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ON CRACK PATHS

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Abstract—Crack paths are discussed from several different points of view. The remarkable reproducibility of crack paths, especially the frequent occurrence of straight cracks, is considered. The general dominance of mode I growth at mixed mode loading (via kinking or tilting of the crack edge) is examined, and it is shown that mode I growth can be suppressed by superposition of a high pressure. Mode I growth is scarcely combineable with mode II or III, because of different micromechanisms, whereas a mixture of modes II and III growth appears to be possible. Directional stability of straight crack paths is also analysed with special reference to the importance of a meaningful definition of the concept itself. Finally crack branching is briefly discussed.

1. INTRODUCTION

THIS PAPER deals mainly with cracks whose paths can be described by a plane curve rather than by a three-dimensional surface.

Experience shows that straight crack paths are very common when the remote stress field is homogeneous. However, available experimental results are essentially limited to mode I growth. Modes II and III are known mostly from earth-quake slipping. Nevertheless, observations made after such events often show conspicuously straight paths.

Even if the loading is such that straight mode I crack growth could be expected for symmetry reasons, the path might be directionally unstable[1–7]. Interaction of originally collinear cracks invariably leads to loss of directional stability, as shown by Melin[6].

Under very general circumstances—nonhomogeneous remote stress field or a non-special crack orientation—an originally straight crack will grow along a curved path, starting in the general case with kinking[8–17].

There has been rather much discussion about criteria that might control the direction of crack growth. Symmetry properties[5–9, 13, 15–23] and physical principles[14, 24, 25] have been used. An account of these criteria is given in [26].

Several mechanisms for branching have been suggested[22, 27–29]. High crack tip velocities seems to be a necessary condition for successful branching[22].

Parts of the contents of this paper are based on investigations carried out at the Division of Solid Mechanics at the Lund Institute of Technology by Drs H. Andersson, H. Bergkvist, Solveig Melin, L. G. Pärletun and P. Ståhle.

2. STRAIGHT CRACKS UNDER LOADING AT PURE MODE I, II OR III

2.1 *Mode I*

A straight crack perpendicular to the largest principal stress is known to grow straight forwards if small scale yielding prevails. This fact cannot easily be explained with reference to the elastic near-tip stress field. With a polar coordinate system r, θ with origin at the crack tip and $\theta = \pm\pi$ along the crack surfaces one has[30]:

$$\sigma_\theta \approx K_I(2\pi r)^{-1/2}(1 - 3\theta^2/8) \quad \text{for} \quad \theta^2 \ll 1 \tag{1}$$

i.e. the maximum at $\theta = 0$ is very flat. With regard to the fact that K_I usually varies several per cent in the same piece of material (depending on the crack tip position) such a flat maximum does not appear to be sufficiently well defined to support a straight path. In addition, the fact th

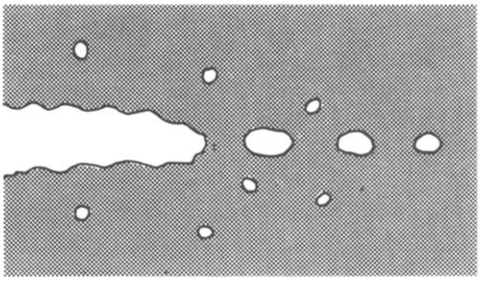


Fig. 1. During slow crack growth micro-separations will be elongated preferably in the crack direction.

$\sigma_r \approx \sigma_\theta$ in front of the crack tip seems to imply that there is no preferred orientation for micro-separations near the tip.

However, the picture becomes different if plastic flow is taken into account. Assuming perfect plasticity in the crack tip neighbourhood, slip line theory for growing cracks shows[31] that the maximum of σ_θ with respect to θ is somewhat less flat, but, most importantly, σ_θ is considerably larger than σ_r in front of the crack tip. In fact

$$\sigma_\theta \approx 1.8\sigma_r \quad \text{for} \quad \theta = 0. \tag{2}$$

Thus micro-separations will be elongated preferably in the crack direction, see Fig. 1 (cf. also [32]). This implies easy coalescence with the main crack and also a strong impetus for straight crack growth.

Since the near-tip part of a crack propagating along a smooth curve can be regarded as being straight, at least under conditions of small scale yielding, the often observed reproducibility even of curved paths (under identical circumstances) appears logical.

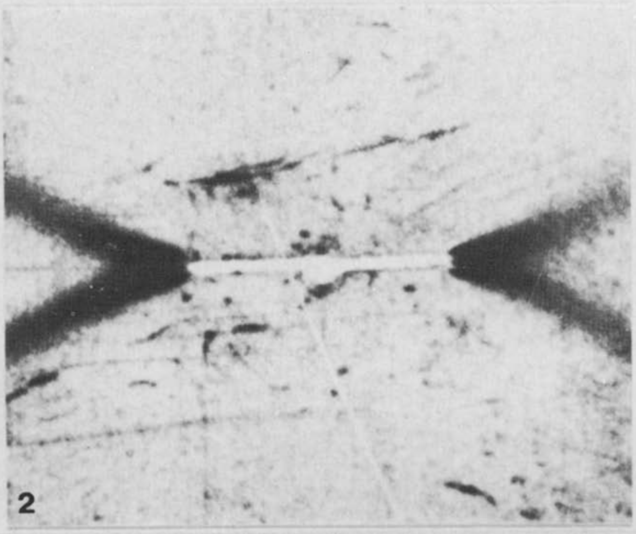
The formation of micro-separations and their growth towards coalescence with the main crack determines the size and shape of the plastic region at small scale yielding. The situation becomes different in some cases of large scale yielding. In thin plates necking regions, traversing ligaments from the crack tips to the boundaries, sometimes appear before unstable crack growth, see Figs 2 and 3. Subsequent crack propagation then proceeds along the mid-line of a necking region, see Figs 4 and 5. Thus the crack path is already stalked out and not governed by near tip processes.

Fatigue crack growth under mode I loading is sometimes observed to follow a zig-zag pattern[12], indicating that the micro-mechanism is slip (i.e. mode II) rather than cleavage, even though from a macroscopic point of view the crack as a whole proceeds in mode I.

2.1 *Mode II*

Theoretically a direction exists in which a straight crack will grow straight forwards under mode II conditions, namely the direction, which maximizes the mode II stress intensity factor, K_{II}. It is very difficult, however, to realize mode II growth under laboratory conditions. Instead the crack grows via a kink. Such is usually the case even when the remote stresses are compressive, see Fig. 6 (cf. also [18]). The kink angle is found to be about 70°, in agreement with theoretical results, assuming crack growth in the direction of maximum mode I stress intensity factor, K_I[17]. Even subsequent crack growth appears to proceed in the direction of maximum K_I, which, after kinking (i.e. during smooth crack growth) coincides with the direction of vanishing K_{II}.

Figure 6 shows one example of results from an experimental investigation with 3 mm thick square (83×83 mm^2) PMMA-plates. Internal cracks were produced by three-point bending, after scratching a line on the surface with a sharp tool, see Fig. 7. The plate was turned upside down after a crack had been produced through-the-thickness, and the procedure was repeated until the crack extended through the plate along the full length of the scratched line. This procedure, which works well also for glass, was described to the author by Professor Yin Xian Chu at the Geophysical Institute of the State Seismological Bureau in Beijing.



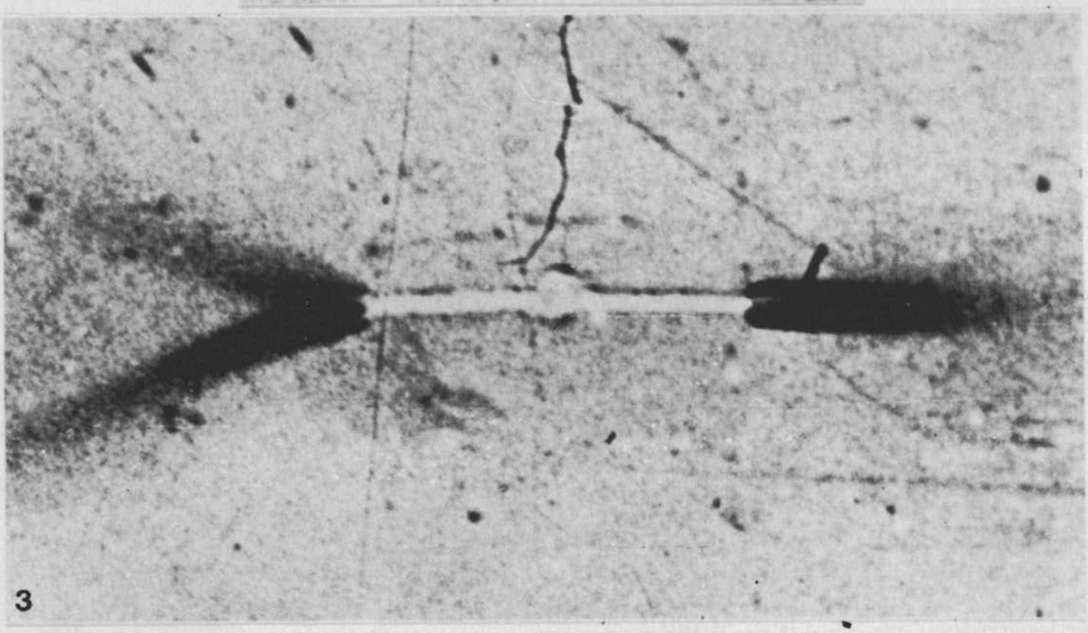
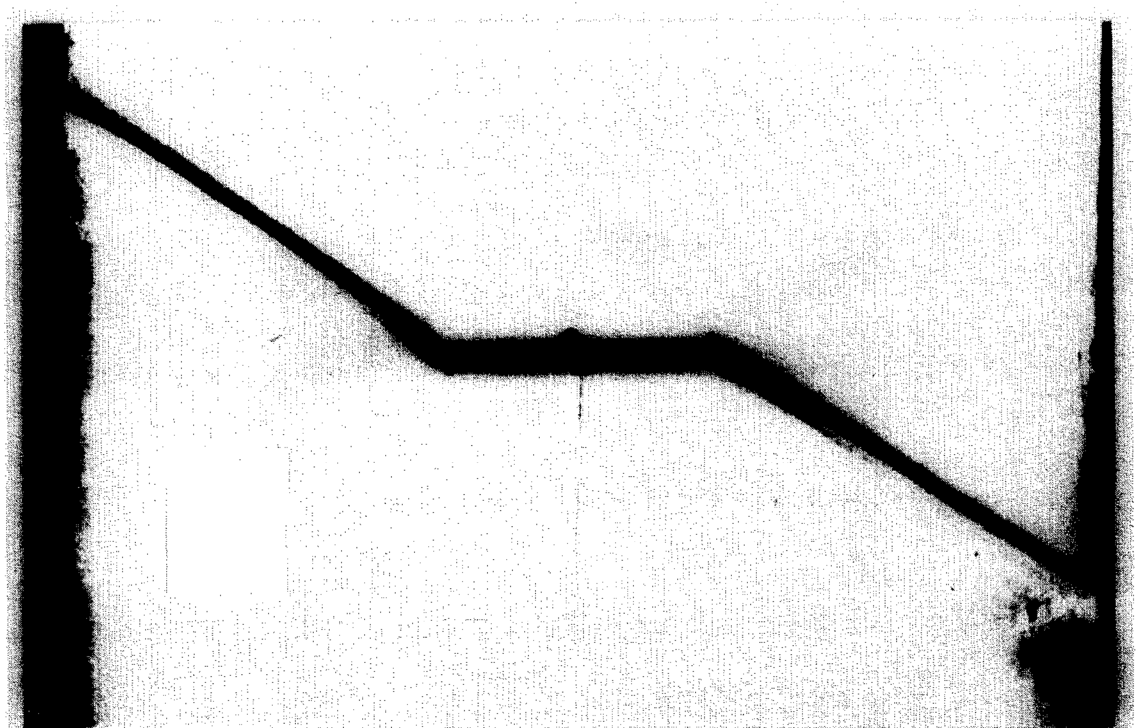


Fig. 2. Symmetric necking in an aluminium sheet.

Fig. 3. Asymmetric necking in an aluminium sheet.



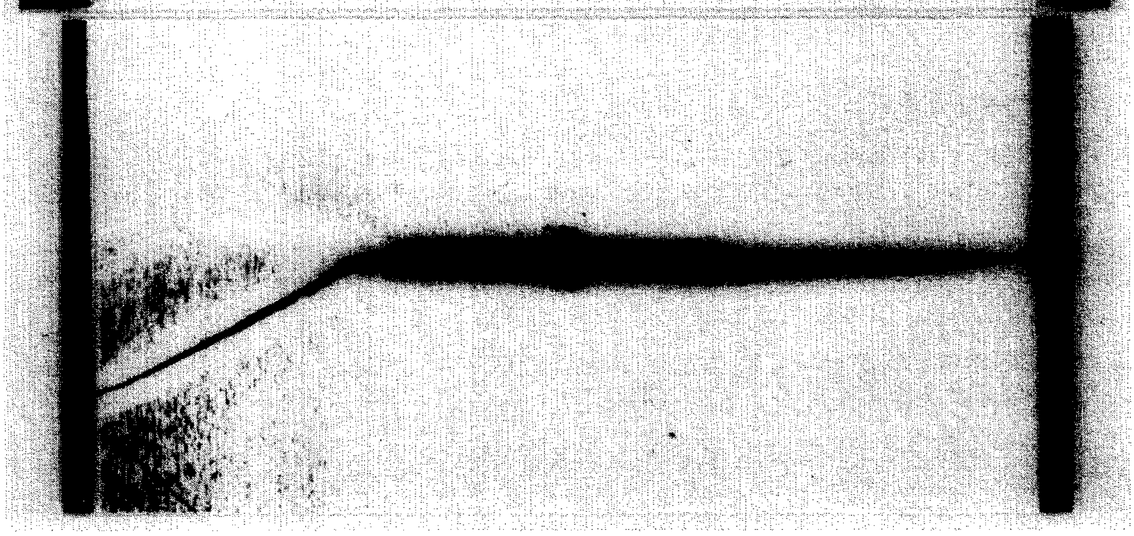


Fig. 4. Anti-symmetric crack propagating along necking regions in an aluminium sheet.

Fig. 5. Asymmetric crack propagating along necking regions in an aluminium sheet.

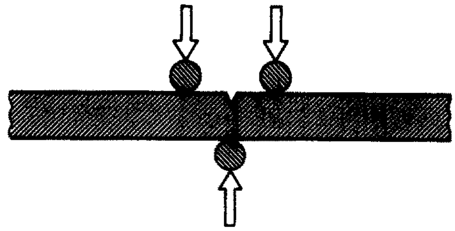


Fig. 7. First stage at manufacturing of an internal crack in a PMMA plate. A roughly semi-elliptical crack will grow from the scratch and traverse the thickness of the plate.

The friction coefficient μ for slip along the crack surfaces was determined by a simple experiment, see Fig. 8. The friction angle was found to be about 19°, which corresponds to $\mu \approx 0.34$. The angle between the crack and the direction of the smallest principal compressive stress was therefore chosen as $(\pi + 2/\tan \mu)/4 \approx 54°$, i.e. the most favourable direction for mode II growth (this direction which maximizes K_{II} coincides with the one given by the Mohr–Coulomb criterion).

In the case shown in Fig. 6 the plate was compressed in the vertical direction only. Since mode I growth via kinking resulted (instead of mode II growth) continued experiments were performed with compression both in vertical (force: V) and horizontal (force: H) directions. A sufficiently high superimposed pressure should provoke mode II growth, according to theoretical results by Melin[17], which can be presented in the way shown in Fig. 9.

If $H = 0$ then mode I crack growth via kinking occurs when $V = V_c$, say. Increasing horizontal load H implies increasing vertical load at initiation of kinking. However, after a certain horizontal load, $H = H_s$, has been reached, mode II growth (straight forwards) takes over. The minimum vertical load, $V = V_s$, for mode II growth might be considerably higher than V_c. How much higher depends on the ratio $\kappa_c = K_{IIc}/K_{Ic}$ between the critical stress intensity factors. From [17] the ratios V_s/V_c and V_s/H_s can be calculated. The results are shown in Fig. 10.

The loads were applied first so that $V = H$ until a certain level $H = H_l$ was reached. Thereupon H was kept constant and V was increased. If kinking occurred then $H_s > H_l$ and if mode II growth occurred then $H_s < H_l$. In this way H_s and also V_s could be determined.

Figure 11 shows an example of mode II growth. The vertical load needed was much higher than V_c. Buckling had to be prevented by means of lateral support. Substantial plastic flow occurred—the plate was somewhat thickened near the crack path. Therefore—and also because of the small ligaments from the tips of the pre-existing crack—a quantitative interpretation based on the theoretical results by Melin[17] (for small scale yielding and large plates) might be

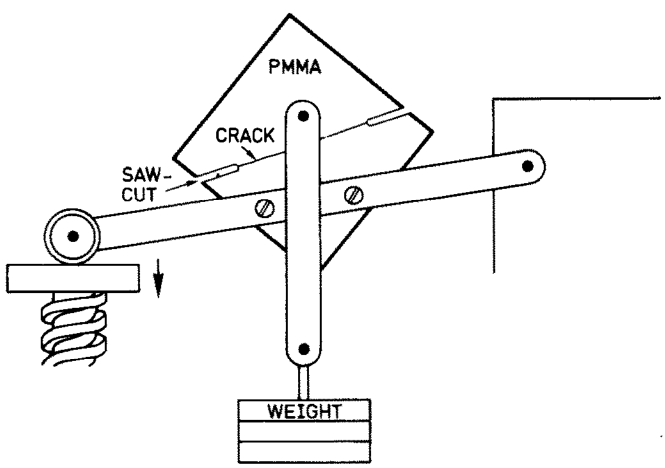


Fig. 8. Determination of the coefficient of friction. An internal crack is first manufactured. Then the ligaments are cut off by saw-cuts in the direction of the crack. The crack surfaces are then put together, oriented horizontally and then subjected to a compressive load. Finally the assembly is slowly tilted until slip occurs and the tilt angle is measured.

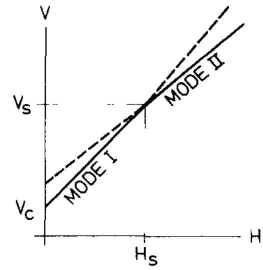


Fig. 9. Schematical representation of biaxial forces (V and H) causing either mode I or mode II growth from a crack favourably oriented for mode II growth.

somewhat uncertain. However, the resulting crack unmistakably occurs under mode II conditions. To the author's knowledge this is the first reported laboratory-produced mode II growth from a pre-existing macroscopic crack under non-cyclic loading. (Apart from earthquake events, macroscopic mode II cracks are otherwise known to follow intensive overall plastic flow, for instance at the central part of a necking region, see e.g. [23].)

V_c could be determined very accurately. Four experiments gave results between 1950 and 2000 N. Five experiments resulting in mode II growth gave $V = 40\,000$ N $\pm$ 6000 N (standard deviation) and $V/H = 3.0 \pm 0.9$. Theses values are regarded to be very close to V_s and V_s/H_s, since a few experiments resulting in mode I growth gave almost as high values of V. Comparison with Fig. 10 gives $\mu \approx 0.46 \pm 0.16$ (relatively close to the direct measurements) and $\kappa_c \approx 2.5 \pm 0.2$.

The results show that straight forwards mode II crack growth is possible if the superimposed

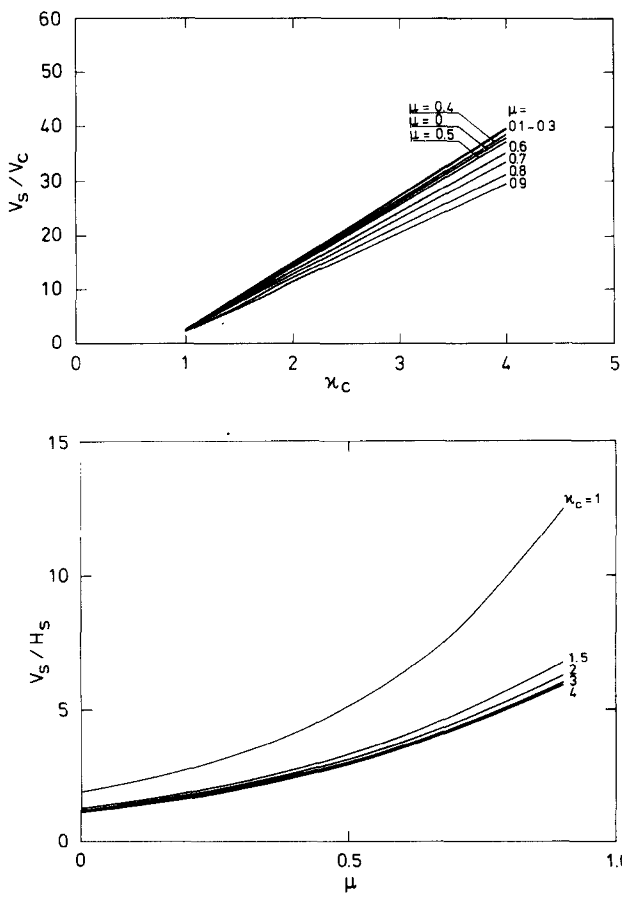
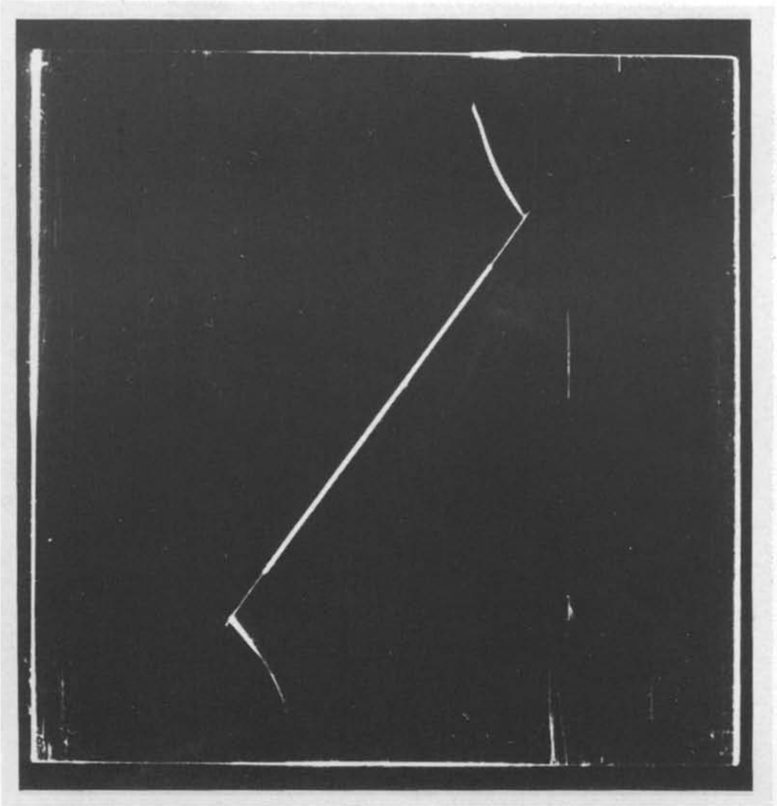


Fig. 10. Representations of V_s/V_c and V_s/H_s from Fig. 9, for different values of the friction coefficient μ and the ratio $\kappa_c = K_{IIc}/K_{Ic}$ between the critical stress intensity factors in modes II and I.



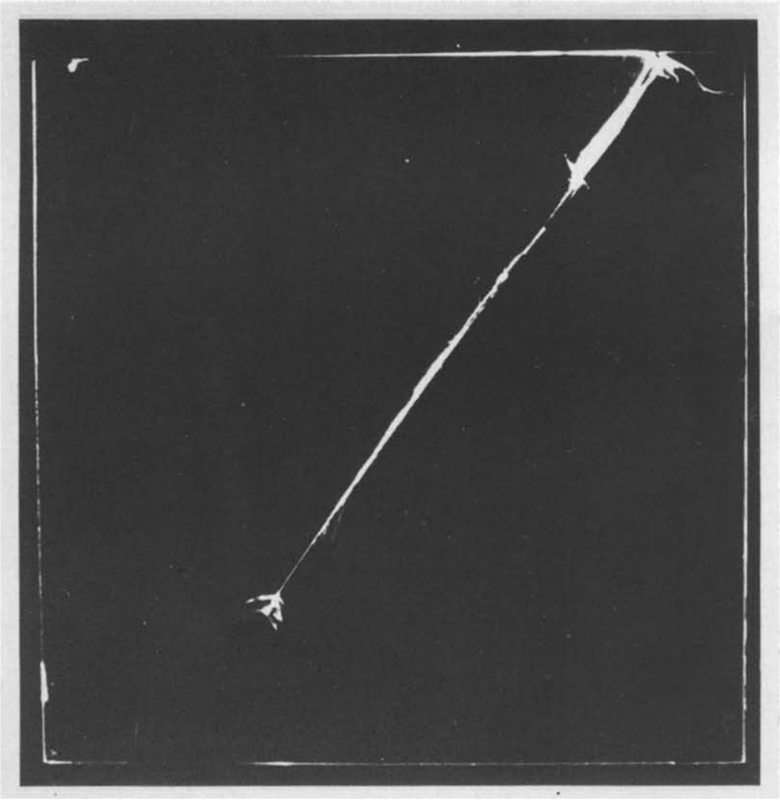
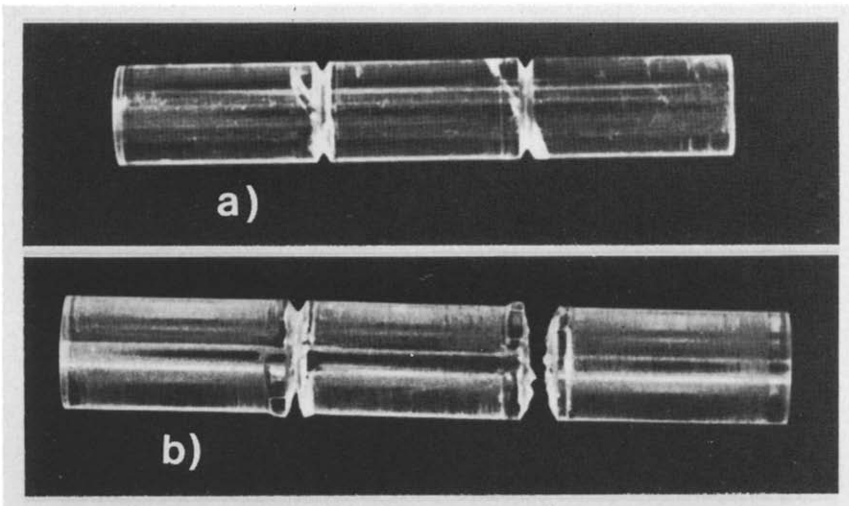


Fig. 6. Crack growth via a kink after uniaxial compressive loading of a PMMA plate.

Fig. 11. Mode II crack growth in PMMA from a pre-existing crack, favourably oriented for mode II growth.



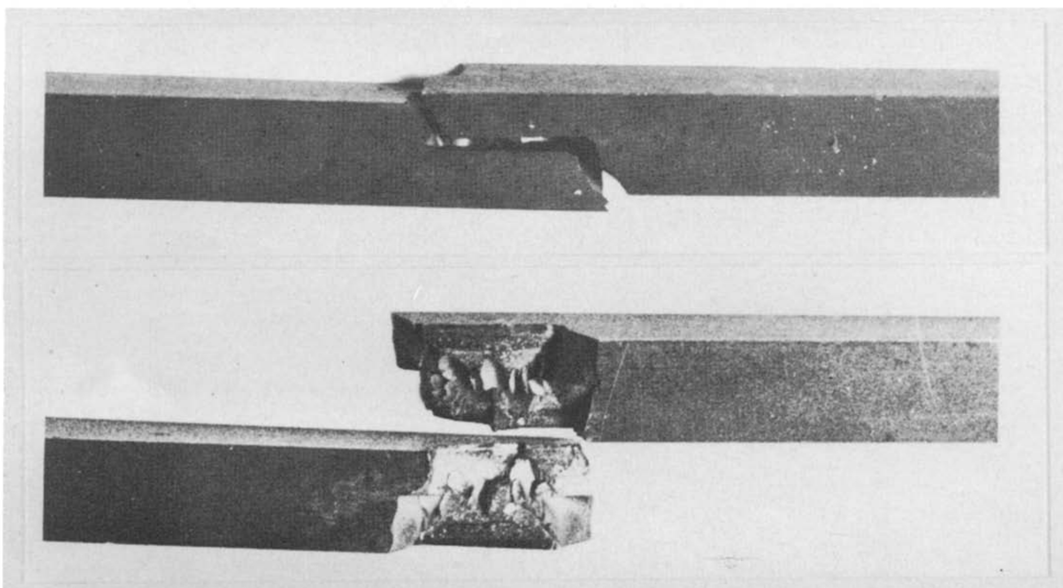


Fig. 12. (a) At no axial load and increasing torque mode I growth develops along a new crack plane (turning eventually into so called spiral fracture). (b) At a high axial load and increasing torque mode III crack growth occurs, even though facets on the crack surfaces indicate signs of mode I growth on a micro-scale.

Fig. 13. Mode III growth with a continuous transition to mode II, interrupted by a final spiral-type fracture (mode I). Torsion-rod with square cross-section, originally situated in a rocking-chair: fatigue.

pressure is high enough. It also indicates that K_{IIc} for PMMA is much higher than K_{Ic}—about 2.5 times higher.

2.3 *Mode III*

Symmetry properties suggest that a straight mode III crack subjected to anti-symmetric remote shear loading will grow straight forwards. However, the difficulties of realizing mode III growth are well known. In certain planes, tilted with respect to the directions of the principal shear stresses, large tensile stresses appear and consequently mode I growth might take over. This is encountered, for instance, at so called spiral fractures at twisting of a rod, even if originally there is a circumferential crack. However, like mode II growth, a sufficiently high superimposed pressure ought to prevent crack opening (i.e. mode I), see Fig. 12. This might be the reason why mode III growth is known from earth-quake events.

Figure 13 shows an example of mode III growth, with some assistance of mode II, interrupted by a final spiral-type mode I fracture. The crack growth before the final fracture was due to fatigue, which probably favours the shear modes, since slip occurs comparatively easily on a small scale.

3. CRACKS UNDER MIXED MODE LOADING

3.1 *Mode I + II*

This combination has been extensively studied experimentally on straight cracks at various angles to the smallest in-plane principal stress, especially when this stress is zero, e.g. [8, 11, 20, 34]. The results invariably show crack growth after kinking (except, of course, when the crack direction coincides with the direction of the smallest principal stress). The same applies to most of the few reported experiments at which at least one principal stress is compressive, an example of which was given in the preceding section.

Melin[16] has analysed some experimental results and found that the kink angle closely and certainly within the experimental accuracy coincides with the angle that maximizes K_I under assumptions of extremely small scale yielding and infinitesimally small kink length. This angle can be calculated with great precision for cases when the crack surfaces are traction free[9, 15], using a method by Khrapkov[21]. It does not coincide with the angle that gives $K_{II} = 0$. Continued crack growth, which occurs slowly (stably) in cases like the one shown in Fig 6, also appears to proceed in the direction that maximizes K_I. (An example of theoretical calculation of such a path is found in [17]). Since the path after kinking is smooth the condition of maximum K_I coincides with the condition of vanishing K_{II}.

If a superimposed pressure is high enough, mode II crack growth may occur, as shown in the preceding section. (The pressure must, however, stay below a certain value, above which neither mode I nor mode II growth is possible, cf. [17]). Such growth would probably occur via kinking if the pre-existing crack had another orientation than the one for which K_{II} is maximum. The kink angle could probably be found from the condition of maximum K_{II} but the lack of experimental data makes such an assumption merely speculative. However, for the special case of straight forwards growth, as in the case shown by Fig. 11 this criterion seems to be applicable.

After studying experimental facts the conclusion is that a crack under mixed mode I + II loading generally grows in the direction that maximizes K_I, except at certain very high superimposed pressures. In these latter cases it appears plausible that the crack grows in the direction that maximizes K_{II}. Theoretical considerations suggest (cf. the preceding section) that the pressure required for mode II instead of mode I depends on κ_c.

One tacit assumption in the discussion so far in this section is that the pre-existing crack is straight. However, the results ought to be applicable also to smoothly curved cracks.

It should be observed that the micro-separation mechanisms in modes I and II are fundamentally different, namely opening and sliding, respectively. Mixed mode loading, therefore should not be expected to lead to some hypothetical mixed mechanism. Micro-separations occur, of course, as a result of local conditions near the crack tip, and their formation and growth will depend upon the directions and strength of the local field. It is therefore not surprising that cracks appear to grow either in the direction that maximizes K_I or in the direction that maximizes K_{II}, not in a direction that maximizes some functional combination of K_I and K_{II}.

Criteria for crack growth direction that are based on concepts borrowed from general physical principles seem to overlook the fact that crack growth is intimately and irrevocably connected with micro-structural, non-continuum processes. Among such criteria is the suggestion that a critical value of the energy release rate from the elastic stress field at small scale yielding governs occurrence and direction of crack growth[8, 14]. In view of the experiments on PMMA described in the preceding section it could lead to very large errors—$\kappa_c = 2$ implies 4 times higher energy release rate at mode II than at mode I, for instance. Other examples of criteria based on general continuum-mechanical concepts without consideration of micro-structural mechanisms are those based on the concept of a generalized force measure of conditions at the crack tip[25] or making use of the strain energy density[24].

3.2 *Mode I + III*

Like the preceding case this is also a question of two fundamentally different micro-separation mechanisms. Therefore it appears plausible that crack growth should occur in either mode I or mode III. If it occurs in mode III, symmetry considerations suggest that the crack extends without kinking On the other hand, pure mode I growth implies change of crack edge direction.

3.3 *Mode II + III*

This is the case at which there is one fundamental type of micro-separation mechanism, only, namely sliding. Mixed mode growth is therefore possible and as demonstrated in Fig. 13, a surface crack can obviously be propagated under mode II conditions in the surface direction, under mode III conditions in the depth direction and under mixed mode II + III conditions in between. In this case, therefore, some functional combination of K_{II} and K_{III} might provide the basis for a criterion of crack growth. Such a combination ought to include other material parameters as well, because of the complicated nature of the dissipative processes. Schematically these processes can be described as formation and growth of micro-separations leading eventually to coalescence with the main crack and determining the size and shape of the plastic region at small scale yielding. The interplay between the process region and the plastic region has been studied by Ståhle[35] under some simple assumptions.

4. DIRECTIONAL STABILITY OF STRAIGHT CRACKS

Experiments[3, 4] have shown that a straight crack under mode I loading from a homogeneous remote stress field grows straight forwards if it is perpendicular to the largest principal stress, but deviates towards such a direction if it is originally perpendicular to the smallest principal stress. Two collinear cracks subjected to uniaxial mode I loading lose directional stability before coalescence—they seem to avoid each other[6]. An example is shown in Fig. 14.

Deviation from a straight path appears to occur smoothly, i.e. without kinking. (Leevers *et al.*[3] report some cases of kinking, but attribute this to preceding rupture of a ligament). Theoretical investigations of directional stability can therefore be performed under the assumption of small changes of the crack direction during growth. Ideally such investigations should take dynamic effects into account, but this can hardly be done in a meaningful way with presently existing methods. Moreover, static and dynamic stress fields do not differ very much before the crack tip velocity is very high[36]. Static analysis should thus suffice for an investigation about directional stability, whereas the crack path after instability might be influenced by dynamic effects. This, however, is less important, since this path anyway depends critically on the disturbance causing the instability.

How should directional stability be defined? For a crack in an infinite plate, situated essentially along the x-axis between $x = -a$ and $x = a$, Melin[6] introduces the quantity $|y(a)/a|$ as a measure of the straightness of the crack. (For simplicity, though not necessary for the analysis, it is assumed that $y(-a) = -y(a)$). Putting $|y(a)/a| = s(a)$ and assuming that the straightness measure for the original crack, $s(a_0)$, is infinitesimally small, the case is defined as directionally unstable if $s(a)/s(a_0)$ increases beyond any predetermined value for sufficiently

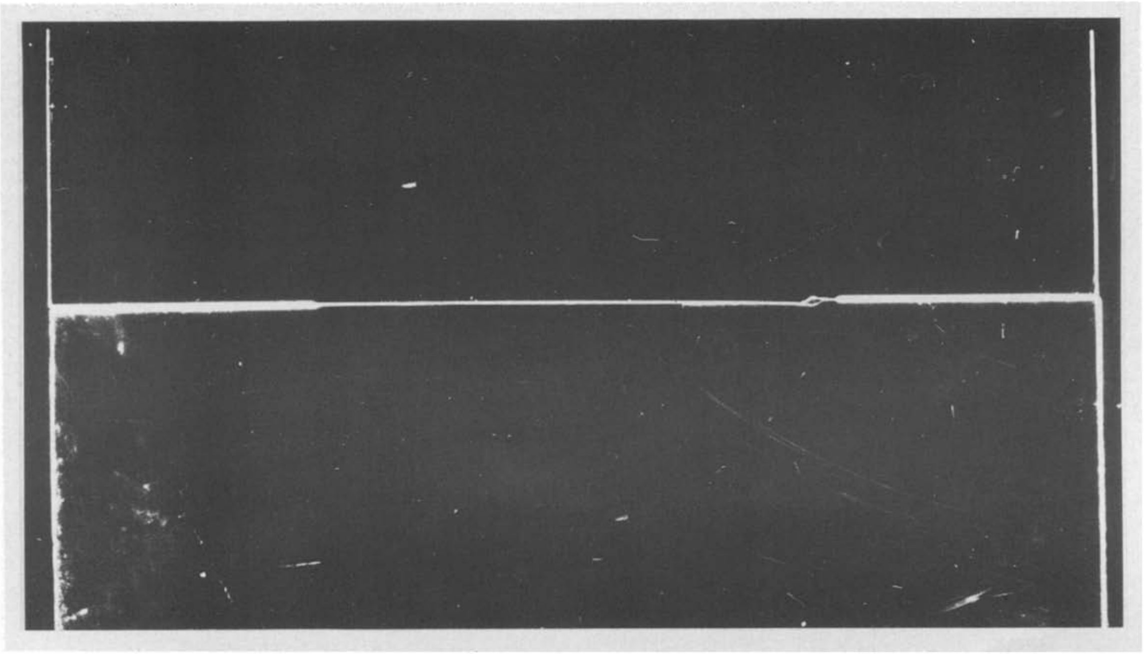


Fig. 14. Crack growth in PMMA, showing how a hardly noticeable deviation of the crack growth direction is successively magnified as the right-going crack tip approaches the pre-existing edge crack at the right. From this edge crack a left-going crack tip emerges, obviously with a tendency to avoid the incoming crack tip from the left.

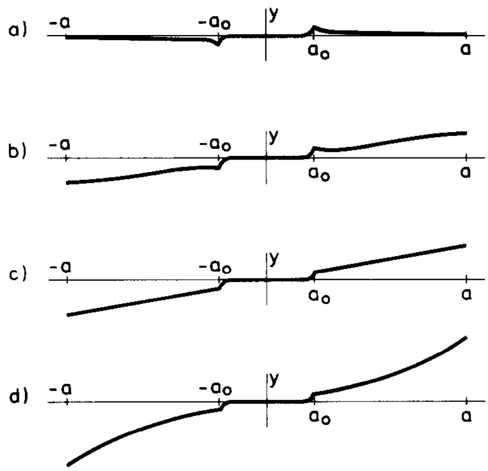


Fig. 15. (a) $\sigma_x^\infty/\sigma_y^\infty \leq 1 - \pi/4$. Stability, at which the crack tips approach $y = 0$. (b) $1 - \pi/4 < \sigma_x^\infty/\sigma_y^\infty < 1$. Stability, at which the crack tips eventually move away from $y = 0$, but so that the crack direction approaches the original one. (c) $\sigma_x^\infty/\sigma_y^\infty = 1$. Indifference. The crack tips move away from $y = 0$ along a straight line. (d) $\sigma_x^\infty/\sigma_y^\infty > 1$. Instability. The crack tips move away from $y = 0$ so that the crack direction deviates more and more from the original one.

large a/a_0 and directionally stable if $s(a)/s(a_0)$ decreases below any predetermined positive value for sufficiently large a/a_0. Between the two extremes directional indifference prevails.

Four different cases could be identified[6] for biaxial loading with the remote stresses σ_y^∞ and σ_x^∞ in the y- and x-directions, respectively. The four cases are shown in Fig. 15. Sufficient and necessary conditions are $\sigma_x^\infty/\sigma_y^\infty < 1$ for directional stability and $\sigma_x^\infty/\sigma_y^\infty > 1$ for instability, i.e. indifference prevails if $\sigma_x^\infty/\sigma_y^\infty = 1$. If $\sigma_x^\infty/\sigma_y^\infty < 1 - \pi/4$, then $|y(a)|$ decreases with increasing a/a_0, but otherwise $|y(a)|$ increases beyond $|y(a_0)|$ for sufficiently large a/a_0.

The crack path for $a > a_0$ depends on the magnitude of the disturbance. If $s(a_0)$ is very small the crack may proceed almost straight forwards a rather long distance even if $\sigma_x^\infty/\sigma_y^\infty > 1$.

Melin used the criterion $K_{II} = 0$ for determination of the crack growth direction. The same criterion was used also by Cotterell and Rice[5] and by Sumi *et al.*[7], although with another approach to the problem in some other respects. In ref.[5] increasing $|y(a)|$ rather than increasing $|y(a)/a|$ seems to have been taken as a definition of directional instability and should therefore have led to the conclusion that directional instability occurs when $\sigma_x^\infty/\sigma_y^\infty > 1 - \pi/4$ instead of $\sigma_x^\infty/\sigma_y^\infty > 1$. The reason why the latter result nevertheless was reported appears in part to be due to a series expansion, interrupted too early[6]. The same criticism applies to [7], even though their definition of instability was somewhat different.

Melin's[6] treatment, therefore, seems to be the first correct analysis of directional stability. She uses an integral equation formulation, based on modelling of the curved crack as a continuous array of dislocations. This method is sometimes superior to complex potential methods at general type crack configurations. It has been used extensively and with great success by Yokobori and co-workers[37–41] for interactions among cracks and between cracks and dislocations, etc. It also was used by Gol'dstein and Salganik[2], who gave explicit expressions for K_I and K_{II} for a slightly curved crack, Their treatment was extended in [6] to an infinite row of almost collinear cracks. The intention was to study the interaction between the cracks. It was shown that directional instability always occurs at uniaxial or biaxial loading. The proof given in [6] assumes a somewhat special form of the initial deviation from straightness, but can be made more general, as shown in Appendix. The result that cracks seem to avoid each other is unexpected from a theoretical point of view: one would rather expect the opposite. This phenomenon is not just an academic curiosity: it influences, for instance, the mechanism of coalescence of micro-cracks near the tip of a main crack, at rapid as well as at slow crack growth.

5. CRACK BRANCHING

Crack branching is a well known expression of directional instability. There are several questions connected with this phenomenon. One concerns the fact that branching usually occurs at higher crack tip velocities—in the neighbourhood of half the Rayleigh velocity or more.

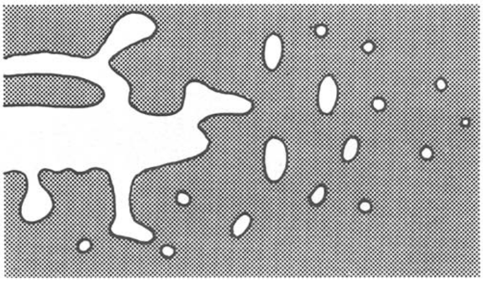


Fig. 16. During high velocity crack growth the micro-separations tend to be elongated in a direction roughly normal to the crack direction and the number of micro-separations is considerably higher than at lower velocities, cf. Fig. 1.

In an interesting analysis Pärletun[22] has shown that successful branching demands a high crack tip velocity (cf. also [42]). At lower velocities any small difference between two branches would be rapidly magnified with the result that the slower branch is quickly arrested.

Pärletun[22] used a quasi-dynamic approach and determined the branching morphology by a static finite element computation, assuming branch growth in mode I (i.e. $K_{II} = 0$). The result showed good agreement with experimentally obtained patterns.

Another question concerns why branching is initiated. It is often experienced that a branch is arrested after a very short growth, indicating unsuccessful branching, see e.g. [43]. By careful examination of the crack tip neighbourhood Ravi-Chandar[44, 45] has shown that attempted but unsuccessful branching takes place repeatedly on a micro-scale before the successful event.

Initiation of attempted or successful branching seems to be related to the micro-separation morphology. Micro-separations appear to grow to a shape favourable for coalescence between a symmetrically situated micro-separation and the main crack at low crack tip velocities, see Fig. 1. At increasing velocity their morphology appears to change considerably, at the same time as their number increases[46, 47], see Fig. 16. This obviously favours coalescence between off-axis micro-separations and the main crack. However, it is difficult to imagine that this mechanism alone would lead to branching instead of a zigzagging path on a micro-scale. Contributing to branching might be the assumed fact that the inner parts of the process region (the cluster of micro-separations near the crack tip) are less accessible to the energy flow from the stress field than the peripheral parts[46, 47].

6. CONCLUSIONS

It appears that crack paths often are astonishingly reproducible. Straight paths in mode I is one example. It is believed that the strong polarity of the plastic field ahead of the crack tip is contributory to the directional constancy.

Modes II and III are very difficult to obtain under laboratory conditions, but are known from earthquake events. Even if a crack is oriented favourably for mode II or III growth, mode I generally takes over after kinking or tilting of the crack edge. Theoretical considerations suggest that mode I growth can be suppressed by superposition of a high pressure. This could actually be demonstrated on a PMMA plate containing an internal crack, favourably oriented for mode II growth and subjected to biaxial loading. The results indicated that the critical mode II stress intensity factor K_{IIc} is much higher than the critical mode I stress intensity factor K_{Ic}. It was estimated that K_{IIc} is about $2.5K_{Ic}$ for PMMA.

Similar results as for mode II were obtained for mode III, i.e. it was demonstrated that a high superposed pressure could be used to suppress mode I growth.

Cracks under mixed mode loading, I and II, grow after kinking and, as it seems, generally in mode I. Therefore the crack path can be predicted by assuming growth in the direction that maximizes K_I. At determination of the (initial) kink angle this direction deviates somewhat from the one for which K_{II} vanishes, but the difference is very small, generally of the order of one degree or less. Moreover, after a small amount of kink growth—a few times the linear extension of the dissipative region—the condition of vanishing K_{II} coincides with the condition of maximum K_I. Since the crack part near the tip can be regarded as straight, except near the point of kinking, this condition could be derived from the experience that a straight mode I crack, favourably oriented for mode I growth, proceeds straight forwards. Only suicidal theories would predict something else, and it is therefore not astonishing that most of the surviving criteria for crack growth are equivalent to the criterion of vanishing K_{II} at prediction of the path for a smoothly curved crack. Differences appear at predictions of the kink angle (especially at compressive loading) and of the transition between mode I and II. The present writer does not believe that criteria based on general continuum-mechanical concepts without consideration of micro-mechanical mechanisms are sound, since the crack tip neighbourhood is a highly discontinuous region.

It appears to be possible to predict directional stability or instability of originally straight mode I cracks under symmetrical biaxial loading, using the criterion of vanishing K_{II} and assuming slow crack growth. It is important that the concept of straightness is clearly defined in order to arrive at a meaningful discussion of directional stability. The fact that the crack tip moves away from the prospective straight path should imply instability only if it takes a direction that deviates more and more from the original one.

Crack branching appears to be due to a combination of two phenomena, both occurring at high crack tip velocities. One is that attempted branching becomes more frequent at higher crack tip velocities, because of an increasing number of micro-separations in combination with changes of their orientations. Another is the strong velocity dependence of the energy release rate at higher velocities, implying less pronounced tendency for quick arrest of the shorter one of the branches.

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APPENDIX

Directional stability at growth of collinear cracks

For an infinite row of collinear cracks, $y = y(x) = -y(-x)$, $|x - nd| < a$, $n = 0, \pm 1, \pm 2, \ldots$ Melin[6] obtains the equation for the path at slow crack growth from $a = a_0$:

$$y'(a) = C(a)\,y(a) - \lambda \int_0^a y'(\xi)\, B(\xi; a)\,\mathrm{d}\xi,\ a \geq a_0 \tag{A.1}$$

where $C(a) = \pi/d \operatorname{cosec}(\pi a/d)$

$$B(\xi; a) = \cos(\pi\xi/2d)[\sin^2(\pi a/2d) - \sin^2(\pi\xi/2d)]^{-1/2}$$

and $\lambda = 2(1 - \sigma_x^\infty/\sigma_y^\infty)/d$.

It is immediately obvious that directional instability occurs if $\lambda \leq 0$[6]. The case $\lambda > 0$ can be dealt with in the following way:

Assume that a disturbance $y_0(x)$ appears in the interval $0 < |x| < a_0$ and that $y_0'(x) \geq 0$.

If $y'(a) \geq 0$ for $a_0 \leq a < d$, then it follows immediately from (A.1) that $y'(a)$ is unbounded. Because, if it is not, a maximum, y'_{max}, exists and then (A.1) gives

$$y'(a) > C(a)\,y(a_0) - \lambda y_{max} \cdot \frac{2d}{\pi}[\sin(\pi a/2d)]^{-1}.$$

Since $C(a)$ is unbounded this is a contradiction.

If $y'(a)$ is not non-negative for $a_0 < a < d$, then assume that $y'(a) < 0$ in the interval $a_0 < a < a_1$. If necessary a_0 has to be redefined (increased) until such a relation holds. Assume further that $y'(a) = 0$ for $a = a_1$ and introduce for $a_0 < a \leq a_1$:

$$R_0(a) = y(a) - \int_0^{a_0} y_0'(\xi)\,d\xi - \int_{a_0}^{a} y'(\xi)\,d\xi \equiv 0.$$

Then, for $a > a_0$, (A.1) can be written in the form

$$\begin{aligned} y'(a) &= y'(a) - \lambda B(a_0;\,a)R_0(a) \\ &= [C(a) - \lambda B(a_0;\,a)]y(a) + \lambda \int_0^{a_0} y_0'(\xi)[B(a_0;\,a) - B(\xi;\,a)]\,d\xi \\ &\quad + \lambda \int_{a_0}^{a} y'(\xi)[B(a_0;\,a) - B(\xi;\,a)]\,d\xi. \end{aligned} \tag{A.2}$$

Both integrands in (A.2) are positive. Therefore, since $y'(a) < 0$, it follows that $y(a) \neq 0$ in the interval $a_0 < a \leq a_1$ and since $y(a_0) > 0$ then $y(a) > 0$ in the interval.

Assume now that $y'(a) > 0$ in the interval $a_1 < a < a_2$, that $y'(a) < 0$ in the interval $a_2 < a < a_3$ and that $y'(a) = 0$ for $a = a_3$. The previous argument can then be repeated to show that $y(a) > 0$ also in the interval $a_1 < a < a_3$. One introduces for $a_1 < a < a_3$:

$$R_2(a) = y(a) - y(a_1) - \int_{a_1}^{a} y'(\xi)\,d\xi \equiv 0$$

and writes (A.1) in the form

$$y'(a) = y'(a) - \lambda B(a_0;\,a)R_0(a_1) - \lambda B(a_2;\,a)R_2(a)$$

which, after introduction of the expression for $y'(a)$ from (A.1) in the right member gives an equation of similar form as (A.2) and it is not difficult to show that $y(a_3) > 0$.

In this way it can be shown that $y(a) > 0$ in the whole interval $a_0 < a < d$. Then it is easy to show that $y'(a) \to \infty$ as $a \to d$, which result also implies that the tacit assumption of the existence of a value $a = a_1$ for which $y'(a) = 0$ is correct if $y'(a)$ is not non-negative for $a_0 \leq a < d$. The existence of other a-values for which $y'(a)$ vanishes is not necessary for the result. (In fact, there are reasons to believe that there is one minimum, only, for $a > a_0$).

The original assumption that $y_0'(x) \geq 0$ can be considerably relaxed. Inspection shows that the only property used for $y_0(x)$ is that the integral

$$\int_0^{a_0} y_0'(\xi)[B(a_0;\,a) - B(\xi;\,a)]\,d\xi$$

is either non-negative or non-positive for $a > a_0$, and this condition is only sufficient, not necessary. It is fulfilled for very general types of disturbances.

Equation (A.1) is valid under the assumption that $y'(x) \ll 1$ and that $d - a \gg |y(a)|$. In reality, therefore, $|y(a)|$ does not continue to increase when a comes very close to d (i.e. the crack tips are very close). Experiments show that $|y(a)|$ decreases when a increases beyond d (i.e. when the crack tips bypass each other).