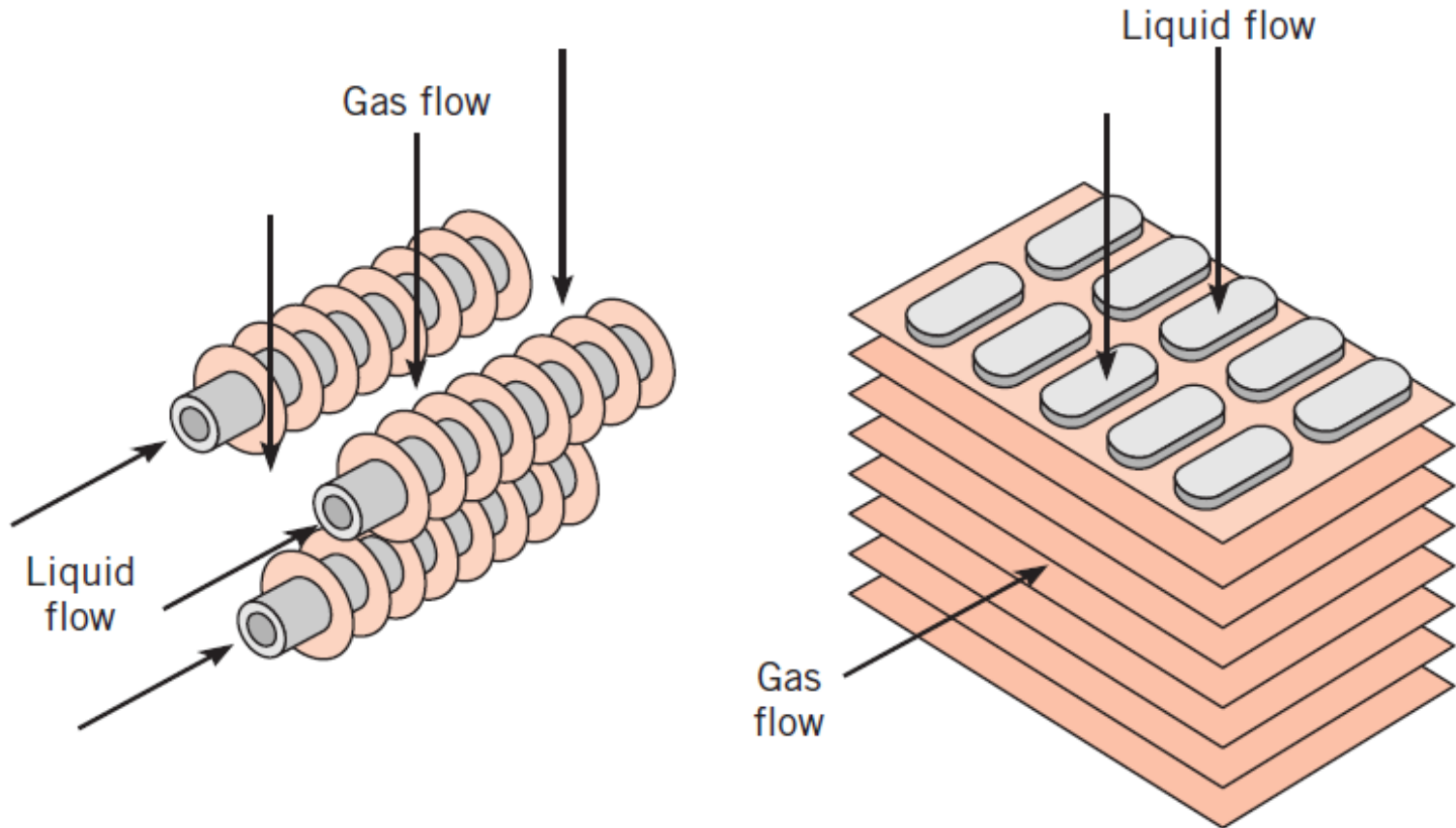


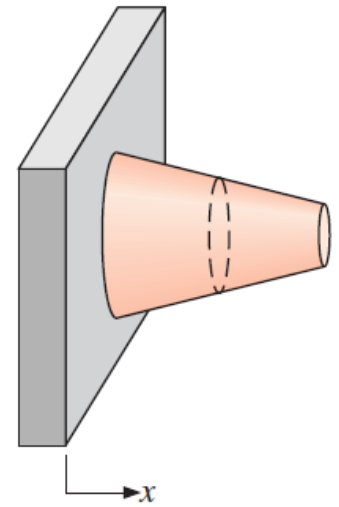
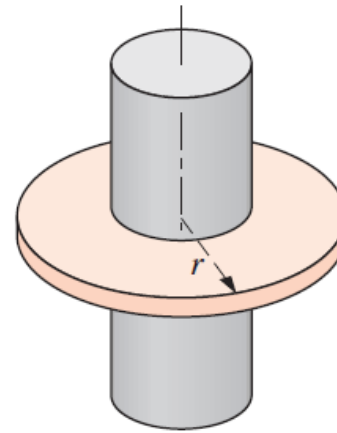
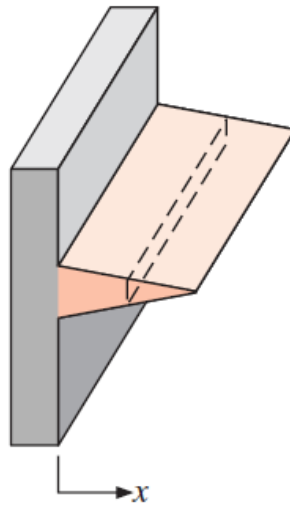
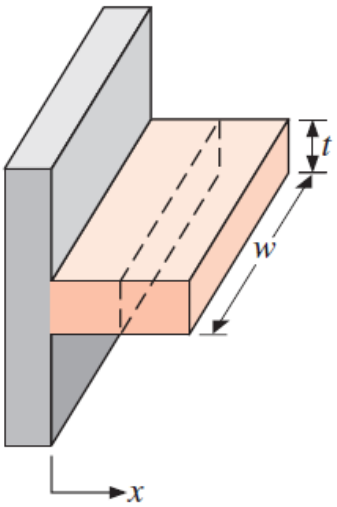
CONDUÇÃO EM ALETAS

Condução em aletas



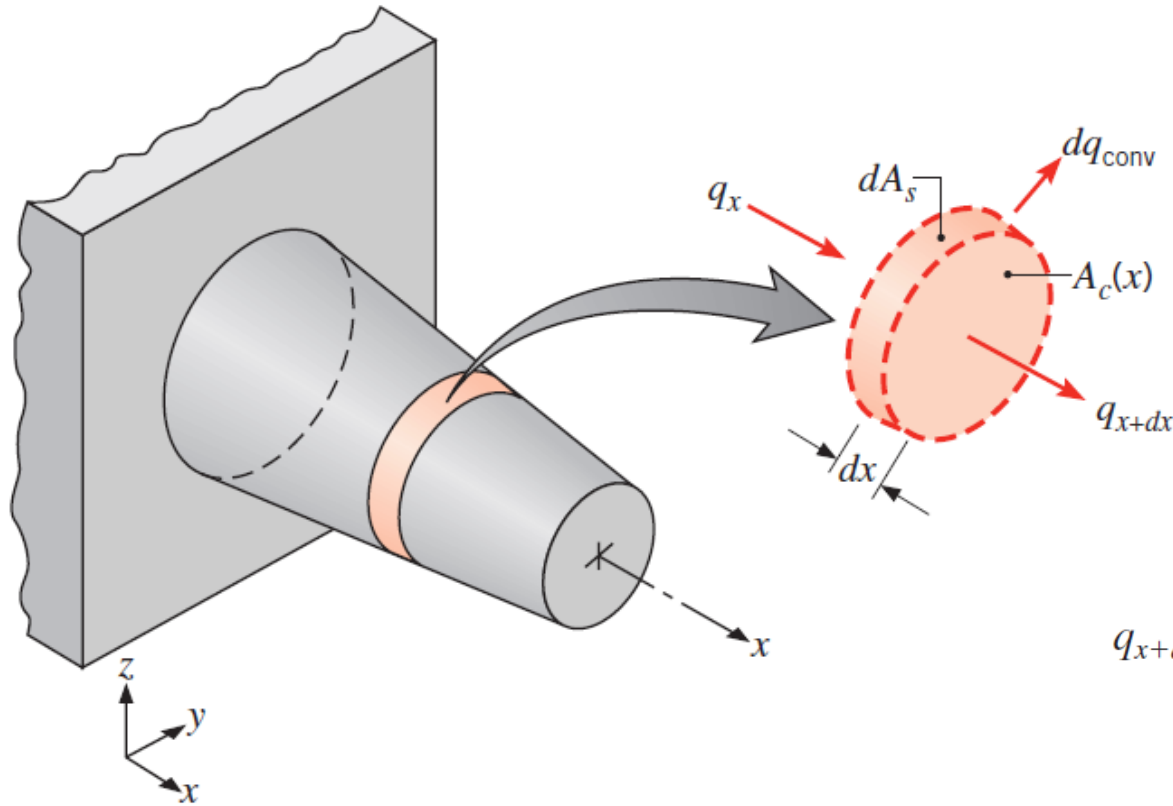
CONDUÇÃO EM ALETAS

Condução em aletas



CONDUÇÃO EM ALETAS

Condução em aletas



$$q_x = q_{x+dx} + dq_{conv}$$

$$q_x = -kA_c \frac{dT}{dx}$$

$$q_{x+dx} = q_x + \frac{dq_x}{dx} dx$$

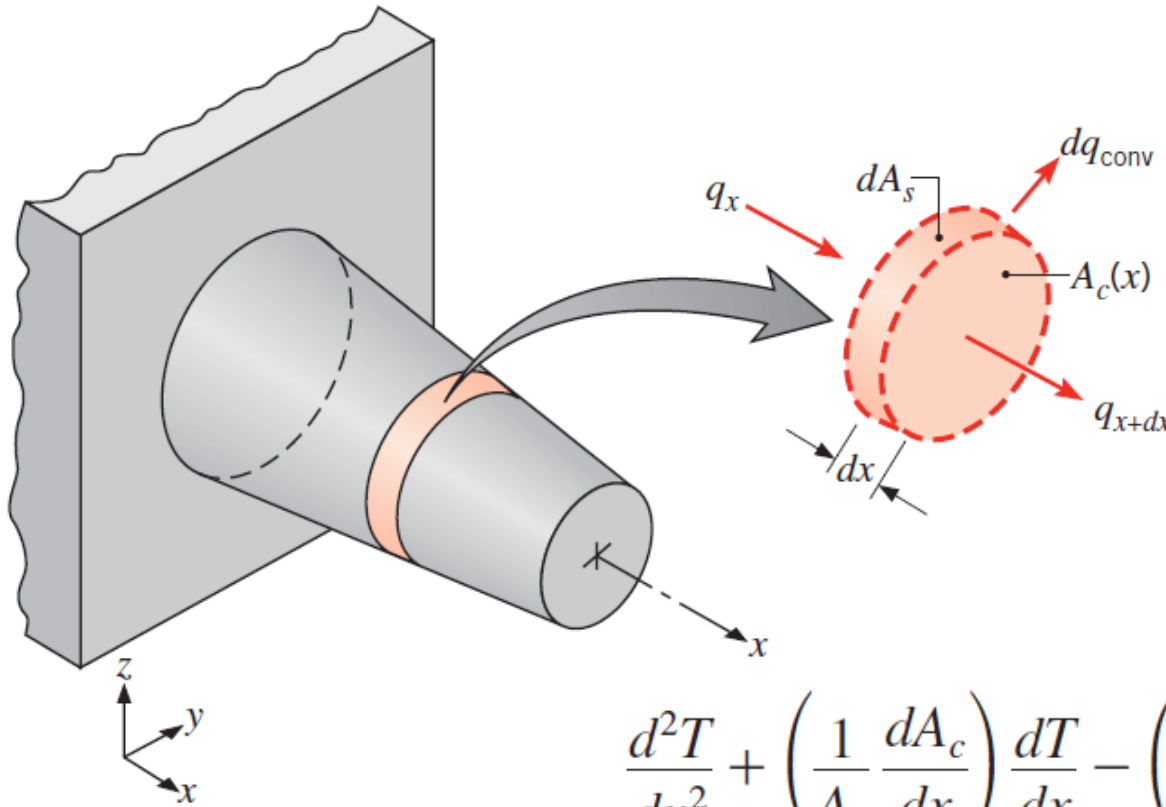
$$q_{x+dx} = -kA_c \frac{dT}{dx} - k \frac{d}{dx} \left(A_c \frac{dT}{dx} \right) dx$$

$$dq_{conv} = hdA_s(T - T_\infty)$$

CONDUÇÃO EM ALETAS

Condução em aletas

$$\frac{d}{dx} \left(A_c \frac{dT}{dx} \right) - \frac{h}{k} \frac{dA_s}{dx} (T - T_\infty) = 0$$

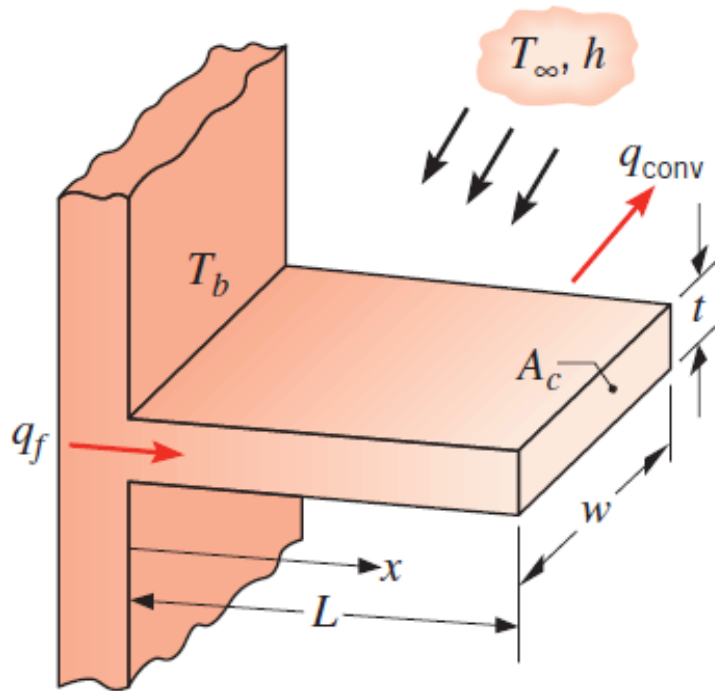


$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c} \frac{dA_c}{dx} \right) \frac{dT}{dx} - \left(\frac{1}{A_c} \frac{h}{k} \frac{dA_s}{dx} \right) (T - T_\infty) = 0$$

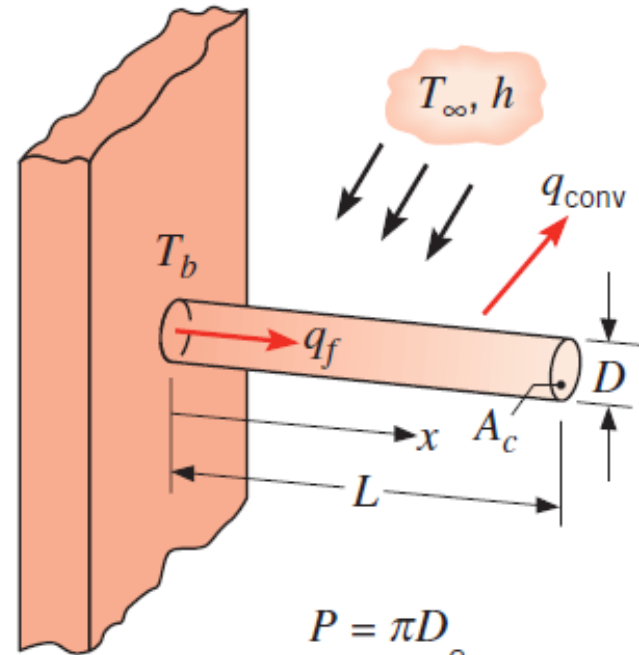
CONDUÇÃO EM ALETAS

Condução em aletas

Aletas com seção transversal constante



$$P = 2w + 2t$$
$$A_c = wt$$



$$P = \pi D$$
$$A_c = \pi D^2/4$$

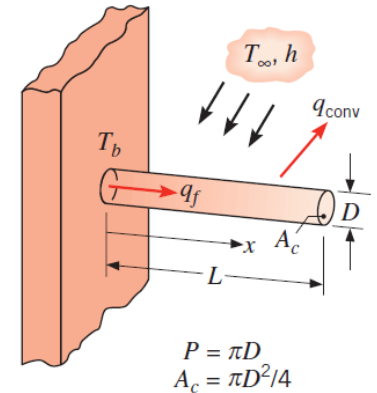
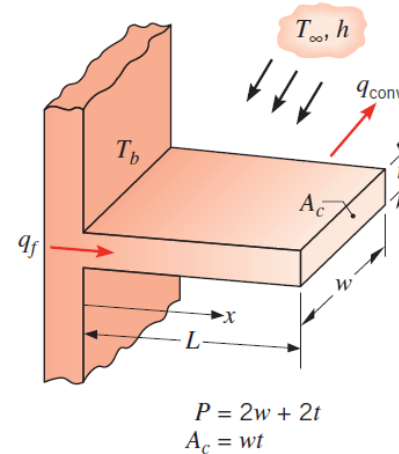
$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c} (T - T_\infty) = 0$$

CONDUÇÃO EM ALETAS

Condução em aletas

Aletas com seção transversal constante

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c} (T - T_\infty) = 0$$



$$\theta(x) \equiv T(x) - T_\infty$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$m^2 \equiv \frac{hP}{kA_c}$$

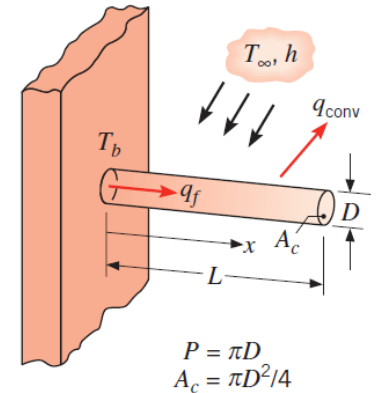
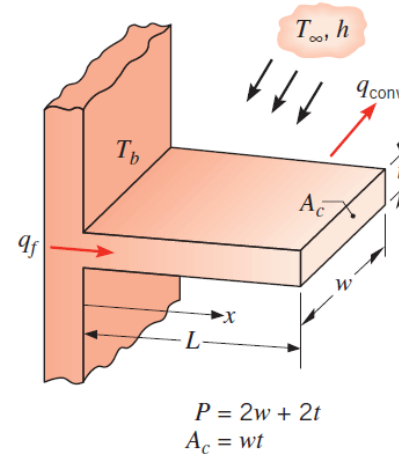
$$q_f = q_b = -kA_c \left. \frac{dT}{dx} \right|_{x=0} = -kA_c \left. \frac{d\theta}{dx} \right|_{x=0}$$

CONDUÇÃO EM ALETAS

Condução em aletas

Aletas com seção transversal constante

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c} (T - T_\infty) = 0$$



$$\theta(x) \equiv T(x) - T_\infty$$

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$$m^2 \equiv \frac{hP}{kA_c}$$

$$q_f = q_b = -kA_c \left. \frac{dT}{dx} \right|_{x=0} = -kA_c \left. \frac{d\theta}{dx} \right|_{x=0}$$

Sugerimos: $\theta(x) = C\epsilon^{-\lambda x}$

CONDUÇÃO EM ALETAS

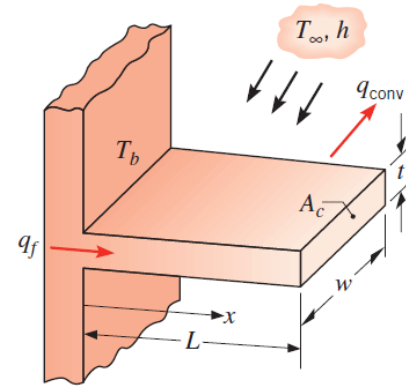
Condução em aletas

Aletas com seção transversal constante

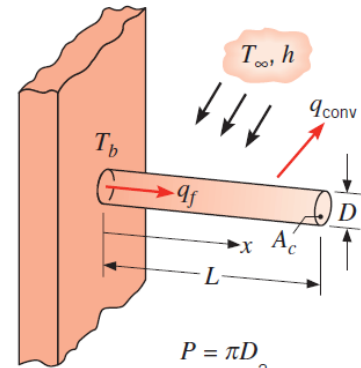
$$\theta(x) = C e^{-mx}$$

$$m^2 \equiv \frac{hP}{kA_c}$$

$$\theta(x) \equiv T(x) - T_\infty$$



$$P = 2w + 2t$$
$$A_c = wt$$



$$P = \pi D$$
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CONDUÇÃO EM ALETAS

Condução em aletas

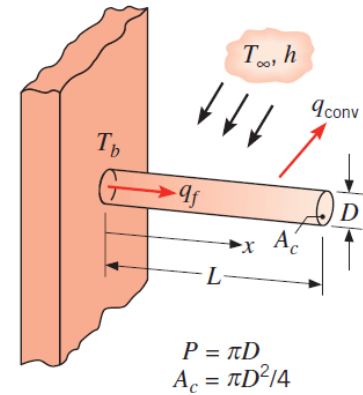
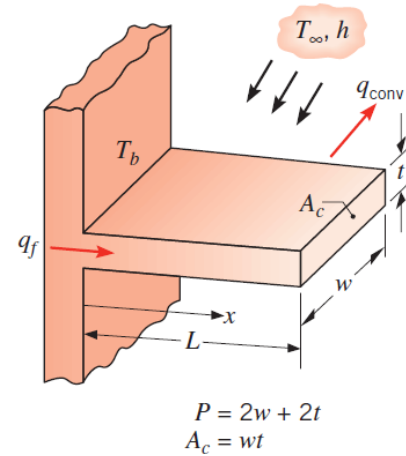
Aletas com seção transversal constante

$$\theta(x) = C e^{-mx}$$

$$m^2 \equiv \frac{hP}{kA_c}$$

$$\theta(x) \equiv T(x) - T_\infty$$

$$\theta_b = \theta(0) = T_b - T_\infty$$



CONDUÇÃO EM ALETAS

Condução em aletas

Aletas com seção transversal constante

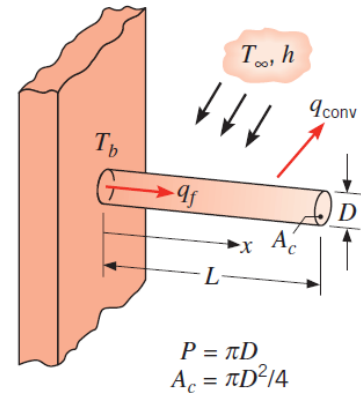
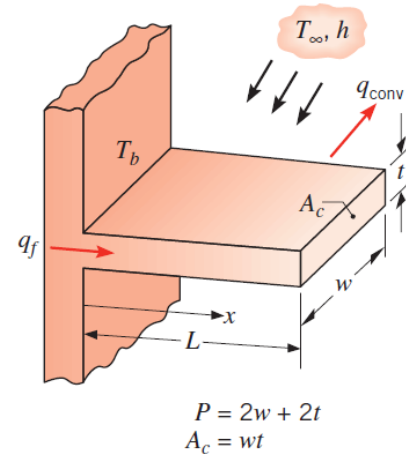
$$\theta(x) = C\epsilon^{-mx}$$

$$m^2 \equiv \frac{hP}{kA_c}$$

$$\theta(x) \equiv T(x) - T_\infty$$

$$\theta_b = \theta(0) = T_b - T_\infty$$

$$\theta(x) = \theta_b \epsilon^{-mx}$$



CONDUÇÃO EM ALETAS

Condução em aletas

Aletas com seção transversal constante

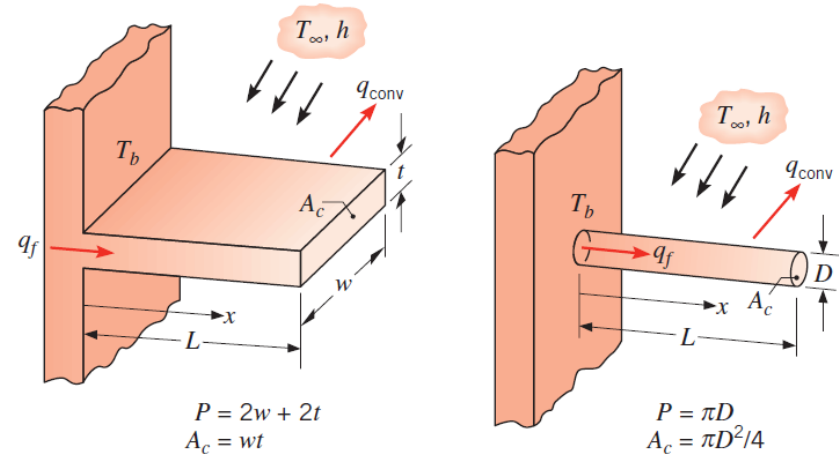
$$\theta(x) = C\epsilon^{-mx}$$

$$m^2 \equiv \frac{hP}{kA_c}$$

$$\theta(x) \equiv T(x) - T_\infty$$

$$\theta_b = \theta(0) = T_b - T_\infty$$

$$\theta(x) = \theta_b \epsilon^{-mx}$$



$$q_f = q_b = -kA_c \left. \frac{dT}{dx} \right|_{x=0} = -kA_c \left. \frac{d\theta}{dx} \right|_{x=0}$$

CONDUÇÃO EM ALETAS

Condução em aletas

Aletas com seção transversal constante

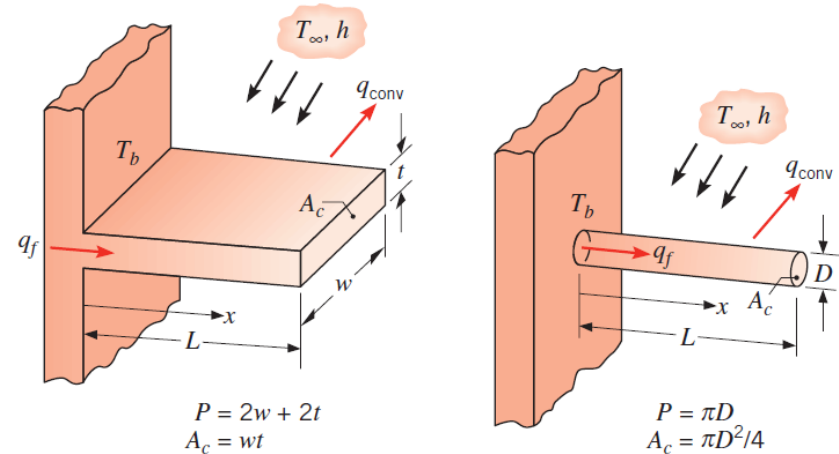
$$\theta(x) = C e^{-mx}$$

$$m^2 \equiv \frac{hP}{kA_c}$$

$$\theta(x) \equiv T(x) - T_\infty$$

$$\theta_b = \theta(0) = T_b - T_\infty$$

$$\theta(x) = \theta_b e^{-mx}$$



$$q_f = q_b = -kA_c \left. \frac{dT}{dx} \right|_{x=0} = -kA_c \left. \frac{d\theta}{dx} \right|_{x=0}$$

$$q_f = \sqrt{hPkA_c} \theta_b$$

CONDUÇÃO EM ALETAS

Condução em aletas

Aletas com seção transversal constante

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx _{x=L}$	$\frac{\cosh m(L-x) + (h/mk) \sinh m(L-x)}{\cosh mL + (h/mk) \sinh mL}$ (3.75)	$M \frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL}$ (3.77)
B	Adiabatic: $d\theta/dx _{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$ (3.80)	$M \tanh mL$ (3.81)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_b) \sinh mx + \sinh m(L-x)}{\sinh mL}$ (3.82)	$M \frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.83)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx} (3.84)	M (3.85)

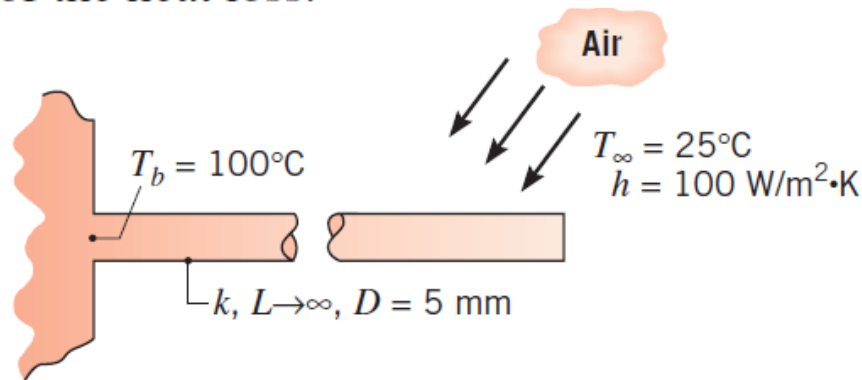
$$\theta \equiv T - T_\infty \quad m^2 \equiv hP/kA_c$$

$$\theta_b = \theta(0) = T_b - T_\infty \quad M \equiv \sqrt{hPkA_c} \theta_b$$

CONDUÇÃO EM ALETAS

A very long rod 5 mm in diameter has one end maintained at 100°C . The surface of the rod is exposed to ambient air at 25°C with a convection heat transfer coefficient of $100\text{ W/m}^2\cdot\text{K}$.

1. Determine the temperature distributions along rods constructed from pure copper, 2024 aluminum alloy, and type AISI 316 stainless steel. What are the corresponding heat losses from the rods?
2. Estimate how long the rods must be for the assumption of *innite length* to yield an accurate estimate of the heat loss.



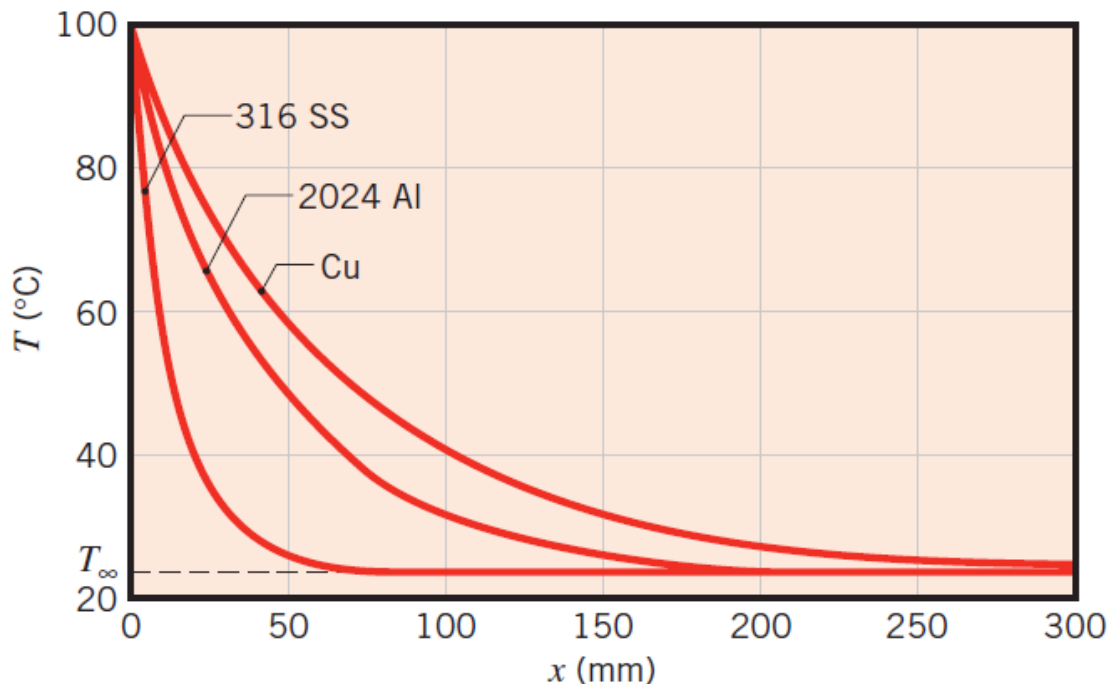
Properties: Table A.1, copper [$T = (T_b + T_{\infty})/2 = 62.5^{\circ}\text{C} \approx 335\text{ K}$]: $k = 398\text{ W/m}\cdot\text{K}$.
Table A.1, 2024 aluminum (335 K): $k = 180\text{ W/m}\cdot\text{K}$. Table A.1, stainless steel, AISI 316 (335 K): $k = 14\text{ W/m}\cdot\text{K}$.

CONDUÇÃO EM ALETAS

$$T = T_{\infty} + (T_b - T_{\infty})e^{-mx}$$

$$m = (hP/kA_c)^{1/2} = (4h/kD)^{1/2}.$$

copper, the aluminum alloy, and the stainless steel, respectively, m are 14.2, 21.2, and 75.6 m^{-1} .



CONDUÇÃO EM ALETAS

$$q_f = \sqrt{hPkA_c} \theta_b$$

Hence for copper,

$$\begin{aligned} q_f &= \left[100 \text{ W/m}^2 \cdot \text{K} \times \pi \times 0.005 \text{ m} \right. \\ &\quad \left. \times 398 \text{ W/m} \cdot \text{K} \times \frac{\pi}{4} (0.005 \text{ m})^2 \right]^{1/2} (100 - 25)^\circ\text{C} \\ &= 8.3 \text{ W} \end{aligned}$$



Similarly, for the aluminum alloy and stainless steel, respectively, the heat rates are $q_f = 5.6 \text{ W}$ and 1.6 W .