Relatório de Resoluções

O código de cada membro pode ser consultado a seguir:

x_{05}: José Soares Jr.	x_{11}: Luca Monaco
x_{06}: Maurício Damião	x_{15}: Rodrigo Melendez
x_{08}: Pedro Lopes Silva	x_{18}: Matheus Cardoso
x_{09}: Rafael Maddalena	x_{20}: Gustavo Zequini

Resolução (‖ Questão: 7.2.1 ‖ Relator: $\mathbf{x}_{06}$ ‖ Revisor: $\mathbf{x}_{18}$ ‖)

According to a study, the demand quantity Q for butter in Stockholm during the period 1925-1937 was related to the price P by the equation $Q \cdot P^{1/2} = 38$. Find dQ/dP by implicit differentiation. Check the answer by using a different method to compute the derivative.

Utilizando o método de diferenciação implícita, temos que:

Isolando Q, chegamos a: $Q = \dfrac{38}{P^{1/2}}$. Utilizaremos em breve este valor.

$$Q \cdot P^{1/2} = 38 \Rightarrow \tag{1}$$

$$Q' \cdot P^{1/2} + \frac{Q}{2P^{1/2}} = 0 \iff \tag{2}$$

$$Q' \cdot P^{1/2} = -\frac{Q}{2P^{1/2}} \iff \tag{3}$$

$$Q' = -\frac{Q}{2P^{1/2} \cdot P^{1/2}} \iff \tag{4}$$

$$Q' = -\frac{\frac{38}{P^{1/2}}}{2P^{1/2} \cdot P^{1/2}} \iff \tag{5}$$

$$Q' = -\frac{19}{P^{3/2}} \tag{6}$$

Conferindo a resposta utilizando outro método:

$$Q = \frac{38}{P^{1/2}} \Rightarrow \tag{7}$$

$$Q' = -\frac{1}{2}\frac{38}{P^{3/2}} \iff \tag{8}$$

$$Q' = -\frac{19}{P^{3/2}} \tag{9}$$

■

Resolução (‖ Questão: 7.2.2 ‖ Relator: $\mathbf{x}_{08}$ ‖ Revisor: $\mathbf{x}_{20}$ ‖)

Consider a profit-maximizing firm producing a single commodity. If the firm gets a fixed price P per unit sold, its profit from selling Q units is $\pi(Q) = P \cdot Q - C(Q)$, where $C(Q)$ is the cost function. Assume that $C'(Q) > 0$ and $C"(Q) > 0$. In Example 8.5.1, it will be shown that $Q = Q^* > 0$ maximizes profits w.r.t. Q provided that:

$P = C'(Q^*)$ (*)

Thus, at the optimum, marginal cost must equal the price per unit.

(a) By implicitly differentiating (*) w.r.t. P, find an expression for dQ^*/dP.

$P = C'(Q^*)$

Tomando a derivada implícita com relação à P vem:

$$1 = C"(Q^*) \cdot \frac{dQ^*}{dP}$$
$$\frac{1}{C"(Q^*)} = \frac{dQ^*}{dP}$$

(b) Comment on the sign of dQ^*/dP.

Como o enunciado diz que $C"(Q) > 0$ teremos que $dQ^*/dP > 0$. Ou seja um aumento do preço leva à um aumento da quantidade ótima produzida.

■

Resolução (‖ Questão: 7.2.3 ‖ Relator: $\mathbf{x}_{09}$ ‖ Revisor: $\mathbf{x}_{05}$ ‖)

Consider the equation $AP^{-\alpha}r^{-\beta} = S$ where A, α, β, and S are positive constants. The left-hand side of the equation expresses the demand for a commodity as a decreasing function of both its price P and the interest rate r. In equilibrium, this demand must equal a fixed supply quantity S.

a) Take natural logarithms of both sides and find dP/dr by implicit differentiation.

Tomando o logarítimo natural:

$$AP^{-\alpha}r^{-\beta} = S \Rightarrow \ln A - \alpha \ln P - \beta \ln r = \ln S$$

Fazendo a derivação implícita:

$$\begin{aligned} -\alpha\frac{P'}{P} - \beta\frac{1}{r} &= 0 \\ -\alpha\frac{P'}{P} &= \frac{\beta}{r} \\ P' = -\frac{P}{\alpha} \cdot \frac{\beta}{r} &= \frac{dP}{dr} \end{aligned}$$

b) How does the equilibrium price react to an increase in the interest rate?

Um aumento na taxa de juros diminui a demanda e o preço cai.

■

Resolução (‖ Questão: 7.2.4 ‖ Relator: $\mathbf{x}_{11}$ ‖ Revisor: $\mathbf{x}_{06}$ ‖)

4. Extending the standard macroeconomic model of Example 7.2.1 for an economy open to international trade gives:

(i) $Y = C + I + X - M$;

(ii) $C = f(Y)$, with $0 < f$, with $0 < f'(Y) < 1$;

(iii) $M = g(Y)$.

Here X is an exogenous constant that denotes exports, whereas M denotes the volume of imports. The function g in (iii) is called an import function, which is assumed to satisfy $0 < g'(Y) < f'(Y)$

a) By inserting (ii) and (iii) into (i), obtain an equation that defines Y as a function of exogenous investment I.

b) Find an expression for dY/dI by implicit differentiation. Discuss the sign of dY/dI.

c) Find an expression for d^2Y/d^2I.

Observação: Modelo macroeconômico do exemplo 7.2.1

A more general version of the standard macroeconomic model for determining national income that we saw in Example 3.2.1 is that: $(i)Y = C + I$; and $(ii)C = f(Y)$. Here, (ii) is the consumption function discussed in Example 4.5.2, whereas (i) states that GDP, Y, is divided up between consumption, C, and investment, I, which is assumed to be exogenous.

a) Substituindo-se os valores dados no enunciado, obtemos:

$$Y = C + I + X - M(i) \tag{10}$$
$$C = f(Y)(ii) \tag{11}$$
$$M = g(Y)(iii) \tag{12}$$
$$Y = f(Y) + I + X - g(Y) \tag{13}$$

b)$Y = f(Y) + I + X - g(Y)$, deriva-se implicitamente esta expressão, obtendo-se assim dY/dI

$$Y = f(Y) + I + X - g(Y) \tag{14}$$
$$Y' = f'(Y) \cdot Y' + 1 + 0 - g'(Y) \cdot Y' \tag{15}$$
$$Y' - f'(Y) \cdot Y' + g'(Y) \cdot Y' = 1 \tag{16}$$
$$Y'(1 - f'(Y) + g'(Y)) = 1 \tag{17}$$
$$Y' = \frac{1}{1 - f'(Y) + g'(Y)} \tag{18}$$

O sinal de dY/dI é positivo, pois seu numerador é positivo e seu denominador também, visto que a subtração $1 - f'(Y)$ é positiva, dado que $f'(Y)$ é maior que 0 e menor que 1 como dito no enunciado e $g'(Y)$ é um valor positivo como também foi dito no enunciado.

c) Para se obter d^2Y/d^2I é preciso derivar implicitamente a derivada encontrada no item b

$$Y' = \frac{1}{1 - f'(Y) + g'(Y)} \tag{19}$$
$$Y'' = \frac{[0 \cdot (1 - f'(Y) + g'(Y))] - [1 \cdot (-f''(Y)Y' + g''(Y)Y')]}{[(1 - f'(Y) + g'(Y)]^2} \tag{20}$$
$$Y'' = \frac{Y'([f''(Y) - g''(Y))}{[(1 - f'(Y) + g'(Y)]^2} \tag{21}$$
$$\tag{22}$$

Trocando Y' por $\dfrac{1}{1-f'(Y)+g'(Y)}$ resultará em:

$$Y'' = \frac{Y'([f''(Y)-g''(Y))}{[(1-f'(Y)+g'(Y)]^2} \tag{23}$$

$$Y'' = \frac{\frac{1}{1-f'(Y)+g'(Y)} \cdot (f''(Y)-g''(Y))}{[(1-f'(Y)+g'(Y)]^2} \tag{24}$$

$$Y'' = \frac{f''(Y)-g''(Y)}{[(1-f'(Y)+g'(Y)]^3} \tag{25}$$

■

Resolução (‖ Questão: 7.2.5 ‖ Relator: $\mathbf{x}_{15}$ ‖ Revisor: $\mathbf{x}_{08}$ ‖)

In part (c) of Example 7.2.2, find an expression for d^2P/dt^2 by differentiating 7.2.3 w.r.t t.

Let 7.2.3 be:

$$f'(P+t)(\frac{dP}{dt}+1) = g'(P)\frac{dP}{dt} \tag{26}$$

Let P be a differentiable function of t. Consider a simplified notation: $\dfrac{dP}{dt} = P'$

$$f'(P+t)(P'+1) = g'(P)P' \tag{27}$$

Now, to find d^2P/dt^2, we must differentiate both sides of (27) w.r.t t.
First applying the chain rule and the product rule for derivatives we have:

$$f''(P+t)\cdot(P'+1)\cdot(P'+1) + f'(P+t)\cdot P'' = g''(P)\cdot P'\cdot P' + g'(P)\cdot P'' \tag{28}$$

Now, we must solve (28) for P'':

$f'(P+1)\cdot P'' - g'(P)\cdot P'' = g''(P)\cdot(P')^2 - f''(P+t)(P'+1)^2 \Rightarrow$

$P''[f'(P+1) - g'(P)] = g''(P)\cdot(P')^2 - f''(P+t)[(P')^2+2P'+1] \Rightarrow$

$P'' = \dfrac{g''(P)\cdot(P')^2 - f''(P+t)[(P')^2+2P'+1]}{f'(P+1)-g'(P)}$

■

Resolução (‖ Questão: 7.2.6 ‖ Relator: $\mathbf{x}_{18}$ ‖ Revisor: $\mathbf{x}_{09}$ ‖)

In Example 7.2.2 we studied a model of supply and demand where a tax is imposed on the consumers. Instead, suppose that the producers have to pay a tax per unit sold that is equal to a fraction t of the sales price P they receive, where $0 < t < 1$. This implies that the equilibrium condition with the tax is

$$f(P) = g(P - tP) \tag{29}$$

We assume that $f' < 0$ and $g' > 0$

a) Differentiate (29) w.r.t. t and find an expression for $\frac{d}{dt}P$.

Assuma que $P = P(t)$ e diferencie a equação com relação a t:

$f'(P)P' = g'(P-tP)(P'-tP'-P) \iff tP'g'(P-tP) - P'g'(P-tP) + f'(P)P' = -Pg'(P-tP) \iff$
$P' = \dfrac{Pg'(P-tP)}{tg'(P-tP) - g'(P-tP) + f'(P)}$

b) Find the sign of $\frac{d}{dt}P$ and give an economic interpretation

Como $0 < t < 1 \iff -1 < t - 1 < 0$ e $f' < 0$ e $g' > 0$, então $P' > 0$, do que se conclui que o preço de venda aumenta conforme aumenta a tarifa "t". Pode-se interpretar que o produtor repassa ao menos parte do aumento dos preços para o consumidor

■