Relatório de Resoluções

O código de cada membro pode ser consultado a seguir:

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x_{06}: Maurício Damião	x_{15}: Rodrigo Melendez
x_{08}: Pedro Lopes Silva	x_{20}: Gustavo Zequini
x_{09}: Rafael Maddalena	

Resolução (‖ Questão: 7.11.1 ‖ Relator: $\mathbf{x}_{11}$ ‖ Revisor: $\mathbf{x}_{09}$ ‖)

1. Let $\alpha_n = \dfrac{3-n}{2n-1}$ and $\beta_n = \dfrac{n^2+2n-1}{3n^2-2}$, for $n = 1, 2, ...$ .Find the following limits:

a)$\lim_{n\to\infty} \alpha_n$

$$\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \frac{3-n}{2n-1} \tag{1}$$

$$\lim_{n\to\infty} \frac{n(\frac{3}{n}-1)}{n(-\frac{1}{n}+2)} \tag{2}$$

$$\lim_{n\to\infty} \frac{\frac{3}{n}-1}{-\frac{1}{n}+2} = -\frac{1}{2} \tag{3}$$

b)$\lim_{n\to\infty} \beta_n$

$$\lim_{n\to\infty} \beta_n = \lim_{n\to\infty} \frac{n^2+2n-1}{3n^2-2} \tag{4}$$

$$\lim_{n\to\infty} \frac{n^2(1+\frac{2}{n}-\frac{1}{n^2})}{n^2(3-\frac{2}{n^2})} \tag{5}$$

$$\lim_{n\to\infty} \frac{1+\frac{2}{n}-\frac{1}{n^2}}{3-\frac{2}{n^2}} = \frac{1}{3} \tag{6}$$

c)$\lim_{n\to\infty}(3\alpha_n + 4\beta_n)$

$$\lim_{n\to\infty}(3\alpha_n + 4\beta_n) \tag{7}$$

$$\lim_{n\to\infty}(3 \cdot -\frac{1}{2} + 4 \cdot \frac{1}{3}) = -\frac{1}{6} \tag{8}$$

d)$\lim_{n\to\infty}(\alpha \cdot \beta_n)$

$$\lim_{n\to\infty}(\alpha \cdot \beta_n) \tag{9}$$

$$\lim_{n\to\infty}(-\frac{1}{2} \cdot \frac{1}{3}) = -\frac{1}{6} \tag{10}$$

e)$\lim_{n\to\infty}(\frac{\alpha_n}{\beta_n})$

$$\lim_{n\to\infty}(\frac{\alpha_n}{\beta_n}) \tag{11}$$

$$\lim_{n\to\infty}\frac{-\frac{1}{2}}{\frac{1}{3}} = -\frac{3}{2} \tag{12}$$

f)$\lim_{n\to\infty}\sqrt{\beta_n - \alpha_n}$

$$\lim_{n\to\infty}\sqrt{\beta_n - \alpha_n} \tag{13}$$

$$\lim_{n\to\infty}\sqrt{\frac{1}{3} - (-\frac{1}{2})} = \sqrt{\frac{5}{6}} \tag{14}$$

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Resolução (‖ Questão: 7.11.2 ‖ Relator: $\mathbf{x}_{15}$ ‖ Revisor: $\mathbf{x}_{11}$ ‖)

Examine the convergence of the sequences whose general terms are as follows:

a) $s_n = 5 - \dfrac{2}{n}$

As $\lim_{n\to\infty}\dfrac{2}{n} = 0$, this implies that $\lim_{n\to\infty} 5 - \dfrac{2}{n} = 5$. Therefore the series converges to 5

b) $s_n = \dfrac{n^2-1}{n}$

Let $\dfrac{n^2-1}{n} = n - \dfrac{1}{n}$.

$\lim_{n\to\infty} n = \infty$ and $\lim_{n\to\infty}\dfrac{1}{n} = 0$ this implies that $\lim_{n\to\infty}\dfrac{n^2-1}{n} = \infty$. Therefore the series diverges.

c) $s_n = \dfrac{3n}{\sqrt{2n^2-1}}$

Let $\dfrac{3n}{\sqrt{2n^2-1}} = \dfrac{3n}{n\sqrt{2-\frac{1}{n^2}}} = \dfrac{3}{\sqrt{2-\frac{1}{n^2}}}$.

As $\lim_{n\to\infty}\dfrac{1}{n^2} = 0$, then $\lim_{n\to\infty}\dfrac{3}{\sqrt{2-\frac{1}{n^2}}} = \dfrac{3}{\sqrt{2}}$. Therefore the series converges to $\dfrac{\sqrt{2}\cdot 3}{2}$.

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Resolução (‖ Questão: 7.11.3 ‖ Relator: $\mathbf{x}_{20}$ ‖ Revisor: $\mathbf{x}_{15}$ ‖) Prove that $e^x = lim_{n\to\infty}(1+\dfrac{x}{n})^n$ for $x > 0$.

$$\lim_{n\to\infty}(1+\frac{x}{n})^n \tag{15}$$

Sendo $\frac{1}{u} = \frac{x}{n} \iff u = \frac{n}{x}$.
Então, quando $n \to \infty$, $u \to \infty$. (Quando n tende ao infinito, u tende ao infinito).

$$\lim_{u \to \infty} (1 + \frac{1}{u})^{u.x} \tag{16}$$

$$= (\lim_{u \to \infty} (1 + \frac{1}{u})^{u})^{x} \tag{17}$$

Como $\lim_{u \to \infty} (1 + \frac{1}{u})^{u} = e$, temos:

$$\lim_{n \to \infty} (1 + \frac{x}{n})^{n} = e^{x} \tag{18}$$

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