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Mohr's Circle of Stress

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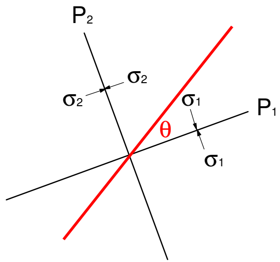
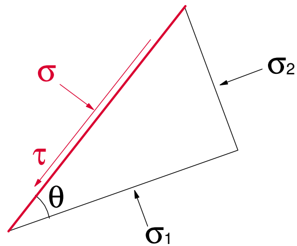
Principal planes and principal stresses

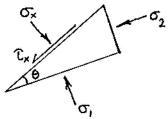
Within any material element (eg soil) there are three orthogonal planes known as *principal planes* on which there are no shear stresses - the normal stresses on those planes are the *principal stresses*. Knowledge of the principal stresses completely defines the stresses on all other planes.





Mohr's circle of stress is a diagram which shows how the normal and shear stresses within a material element (eg soil) vary with orientation.

Consider an element subjected to a 2-D state of stress :



Here σ_1 and σ_2 are major and minor principal stresses – how are they related to τ_x and σ_x ?

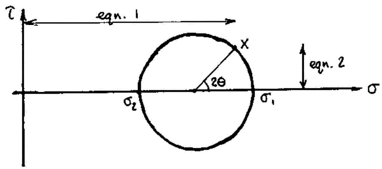
Note sign convention : compressive stresses are +ve and τ is +ve when causing anticlockwise rotation
θ is +ve when measured anticlockwise from major principal plane on which σ_1 acts.

Resolving forces gives :

$$\sigma_x = (\sigma_1 + \sigma_2)/2 + \frac{(\sigma_1 - \sigma_2)}{2}\cos 2\theta \qquad (1)$$

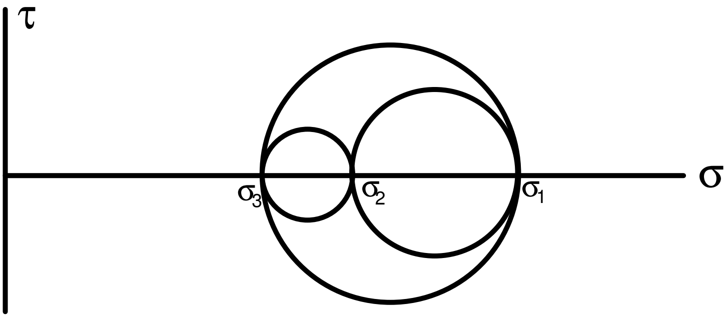
$$\tau_x = \frac{(\sigma_1 - \sigma_2)}{2}\sin 2\theta \qquad (2)$$

Show variation of τ, σ with θ on a diagram



Mohr's Circle of Stress

In general we need to consider 3-D states of stress. The diagram below shows three Mohr's circles for stresses acting on three sets of planes, each set containing one of the principal stress directions. In practice we often limit consideration to planes containing the σ_2 direction and plot only the biggest Mohr circle through σ_1 and σ_3.



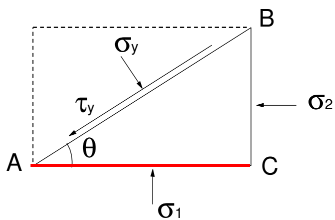
By convention $\sigma_1 > \sigma_2 > \sigma_3$

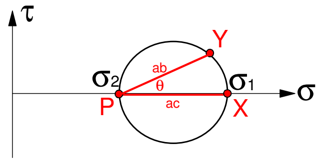
Pole Construction

This is a simple graphical technique which can be used to find the state of stress on one plane of an element once you have constructed the Mohr's circle.

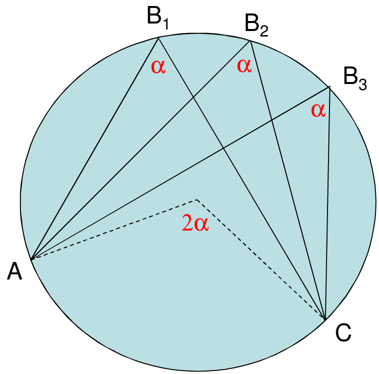
Steps involved :

- Draw the element and Mohr's circle on the same sheet (see below)
- Identify a plane (say AC) on which you know the stresses and mark on Mohr's circle (here point X at 1)
- Draw a line (ac) through X parallel to plane (AC) - where it cuts the circle that is pole P
- Draw a line (ab) through P parallel to plane (AB) on which you wish to know state of stress - where it cuts the circle that is stress state required (Y).

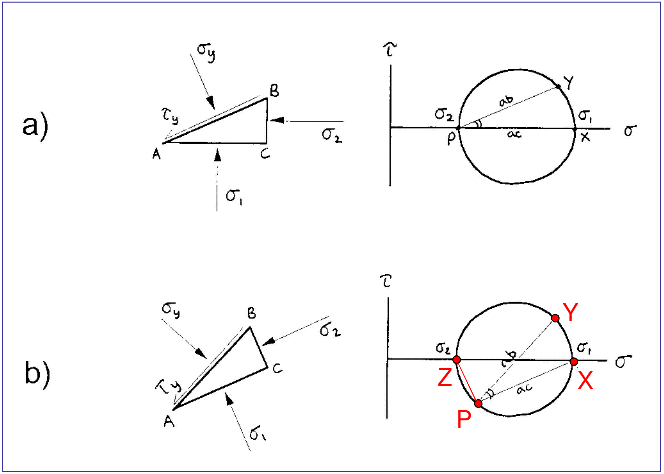




Geometry of Circle



Although location of pole P varies with relative orientation the point Y does not. The technique can obviously be used to find the orientation of planes on which the state of stress is known.

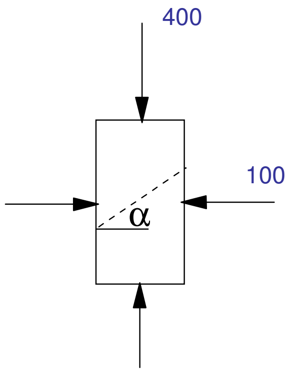


Note:

Although in the examples given here the orientation of the elements is vertical/horizontal the pole method works very well for any orientation of the soil element.

In principle the calculation method may also be used for any orientation but in practice most students find such problems very hard.

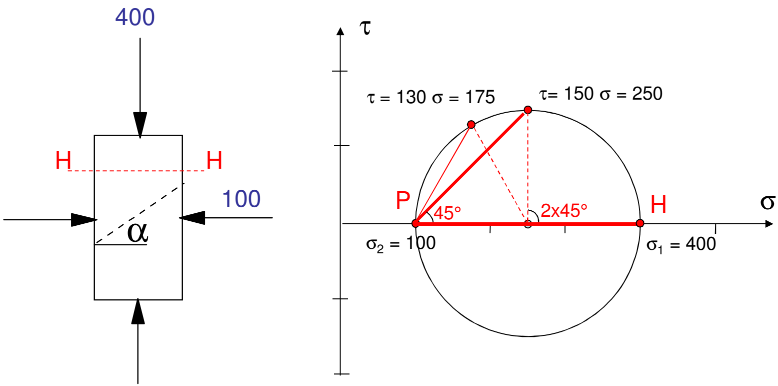
Mohr's Circle Example 1



What are stresses on planes inclined at α = 45° and 60° ?

Mohr's Circle Example 1

What are stresses on planes inclined at α = 45° and 60° ?



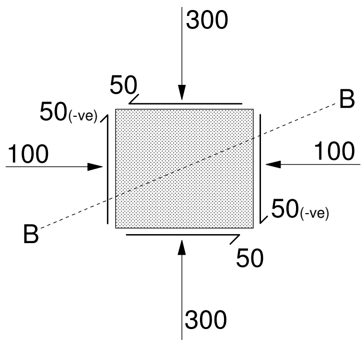
similarly for the 60° plane

Mohr's Circle Example 2

For the soil element shown, find:

i) the magnitude of the principal stresses and the orientation of the major principal plane, and

ii) the stresses acting on the plane BB inclined at 20°.



Mohr's Circle Example 2

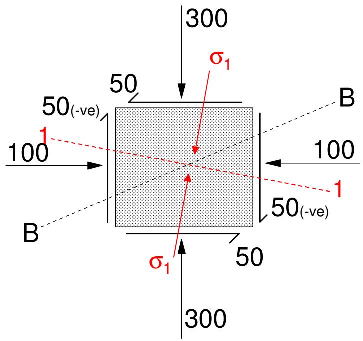
What are magnitudes of σ_1 and σ_2, the orientation of plane 1-1, and the stresses on plane BB inclined at α = 20° to the horizontal?

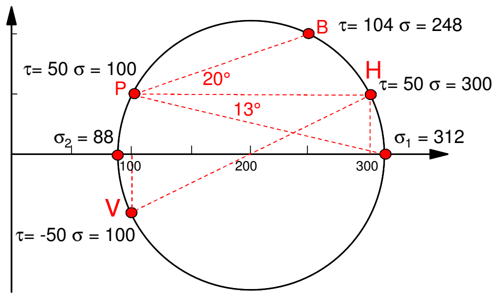
First draw the Mohr's circle through points H and V and find σ_1 and σ_2

Next find the pole P by drawing from H or V.

Next find the orientation of 1-1 plane by drawing from pole P.

Next find the stresses on plane B-B.





Plane BB is inclined at 20° to the horizontal

Mean stress = (300+100)/2 = 200 kPa

Radius = $\sqrt{((300-200)^2 + 50^2)}$ = 111.8 kPa

Mohr's Circle Example 3

Worked solution to be given out after students attempt the problem

Mohr's circle analysis step by step

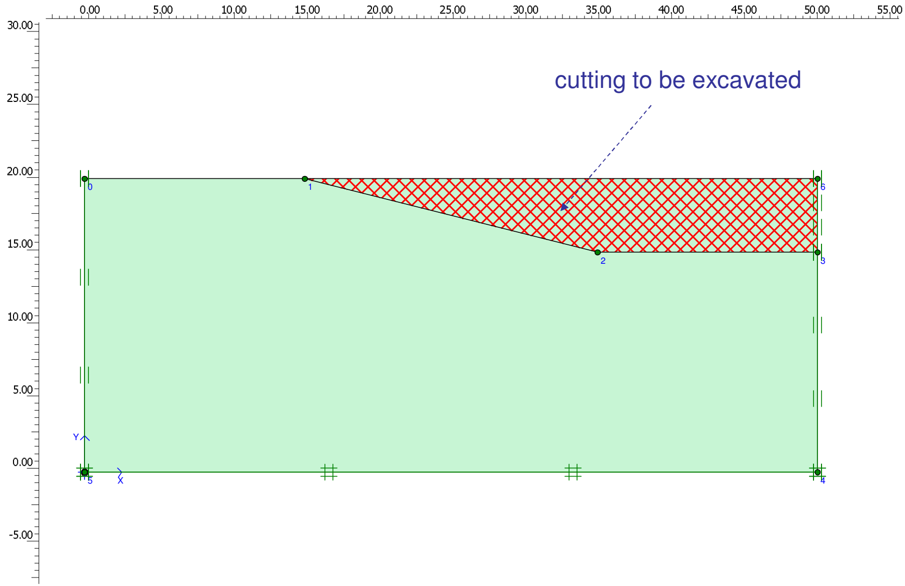
a) using pole construction

b) by calculation

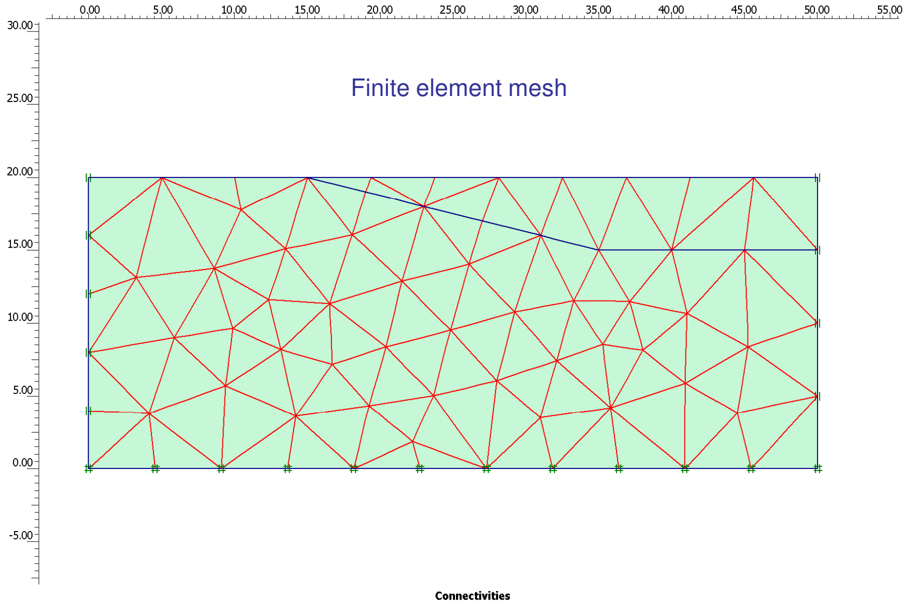
A plane strain finite element analysis (2D-FEA) is made in order to predict the stress conditions at various critical locations of a proposed excavation. The results show the stresses at the end of construction.

You are required to draw the Mohr's circle for an element at a critical location and to determine the stresses and orientations on critical planes.

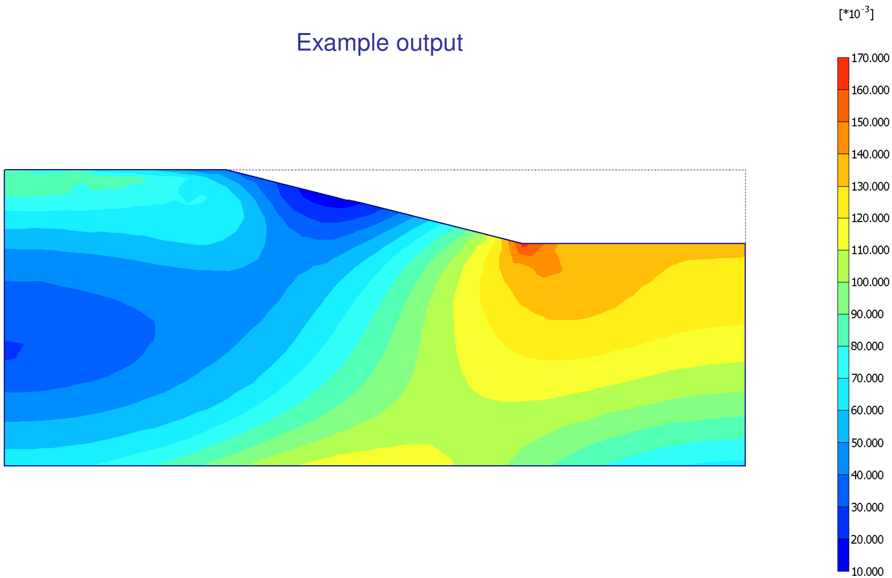
Finite element analysis of a road cutting



Finite element mesh

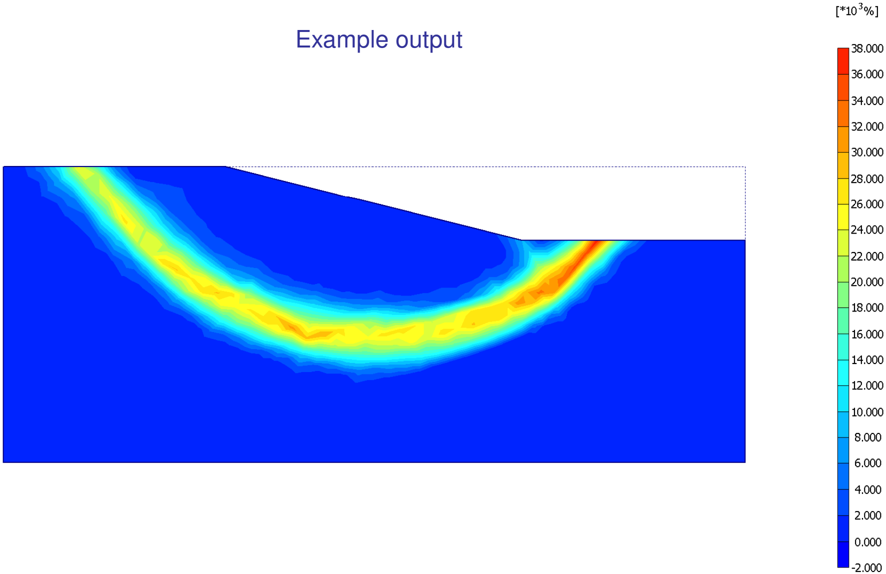


Example output



Relative shear stresses
Extreme relative shear stress 0.16

Example output

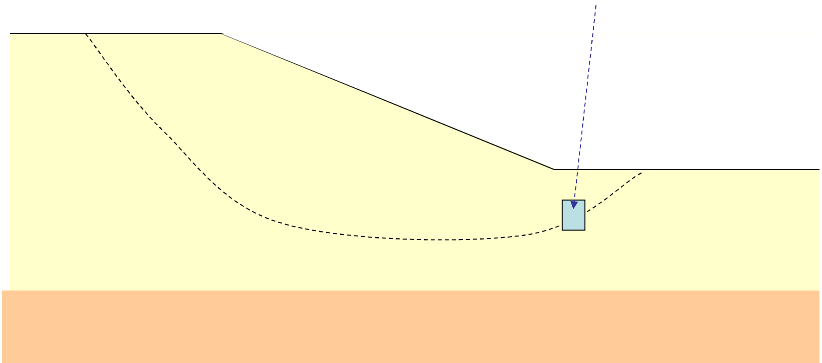


Shear strains

Extreme shear strain 37.77*10^3 %

Incipient failure ???

Examine stresses here
Results of 2D-FEA available (next slide)



A plane strain finite element analysis is made in order to predict the stress conditions at various critical locations of a proposed excavation. The results show that the total normal and shear stresses (in kPa) at one such critical location, at the end of construction, will be as shown below. The pore pressure is 60 kPa.

Carefully draw the Mohr's circle of *effective* stress to scale on graph paper for the soil at this location, and determine (graphically or otherwise) :

(i) the magnitudes of the effective principal stresses and orientations (relative to the horizontal) of the principal planes;

(ii) the maximum stress ratio τ/σ' in this element of soil, and the orientations of the planes on which it is a maximum.