Then with the transmission loss *not* coordinated, we would have

$$P_1 + P_2 - 5 \times 10^{-4} P_1^2 = 438.75 \qquad (2)$$

Solving (*1*) and (*2*) simultaneously yields $P_1 = 417$ MW and $P_2 = 108.5$ MW.

Comparing these results with the results of Problem 9.13, we see that the load on plant 1 is increased from 350 MW to 417 MW; hence its fuel cost increases by

$$\int_{350}^{417} (0.01P_1 + 10)\, dP_1 = \$926.945/\text{h}$$

The load on plant 2 is decreased from 150 MW to 108.5 MW; hence its fuel cost decreases by

$$-\int_{150}^{108.5} (0.02P_2 + 12)\, dP_2 = \$605.277/\text{h}$$

The saving with loss coordination is thus 926.945 − 605.277 = \$321.67/h.

9.15 For the system shown in Fig. 9-3, what is the minimum open-loop gain such that the steady-state error Δe_{ss} does not exceed 1 percent?

From Fig. 9-3,

$$\frac{\Delta e}{\Delta V_{ref}} = \frac{1}{1 + G(s)} \qquad (1)$$

Substituting (*9.24*) in (*1*) and setting $s = 0$ (for the steady state) yield

$$\Delta e_{ss} = \frac{(\Delta V_{ref})_{ss}}{1 + k} \quad \text{or} \quad 1 + k = \frac{(\Delta V_{ref})_{ss}}{\Delta e_{ss}} \qquad (2)$$

The condition of the problem implies that the right side of (*2*) is not less than 100. Hence,

$$1 + k \geq 100$$

and $k \geq 99$.

9.16 Obtain the form of the dynamic response of the system of Fig. 9-3 to a step change in the reference input voltage.

From Fig. 9-3,

$$\Delta V_t(t) = \mathcal{L}^{-1}\left[\frac{G(s)}{1 + G(s)} \Delta V_{ref}(s)\right] \qquad (1)$$

Where $G(s)$ is given in (*9.24*). The response of the system will depend on the characteristic roots of the equation

$$1 + G(s) = 0 \qquad (2)$$

If the roots s_1, s_2, and s_3 are real and distinct, then the response will include the transient components $A_1 e^{s_1 t}$, $A_2 e^{s_2 t}$, and $A_3 e^{s_3 t}$. However, if (*2*) has a pair of complex conjugate roots $s_1, s_2 = \sigma \pm j\omega$, then the dynamic response will be of the form $Ae^{\sigma t} \sin(\omega t + \phi)$.

9.17 Assume that there are no changes occurring in the reference power setting of a turbine-governor system (that is, the system is operating in the steady state), and the frequency-power relationship of the turbine governor is that represented graphically in Fig. 9-7. Determine the regulation constant R.

In (*9.26*), we see that, with $\Delta P_{ref} = 0$, R is the negative of the slope of the f versus P_m curve, plotted in per-unit values. Hence, from Fig. 9-7,

$$R = \frac{\Delta f}{\Delta P_m} = \frac{0.01}{-0.2} = -0.05 \text{ pu}$$

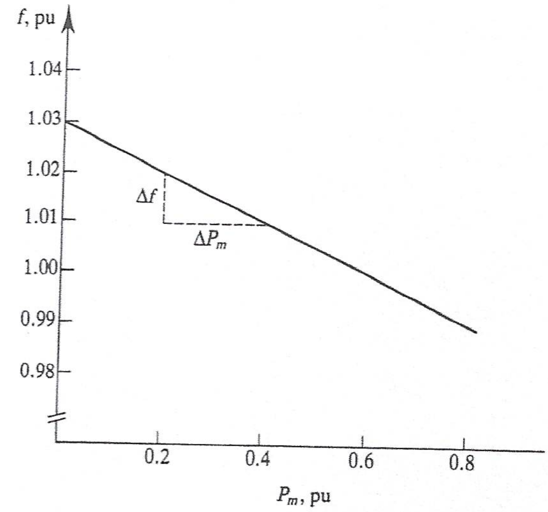

Fig. 9-7.

9.18 For a certain turbine-generator set, $R = 0.04$ pu, based on the generator rating of 100 MVA and 60 HZ. The generator frequency decreases by 0.02 Hz, and the system adjusts to steady-state operation. By how much does the turbine output power increase?

The per-unit frequency change is

$$\text{Per-unit } \Delta f = \frac{\Delta f}{f_{\text{base}}} = \frac{-0.02}{60} = -3.33 \times 10^{-4} \text{ pu}$$

Then (*9.26*) yields

$$\text{Per-unit } \Delta P_m = -\frac{1}{0.04}(-3.33 \times 10^{-4}) = 8.33 \times 10^{-3} \text{ pu}$$

The actual increase in output power is then

$$\Delta P_m = (8.33 \times 10^{-3})(100) = 0.833 \text{ MW}$$

9.19 An area includes two turbine-generator units, rated at 500 and 750 MVA and 60 Hz, for which $R_1 = 0.04$ pu and $R_2 = 0.05$ pu based on their respective ratings. Each unit carries a 300-MVA steady-state load. The load on the system suddenly increases by 250 MVA. (*a*) Calculate β on a 1000-MVA base. (*b*) Determine Δf on a 60-Hz base and in hertz.

(*a*) We can change the bases of the R values with the formula

$$R_{\text{new}} = R_{\text{old}} \frac{S_{\text{base(new)}}}{S_{\text{base(old)}}}$$

Thus

$$R_{1(\text{new})} = (0.04)\frac{1000}{500} = 0.08 \text{ pu}$$

and

$$R_{2(\text{new})} = (0.05)\frac{1000}{750} = 0.067 \text{ pu}$$

Now, from (*9.30*),

$$\beta = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{0.08} + \frac{1}{0.067} = 27.5 \text{ pu}$$

(*b*) The per-unit increase in the load is 250/1000 = 0.25 pu. From (*9.29*), with $\Delta P_{\text{ref(total)}} = 0$ for steady-state conditions,

$$\Delta f = \frac{-1}{\beta}\Delta P_m = -\frac{1}{27.5}0.25 = -9.091 \times 10^{-3} \text{ pu}$$

Also,

$$\Delta f = -9.091 \times 10^{-3} \times 60 = -0.545 \text{ Hz}$$

9.20 For areas 1 and 2 in a 60-Hz power system, $\beta_1 = 400$ MW/Hz and $\beta_2 = 250$ MW/Hz. The total power generated in each of these areas is, respectively, 1000 MW, and 750 MW. While each area is generating power at the steady state with $\Delta P_{\text{tie1}} = \Delta P_{\text{tie2}} = 0$, the load in area 1 suddenly increases by 50 MW. Determine the resulting Δf, (*a*) without LFC and (*b*) with LFC. Neglect all losses.

(*a*) From (*9.29*), since $\Delta P_{\text{ref(total)}} = 0$ without LFC,

$$50 = -(400 + 250)\,\Delta f$$

from which $\Delta f = -0.0769$ Hz.
(*b*) With LFC, in the steady state, (*9.27*) implies that $ACE_1 = ACE_2 = 0$; otherwise, the LFC given by (*9.27*) would be changing the reference power settings of the governors on LFC. Also, the sum of the net tie-line flows, $\Delta P_{\text{tie1}} + \Delta P_{\text{tie2}}$, is zero (neglecting losses). So

$$ACE_1 + ACE_2 = 0 = (B_1 + B_2)\,\Delta f$$

and $\Delta f = 0$, since $B_1 + B_2 \neq 0$.

Supplementary Problems

9.21 A graph of fuel input versus power output for a certain plant is given in Fig. 9-8. Determine the fuel requirements at (*a*) 120 MW and (*b*) 560 MW output power.

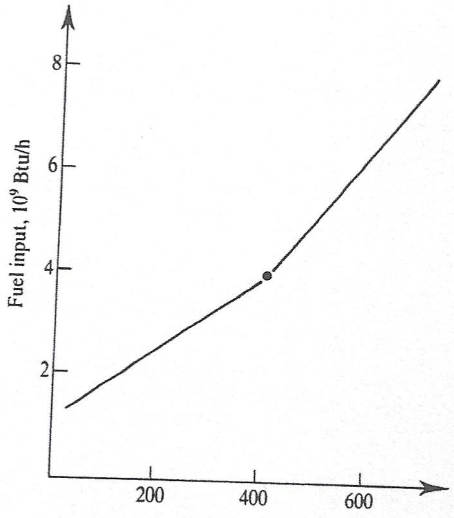

Fig. 9-8.

Ans. (*a*) 16.67×10^6 Btu/MWh; (*b*) 10.71×10^6 Btu/MWh

9.22 (*a*) For the plant of Problem 9.21, determine the fuel requirement at the maximum-efficiency operating point. (*b*) What is the power output at that point?

Ans. (*a*) 10×10^6 Btu/MWh; (*b*) 400 MW

If the discontinuity occurs at the middle of an interval, then for that interval

$$P_a = P_i - \text{output during the fault} \tag{10.27}$$

For this case, at the beginning of the interval immediately following the clearing of the fault, P_a is given by

$$P_a = P_i - \text{output after the fault is cleared} \tag{10.28}$$

Finally, if the discontinuity occurs neither at the beginning nor at the middle of an interval, P_a may still be evaluated from (*10.26*) through (*10.28*).

Algorithm for the Iterations

Returning now to (*10.25*), we see that δ_1 gives us one point on the swing curve. The algorithm for the iterative process is as follows:

$$P_{a(n-1)} = P_i - P_{e(n-1)} \tag{10.29}$$

$$P_{e(n-1)} = \frac{|E|\,|V|}{X}\sin\delta_{(n-1)} \tag{10.30}$$

$$\alpha_{(n-1)} = \frac{P_{a(n-1)}}{M} \tag{10.31}$$

$$\Delta\omega_{r(n)} = \alpha_{(n-1)}\,\Delta t \tag{10.32}$$

$$\omega_{r(n)} = \omega_{r(n-1)} + \alpha_{(n-1)}\,\Delta t \tag{10.33}$$

$$\Delta\delta_{(n)} = \Delta\delta_{(n-1)} + \frac{P_{a(n-1)}}{M}(\Delta t)^2 \tag{10.34}$$

$$\delta_{(n)} = \delta_{(n-1)} + \Delta\delta_{(n)} \tag{10.35}$$

The use of this algorithm in conjunction with the equal-area criterion provides the critical clearing angle and the corresponding critical clearing time.

Solved Problems

10.1 The inertia constant H for a 60-Hz, 100-MVA hydroelectric generator is 4.0 MJ/MVA. How much kinetic energy is stored in the rotor at synchronous speed? If the input to the generator is suddenly increased by 20 MVA, what acceleration is imparted to the rotor?

The energy stored in the rotor at synchronous speed is given by (*10.1*) and is

$$GH = 100 \times 4 = 400 \text{ MJ}$$

The rotor acceleration $d^2\delta/dt^2$ is given by (*10.7*) with $P_a = 20$ MVA of accelerating power and with M as determined from (*10.3*). Thus, (*10.3*) yields

$$M = \frac{GH}{180f} = \frac{400}{180 \times 60} = \frac{1}{27}$$

and (*10.7*) becomes

$$\frac{1}{27}\frac{d^2\delta}{dt^2} = 20$$

so $d^2\delta/dt^2 = 20 \times 27 = 540°/\text{s}^2$.

10.2 In Section 10.2 we noted that machinery manufacturers generally supply the value of WR^2.

Derive a relationship between H and WR^2 for a machine whose rating is S_{mach} MVA.

The kinetic energy of rotation of the rotor at synchronous speed is

$$KE = \frac{1}{2}\frac{WR^2}{32.2}\left(\frac{2\pi n}{60}\right)^2 \quad \text{(in foot-pounds)}$$

where n is the rotor speed in revolutions per minute. Since 550 ft · lb/s = 746 W, 1 ft · lb = 746/550 J. Converting foot-pounds to megajoules and dividing the last equation by the machine rating in megavoltamperes, we obtain

$$H = \frac{\left(\frac{746}{550} \times 10^{-6}\right)\left(\frac{1}{2}\frac{WR^2}{32.2}\right)\left(\frac{2\pi n}{60}\right)^2}{S_{mach}}$$

$$= \frac{2.31 \times 10^{-10} WR^2 n^2}{S_{mach}} \qquad (1)$$

10.3 A 1500-MVA, 1800-rev/min synchronous generator has $WR^2 = 6 \times 10^6$ lb · ft². Find the inertia constant H of the machine relative to a 100-MVA base.

From (1) of Problem 10.2,

$$H = \frac{(2.31 \times 10^{-10})(6 \times 10^6)(1800)^2}{1500} = 2.994 \text{ MJ/MVA}$$

Relative to a 100-MVA base, then,

$$H = 2.994 \times \frac{1500}{100} = 44.91 \text{ MJ/MVA}$$

10.4 A 500-MVA synchronous machine has $H_1 = 4.6$ MJ/MVA, and a 1500-MVA machine has $H_2 = 3.0$ MJ/MVA. The two machines operate in parallel in a power station. What is the equivalent H constant for the two, relative to a 100-MVA base?

The total kinetic energy of the two machines is

$$KE = 4.6 \times 600 + 3 \times 1500 = 6800 \text{ MJ}$$

Thus, the equivalent H relative to a 100-MVA base is

$$H = \frac{6800}{100} = 68 \text{ MJ/MVA}$$

10.5 For a certain lagging-power-factor load, the sending-end and receiving-end voltages of a short transmission line of impedance $R + jX$ are equal. Determine the ratio X/R so that maximum power is transmitted over the line under steady-state conditions.

From the phasor diagram of Fig. 10-6, we may write

$$\begin{aligned} V_S &= V_R + I(\cos\phi - j\sin\phi)(R + jX) \\ &= (V_R + IR\cos\phi + IX\sin\phi) + j(IX\cos\phi - IR\sin\phi) \\ R(V_S\cos\delta) &= (V_R + IR\cos\phi + IX\sin\phi)R \\ X(V_S\sin\delta) &= (IX\cos\phi - IR\sin\phi)X \end{aligned}$$

Combining these equations and letting $Z^2 = R^2 + X^2$, we get

$$V_S(R\cos\delta + X\sin\delta) = RV_R + IZ^2\cos\phi$$

or

$$I\cos\phi = \frac{V_S}{Z^2}(R\cos\delta + X\sin\delta) - \frac{RV_R}{Z^2}$$

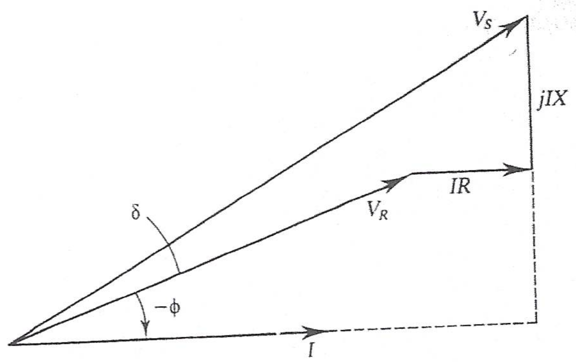

Fig. 10-6.

Hence, we have

$$P_R = V_R I \cos\phi = \frac{V_R V_S}{Z^2}(R\cos\delta + X\sin\delta) - \frac{RV_R^2}{Z^2} \qquad (1)$$

Now let $\tan\beta = X/R$; then (1) becomes

$$P_R = \frac{V_R V_S}{Z}\cos(\beta - \delta) - \frac{RV_R^2}{Z^2}$$

For maximum power $\beta = \delta$, and so

$$P_{R(\max)} = \frac{V_R V_S}{\sqrt{R^2 + X^2}} - \frac{RV_R^2}{R^2 + X^2} \qquad (2)$$

and

$$\frac{dP_{R(\max)}}{dX} = 0$$

Thus

$$\left(\frac{V_S}{V_R}\right)^2 (R^2 + X^2) = 4R^2$$

and since $V_S = V_R$, we have $X/R = \sqrt{3}$.

10.6 The sending-end and receiving-end voltages of a transmission line at a 100-MW load are equ at 115 kV. The per-phase line impedance is $(4 + j7)\ \Omega$. Calculate the maximum steady-sta power that can be transmitted over the line.

Since $V_R = V_S = 115{,}000/\sqrt{3} = 66{,}400$, we have, from (2) of Problem 10.5,

$$P_{R(\max)} = \frac{V_R V_S}{\sqrt{R^2 + X^2}} - \frac{RV_R^2}{R^2 + X^2}$$

$$= \left[\frac{(66.4)^2}{\sqrt{4^2 + 7^2}} - \frac{4(66.4)^2}{4^2 + 7^2}\right]10^6 = 275.5\ \text{MW/phase}$$

$$= 826.5\ \text{MW total}$$

10.7 A synchronous generator, capable of developing 500 MW of power, operates at a power a of 8°. By how much can the input shaft power be increased suddenly without loss of stabi

Initially, at $\delta_0 = 8°$, the electromagnetic power being developed is

$$P_{e0} = P_{\max}\sin\delta_0 = 500\sin 8° = 69.6\ \text{MW}$$

Let δ_m (Fig. 10-7) be the power angle to which the rotor can swing before losing synchronism. The equal-area criterion requires that (10.12) be satisfied (with δ_m replacing δ_2). From Fig.

$\delta_m = \pi - \delta_1$, so (*10.12*) yields

$$(\pi - \delta_1 - \delta_0)\sin\delta_1 + \cos(\pi - \delta_1) - \cos\delta_0 = 0$$

or

$$(\pi - \delta_1 - \delta_0)\sin\delta_1 - \cos\delta_1 - \cos\delta_0 = 0 \qquad (1)$$

Substituting $\delta_0 = 8° = 0.13885$ rad in (*1*) gives

$$(3 - \delta_1)\sin\delta_1 - \cos\delta_1 - 0.99 = 0$$

This yields $\delta_1 = 50°$, for which the corresponding after-the-fault electromagnetic power is

$$P_{ef} = P_{\max}\sin\delta_1 = 500\sin 50° = 383.02\ \text{MW}$$

The initial power developed by the machine was 69.6 MW. Hence, without loss of stability, the system can accommodate a sudden increase of

$$P_{ef} - P_{e0} = 383.02 - 69.6 = 313.42\ \text{MW}$$

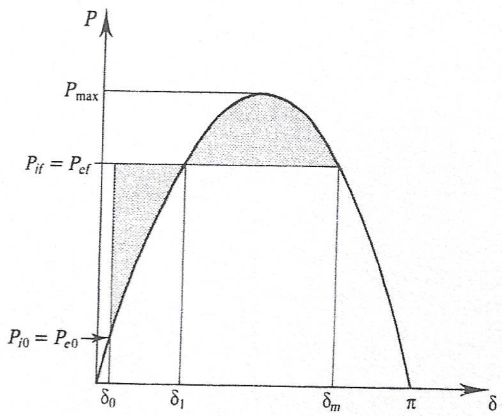

Fig. 10-7.

10.8 Determine the maximum additional load that could suddenly be taken on by the transmission line of Problem 10.6 without losing stability.

If we neglect the resistance, then the initial (maximum) power P_0 is

$$P_0 = \frac{V_S V_R}{X}\sin\delta_0 = P_{R(\max)}\sin\delta_0$$

From (*1*) of Problem 10.7,

$$(\pi - \delta_1 - \delta_0)\sin\delta_1 - \cos\delta_1 - \cos\delta_0 = 0 \qquad (1)$$

We have

$$P_0 = \tfrac{1}{3}(100) = 33.33\ \text{MW}$$

and

$$P_{R(\max)} = \frac{1}{3}\frac{115^2}{7}10^6 = 629.76\ \text{MW}$$

so

$$\delta_0 = \sin^{-1}\frac{33.33}{629.76} = 3° = 0.052\ \text{rad}$$

Then (*1*) becomes

$$(\pi - \delta_1 - 0.052)\sin\delta_1 - \cos\delta_1 - \cos 3° = 0$$

which yields $\delta_1 = 47.8°$. Hence the system will remain stable for an increase in load of up to

$$P_{R(\max)}\sin\delta_1 - P_0 = 629.76\sin 47.8° - 33.33 = 433.2\ \text{MW/phase}$$
$$= 1299.6\ \text{MW total}$$

10.9 A synchronous generator is operating at an infinite bus and supplying 0.45 pu of its maximum power capacity. A fault occurs, and the reactance between the generator and the line becomes four times its value before the fault. The maximum power that can be delivered after the fault is cleared is 70 percent of the original maximum value. Determine the critical clearing angle.

Let

$$x_1 = \frac{P_{max} \text{ during the fault}}{P_{max} \text{ before the fault}}$$

$$x_2 = \frac{P_{max} \text{ after the fault}}{P_{max} \text{ before the fault}}$$

$$\delta_0 = \text{power angle at the time of the fault}$$

$$\delta_c = \text{power angle when fault is cleared}$$

$$\delta_m = \text{maximum angle of swing}$$

Then the equal-area criterion, $A_1 = A_2$ in Fig. 10-8, gives us

$$P_s(\delta_c - \delta_0) - \int_{\delta_0}^{\delta_c} x_1 P_{max} \sin \delta \, d\delta = \int_{\delta_c}^{\delta_m} x_2 P_{max} \sin \delta \, d\delta - P_s(\delta_m - \delta_c)$$

Hence,

$$\cos \delta_c = \frac{1}{x_2 - x_1}\left[\frac{P_s}{P_{max}}(\delta_m - \delta_0) + x_2 \cos \delta_m - x_1 \cos \delta_0\right] \quad (1)$$

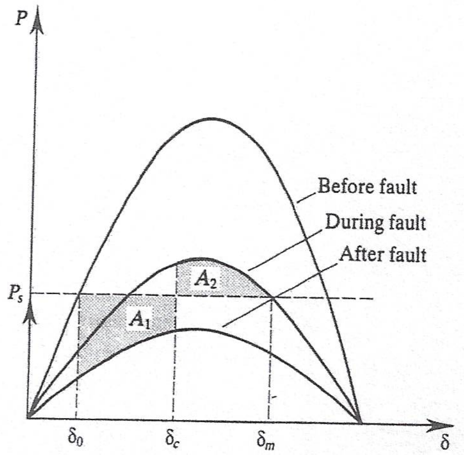

Fig. 10-8.

Initially, the generator is supplying 0.45 pu of P_{max}. Thus,

$$P_s = 0.45 P_{max} = P_{max} \sin \delta_0$$

from which $\delta_0 = \sin^{-1} 0.45 = 26.74°$. Now $P_{max} = EV/X$. When the fault occurs, X becomes $4X$, so that

$$x_1 P_{max} \sin \delta_m = \frac{EV}{4X} \sin \delta_m = \tfrac{1}{4} P_{max} \sin \delta_m$$

so that $x_1 = 0.25$.

After the fault, with $x_2 = 0.70$, we have

$$P_s = x_2 P_{max} \sin \delta'_m$$

from which

$$\delta'_m = \sin^{-1} \frac{P_s}{x_2 P_{max}} = \sin^{-1} \frac{0.45 P_{max}}{0.70 P_{max}} = 40°$$

Then $\delta_m = 90° + \delta'_m = 130°$ (see Fig. 10-8), and

$$\delta_m - \delta_0 = 130° - 26.74° = 103.26° \quad \text{or} \quad 1.8019 \text{ rad}$$

Hence, from (*1*),

$$\cos\delta_c = \frac{1}{0.70 - 0.25}[0.45(1.8019) + 0.70\cos 130° - 0.25\cos 26.74°] = 0.3059$$

so that $\delta_c = \cos^{-1} 0.3059 = 72.2°$.

10.10 A 100-MVA, two-pole, 60-Hz generator has a moment of inertia of $50 \times 10^3\ \text{kg}\cdot\text{m}^2$. What is the energy stored in the rotor at the rated speed? What is the corresponding angular momentum? Determine the inertia constant H.

The stored energy is

$$\text{KE(stored)} = \frac{1}{2}J\omega_m^2 = \frac{1}{2}(50 \times 10^3)\left(\frac{2\pi \times 3600}{60}\right)^2 = 3553 \text{ MJ}$$

Then

$$H = \frac{\text{KE(stored)}}{\text{MVA}} = \frac{3553}{100} = 35.53 \text{ MJ/MVA}$$

$$M = \frac{GH}{180f} = \frac{(100)(35.53)}{(180)(60)} = 0.329 \text{ MJ}\cdot\text{rad/s}$$

10.11 The input to the generator of Problem 10.10 is suddenly increased by 25 MW. Determine the rotor acceleration.

From Problems 10.10 and 10.1,

$$0.329\ddot{\delta} = 25$$

Thus,

$$\ddot{\delta} = \frac{25}{0.329} = 76°/\text{s}^2$$

10.12 Assuming the acceleration calculated in Problem 10.11 remains constant for twelve cycles, calculate the change in the power angle and the speed that occurs during those twelve cycles.

Twelve cycles are equivalent to $12/60 = 0.2$ s. During that time, δ changes by $\frac{1}{2}(471.25)(0.2)^2 =$ 9.425 electrical degrees. Now

$$\ddot{\delta} = 60 \times \frac{471.25}{360} = 78.5 \text{ rev/min/s}$$

so the rotor speed at the end of the twelve cycles is $3600 + 78.5 = 3678.5$ rev/min.

10.13 A 60-Hz generator, connected directly to an infinite bus operating at a voltage of $1\underline{/0°}$ pu, has a synchronous reactance of 1.35 pu. The generator no-load voltage is 1.1 pu, and its inertia constant H is 4 MJ/MVA. The generator is suddenly loaded to 60 percent of its maximum power limit; determine the frequency of the resulting natural oscillations of the generator rotor.

We find δ_0 using $\sin\delta_0 = P_e/P_i = 0.6$, which gives $\delta_0 = 36.87°$. Then

$$\left.\frac{\partial P_e}{\partial\delta}\right|_{36.87} = \frac{1.1 \times 1}{1.35}\cos 36.87 = 0.6518 \text{ pu/rad}$$

Also, we have

$$M = \frac{H}{\pi f} = \frac{4}{\pi \times 60} \text{ pu s}^2/\text{rad}$$

$$\text{Frequency of oscillation} = \sqrt{\frac{(\partial P_e/\partial\delta)_{36.87}}{M}}$$

$$= \sqrt{\frac{\pi \times 60 \times 0.6518}{4}} = 5.5\ \text{rad/s} = 0.882\ \text{Hz}$$

10.14 Derive (*10.19*).

Since $\delta = \delta_1 - \delta_2$,

$$\ddot{\delta} = \ddot{\delta}_1 - \ddot{\delta}_2 \tag{1}$$

From (*1*), (*10.17*), and (*10.18*), we have

$$\ddot{\delta}_1 - \ddot{\delta}_2 = \frac{1}{M_1}(P_{i1} - P_{e1}) - \frac{1}{M_2}(P_{i2} - P_{e2}) = \ddot{\delta} \tag{2}$$

Multiplying both sides of (*2*) by $M_1M_2/(M_1 + M_2)$ yields

$$\frac{M_1M_2}{M_1 + M_2}\ddot{\delta} = \frac{1}{M_1 + M_2}[(M_2P_{i1} - M_1P_{i2}) - (M_2P_{e1} - M_1P_{e2})]$$

$$= \frac{M_2P_{i1} - M_1P_{i2}}{M_1 + M_2} - \frac{M_2P_{e1} - M_1P_{e2}}{M_1 + M_2}$$

or

$$M\ddot{\delta} = P_i - P_e$$

which is the same as (*10.19*).

10.15 The kinetic energy stored in the rotor of a 50-MVA, six-pole, 60-Hz synchronous machine is 200 MJ. The input to the machine is 25 MW at a developed power of 22.5 MW. Calculate the accelerating power and the acceleration.

The accelerating power is

$$P_a = P_i - P_e = 25 - 22.5 = 2.5\ \text{MW}$$

Now, also,

$$H = \frac{\text{KE(stored)}}{\text{machine rating in MVA}} = \frac{200}{50} = 4$$

and, from (*10.3*),

$$M = \frac{GH}{180f} = \frac{50 \times 4}{180 \times 60} = 0.0185\ \text{MJ}\cdot\text{s/degree}$$

$$= 1.06\ \text{MJ}\cdot\text{s/rad}$$

Finally, from (*10.7*),

$$\ddot{\delta} = \frac{2.5}{1.06} = 2.356\ \text{rad/s}^2$$

10.16 If the acceleration of the machine of Problem 10.15 remains constant for ten cycles, what is the power angle at the end of the ten cycles?

From Problem 10-15, $\ddot{\delta} = 2.356$. Integration with respect to t yields

$$\dot{\delta} = 2.356t + C_1$$

Since $\dot{\delta} = 0$ at $t = 0$, $C_1 = 0$. A second integration now gives

$$\delta = 1.178t^2 + C_2$$

At $t = 0$, let $\delta = \delta_0$ (the initial power angle). Then

$$\delta = 1.178t^2 + \delta_0$$

At 60 Hz, the time required for ten cycles is $t = \frac{1}{6}$ s. For this value of t,

$$\delta = 1.178(\tfrac{1}{6})^2 + \delta_0 = (0.0327 + \delta_0)\ \text{rad}$$

10.17 The generator of Problem 10.15 has an internal voltage of 1.2 pu and is connected to an infinite bus operating at a voltage of 1.0 pu through a 0.3-pu reactance. A three-phase short circuit occurs on the line. Subsequently, circuit breakers operate and the reactance between the generator and the bus becomes 0.4 pu. Calculate the critical clearing angle.

Before the fault,

$$P_{max} = \frac{1.2 \times 1.0}{0.3} = 4.0\ \text{pu}$$

During the fault,

$$P_{max2} = 0$$

and $k_1 = 0$ for use in (*10.14*). After the fault is cleared,

$$P_{max3} = \frac{1.2 \times 1.0}{0.4} = 3.0\ \text{pu}$$

and $k_2 = 3.0/4.0 = 0.75$ for use in (*10.14*).

The initial power angle δ_0 is given by $4 \sin \delta_0 = 1.0$, from which $\delta_0 = 0.2527$ rad. Define $\delta'_m = \pi - \delta_m$ (see Fig. 10-4). The angle δ_m in (*10.14*) is obtained from

$$\sin \delta'_m = \frac{1}{3.0} \quad \text{and} \quad \delta_m = \pi - \delta'_m$$

from which $\delta_m = 2.8$ rad. Substituting k_1, k_2, δ_0 and δ_m in (*10.14*) yields

$$\cos \delta_c = \frac{1}{0.75}[(2.8 - 0.2527)0.25 - 0 + 0.75 \cos 2.8] = -0.093$$

from which $\delta_c = 95.34°$.

10.18 Using the step-by-step algorithm, plot the swing curve for the machine of Problem 10.17.

The per-unit value of the angular momentum, based on the machine rating, is

$$M = \frac{1.0 \times 4}{180 \times 60} = 3.7 \times 10^{-4}\ \text{pu}$$

From (*10.26*), we have

$$P_a(0+) = \frac{1.0 - 0.0}{2} = 0.5$$

From (*10.21*),

$$\alpha(0+) = \frac{0.5}{3.7 \times 10^4} = 1351°/\text{s}$$

From (*10.22*) with $\Delta t = 0.05$ s,

$$\Delta\omega_{r(1)} = 1351 \times 0.05 = 67.55°/\text{s}$$

From (*10.23*),

$$\omega_{r(1)} = 0 + 67.55 = 67.55°/\text{s}$$

From (*10.24*),

$$\Delta\delta_{(1)} = 67.55 \times 0.05 = 3.3775°$$

Finally, from (*10.25*), with $\delta_0 = 14.4775°$ as determined in Problem 10.17,

$$\delta_{(1)} = 14.4775 + 3.3775 = 17.855°$$

For the second interval, (*10.29*) and (*10.31*) to (*10.35*) give us

$$P_{a(1)} = 1.0 - 0.0 = 1.0$$

$$\alpha_{(1)} = \frac{1.0}{3.7 \times 10^4} = 2702°/\text{s}$$

$$\Delta\omega_{r(2)} = 2702 \times 0.05 = 135.1°$$

$$\omega_{r(2)} = \omega_{r(1)} + \Delta\omega_{r(2)} = 67.55 + 135.1 = 202.65°/\text{s}$$

$$\Delta\delta_{(2)} = \omega_{r(2)}\,\Delta t = 202.65 \times 0.05 = 10.1325°$$

$$\delta_{(2)} = \delta_{(1)} + \Delta\delta_{(2)} = 17.855 + 10.1325 = 27.9875°$$

Since α and $\Delta\omega_r$ do not change during succeeding intervals, we have

$$\omega_{r(3)} = \omega_{r(2)} + \Delta\omega_{r(3)} = 337.75°/\text{s}$$

$$\Delta\delta_{(3)} = \omega_{r(3)}\,\Delta t = 337.75 \times 0.05 = 16.8875°$$

$$\delta_{(3)} = \delta_{(2)} + \Delta\delta_{(3)} = 44.875°$$

and so on. In this way we obtain the following table of values, from which Fig. 10-9 is plotted:

t, s	δ, degrees
0.0	14.48
0.05	17.85
0.10	27.99
0.15	44.88
0.20	68.52
0.25	98.92

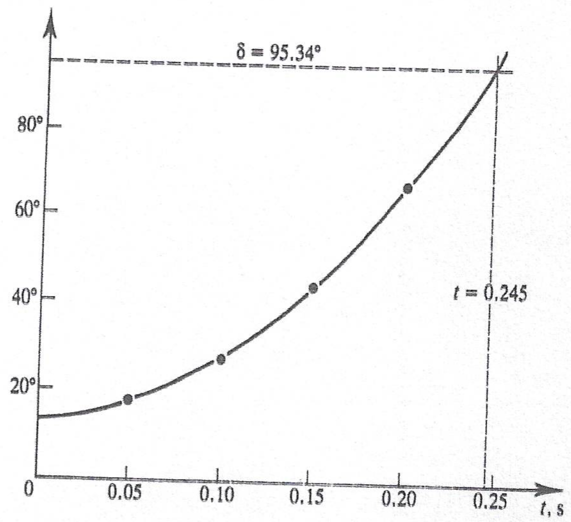

Fig. 10-9.

10.19 From the results of Problems 10.17 and 10.18, find the critical clearing time in cycles for an appropriately set circuit breaker.

From Problem 10.17, $\delta_c = 95.34°$. For this critical clearing angle, Fig. 10-9 gives $t = 0.245$ s. Hence the fault must be cleared within $60 \times 0.245 = 14.7$ cycles.

Supplementary Problems

10.20 The inertia constant H of a 150-MVA, six-pole, 60-Hz synchronous machine is 4.2 MJ/MVA. Determine the value of WR^2 in lb · ft^2.

Ans. 1,893,939 lb · ft^2

10.21 The generator of Problem 10.20 is running at synchronous speed in the steady state. (*a*) What kinetic energy is stored in the rotor? (*b*) If the accelerating power due to a transient change is 28 MW, calculate the rotor acceleration.

Ans. (*a*) 630 MJ; (*b*) 480°/s^2

10.22 A 300-MVA, 1200-rpm synchronous machine has $WR^2 = 3.6 \times 10^6$ lb · ft^2. Calculate H for the machine (*a*) on its own base and (*b*) on a 100-MVA base.

Ans. (*a*) 3.99 MJ/MVA; (*b*) 11.97 MJ/MVA

10.23 A 100-MVA generator has $H = 4.2$ MJ/MVA, and 250-MVA machine, operating in parallel with the first, has $H = 3.6$ MJ/MVA. Calculate the equivalent inertia constant H for the two machines on a 50-MVA base.

Ans. 26.4 MJ/MVA

10.24 The moment of inertia of a 50-MVA, six-pole, 60-Hz generator is 20×10^3 kg · m^2. Determine H and M for the machine.

Ans. 3.15 MJ/MVA; 0.0146 MJ · s/degree

10.25 A synchronous motor develops 30 percent of its rated power for a certain load. The load on the motor is suddenly increased by 150 percent of the original value. Neglecting all losses, calculate the maximum power angle on the swing curve.

Ans. 40°

10.26 A 100-MVA synchronous generator supplies 62.5 MVA of power at 0.8 lagging power factor. The reactance between the load and the generator is normally 1.0 pu, but it increases to 3.0 pu because of a sudden three-phase short circuit. The fault is subsequently cleared and the generator then supplies 43.75 MVA at 0.8 lagging power factor. Determine the critical clearing angle.

Ans. 68.58°

10.27 A synchronous generator supplies its rated power to an infinite bus at a voltage of 1.0 pu. The reactance between the generator and the line, normally 0.825 pu, increases to 0.95 pu because of a fault. Find the critical clearing angle.

Ans. 58.73°

10.28 For the generator of Problem 10.15, determine the rotor speed in revolutions per minute at the end of ten cycles.

Ans. 1203.75 rev/min

10.29 A motor delivers 0.25 pu of its rated power while operating from an infinite bus. If the load on the motor is suddenly doubled, determine δ_m based on the equal-area criterion. Neglect all losses.

Ans. 45°

10.30 The inertia constant M of a synchronous machine is 4.45×10^{-4} pu. The machine operates at a steady-state power angle of 24.7°. Because of a fault, the power angle changes to a value given by the swing equation $\ddot{\delta} = 0.314$ pu. Using the step-by-step algorithm, plot the swing curve and use it to determine the maximum value of the power angle.

Ans. 67°

10.31 The $ABCD$ constants for the nominal-Π circuit representation of a transmission line are $A = D = 0.9\angle 0.3°$, $B = 82.5\angle 76°\ \Omega$, and $C = 0.0005\angle 90°$ S. What is the maximum power that can be transmitted over the line without making the system unstable if $|V_S| = |V_R| = 110$ kV?

Ans. 114.09 MW

10.32 Sketch the power-angle diagram for the line of Problem 10.31 when that line is represented by (*a*) an approximate series circuit and (*b*) a series reactance only. Determine the maximum power transmitted in each case.

Ans. (*a*) 111.18 MW; (*b*) 151.16 MW

10.33 The per-unit reactances for a given system are shown in Fig. 10-10. Unit power is being delivered to the receiving-end bus of the system at unity power factor and unit voltage. A three-phase short circuit occurs at F, the receiving end of one of the lines. Find the critical clearing angle.

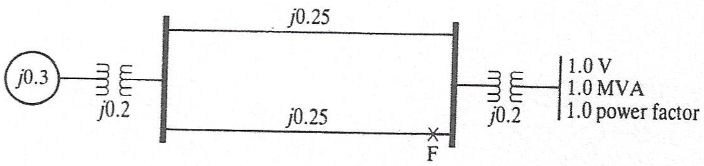

Fig. 10-10.

Ans. 59°

10.34 A 50-MVA, 33-kV, three-phase, four-pole, 60-Hz synchronous generator delivers 40 MW of power to an infinite bus through a total reactance of 0.55 pu. Because of a sudden fault, the reactance of the transmission line changes to 0.5 pu. The inertia constant of the machine 4.806 MJ/MVA. Sketch the swing curve during the fault, assuming that the voltage at the infinite bus is 1.0 pu and that behind the transient reactance is 1.05 pu. The transient reactance of the machine is 0.4 pu.

Ans. Fig. 10-11

10.35 In a plant, two synchronous machines swing together. The inertia constants of the machines are H_1 and H_2, their MVA ratings are S_1 and S_2, the per-unit mechanical power inputs to the two units are P_{m1} and P_{m2}, and P_{e1} and P_{e2} are respectively the electrical power developed by the machines. Obtain an equivalent swing equation for the two-machine system in terms of inertia constants referred to a common base, the per-unit synchronous frequency ω_s in radians per second, the per-unit electrical frequency in radians per second, and the given values of per-unit power.

Ans. $\dfrac{2}{\omega_s}(H_1 + H_2)\omega_{pu}(t)\ddot{\delta} = P_{m1} + P_{m2} - (P_{e1} + P_{e2})$

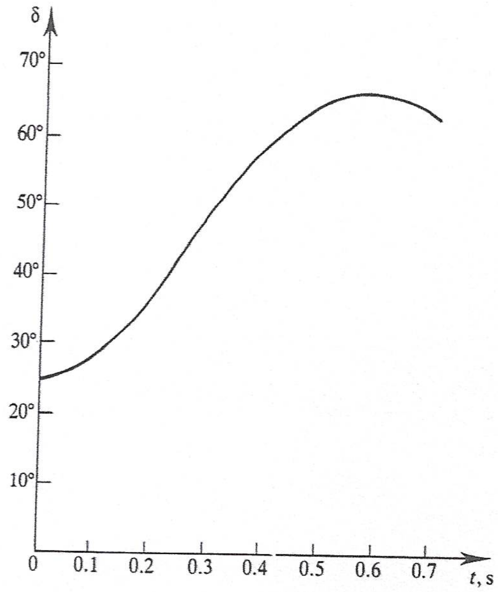

Fig. 10-11.

10.36 During a fault lasting 0.05 s, the swing equation for a 60-Hz maching was, for per-unit values,

$$\ddot{\delta} = \frac{5\pi}{3} \qquad 0 \leq t \leq 0.05 \text{ s}$$

The initial power angle was 0.418 rad. When the fault was cleared, the developed electrical power became 2.46 sin δ per unit. Determine (*a*) the maximum power angle and (*b*) whether or not the machine remained stable.

Ans. (*a*) 156°; (*b*) remained stable

10.37 Calculate the critical clearing time in cycles for the machine of Problem 10.37.

Ans. 11.5 cycles

10.38 Rework Problems 10.36 and 10.37 using a numerical method.