Relatório de Resoluções

O código de cada membro pode ser consultado a seguir:

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Resolução (‖ Questão: 7.7.1 ‖ Relator: $\mathbf{x}_{11}$ ‖ Revisor: $\mathbf{x}_{?}$ ‖)

1. Find the elasticities of the functions given by the following formulas:

a)$3x^{-3}$
Para este e os outros itens adotaremos a seguinte fórmula para calcular a elasticidade:

$$El_x f(x) = \frac{x}{f(x)} \cdot f'(x) \tag{1}$$

Continuando..

$$f(x) = 3x^{-3} \tag{2}$$
$$f'(x) = -9x^{-4} \tag{3}$$
$$El_x f(x) = \frac{x}{3x^{-3}} \cdot -9x^{-4} \tag{4}$$
$$El_x f(x) = \frac{x^4}{3} \cdot -9x^{-4} = -3 \tag{5}$$

b)$-100x^{100}$

$$f(x) = -100x^{100} \tag{6}$$
$$f'(x) = -10000x^{99} \tag{7}$$
$$El_x f(x) = \frac{x}{-100x^{100}} \cdot -10000x^{99} = \frac{-10000x^{100}}{-100x^{100}} = 100 \tag{8}$$

c)$\sqrt{x} = x^{\frac{1}{2}}$

$$f(x) = \sqrt{x} \tag{9}$$
$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} \tag{10}$$
$$El_x f(x) = \frac{x}{x^{\frac{1}{2}}} \cdot \frac{1}{2}x^{-\frac{1}{2}} \tag{11}$$
$$El_x f(x) = x^{\frac{1}{2}} \cdot \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2} \tag{12}$$

d) $\frac{A}{x\sqrt{x}} = \frac{A}{x^{\frac{3}{2}}}$, sendo A constante

$$f(x) = \frac{A}{x\sqrt{x}} \quad (13)$$

$$f'(x) = -\frac{3}{2}Ax^{-\frac{5}{2}} \quad (14)$$

$$El_x f(x) = \frac{x}{\frac{A}{x^{\frac{3}{2}}}} \cdot -\frac{3}{2}Ax^{-\frac{5}{2}} \quad (15)$$

$$El_x f(x) = \frac{x^{\frac{5}{2}}}{A} \cdot -\frac{3}{2}Ax^{-\frac{5}{2}} = -\frac{3}{2} \quad (16)$$

■

Resolução (|| Questão: 7.7.2 || Relator: $\mathbf{x}_{15}$ || Revisor: $\mathbf{x}_{20}$ ||)

A study of transport economics uses the relation $T = 0.4K^{1.06}$, where K is expenditure on building roads, and T is a measure of traffic volume. Find the elasticity of T w.r.t. K. In this model, if expenditure increases by 1%, by what percentage (approximately) does traffic volume increase?

As the expenditure-elasticity of the traffic volume is equal to:

$$El_K T = \frac{K}{0.4K^{1.06}} \cdot \frac{dT}{dK} \quad (17)$$

As $\frac{dT}{dK} = 0.4 \cdot 1.06K^{0.06}$, we substitute in the elasticity formula.

$$El_K T = \frac{K}{0.4K^{1.06}} \cdot \frac{dT}{dK} = \frac{K}{0.4K^{1.06}} \cdot 0.4 \cdot 1.06K^{0.06} = 1.06 \quad (18)$$

Hence, the expenditure-elasticity of the traffic volume is equal to 1.06, for any K. As the value of the elasticity is approximately the percentage of change in the traffic volume due a 1% change in the expenditure building, if expenditure increases by 1%, traffic volume increases by 1.06%.

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Resolução (|| Questão: 7.7.3 || Relator: $\mathbf{x}_{18}$ || Revisor: $\mathbf{x}_{05}$ ||)

A study of Norway's State Railways revealed that, for rides up to 60 km, the price elasticity of the volume of traffic was approximately -0.4.

a) According to this study, what is the consequence of a 10% increase in fares?

De acordo com o estudo, um aumento de 10% no preço da viagem teria como efeito uma queda de 4% na quantidade de viagens

b) The corresponding elasticity for journeys over 300 km was calculated to be approximately -0.9. Can you think of a reason why this elasticity is larger in absolute value than the previous one?

Distâncias acima de 300 km são consideravelmente mais longas que viagens até 60 km, tal que os indivíduos podem realizar essas viagens de avião. Pode-se acrescentar o fato de dificilmente uma viagem desse tipo ser rotineira, de forma que um aumento do custo faria pessoas suspenderem ou postergarem a viagem.

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Resolução (‖ Questão: 7.7.4 ‖ Relator: $\mathbf{x}_{20}$ ‖ Revisor: $\mathbf{x}_{06}$ ‖)

Use definition (7.7.2) to find $El_x y$ for the following functions, where a and p are constants:

(a) $y = e^{ax}$

(b) $y = \ln x$

(c) $y = x^p.e^{ax}$

(d) $y = x^p \ln x$

Definição (7.7.2):

Considerando f diferenciável em x e $f(x) \neq 0$, a elasticidade de f em relação a x é dada por:

$$El_x f(x) = \frac{x}{f(x)}.f'(x) \tag{19}$$

(a) $El_x y = \dfrac{x}{e^{ax}}.a.e^{ax} = ax$

(b) $El_x y = \dfrac{x}{\ln x}.\dfrac{1}{x} = \dfrac{1}{\ln x}$

(c) $El_x y = \dfrac{x}{x^p e^{ax}}(px^{p-1}e^{ax} + x^p a e^{ax}) = p + ax$

(d) $El_x y = \dfrac{x}{x^p \ln x}(px^{p-1}\ln x + x^p.\frac{1}{x}) = p + \dfrac{1}{\ln x}$

■

Resolução (‖ Questão: 7.7.5 ‖ Relator: $\mathbf{x}_{05}$ ‖ Revisor: $\mathbf{x}_{09}$ ‖)

Prove that $\text{El}_x(f(x)^p) = p\text{El}_x f(x)$, where p is a constant.

Como visto no capítulo, elasticidade:
Se f é diferenciavel em x e $f(x) \neq 0$, a elasticidade de f em relação a x é:

$$\text{El}_x f(x) = \frac{x}{f(x)} f'(x)$$

Assim, resolvendo a equação original:
$\text{El}_x((f(x))^p) = \dfrac{x}{(f(x))^p} p(f(x))^{p-1} f'(x) = p\dfrac{x}{f(x)} f'(x) = p\text{El}_x f(x)$ ■

Resolução (‖ Questão: 7.7.6 ‖ Relator: $\mathbf{x}_{06}$ ‖ Revisor: $\mathbf{x}_{11}$ ‖)

The demand D for apples in the USA during the period 1927 to 1941, as a function of income r, was estimated as $D = Ar^{1.23}$, where A is a constant. Find and interpret the income elasticity of demand, or *Engel elasticity*, defined as the elasticity of D *w.r.t* r.

Calculando a elasticidade da função D, temos que:
$\text{El}_r D = \dfrac{r}{Ar^{1.23}} \cdot (1.23) \cdot r^{0.23} = 1.23$

Vale dizer que assumimos $r > 0$

Isso signfica que, para cada aumento de 1% na renda, a demanda por maçãs aumenta em 1.23%.

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Resolução (‖ Questão: 7.7.7 ‖ Relator: $\mathbf{x}_{08}$ ‖ Revisor: $\mathbf{x}_{15}$ ‖)

A study of the transportation systems in 37 American cities estimated the average travel time to work, m (in minutes), as a function of the number of inhabitants, N. They found that $m = e^{-0.02}N^{0.19}$. Write the relation in log-linear form. What is the value of m when $N = 480000$?

Escrevendo a relação na forma log-linear temos:

$\ln m = \ln e^{-0.02}N^{0.19}$

$\ln m = \ln e^{-0.02} + \ln N^{0.19}$

$\ln m = -0,02 \ln e + 0,19 \ln N$

$\ln m = -0,02 + 0,19 \ln N$

Quando $N = 480000$ temos:

$\ln m = -0,02 + 0,19 \ln 480000$

$\ln m = -0,02 + 0,19 \cdot 13,0815$

$\ln m = 2,4654$

$e^{\ln m} = e^{2,4654}$

$m = 11,7681$

■

Resolução (‖ Questão: 7.7.8 ‖ Relator: $\mathbf{x}_{09}$ ‖ Revisor: $\mathbf{x}_{18}$ ‖)

Mostre que, ao calcular elasticidades:

a) Constantes multiplicativas desaparecem: $\text{El}_x(Af(x)) = \text{El}_x f(x)$.

Considerando $g(x) = Af(x)$ e $g'(x) = Af'(x)$ e aplicando $g(x)$ na fórmula de elasticidade, temos:

$$\text{El}_x g(x) = \frac{x}{Af(x)} \cdot Af'(x) = \frac{xf'(x)}{f(x)} = \text{El}_x f(x)$$

b) Constantes aditivas *não* desaparecem: $\text{El}_x(A + f(x)) = \dfrac{f(x)\text{El}_x f(x)}{A + f(x)}$.

Considerando $g(x) = A + f(x)$ e $g'(x) = f'(x)$ e aplicando $g(x)$ na fórmula de elasticidade, temos:

$$\text{El}_x g(x) = \frac{x}{A + f(x)} \cdot f'(x) = \frac{x}{A + f(x)} \cdot \frac{f(x) \cdot f'(x)}{f(x)} = \frac{f(x)}{A + f(x)} \cdot \frac{xf'(x)}{f(x)} = \frac{f(x)\text{El}_x f(x)}{A + f(x)}$$

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Resolução (‖ Questão: 7.7.9 ‖ Relator: $\mathbf{x}_{11}$ ‖ Revisor: $\mathbf{x}_{20}$ ‖)

9. Prove that if f and g are positive-valued differentiable functions of x and A is a constant, then the following rules hold. Here we write, for instance, $El_x f$ instead of $El_x f(x)$.

a) $El_x A = 0$

$$El_x A = \frac{x}{A} \cdot 0 = 0 \tag{20}$$

b) $El_x(fg) = El_x f + El_x g$

$$El_x(fg) = \frac{x}{fg} \cdot [(f'g) + (fg')] \tag{21}$$

$$El_x(fg) = \frac{xf'g}{fg} + \frac{xfg'}{fg} \tag{22}$$

$$El_x(fg) = \frac{xf'}{f} + \frac{xg'}{g} \tag{23}$$

$$El_x(fg) = El_x f + El_x g \tag{24}$$

c) $El_x(f/g) = El_x f - El_x g$

$$El_x(f/g) = \frac{x}{f/g} \cdot [(f'/g) + (f \cdot \frac{-g'}{g^2})] \tag{25}$$

$$El_x(f/g) = \frac{\frac{xf'}{g}}{f/g} - \frac{\frac{xfg'}{g^2}}{f/g} \tag{26}$$

$$El_x(f/g) = (\frac{xf'}{g} \cdot \frac{g}{f}) - (\frac{xfg'}{g^2} \cdot \frac{g}{f}) \tag{27}$$

$$El_x(f/g) = \frac{xf'}{f} - \frac{xg'}{g} \tag{28}$$

$$El_x(f/g) = El_x f - El_x g \tag{29}$$

d) $El_x(f+g) = \dfrac{fEl_x f + gEl_x g}{f+g}$

$$El_x(f+g) = \frac{x}{f+g} \cdot (f' + g') \tag{30}$$

$$El_x(f+g) = \frac{x}{f+g} \cdot (\frac{ff'}{f} + \frac{gg'}{g}) \tag{31}$$

$$El_x(f+g) = \frac{1}{f+g} \cdot [(\frac{xf'f}{f}) + (\frac{xg'g}{g})] \tag{32}$$

$$El_x(f+g) = \frac{1}{f+g} \cdot [fEl_x f + gEl_x g] \tag{33}$$

$$El_x(f+g) = \frac{fEl_x f + gEl_x g}{f+g} \tag{34}$$

e)$El_x(f-g) = \frac{fEl_xf - gEl_xg}{f-g}$

$$El_x(f-g) = \frac{x}{f-g} \cdot (f' - g') \tag{35}$$

$$El_x(f-g) = \frac{x}{f-g} \cdot (\frac{ff'}{f} - \frac{gg'}{g}) \tag{36}$$

$$El_x(f-g) = \frac{1}{f-g} \cdot [(\frac{xf'f}{f}) - (\frac{xg'g}{g})] \tag{37}$$

$$El_x(f-g) = \frac{1}{f-g} \cdot [fEl_xf - gEl_xg] \tag{38}$$

$$El_x(f-g) = \frac{fEl_xf - gEl_xg}{f-g} \tag{39}$$

f)$El_xf(g(x)) = El_uf(u)El_xg$, sendo $g(x) = u$

$$El_xf(g(x)) = \frac{x}{f(g(x))} \cdot f'(g(x))g'(x) \tag{40}$$

$$El_xf(g(x)) = \frac{x}{f(u)} \cdot f'(u)g'(x) \tag{41}$$

$$El_xf(g(x)) = \frac{x}{f(u)} \cdot f'(u)g'(x) \cdot \frac{g(x)}{g(x)} \tag{42}$$

$$El_xf(g(x)) = \frac{g(x)f'(u)}{f(u)} \cdot \frac{xg'(x)}{g(x)} \tag{43}$$

$$El_xf(g(x)) = El_uf(u)El_xg \tag{44}$$

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Resolução (‖ Questão: 7.7.10 ‖ Relator: $\mathbf{x}_{15}$ ‖ Revisor: $\mathbf{x}_{05}$ ‖)

[HARDER] Use the rules of Exercise 9 to evaluate the following:

a) $El_x(-10x^{-5})$

As (7.7.3) suggests:

$$f(x) = Ax^b \Rightarrow El_xf(x) = b \tag{45}$$

Therefore

$$El_x(-10x^{-5}) = -5 \tag{46}$$

b) $El_x(x + x^2)$

As exercise 9.d evaluate:

$$El_x(f+g) = \frac{fEl_xf + gEl_xg}{f+g} \tag{47}$$

Considering f and g as differentiable functions of x

Let $f = x$ and $g = x^2$, then, the elasticity of $(x + x^2)$ is:

$$El_x(x+x^2) = \frac{xEl_xx + x^2El_xx^2}{x+x^2} \tag{48}$$

We know that $El_x x = \frac{x}{x} \cdot \frac{dx}{dx} = 1$

And that $El_x x^2 = 2$

Substituting in the elasticity formula we have:

$$El_x(x + x^2) = \frac{x + 2x^2}{x + x^2} \tag{49}$$

c) $El_x(x^3 + 1)^{10}$

As exercise 9.f evaluate:

$$El_x f(g(x)) = El_u f(u) \cdot El_x u \tag{50}$$

Considering $u = g(x)$

Let $f = u^{10}$ and $g = x^3 + 1$, then, the elasticity of $(x^3 + 1)^{10}$ is:

$$El_x(x^3 + 1)^{10} = El_u u^{10} \cdot El_x(x^3 + 1) \tag{51}$$

As $El_u u^{10} = 10$

And $El_x(x^3 + 1) = \frac{x}{x^3 + 1} \cdot 3x^2 = \frac{3x^3}{x^3 + 1}$

Substituting in the elasticity formula we have:

$$El_x(x^3 + 1)^{10} = 10 \cdot \frac{3x^3}{x^3 + 1} = \frac{30x^3}{x^3 + 1} \tag{52}$$

d) $El_x(El_x(5x^2))$

We know that $El_x 5x^2 = 2$, as (7.7.3) suggests, then we want to calculate:

$$El_x(2) \tag{53}$$

As exercise 9.a suggests, the elasticity of a constant is 0. therefore:

$$El_x(2) = 0 \tag{54}$$

e) $El(1 + x^2)$

As exercise 9.d evaluate:

$$El_x(f + g) = \frac{f El_x f + g El_x g}{f + g} \tag{55}$$

Considering f and g as differentiable functions of x

Let $f = 1$ and $g = x^2$, then, the elasticity of $(1 + x^2)$ is:

$$El_x(1 + x^2) = \frac{1 El_x 1 + x^2 El_x x^2}{1 + x^2} \tag{56}$$

We know that $El_x 1 = 0$

And that $El_x x^2 = 2$

Substituting in the elasticity formula we have:

$$El_x(1 + x^2) = \frac{2x^2}{1 + x^2} \tag{57}$$

f) $El_x(\frac{x-1}{x^5+1})$

As exercise 9.c evaluate:

$$El_x(f/g) = El_x f - El_x g \tag{58}$$

Considering f and g as differentiable functions of x

Let $f = x - 1$ and $g = x^5 + 1$, then, the elasticity of $(\frac{x-1}{x^5+1})$ is:

$$El_x(\frac{x-1}{x^5+1}) = El_x(x-1) - El_x(x^5+1) \tag{59}$$

We know that $El_x(x-1) = \frac{x}{x-1} \cdot 1 = \frac{x}{x-1}$

And that $El_x(x^5+1) = \frac{x}{x^5+1} \cdot 5x^4 = \frac{5x^5}{x^5+1}$

Substituting in the elasticity formula we have that:

$$El_x(\frac{x-1}{x^5+1}) = \frac{x}{x-1} - \frac{5x^5}{x^5+1} \tag{60}$$

■