Relatório de Resoluções

O código de cada membro pode ser consultado a seguir:

x_{05}: José Soares Jr.	x_{11}: Luca Monaco
x_{06}: Maurício Damião	x_{15}: Rodrigo Melendez
x_{08}: Pedro Lopes Silva	x_{20}: Gustavo Zequini
x_{09}: Rafael Maddalena	

Resolução (‖ Questão: 7.12.1 ‖ Relator: $\mathbf{x_{05}}$ ‖ Revisor: $\mathbf{x_{06}}$ ‖)

Use l'Hôpital's rule to find.
L'Hôpital's rule:
Se $f(a) = g(a) = 0$ e $g'(a) \neq 0$, então

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)} \tag{1}$$

a) $\lim_{x\to 3} \dfrac{3x^2 - 27}{x - 3} = \text{"}0/0\text{"} = \lim_{x\to 3} \dfrac{6x}{1} = 18$

b) $\lim_{x\to 0} \dfrac{e^x - 1 - x - \frac{1}{2}x^2}{3x^3} = \text{"}0/0\text{"} = \lim_{x\to 0} \dfrac{e^x - 1 - x}{9x^2} = \text{"}0/0\text{"} = \lim_{x\to 0} \dfrac{e^x - 1}{18x} = \text{"}0/0\text{"} = \lim_{x\to 0} \dfrac{e^x}{18} = \dfrac{1}{18}$

c) $\lim_{x\to 0} \dfrac{e^{-3x} - e^{-2x} + x}{x^2} = \text{"}0/0\text{"} = \lim_{x\to 0} \dfrac{-3e^{-3x} + 2e^{-2x} + 1}{2x} = \text{"}0/0\text{"} = \lim_{x\to 0} \dfrac{9e^{-3x} - 4e^{-2x}}{2} = \dfrac{5}{2}$

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Resolução (‖ Questão: 7.12.2 ‖ Relator: $\mathbf{x_{06}}$ ‖ Revisor: $\mathbf{x_{08}}$ ‖)

Find the limits:

a) $\lim_{x\to a} \dfrac{x^2 - a^2}{x - a}$

$\lim_{x\to a} \dfrac{x^2 - a^2}{x - a} = \lim_{x\to a} \dfrac{(x + a)(x + a)}{(x - a)} = \lim_{x\to a}(x + a) = 2a$

b) $\lim_{x\to 0} \dfrac{2\sqrt{1 + x} - 2 - x}{2\sqrt{1 + x + x^2} - 2 - x} = 0/0$

Usando L'Hôpital:

Se $f(a) = g(a) = 0$ e $g'(a) \neq 0$, então

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)} \tag{2}$$

No nosso caso, temos que $a = 0$, $f(x) = 2\sqrt{1+x} - 2 - x$ e $g(x) = 2\sqrt{1+x+x^2} - 2 - x$

$f(x) = 2\sqrt{1+x} - 2 - x \Rightarrow f'(x) = \dfrac{1}{\sqrt{1+x}} - 1$

$g(x) = 2\sqrt{1+x+x^2} - 2 - x \Rightarrow g'(x) = \dfrac{1}{\sqrt{1+x+x^2} \cdot (2x+1)} - 1$

$$\lim_{x\to 0} \frac{2\sqrt{1+x} - 2 - x}{2\sqrt{1+x+x^2} - 2 - x} = \lim_{x\to 0} \frac{\dfrac{1}{\sqrt{1+x}} - 1}{\dfrac{1}{\sqrt{1+x+x^2} \cdot (2x+1)} - 1} = 0/0$$

Usando L'Hôpital novamente, temos que:

$f'(x) = \dfrac{1}{\sqrt{1+x}} - 1 \Rightarrow f''(x) = \left(\dfrac{-1}{2}\right) \cdot \dfrac{1}{(1+x)^{3/2}}$

$$g'(x) = \frac{1}{\sqrt{1+x+x^2} \cdot (2x+1)} - 1 \Rightarrow g''(x) = \frac{2(\sqrt{1+x+x^2}) - (2x+1)\left(\dfrac{1}{2}\right)(1+x+x^2)^{-1/2}(2x+1)}{(1+x+x^2)}$$

Portanto,

$$\lim_{x\to 0} \frac{2\sqrt{1+x} - 2 - x}{2\sqrt{1+x+x^2} - 2 - x} = \lim_{x\to 0} \frac{\dfrac{1}{\sqrt{1+x}} - 1}{\dfrac{1}{\sqrt{1+x+x^2} \cdot (2x+1)} - 1} =$$

$$\lim_{x\to 0} \frac{\left(\dfrac{-1}{2}\right) \cdot \dfrac{1}{(1+x)^{3/2}}}{\dfrac{2(1+x+x^2)^{1/2} - (2x+1)^2\left(\dfrac{1}{2}\right)(1+x+x^2)^{-1/2}}{(1+x+x^2)}} = \frac{(-1/2)}{(2-1/2)} = (-1/2)\cdot(2/3) = \frac{-1}{3}$$

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Resolução (‖ Questão: 7.12.3 ‖ Relator: $\mathbf{x}_{08}$ ‖ Revisor: $\mathbf{x}_{09}$ ‖)

Use l'Hôpital's rule to find the following limits:

(a) $\lim_{x\to 1} \dfrac{x-1}{x^2-1} = \lim_{x\to 1} \dfrac{1}{2x} = \dfrac{1}{2}$

(b) $\lim_{x\to -2} \dfrac{x^3+3x^2-4}{x^3+5x^2+8x+4} = \lim_{x\to -2} \dfrac{3x^2+6x}{3x^2+10x+8} = \lim_{x\to -2} \dfrac{6x+6}{6x+10} = \dfrac{6(-2)+6}{6(-2)+10} = \dfrac{-6}{-2} = 3$

(c) $\lim_{x\to 2} \dfrac{x^4-4x^3+6x^2-8x+8}{x^3-3x^2+4} = \lim_{x\to 2} \dfrac{4x^3-12x^2-12x-8}{3x^2-6x} = \lim_{x\to 2} \dfrac{12x^2-24x-12}{6x-6} =$
$\dfrac{12(2)^2-24(2)-12}{6(2)-6} = \dfrac{48-36}{6} = \dfrac{12}{6} = 2$

(d) $\lim_{x\to 1} \dfrac{\ln x - x + 1}{(x-1)^2} = \lim_{x\to 1} \dfrac{\frac{1}{x} - 1}{2(x-1)(1)} = \lim_{x\to 1} \dfrac{-\frac{1}{x^2}}{2} = -\dfrac{1}{2}$

(e) $\lim_{x\to 1} \dfrac{1}{x-1} \ln\left(\dfrac{7x+1}{4x+4}\right) = \lim_{x\to 1} (\dfrac{4x+4}{7x+1}) \cdot \dfrac{7(4x+4)-4(7x+1)}{(4x+4)^2} \cdot \dfrac{1}{1} =$
$(\dfrac{4(1)+4}{7(1)+1}) \cdot \dfrac{7(4(1)+4)-4(7(1)+1)}{(4(1)+4)^2} = \dfrac{8}{8} \cdot \dfrac{56-32}{64} = \dfrac{24}{64} = \dfrac{3}{8}$

(f) $\lim_{x\to 1} \dfrac{x^x - x}{1 - x + \ln x} = \lim_{x\to 1} \dfrac{e^{x\ln x} - e^{\ln x}}{1 - x + \ln x} = \lim_{x\to 1} \dfrac{(\ln x + 1)e^{x\ln x} - \frac{e^{\ln x}}{x}}{-1 + \dfrac{1}{x}} =$

$\lim_{x\to 1} \dfrac{\frac{e^{x\ln x}}{x} + (\ln x + 1)^2 e^{x\ln x}}{\frac{-1}{x^2}} = \dfrac{1+1}{-1} = -2$

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Resolução (‖ Questão: 7.12.4 ‖ Relator: $\mathbf{x}_{09}$ ‖ Revisor: $\mathbf{x}_{11}$ ‖)

Encontre os seguintes limites:

a) $\lim_{x\to\infty} \dfrac{lnx}{\sqrt{x}}$

$$\lim_{x\to\infty} \frac{lnx}{\sqrt{x}} = \lim_{x\to\infty} \frac{1/x}{(1/2)x^{-1/2}} = \lim_{x\to\infty} \frac{2x^{1/2}}{x} = \lim_{x\to\infty} \frac{2}{x^{1/2}} = 0$$

b) $\lim_{x\to 0^+} xlnx$

$$\lim_{x\to 0^+} xlnx = \lim_{x\to 0^+} \frac{lnx}{1/x} = \lim_{x\to 0^+} \frac{1/x}{-x^{-2}} = \lim_{x\to 0^+} \frac{-x^2}{x} = 0$$

c) $\lim_{x\to 0^+}(xe^{1/x} - x)$

Considerando $1/x = u$

$$\lim_{x\to 0^+}(xe^{1/x} - x) = \lim_{u\to\infty}(e^u/u - 1/u) = \lim_{u\to\infty} \frac{e^u - 1}{u} = \lim_{u\to\infty} \frac{e^u}{1} = \infty$$

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Resolução (‖ Questão: 7.12.5 ‖ Relator: $\mathbf{x}_{11}$ ‖ Revisor: $\mathbf{x}_{15}$ ‖)

5. Find the error in the following line of reasoning:

$$\lim_{x\to 1} \frac{x^2 + 3x - 4}{2x^2 - 2x} = \lim_{x\to 1} \frac{2x+3}{4x-2} = \lim_{x\to 1} \frac{2}{4} = \frac{1}{2} \tag{3}$$

What is the correct value of the first limit?

O erro está no segundo limite, pois ele não é da forma "0/0" como o primeiro, mas mesmo assim foi aplicada a regra de L'Hôpital.

A forma correta seria:

$$\lim_{x\to 1} \frac{x^2 + 3x - 4}{2x^2 - 2x} = \lim_{x\to 1} \frac{2x+3}{4x-2} = \frac{5}{2} \tag{4}$$

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Resolução (‖ Questão: 7.12.6 ‖ Relator: $\mathbf{x}_{15}$ ‖ Revisor: $\mathbf{x}_{20}$ ‖)

With $B > 0$ and $y > 0$, find $\lim v \to 0^+ \dfrac{1 - (1 + v^B)^{-y}}{v}$

$\lim_{v\to 0^+} \dfrac{1 - (1 + v^B)^{-y}}{v} = \dfrac{0}{0}$

Then, aplying L'Hospital's rule we have:

$\lim_{v\to 0^+} \dfrac{1-(1+v^B)^{-y}}{v} = \lim_{v\to 0^+} y(1+v^B)^{-(y+1)}Bv^{B-1}$

For $B = 1$

$$\lim_{v\to 0^+} y(1+v^1)^{-(y+1)}1\cdot v^{1-1} = \lim_{v\to 0^+} y(1+v)^{-(y+1)} = y \tag{5}$$

For $B > 1$

$$\lim_{v\to 0^+} y(1+v^B)^{-(y+1)}Bv^{B-1} = 0 \tag{6}$$

And for $B < 1$

$$\lim_{v\to 0^+} y(1+v^B)^{-(y+1)}Bv^{B-1} = \infty \tag{7}$$

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Resolução (‖ Questão: 7.12.7 ‖ Relator: $\mathbf{x}_{20}$ ‖ Revisor: $\mathbf{x}_{05}$ ‖) In the context of Examples 7.1.5 and 7.1.8, the family of *CES* utility functions is given by:

$$u(c) = \begin{cases} \dfrac{c^{1-\rho}-1}{1-\rho}, & \text{if } \rho \neq 1 \\ \ln c, & \text{if } \rho = 1 \end{cases}$$

for all $c > 0$. Use l'Hôpital's rule to show that: $\lim_{\rho\to 1} \dfrac{c^{1-\rho}-1}{1-\rho} = \ln c$. In this sense, the family is "continuous in ρ".

Aplicando l'Hôpital em $\lim_{\rho\to 1} \dfrac{c^{1-\rho}-1}{1-\rho}$, uma vez que $\lim_{\rho\to 1} \dfrac{c^{1-\rho}-1}{1-\rho} = \dfrac{0}{0}$:

$$\lim_{\rho\to 1} \frac{c^{1-\rho}-1}{1-\rho} = \lim_{\rho\to 1} \frac{-c^{1-\rho}\ln(c)}{-1} = \frac{-c^{1-1}\cdot\ln(c)}{-1} = \frac{-1.\ln(c)}{-1} = \ln(c) \tag{8}$$

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Resolução (‖ Questão: 7.12.9 ‖ Relator: $\mathbf{x}_{06}$ ‖ Revisor: $\mathbf{x}_{09}$ ‖)

Suppose that $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \pm$"$\infty/\infty$" $= L \neq 0$ where f and g are differentiable functions whose derivatives $f'(x)$ and $g'(x)$ converge to non-zero limits as x tends to a. By applying l'Hôpital's rule to the equivalent limit, $\lim_{x\to a} \dfrac{\frac{1}{g(x)}}{\frac{1}{f(x)}} =$ "0/0", show that one has $L = \lim_{x\to a} \dfrac{f'(x)}{g'(x)}$ provided this limit exists.

$L = \lim_{x\to a} \dfrac{\frac{1}{g(x)}}{\frac{1}{f(x)}} = \lim_{x\to a} \dfrac{(g(x))^{-1}}{(f(x))^{-1}} = 0/0$

Usando a regra de l'Hôpital, dada por: Se $f(a) = g(a) = 0$ e $g'(a) \neq 0$, então

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)} \tag{9}$$

$L = \lim_{x\to a} \dfrac{(g(x))^{-1}}{(f(x))^{-1}} = \lim_{x\to a} \dfrac{(-1)(g(x))^{-2}(g'(x))}{(-1)(f(x))^{-2}(f'(x))} = \lim_{x\to a} \dfrac{g'(x)}{f'(x)} \cdot \left(\lim_{x\to a} \dfrac{f(x)}{g(x)}\right)^2$

Substituindo $\lim_{x\to a} \dfrac{f(x)}{g(x)}$ por L, temos:

$L = \lim_{x\to a} \dfrac{g'(x)}{f'(x)} \cdot (L)^2 \Rightarrow \lim_{x\to a} \dfrac{g'(x)}{f'(x)} = \dfrac{1}{L}$

$\dfrac{\lim_{x\to a} g'(x)}{\lim_{x\to a} f'(x)} = \dfrac{1}{L} \Rightarrow \dfrac{\lim_{x\to a} f'(x)}{\lim_{x\to a} g'(x)} = L$

Válido já que $\lim_{x\to a} g'(x)$ e $\lim_{x\to a} f'(x) \neq 0$, como diz o enunciado.

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