Relatório de Resoluções

O código de cada membro pode ser consultado a seguir:

x_{05}: José Soares Jr.	x_{11}: Luca Monaco
x_{06}: Maurício Damião	x_{15}: Rodrigo Melendez
x_{08}: Pedro Lopes Silva	x_{18}: Matheus Cardoso
x_{09}: Rafael Maddalena	x_{20}: Gustavo Zequini

Resolução (‖ Questão: 6.R.1 ‖ Relator: $\mathbf{x_{20}}$ ‖ Revisor: $\mathbf{x_{05}}$ ‖) Let $f(x) = x^2 - x + 2$. Show that $\dfrac{f(x+h) - f(x)}{h} = 2x - 1 + h$, and use this result to find $f'(x)$.

$$\frac{f(x+h) - f(x)}{h} \tag{1}$$

Substituindo a função do enunciado:

$$\frac{(x+h)^2 - (x+h) + 2 - (x^2 - x + 2)}{h} = \tag{2}$$

$$= \frac{x^2 + 2xh + h^2 - x - h + 2 - x^2 + x - 2}{h} = \frac{2xh + h^2 - h}{h} \tag{3}$$

$$\frac{h(2x + h - 1)}{h} = 2x - 1 + h \tag{4}$$

Para achar a derivada de $f(x)$ isto é $f'(x)$, faremos $\lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$:

$$\lim_{h\to 0} \frac{(x+h)^2 - (x+h) + 2 - (x^2 - x + 2)}{h} = \tag{5}$$

$$\lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x - h + 2 - x^2 + x - 2}{h} = \frac{2xh + h^2 - h}{h} \tag{6}$$

$$\lim_{h\to 0} \frac{h(2x + h - 1)}{h} \tag{7}$$

$$\lim_{h\to 0} 2x - 1 + h = 2x - 1 \tag{8}$$

■

Resolução (‖ Questão: 6.R.2 ‖ Relator: $\mathbf{x_{05}}$ ‖ Revisor: $\mathbf{x_{08}}$ ‖)

Let $f(x) = -2x^3 + x^2$. Compute $\frac{f(x+h)-f(x)}{h}$, and find $f'(x)$:

$f(x+h) = -2(x+h)^3 + (x+h)^2 = -2(x^3 + 3x^2h + 3xh^2 + h^3) + x^2 + 2xh + h^2 = -2x^3 - 6x^2h - 6xh^2 - 2h^3 + x^2 + 2xh + h^2$

$f(x+h) - f(x) = -2x^3 - 6x^2h - 6xh^2 - 2h^3 + x^2 + 2xh + h^2 - (-2x^3 + x^2) = -6x^2h - 6xh^2 - 2h^3 + 2xh + h^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{-6x^2h - 6xh^2 - 2h^3 + 2xh + h^2}{h} = -6x^2 - 6xh - 2h^2 + 2x + h$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} -6x^2 - 6xh - 2h^2 + 2x + h \Longrightarrow f'(x) = -6x^2 + 2x \quad \blacksquare$$

Resolução (‖ Questão: 6.R.3 ‖ Relator: $\mathbf{x}_{06}$ ‖ Revisor: $\mathbf{x}_{09}$ ‖)

Compute the first- and second-order derivatives of the following functions

a) $y = 2x - 5$

$y = 2x - 5 \Rightarrow y' = 2 \Rightarrow y'' = 0$

b) $y = \dfrac{x^9}{3}$

$y = \dfrac{x^9}{3} \Rightarrow y' = 3x^8 \Rightarrow y'' = 24x^7$

c) $y = 1 - \dfrac{x^{10}}{10}$

$y = 1 - \dfrac{x^{10}}{10} \Rightarrow y' = -x^9 \Rightarrow y'' = -9x^8$

d) $y = 3x^7 + 8$

$y = 3x^7 + 8 \Rightarrow y' = 21x^6 \Rightarrow y'' = 126x^5$

e) $y = \dfrac{x-5}{10} = \dfrac{x}{10} - \dfrac{5}{10}$

$y = \dfrac{x}{10} - \dfrac{5}{10} \Rightarrow y' = \dfrac{1}{10} \Rightarrow y'' = 0$

f) $y = x^5 - x^{-5}$

$y = x^5 - x^{-5} \Rightarrow y' = 5x^4 + 5x^{-6} \Rightarrow y'' = 20x^3 - 30x^{-7}$

g) $y = \dfrac{x^4}{4} + \dfrac{x^3}{3} + \dfrac{5^2}{2}$

$y = \dfrac{x^4}{4} + \dfrac{x^3}{3} + \dfrac{5^2}{2} \Rightarrow y' = x^3 + x^2 \Rightarrow y'' = 3x^2 + 2x$

h) $y = \dfrac{1}{x} + \dfrac{1}{x^3}$

$y = \dfrac{1}{x} + \dfrac{1}{x^3} \Rightarrow y' = -\dfrac{1}{x^2} - \dfrac{3}{x^4} \Rightarrow y'' = \dfrac{2}{x^3} + \dfrac{12}{x^5}$

$\blacksquare$

Resolução (‖ Questão: 6.R.4 ‖ Relator: $\mathbf{x}_{08}$ ‖ Revisor: $\mathbf{x}_{11}$ ‖)

Let C(Q) denote the cost of producing Q units per month of a commodity. What is the interpretation of $C'(1000) = 25$? Suppose the price obtained per unit is fixed at 30 and that the current output per month is 1000. Is it profitable to increase production?

Sabemos que $C'(1000) \approx C(1001) - C(1000)$, ou seja o custo marginal quando $Q = 1000$ é o custo de produção por unidade quando se produz uma quantidade ligeiramente maior que 1000 unidades.

É rentável aumentar a produção. Isso porque quando $C'(1000) = 25$, se produzirmos uma unidade a mais, o lucro por essa unidade a mais será de $P - Cmg = 30 - 25 = 5$, sendo assim lucrativo produzir essa unidade a mais.

■

Resolução (‖ Questão: 6.R.5 ‖ Relator: $\mathbf{x}_{09}$ ‖ Revisor: $\mathbf{x}_{15}$ ‖)

Para cada uma das funções abaixo, encontre a equação da tangente do gráfico no ponto especificado.

a) $y = -3x^2$ em $x = 1$

Para encontrar o valor de y quando $x = 1$:

$$y = -3(1)^2 = -3$$

Tomando a derivada de y:

$$y' = -6x$$

Para encontrar o valor de y' quando $x = 1$:

$$y' = -6(1) = -6$$

Para encontrar a equação da tangente:

$$\begin{aligned} y - (-3) &= (x-1)(-6) \\ y &= -6x + 6 - 3 \\ y &= -6x + 3 \end{aligned}$$

b) $y = \sqrt{x} - x^2$ em $x = 4$

Para encontrar o valor de y quando $x = 4$:

$$y = \sqrt{4} - 4^2 = 2 - 16 = -14$$

Tomando a derivada de y:

$$y' = \frac{1}{2\sqrt{x}} - 2x$$

Para encontrar o valor de y' quando $x = 4$:

$$y' = \frac{1}{2\sqrt{4}} - 2 \cdot 4 = \frac{1}{4} - 8 = -\frac{31}{4}$$

Para encontrar a equação da tangente:

$$\begin{aligned} y - (-14) &= (x-4)(-31/4) \\ y &= -\frac{31x}{4} + 31 - 14 \\ y &= \frac{31x}{4} + 17 \end{aligned}$$

c) $y = \dfrac{x^2 - x^3}{x+3}$ em $x = 1$

Para encontrar o valor de y quando $x = 1$:

$$y = \frac{1^2 - 1^3}{1+3} = \frac{1-1}{1+3} = 0$$

Tomando a derivada de y:

$$y' = \frac{(2x - 3x^2)(x+3) - (x^2 - x^3)}{(x+3)^2}$$

Para encontrar o valor de y' quando $x = 4$:

$$y' = \frac{(2 \cdot 1 - 3(1)^2)(1+3) - ((1)^2 - (1)^3)}{(1+3)^2} = -\frac{4}{16} = -\frac{1}{4}$$

Para encontrar a equação da tangente:

$$y - (0) = (x-1)(-1/4)$$
$$y = -\frac{x}{4} + \frac{1}{4}$$

■

Resolução (‖ Questão: 6.R.6 ‖ Relator: $\mathbf{x}_{11}$ ‖ Revisor: $\mathbf{x}_{18}$ ‖)

6. Let $A(x)$ denote the dollar cost of building a house with a floor area of x square metres. What is the interpretation of $A'(100) = 250$?

Tendo em vista que $A(100)$ é o custo de se construir uma casa com área construída de $100m^2$, então $A'(100)$ é o valor adicional gasto ao aumentar a área construída da casa para $100m^2$, no caso cerca de 250 dólares.

■

Resolução (‖ Questão: 6.R.7 ‖ Relator: $\mathbf{x}_{15}$ ‖ Revisor: $\mathbf{x}_{20}$ ‖)

Differentiate the following functions:

a) $f(x) = x(x^2 + 1)$

Using the product rule for derivatives:

$f(x) = x(x^2+1) \Rightarrow f'(x) = (x^2+1) + x \cdot 2x = 3x^2 + 1$

b) $g(w) = w^{-5}$

$g(w) = w^{-5} \Rightarrow g'(w) = -5w^{-6}$

c) $h(y) = y(y-1)(y+1)$

$h(y) = y(y-1)(y+1) \Rightarrow h(y) = y(y^2-1) \Rightarrow h(y) = y^3 - y \Rightarrow h'(y) = 3y^2 - 1$

d) $G(t) = \dfrac{2t+1}{t^2+3}$

$$G(t) = \frac{2t+1}{t^2+3} \Rightarrow G'(t) = \frac{2(t^2+3) - (2t+1)2t}{(t^2+3)^2} = \frac{2t^2 + 6 - 4t^2 - 2t}{(t^2+3)^2} = \frac{-2t^2 - 2t + 6}{(t^2+3)^2}$$

e) $f(x) = \dfrac{2x}{x^2+2}$

$f(x) = \dfrac{2x}{x^2+2} \Rightarrow f'(x) = \dfrac{2(x^2+2) - 2x \cdot 2x}{(x^2+2)^2} = \dfrac{-2x^2+4}{(x^2+2)^2}$

f) $F(s) = \dfrac{s}{s^2+s-2}$

$F(s) = \dfrac{s}{s^2+s-2} \Rightarrow F'(s) = \dfrac{s^2+s-2-s(2s+1)}{(s^2+s-2)^2} = \dfrac{-(s^2+2)}{(s^2+s-2)^2}$

■

Resolução (‖ Questão: 6.R.8 ‖ Relator: $\mathbf{x}_{18}$ ‖ Revisor: $\mathbf{x}_{05}$ ‖)

a) $\frac{d}{da}(a^2t - t^2)$

A derivada será:

$2at$

b) $\frac{d}{dt}(a^2t - t^2)$

A derivada será:

$a^2 - 2t$

c) $\frac{d}{d\varphi}(x\varphi^2 - \sqrt{\varphi})$

A derivada será:

$2x\varphi - \dfrac{1}{2\sqrt{\varphi}}$

■

Resolução (‖ Questão: 6.R.9 ‖ Relator: $\mathbf{x}_{20}$ ‖ Revisor: $\mathbf{x}_{06}$ ‖)

Use the chain rule to find dy/dx for the following:

(a) $y = 10u^2$ where $u = 5 - x^2$.

(b) $y = \sqrt{u}$ where $u = \dfrac{1}{x} - 1$

(a)

$$y' = 20.u.u'(x) \tag{9}$$
$$y' = 20(5-x^2).(-2x) = 20(2x^3 - 10x) \tag{10}$$
$$y' = 40x^3 - 200x \tag{11}$$

(b)

$$y' = \frac{1}{2\sqrt{u}}.u'(x) \tag{12}$$
$$y' = \frac{1}{2\sqrt{(1/x)-1}}. - 1.x^{-2} \tag{13}$$
$$y' = -\frac{1}{2x^2\sqrt{(1/x)-1}} \tag{14}$$

■

Resolução (‖ Questão: 6.R.10 ‖ Relator: $\mathbf{x}_{05}$ ‖ Revisor: $\mathbf{x}_{09}$ ‖)

Compute the following:

a) $\frac{\mathrm{d}Z}{\mathrm{d}t}$, when $Z = (u^2 - 1)^3$ and $u = t^3$

Reunindo as duas equações temos: $Z = (t^6 - 1)^3$, para derivarmos essa equação em relação a t, utilizaremos a regra da cadeia, ao qual possui a formula geral $f'(g(x))g'(x)$. Portanto vamos considerar:

$f(g(x)) = f(g(t)) = (t^6 - 1)^3$ e $g(x) = g(t) = t^6 - 1$

Assim, utilizando-se da forma geral iremos achar $f'(g(t))g'(t)$:

$f'(g(t)) = 3(t^6 - 1)^2$

$g'(t) = 6t^5$

$\therefore f'(g(t))g'(t) = 18t^5(t^6 - 1)^2$

b) $\frac{\mathrm{d}K}{\mathrm{d}t}$, when $K = \sqrt{L}$ and $L = 1 + \frac{1}{t}$

Refazendo o mesmo procedimento do item a):

$f(g(x)) = f(g(t)) = \sqrt{1 + \frac{1}{t}}$ e $g(x) = g(t) = 1 + \frac{1}{t}$

$f'(g(t)) = \frac{1}{2}(1 + \frac{1}{t})^{-\frac{1}{2}}$

$g'(t) = -t^{-2}$

$\therefore f'(g(t))g'(t) = -\frac{1}{2t^2}(1 + \frac{1}{t})^{-\frac{1}{2}}$

■

Resolução (‖ Questão: 6.R.11 ‖ Relator: $\mathbf{x}_{06}$ ‖ Revisor: $\mathbf{x}_{11}$ ‖)

If $a(t)$ and $b(t)$ are positive valued differentiable functions of t, and if A, α and β are constants, find expressions for $\dot{x}/x$ where:

a) $x = a(t)^2 \cdot b(t)$

$$\Rightarrow lnx = ln(a(t)^2 \cdot b(t)) \tag{15}$$
$$\iff lnx = 2 \cdot ln(a(t)) + ln(b(t)) \tag{16}$$
$$\Rightarrow \dot{x}/x = \frac{2}{a(t)} \cdot \dot{a}(t) + \frac{1}{(b(t))} \cdot \dot{b}(t) \tag{17}$$
$$\iff \dot{x}/x = \frac{2 \cdot \dot{a}(t)}{a(t)} + \frac{1 \cdot \dot{b}(t)}{b(t)} \tag{18}$$

b) $x = A \cdot a(t)^\alpha \cdot b(t)^\beta$

$$\Rightarrow lnx = ln(A \cdot a(t)^{\alpha} \cdot b(t)^{\beta}) \tag{19}$$
$$\iff lnx = ln(A) + ln(a(t)^{\alpha}) + ln(b(t)^{\beta})) \tag{20}$$
$$\iff lnx = ln(A) + \alpha \cdot ln(a(t)) + \beta \cdot ln(b(t)) \tag{21}$$
$$\Rightarrow \dot{x}/x = \alpha \cdot \frac{1}{a(t)} \cdot \dot{a}(t) + \beta \cdot \frac{1}{b(t)} \cdot \dot{b}(t) \tag{22}$$
$$\iff \dot{x}/x = \frac{\alpha \cdot \dot{a}(t)}{a(t)} + \frac{\beta \cdot \dot{b}(t)}{b(t)} \tag{23}$$

c) $x = A \cdot [a(t)^{\alpha} + b(t)^{\beta}]^{\alpha+\beta}$

$$x = A \cdot [a(t)^{\alpha} + b(t)^{\beta}]^{\alpha+\beta} \tag{24}$$
$$\Rightarrow lnx = ln(A \cdot [a(t)^{\alpha} + b(t)^{\beta}]^{\alpha+\beta}) \tag{25}$$
$$\iff lnx = ln(A) + ln([a(t)^{\alpha} + b(t)^{\beta}]^{\alpha+\beta}) \tag{26}$$
$$\iff lnx = ln(A) + (\alpha+\beta) \cdot ln(a(t)^{\alpha} + b(t)^{\beta}) \tag{27}$$
$$\Rightarrow \dot{x}/x = (\alpha+\beta) \cdot \frac{1}{a(t)^{\alpha} + b(t)^{\beta}} \cdot [\alpha \cdot a(t)^{\alpha-1} \cdot \dot{a}(t) + \beta \cdot b(t)^{\beta-1} \cdot \dot{b}(t)] \tag{28}$$
$$\iff \dot{x}/x = (\alpha+\beta) \cdot \frac{[\alpha \cdot a(t)^{\alpha-1} \cdot \dot{a}(t) + \beta \cdot b(t)^{\beta-1} \cdot \dot{b}(t)]}{a(t)^{\alpha} + b(t)^{\beta}} \tag{29}$$

■

Resolução (‖ Questão: 6.R.12 ‖ Relator: $\mathbf{x}_{08}$ ‖ Revisor: $\mathbf{x}_{15}$ ‖)

If $R = S^{\alpha}$, $S = 1 + \beta K^{\gamma}$, and $K = At^{p} + B$, find an expression for dR/dt

.

Temos que $R = (1 + \beta(At^{p} + B)^{\gamma})^{\alpha}$

$\frac{dR}{dT} = \frac{dR}{dS}\frac{dS}{dK}\frac{dK}{dt} = (\alpha \cdot S^{\alpha-1}) \cdot (\beta \cdot \gamma \cdot K^{\gamma-1}) \cdot (Apt^{p-1}) = (\alpha \cdot (1 + \beta(At^{p} + B)^{\gamma})^{\alpha-1}) \cdot (\beta \cdot \gamma \cdot (At^{p} + B)^{\gamma-1}) \cdot (Apt^{p-1})$

■

Resolução (‖ Questão: 6.R.13 ‖ Relator: $\mathbf{x}_{09}$ ‖ Revisor: $\mathbf{x}_{18}$ ‖)

Encontre as derivadas das seguintes funções, onde a, b, p e q são constantes.

a) $h(L) = (L^{a} + b)^{p}$

$$h(L) = (L^{a} + b)^{p}$$
$$h'(L) = p(L^{a} + b)^{p-1} \cdot aL^{a-1}$$

b) $C(Q) = aQ + bQ^{2}$

$$C(Q) = aQ + bQ^{2}$$
$$C'(Q) = a + 2bQ$$

c) $P(x) = (ax^{1/q} + b)^q$

$$P(x) = (ax^{1/q} + b)^q$$
$$P'(x) = q(ax^{1/q} + b)^{q-1} \cdot \frac{a}{q} x^{(1/q)-1}$$
$$P'(x) = ax^{(1-q)/q}(ax^{1/q} + b)^{q-1}$$

■

Resolução (‖ Questão: 6.R.14 ‖ Relator: $\mathbf{x}_{11}$ ‖ Revisor: $\mathbf{x}_{20}$ ‖)

14. Find the first derivatives of:

a) $y = -7e^x$

$$y' = -7e^x \tag{30}$$

b) $y = e^{-3x^2}$

$$y = e^{-3x^2} \tag{31}$$
$$y' = e^{-3x^2} \cdot -6x \tag{32}$$

c) $y = \dfrac{x^2}{e^x}$

$$y = \frac{x^2}{e^x} \tag{33}$$
$$y = x^2 \cdot e^{-x} \tag{34}$$
$$y' = (2x \cdot e^{-x}) + (x^2 \cdot -e^{-x}) \tag{35}$$
$$y' = xe^{-x}(2 - x) \tag{36}$$

d) $y = e^x \ln(x^2 + 2)$

$$y = e^x \ln\big(x^2 + 2\big) \tag{37}$$
$$y' = [e^x \cdot \ln\big(x^2 + 2\big)] + [e^x \cdot \frac{2x}{x^2 + 2}] \tag{38}$$
$$y' = e^x[\ln\big(x^2 + 2\big) + \frac{2x}{x^2 + 2}] \tag{39}$$

e) $y = e^{5x^3}$

$$y = e^{5x^3} \tag{40}$$
$$y' = 15x^2 \cdot e^{5x^3} \tag{41}$$

f) $y = 2 - x^4 e^{-x}$

$$y = 2 - x^4 e^{-x} \tag{42}$$
$$y' = (-4x^3 \cdot e^{-x}) + (-x^4 \cdot -e^{-x}) \tag{43}$$
$$y' = -4x^3 e^{-x} + x^4 e^{-x} \tag{44}$$
$$y' = x^3 e^{-x}(-4 + x) \tag{45}$$

g) $y = (e^x + x^2)^{10}$

$$y = y = (e^x + x^2)^{10} \tag{46}$$
$$y' = 10(e^x + x^2)^9 \cdot (e^x + 2x) \tag{47}$$

h) $y = \ln(\sqrt{x} + 1)$

$$y = \ln\big(\sqrt{x} + 1\big) \tag{48}$$
$$y' = \frac{1}{\sqrt{x} + 1} \cdot \frac{1}{2\sqrt{x}} \tag{49}$$
$$y' = \frac{1}{(\sqrt{x} + 1)(2\sqrt{x})} \tag{50}$$

■

Resolução (‖ Questão: 6.R.15 ‖ Relator: $\mathbf{x}_{15}$ ‖ Revisor: $\mathbf{x}_{05}$ ‖)

Find the intervals where the following functions are increasing:

a) $y = (\ln x)^2 - 4$

If we want to find where the function is increasing, we must find the values of x which make the derivative of the function positive.

As: $y = (\ln x)^2 - 4 \Rightarrow y' = 2\ln x \cdot \dfrac{1}{x} = \dfrac{2\ln x}{x}$

Now we shall analyse the sign of the function y'. Considering that y' is the ratio between $2\ln x$ and x, we must use the signal diagram.

$2\ln x$	$*$	$-$	$+$
x	$-$	$+$	$+$
$\dfrac{2\ln x}{x}$	$*$	$-$	$+$
		0	1

Where * denotes undefined, because the logarithm of a negative number is not defined for real numbers.

As we can see, the derivative of the function $y = (\ln x)^2 - 4$ is positive when x is bigger than 1, therefore the function is increasing when $x \in [1, +\infty[$. The interval is inclusive at 1 because a function is increasing in the interval that makes its derivative equal or bigger than 0, and when x is equal to 1, the derivative is equal to 0.

b) $y = \ln[e^x + e^{-x}]$

If we want to find where the function is increasing, we must find the values of x which make the derivative of the function positive.

We use the following derivatives:

$y = \ln x \Rightarrow y' = \dfrac{1}{x}$

$y = e^{-x} \Rightarrow y' = -e^{-x}$

And the chain rule for derivatives to find:

$y = \ln[e^x + e^{-x}] \Rightarrow y' = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

As the function $e^x > 0$ and $e^{-x} > 0$, $\forall\ x \in \mathbf{R}$, the denominator of the derivative, $e^x + e^{-x}$ is a sum of two positives, thus is always positive.

Now, we shall analyse which values of x makes the numerator, $e^x - e^{-x}$, positive, and therefore, the derivative itself.

$e^x - e^{-x} > 0 \Rightarrow e^x > e^{-x} \Rightarrow \ln e^x > \ln e^{-x} \Rightarrow x > -x$

As x needs to be bigger than its inverse, x must be a positive number, so that the derivative will be positive, and the function will be increasing.

Therefore, the interval where the function is increasing is $[0, +\infty[$. The interval is inclusive at 0 because a function is increasing in the interval that makes its derivative equal or bigger than 0, and when x is equal to 0, the derivative is equal to 0.

c) $y = x - \dfrac{3}{2}\ln[x^2 + 2]$

If we want to find where the function is increasing, we must find the values of x which make the derivative of the function positive.

$y = x - \dfrac{3}{2}\ln[x^2 + 2] \Rightarrow y' = 1 - \dfrac{3}{2}\dfrac{2x}{x^2 + 2} \Rightarrow y' = 1 - \dfrac{3x}{x^2 + 2}$

Therefore, we want to find which values of x makes $1 - \dfrac{3x}{x^2 + 2} > 0$

$1 - \dfrac{3x}{x^2 + 2} > 0 \Rightarrow 1 > \dfrac{3x}{x^2 + 2} \Rightarrow x^2 + 2 > 3x \Rightarrow x^2 + 2 - 3x > 0 \Rightarrow (x - 2)(x - 1) > 0$

Analysing the sign of the product $(x - 2)(x - 1)$ using the sign diagram:

$x - 2$	$-$	$-$	$+$
$x - 1$	$-$	$+$	$+$
$(x - 2)(x - 1)$	$+$	$-$	$+$
	1	2	

We conclude that the derivative of the function $y = x - \dfrac{3}{2}\ln[x^2 + 2]$ is positive when $x < 1$ or $x > 2$, and therefore, the function is increasing in the interval $]-\infty, 1] \cup [2, +\infty[$. The interval is inclusive at 1 and 2 because a function is increasing in the interval that makes its derivative equal or bigger than 0, and when x is equal to 1 or 2, the derivative is equal to 0.

■

Resolução (‖ Questão: 6.R.16 ‖ Relator: $\mathbf{x}_{18}$ ‖ Revisor: $\mathbf{x}_{06}$ ‖)

a) Suppose $\pi(Q) = QP(Q) - cQ$, where P is a differentiable function and c is a constant. Find an expression for $\frac{d}{dQ}\pi$.

   $\frac{d}{dQ}\pi = P(Q) + Q(P'(Q)) - c$

b) Suppose $\pi(L) = PF(L) - wL$, where F is a differentiable function and P and w are constants. Find an expression for $\frac{d}{dL}\pi$.

   $\frac{d}{dQ}\pi = PF'(L) - w$

   ■