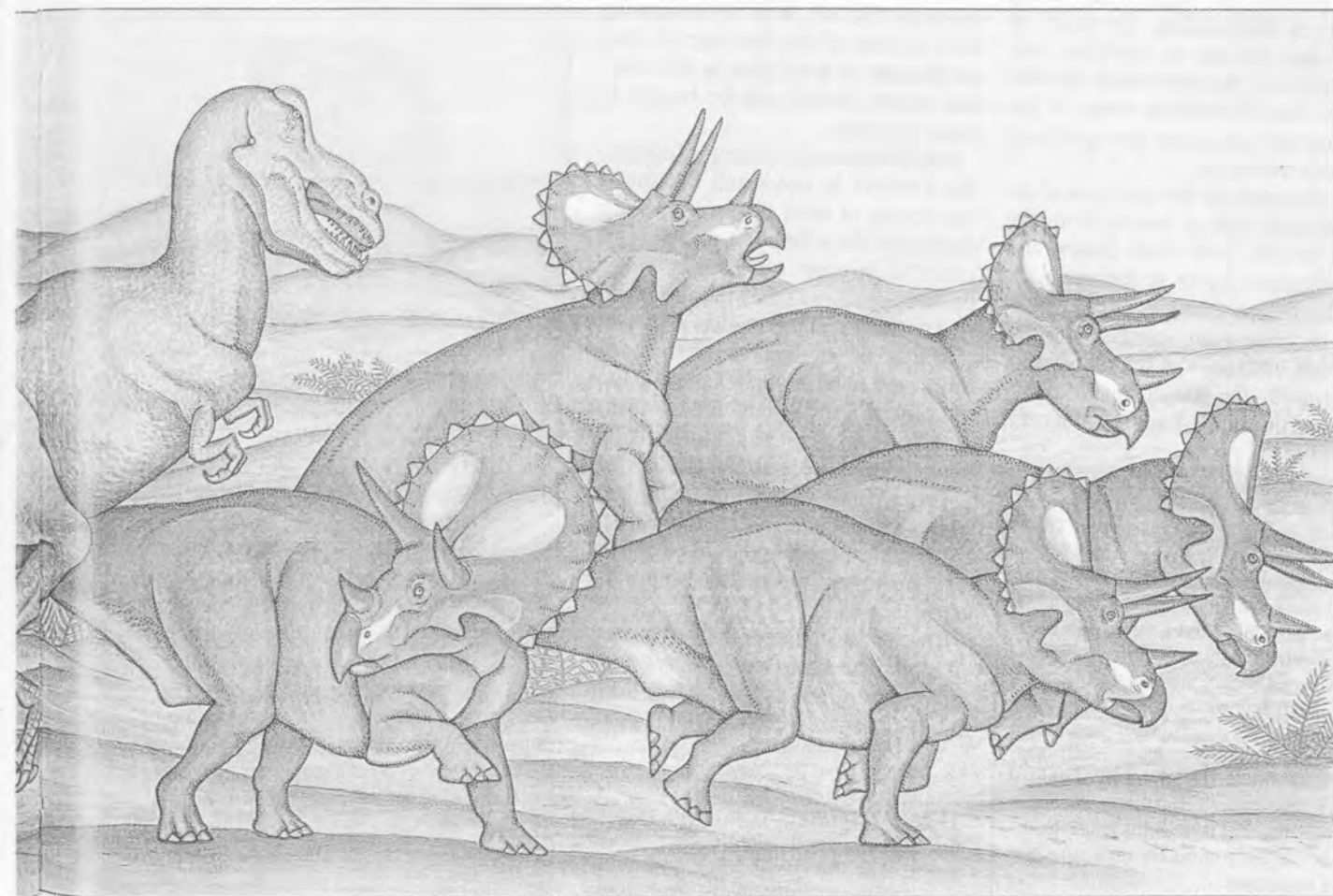
To apply this observation about support to the movement of dinosaurs, we need information about modern animals—particularly about the effects of size differences on their movement. I had been researching the mechanics of running and jumping in frogs, dogs, kangaroos and other animals for several years, when I was invited to speak at a conference about the impact of size on animal locomotion. So I set out to develop a theory of running that would account for size differences and soon noticed that an expression kept cropping up in my equations. This was v^2/gl , where v is running speed, g is the acceleration of gravity (9.8 meters per second squared) and l is leg length. This expression held the key that eventually made it possible to assess the athletic abilities of dinosaurs and even to calculate their running speeds.

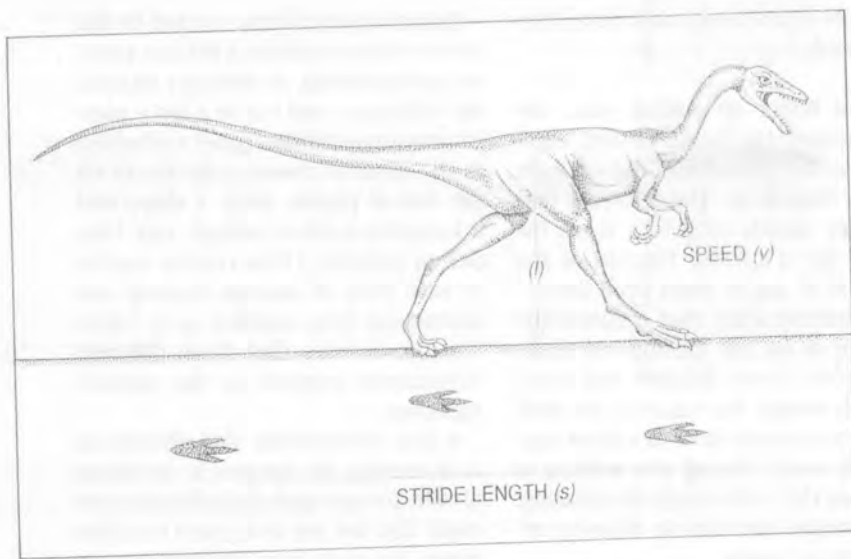
Although I did not realize it at first, v^2/gl falls into a class of expressions first used by a 19th-century naval architect named William Froude. Froude wanted to estimate the power needed to propel newly designed ships by making tests on small scale models before the ship itself was built. (In doing so, he hoped to avoid expensive mistakes.) Since the bow wave that

is pushed in front of a ship causes much of the resistance to the vessel, Froude realized that the height of the bow wave in his models must be proportionate to that of a real wave against the prow of a real ship. To achieve this wave size, he showed that the model must be tested at a specific speed, giving rise to an expression now named after him: a Froude number.

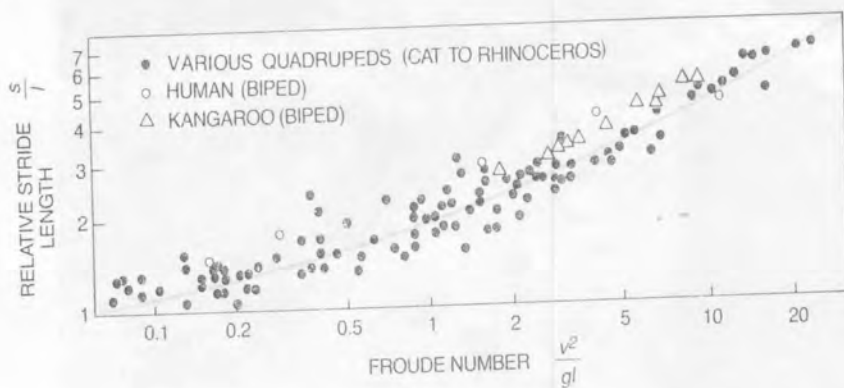
In the realm of shipbuilding, the model's Froude number (v^2/gl) must be the same as the Froude number for the real ship traveling at its normal speed in order for the test to be useful. In Froude's calculations, of course, l was not leg length but the length of the hull of the ship.

Froude's experiment illustrated a special case for a more general rule—if it had not, it would have been of no use to me, since ships are not much like dinosaurs. The rule involves the concept of dynamic similarity, which can be thought of as an extension of geometric similarity. As we saw, two shapes are geometrically similar if one can become identical to the other by a uniform change in the scale of length. (Height and width, of course, have to be scaled up in proportion.) Likewise, two motions are dynamically similar if one can become identical to the other by uniform changes in the scales of





STRIDE LENGTH is the distance between two successive prints from the same foot. *Compsognathus*, a carnivore the size of a contemporary chicken, is depicted here.



FROUDE NUMBERS for kangaroos, humans and quadrupeds, such as rhinoceroses, are plotted against the animals' relative stride lengths. The numbers increase logarithmically—so that the difference between Froude numbers 0.1 and 20 is clear.

length, time and force. Two animals of different sizes, for example, may be dynamically similar if they both are trotting or galloping.

The general rule that applies to bow waves, running animals and all other systems involving gravity reveals that dynamic similarity is possible only when Froude numbers are equal. Looking at the energy of these systems clarifies this point. In the rise and fall of a wave, or of a limb, energy is converted back and forth between kinetic and potential forms. Two systems can be dynamically similar only if they have the same ratio of kinetic energy (equal to $1/2 mv^2$ for a body of mass m moving at speed v) to potential energy (equal to mgh , where h is height). This ratio is $v^2/2gh$, proportional to a Froude number.

With the formula in hand, I was able to extend the observations from shipbuilding to other aspects of dynamic similarity. I hypothesized that geometrically similar animals of different sizes would run in approximately

dynamically similar fashion whenever their speeds made their Froude numbers equal. Precise dynamic similarity could not be expected, because animals of different sizes are not quite the same shape, and strict dynamic similarity requires strict geometric similarity.

The theory proved generally true. For instance, one of the predictions of the hypothesis was that animals of different sizes would use similar gaits when their Froude numbers are equal. Indeed, ferrets and rhinoceroses change from trotting to galloping at very different speeds, but in each case the Froude number is about the same. Ferrets alter their gait when they start to travel at 1.5 meters per second. Because their hip height is 0.09 meters, the expression of the Froude number is 1.5 squared divided by 0.09 times g (or 9.8 meters per second squared). The result is 2.55. Similarly, a rhinoceros changes from a trot to a gallop at 5.5 meters per second, and its hip height is 1.2 meters—when the calculations

are made, the Froude number is 2.57.

Another prediction involved stride length, the distance between successive footprints of the same foot. The faster animals run, the longer their stride. The hypothesis held that when their Froude numbers were equal, different animals would take strides in the same proportion to their leg length—because of their dynamic similarity. Therefore, a graph of relative stride length (that is, stride length divided by leg length) plotted against Froude number should be the same for cats and camels, ferrets and rhinoceroses [see lower illustration at left].

This relationship, however, turns out to be true only for mammals the size of a house cat and larger. It does not hold as well for smaller mammals such as rats because they run in an often peculiar, crouched manner, quite different from the bigger mammals. The graph also shows that the relation between relative stride length and Froude number is similar for bipeds such as people and kangaroos and for quadrupeds such as dogs and horses—demonstrating that these two forms of movement share some dynamic similarity. (To that extent, you run the way the hindquarters of a small pony move.)

The graph proved to be even more illuminating because it soon occurred to me that I could use it to estimate the speed of dinosaurs from the stride lengths shown by their footprints. Rather surprisingly, great numbers of dinosaur footprints have survived as impressions in mud that turned to stone [see "The Footprints of Extinct Animals," by David J. Mossman and William A. S. Sarjeant; SCIENTIFIC AMERICAN, March 1983]. These tracks show that dinosaurs walked with their feet directly under their bodies like mammals and birds, not sprawled out to either side in the manner of modern reptiles. Consequently, the relation between relative stride length and Froude number for mammals should also apply to dinosaurs. In contrast, estimating dinosaur speed by comparing these giants with modern reptiles would not be appropriate.

The largest-known footprints, with hind feet measuring 1.3 meters in diameter, have been found in Spain. Tracks of slightly smaller prints lie in other parts of the world—even in Yorkshire, England. The best known of the smaller prints—measuring 0.9 to 1.0 meter in diameter—were discovered in Texas. The size and shape of these footprints suggest that sauropods, huge long-necked, long-tailed herbivores, roamed this region. Three-toed bipeds similar to *Tyrannosaurus*

made other tracks nearby. One famous trail shows that these two Texans met: a sauropod and a tyrannosaurlike biped traveled along the same path [see *illustration on this page*]. Was this dramatic chase leading to a kill? Were the animals heaving themselves along slowly and with difficulty, or did they rush past at earth-shaking speeds?

Using Froude numbers and stride lengths measured from these and similar trackways, I made my first efforts to infer the speed of dinosaurs. Since leg length can be estimated from the size of the footprints—footprints should be about one quarter of leg length—relative stride length could also be calculated. And once relative stride length was established, I used the graph to find the corresponding Froude numbers. Then, armed with the leg length, I could calculate speed. The results may not be very accurate, however, because the data points on the graph show a fair amount of scatter and because we are using data from modern animals to estimate speeds for dinosaurs.

By this account, the speed of the large dinosaurs was unimpressive. All known footprints of large sauropods seem to show speeds of about one meter per second, a slow walking speed for humans that seems painfully slow for animals with three-meter hind legs. None of the footprints of very large bipedal dinosaurs show speeds above 2.2 meters per second, the pace of a fast human walk.

Although most of the footprints of big dinosaurs seem to show walking speeds, many footprints of smaller ones record running. The fastest tracks were made in Texas by a biped of probably a little more than half a metric ton, roughly the mass of a racehorse, and by another somewhat smaller one. Both sets of footprints indicate a speed of 12 meters per second, which is higher than the peak speed of 11 meters per second reached by the best human sprinters—but well below the speeds of 15 to 17 meters per second at which horse races are generally won.

The lack of running footprints of huge dinosaurs, however, does not show that they could not run but merely that they usually walked—at least on surfaces in which footprints were likely to be preserved. If you were to go out on a snowy day, for example, and measure human footprints, you would probably find only short strides, indicating walking speeds, but you would be wrong to conclude that people cannot run. Clearly, a different approach was needed to judge how athletically large

dinosaurs could have moved when they really tried.

The faster an animal runs, the greater the forces its feet exert on the ground and the stronger its legs need to be. The reason is that at higher speeds each foot is on the ground for a smaller fraction of the stride, so it has to exert peak force—the maximum force that occurs while the foot is on the ground—to make a complete stride balance and carry the body weight. For instance, the peak force on a human foot rises from simple body weight during slow walking to 3.5 times the body weight in sprinting. Fast running and athletic behavior require strong bones.

Examining the forces exerted by the feet of various creatures led to a greater understanding of dinosaur motion. My colleagues and I used a force plate (a pressure-sensitive panel embedded in the floor) to measure the forces on the feet of people, dogs, a sheep and a kangaroo as they walked, ran, hopped or jumped. These results, together with films of animals running and anatomical data, enabled us to calculate the stresses that these different movements imposed on the animals' leg bones.

A few assumptions also allowed us to determine the stresses in the bones of wild animals such as buffalo that we could film but not bring into the laboratory. We could have applied the same



DINOSAUR TRACKS provide a record of stride length and speed. A small, three-toed carnivore may have pursued a larger sauropod along this Texan trail. This pair of footprints was discovered by Ronald T. Bird at Paluxy Creek in 1944.

approach to dinosaurs to find out how well their bones would have withstood the stresses of running, but reconstructing their patterns of movement would have involved elaborate calculations and a good deal of imagination. I preferred a quicker and easier way, again using the concept of dynamic similarity and insights from structural engineering.

Forces act on the ends of bones (at the joints) setting up stresses in the bone shaft. These forces can be broken down into their components: axial force (F_{ax}) acts along the long axis of the bone, and transverse force (F_{trans}) acts at right angles to it [see box on opposite page]. Taken alone, F_{ax} sets up a uniform stress $-F_{ax}/A$, where A is a cross section of the bone and the minus sign indicates a compressive stress.

Added to this force are the stresses caused by F_{trans} . These transverse stresses vary across the thickness of the bone from $-F_{trans}x/Z$ at one end of the bone to $+F_{trans}x/Z$ at the other. In these expressions, x is the distance of the cross section from the end of the bone, and Z is the section modulus, a geometric property of the cross section. (Engineering textbooks explain how to determine the section modulus, for those interested.)

Too great a stress will break a bone. Calculations for the leg bones of running and jumping modern animals showed that the stress $F_{trans}x/Z$ was generally much greater than the stress F_{ax}/A . This difference tells us that transverse forces are a much more serious threat to bones than axial ones: it is much easier to break a stick or a bone or any other long, thin bar by

transverse forces than it is to break it by axial ones. Therefore, since we need only rough estimates of bone stresses, we can ignore the less important F_{ax} and consider only F_{trans} .


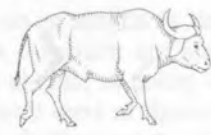




For animals running in dynamically similar fashion, all the forces acting on the bones are proportional to body weight, W , because the bones have to support that weight. This rule includes transverse forces on the leg bones: the stresses they cause ($F_{trans}x/Z$) are proportional to Wx/Z . Now imagine two similar animals of different sizes moving in a dynamically similar fashion. The stresses in the leg bones will be less in whichever animal has lower values of Wx/Z , indicating that its bones are strong enough to do more athletic things than the other animal's bones.

By changing the expression just slightly, we can arrive at a value for a strength indicator that is more straightforward to use: the reciprocal Z/Wx . Thus, the greater the value of Z/Wx is for the bones of the animal, the more athletic we can expect that animal to be. To be even more precise, we can use Z/aWx , where a is the fraction of body weight supported by the forelegs or hind legs, as appropriate. Including this weight distribution enables us to make meaningful comparisons between, for example, elephants and the brontosaur *Apatosaurus*—a massive herbivore dinosaur—since elephants carry the major part of their weight on the forelegs and *Apatosaurus* on the hind legs.

Once we had determined a strength indicator for the principal leg bones of large modern mammals and of dinosaurs, we were able to conclude that a dinosaur could have been as athletic as any modern mammal with similar strength indicators. This conclusion depended on the assumption that the bones of different animals can stand about the same stresses. And tests on samples of bones from birds and mammals show that this is indeed roughly true. We cannot check these stresses on dinosaur bones, however, since their properties have been altered by decay and fossilization.

The conclusion that high strength indicators imply agility also depended on the assumption that evolution has adjusted the strengths of bones of different animals to give them equal safety factors. The safety factor of a structure is described as the force needed to break it, divided by the maximum force expected to act on it in normal use. (Engineers generally design structures with safety factors of two or more to make disaster unlikely.)

Bone Strength in Large Animals

	BODY MASS (METRIC TONS)	STRENGTH INDICATOR		
		FEMUR	TIBIA	HUMERUS
AFRICAN ELEPHANT 				
AFRICAN BUFFALO 				
APATOSAURUS 				
DIPLODOCUS 			NO DATA	NO DATA
TRICERATOPS 			NO DATA	
TYRANNOSAURUS 	7.6	8	NO DATA	NO DATA

The process of arriving at the values for the strength indicators for the dinosaurs is not as straightforward as the conclusions. The first step is to calculate the value of the section modulus, or Z , of their bones. To do this, I needed detailed measurements of cross sections at known distances, x , from the end of the bones. (The section modulus, although complicated to arrive at, simply takes account of the areas of all parts of the cross section and their distances from the middle of the bone, or girder.) I was able to obtain values for x from accurate drawings published by earlier paleontologists.

Now all I was missing to make my calculation of the strength indicators complete was the weight, W , of the dinosaur and, if it was a quadruped, the fraction, a , of that weight carried by each pair of feet. (I have already explained how dinosaur weights are estimated from the volumes of models.)

To calculate the distribution of weight between the feet, I had to find the center of gravity of each dinosaur. By suspending each model from a thread—first from its nose so that it hung vertically and then from its back so it hung horizontally—I could arrive at this center. Each model was photographed in both positions, and care was taken to ensure that the axis of the camera was perpendicular to the plane from which the scaled plastic replica hung. (For such an exercise, monofilament nylon is superior to spun thread, which unwinds, thus making the model spin.) Because the dinosaurs hung with their centers of gravity directly below the point of support, the thread pointed toward the center of gravity in each photograph. By superimposing two photographs, I could discover the center of gravity.

One slight complication in this technique was easily remedied. Unlike real dinosaurs, which would have had air in

their lungs and dense bones running through less dense flesh, the models were solid plastic of uniform density. In the real animals the bones were distributed along the whole length of the body, so they probably did not affect the position of the center of gravity too much—but the air was all near the front end of the trunk. I compensated for these different densities either by calculation (assuming lungs of about the same fraction of body volume as in modern reptiles and mammals) or by boring a hole representing the approximate volume where the lungs would have been.

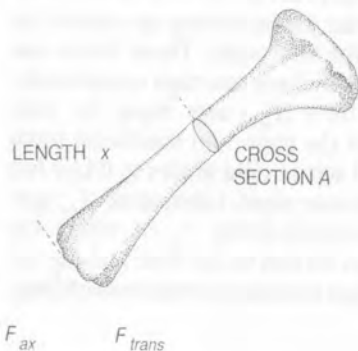
Correction for the lungs moved the center of mass only a little bit, less than 20 centimeters in a 20-meter sauropod. Once I knew, or thought I knew, the dinosaur's center of mass, I could divide its weight appropriately between the forefeet and hind feet. (For example, if the center of mass was two meters behind the front feet and one meter in front of the hind ones, the hind feet would have carried two thirds of the weight.)

All these procedures and data, taken together, supplied the information we needed to calculate the strength indicators Z/aWx for different dinosaur limbs [see table on opposite page]. Remember that obtaining larger values from this formula implies more agility for the animal. The computations enabled us to compare elephants with *Apatosaurus*, the very large sauropod dinosaur commonly known as the brontosaurus. The strength indicators ranging from seven to 11 for the various leg bones of an African elephant are not very different from the values of six to 14 for *Apatosaurus*—whose bones were bigger but similarly proportioned.

This comparison suggests that despite its huge size, *Apatosaurus* could

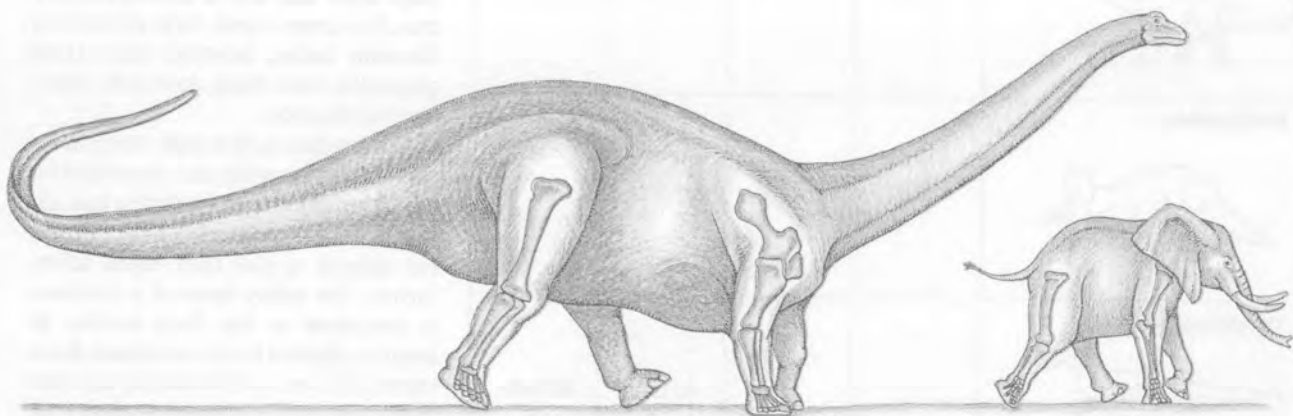
How Forces Act on Bones

When an animal moves, forces are exerted on the ends of its bones and cause stresses in the bone shaft. These forces can be broken down into different components: the axial force (F_{ax}) and the transverse force (F_{trans}). In the

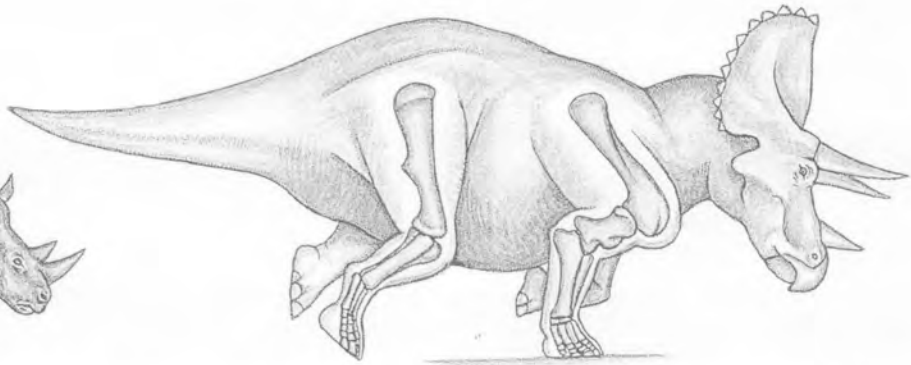
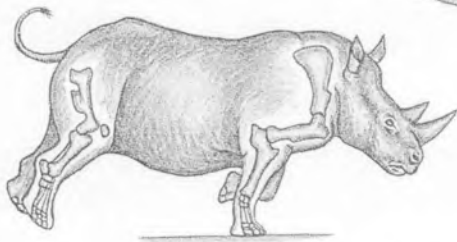
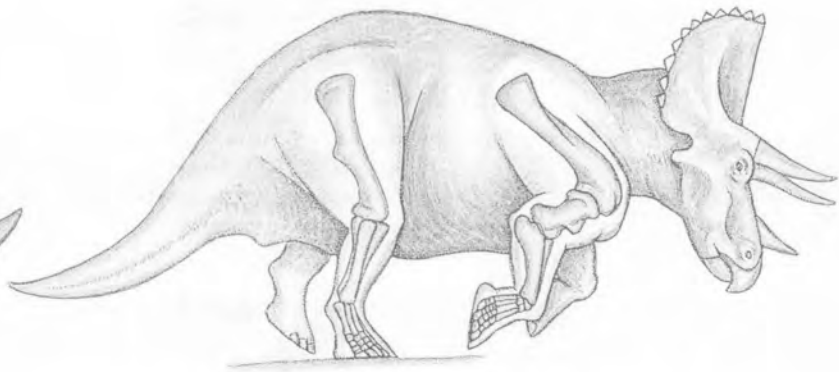
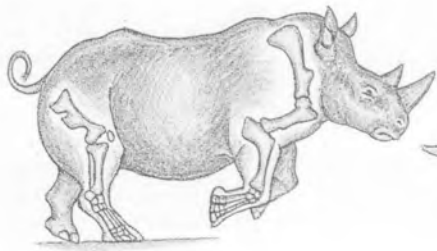


cross section (area A) of the diagram, F_{ax} produces a stress, $-F_{ax}/A$ (the stress is negative because it is compressive). This stress is weaker than that stress produced by F_{trans} : $F_{trans}x/Z$. (Z represents the section modulus, an engineering term, which describes properties of the cross section.)

have been about as athletic as an elephant. Elephants can manage a slow run but cannot gallop or jump (a narrow ditch is enough to keep them in a zoo enclosure). It seems likely that the same was true of *Apatosaurus*. Unfortunately, it is difficult to be specific about speeds because most published elephant speeds are subjective estimates, and anyone who has had an alarming experience with an elephant is likely to exaggerate. One film of a young elephant running shows a speed



ELEPHANT AND APATOSAURUS, a plant-eating dinosaur, both have approximately the same strength indicators for their leg bones. This similarity means that the extinct giant probably ran much the same way our contemporary does.



TRICERATOPS may have moved like the White rhinoceros, which was sketched from a film, is galloping at seven meters per second—a modern, horned herbivore. The White rhinoceros here, per second—which is about the speed of a fast human run.

of about five meters per second (a slowish speed for a human runner). To reach the same Froude number on its longer legs, *Apatosaurus* would have had to run at seven meters per second—slightly brisker than an elephant.

Diplodocus, a more slender sauropod, seems to have a lower strength indicator, at least for the femur (the only bone for which I found a cross section). Therefore, it was probably less agile, able to walk on land with no need to be buoyed up by water but possibly not able to run. Unfortunately, considerable doubt exists about the mass both of *Diplodocus* and of the well-known horned *Triceratops* because measurements have been made in each case on a skinny model and a stout one—it seems uncertain which model is more accurate.

The strength indicators for the *Triceratops* are higher than for either of the two sauropods and fall between elephant values and values for more athletic animals such as the African buffalo. These findings suggest that *Triceratops* may have been more athletic than elephants, possibly able to gallop like buffalo and rhinoceroses. I have a film of an adult White rhinoceros of about two tons, galloping in a large zoo enclosure at seven meters per second while being pursued by a

vehicle. For *Triceratops* to have reached that same Froude number—although it is doubtful whether it could have—it would have had to travel at nine meters per second.

The conclusions reached for *Apatosaurus*, *Diplodocus* and *Triceratops* are already tentative, and we must be even more cautious about *Tyrannosaurus* because all modern bipeds are so very much smaller. Indeed, no contemporary biped moves its legs in a way that seems likely for its structure. We can only note that the strength indicator of a *Tyrannosaurus* femur is low, in the elephant range.

The calculations that allowed me to assess the agility of dinosaurs are firmly based in physics and engineering. Especially useful to me was the concept of dynamic similarity, which had its origins in shipbuilding but has become immensely important in aerodynamics, heat engineering and other branches of physical science. And the theory of stresses in beams (the section modulus), which is constantly used by structural engineers, provided further insights.

Both sets of theory enabled me to arrive at dinosaur strengths and speeds, but it would be foolish to claim that the calculations are accurate. I hope I

have made clear some of the sources of error. These estimates give the impression that although large dinosaurs walked slowly, most were capable of quite a quick run and none needed to live in water or to rely on buoyancy for support.

The evidence from footprints suggests that if we had been alive at the time (and had had the nerve), we could have strolled alongside a walking sauropod or tyrannosaur, keeping up with it without difficulty. The calculations derived from bone dimensions suggest that large sauropods may have been as agile as elephants and that *Triceratops* may have been a little more athletic. I think I am probably fast enough to outrun a pursuing tyrannosaur, but, perhaps fortunately, I am unlikely to have to try.

FURTHER READING

- MECHANICS OF POSTURE AND GAIT OF SOME LARGE DINOSAURS. R. McN. Alexander in *Zoological Journal of the Linnean Society*, Vol. 83, No. 1, pages 1-25; February 1985.
- DYNAMICS OF DINOSAURS AND OTHER EXTINCT GIANTS. R. McNeill Alexander. Columbia University Press, 1989.
- DINOSAUR TRACKS. Tony Thulborn. Chapman and Hall, 1990.